

1/8/12

Lecture 2

Note Title

31-07-2012

Topics covered :

- Linear networks overview (LTI)
- Non linear networks
- Notion of incremental linearity to analyse nonlinear networks (under certain conditions)
- Non linear elements

* Diodes

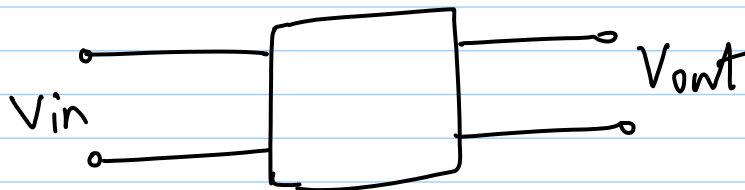
* MOSFET

* BJT

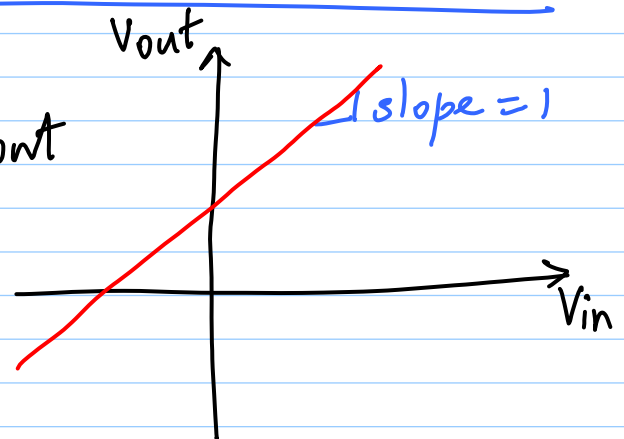
Blackbox
treatment

→ Design of amplifiers using MOSFETs & BJTs

→ Negative Feedback



* Is this linear?



Linear :

$$x_1 \rightarrow y_1$$

$$x_2 \rightarrow y_2$$

$$ax_1 + bx_2 \rightarrow ay_1 + by_2$$

$$\forall a, b, x_1, x_2$$

i.e. Superposition holds true

e.g. $x_1 = 0, y_1 = 2$

$$x_2 = 1, y_2 = 3$$

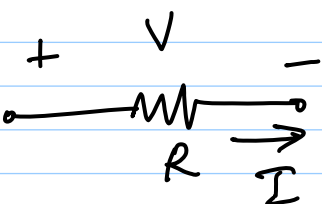
$$x_1 + x_2 = 1 \Rightarrow y_1 + y_2 = 5 (\neq 3)$$

* Input = 0 $\Rightarrow$ Output = 0 is a necessary condition for linearity

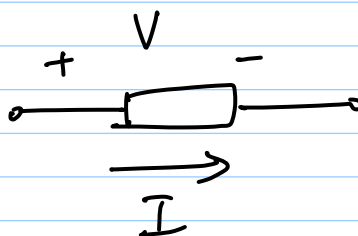
* Impulse response $\Rightarrow$ defines LTI system
but — all practical systems are
NON LINEAR

Linear elements : R, L, C

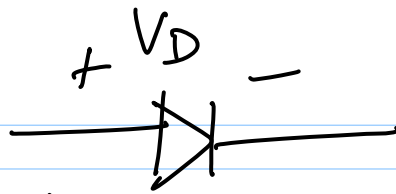
2-T element defined by $V-I$ relationship

e.g.  $I = \frac{V}{R}$

Similarly $L, C \rightarrow$ diff. / integral relationship
(still linear)

 $I = f(V)$

Diode



anode $\rightarrow$ cathode

I_D

from SSD theory, you can show that

$$I_D = I_s \left\{ \exp\left(\frac{V_D}{V_t}\right) - 1 \right\}$$

where I_s = Saturation current

V_t = thermal voltage

$$= \frac{kT}{q} \approx 25 \text{ mV @ RT}$$

$V_D > 0 \Rightarrow$ forward biased

$V_D < 0 \Rightarrow$ reverse biased

for $V_D \gg \text{few } V_T$,

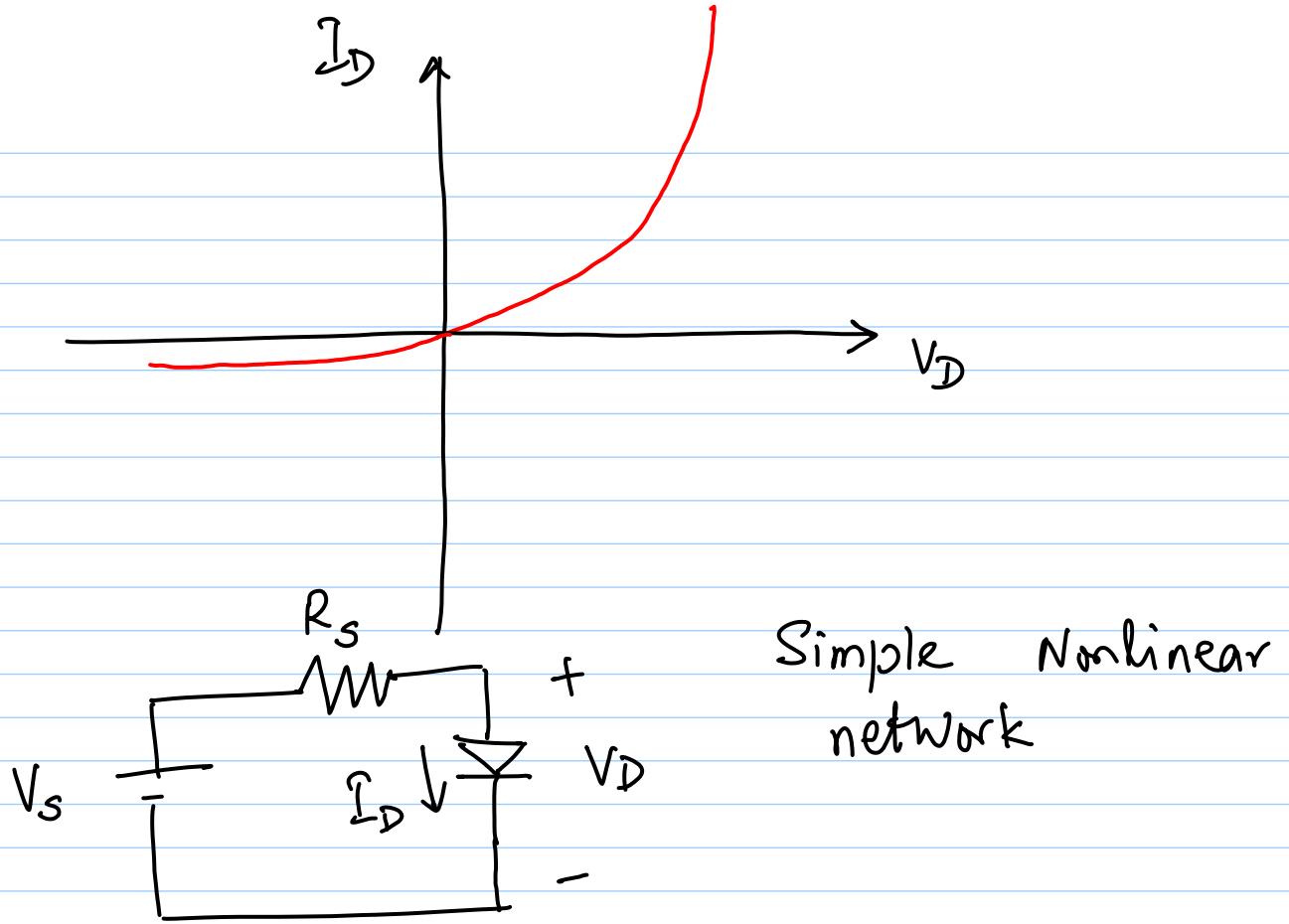
$$\exp\left(\frac{V_D}{V_t}\right) \gg 1$$

$$\Rightarrow I_D \approx I_s \exp\left(\frac{V_D}{V_t}\right)$$

$$\text{or } V_D = V_T \ln\left(\frac{I_D}{I_s}\right)$$

for $\frac{V_D}{V_t} \ll 0$, $\exp\left(\frac{V_D}{V_t}\right) \ll 1$

$$\Rightarrow I_D \approx -I_s$$



KVL

$$V_S = V_{R_S} + V_D$$

$$= I_D R_S + V_D$$

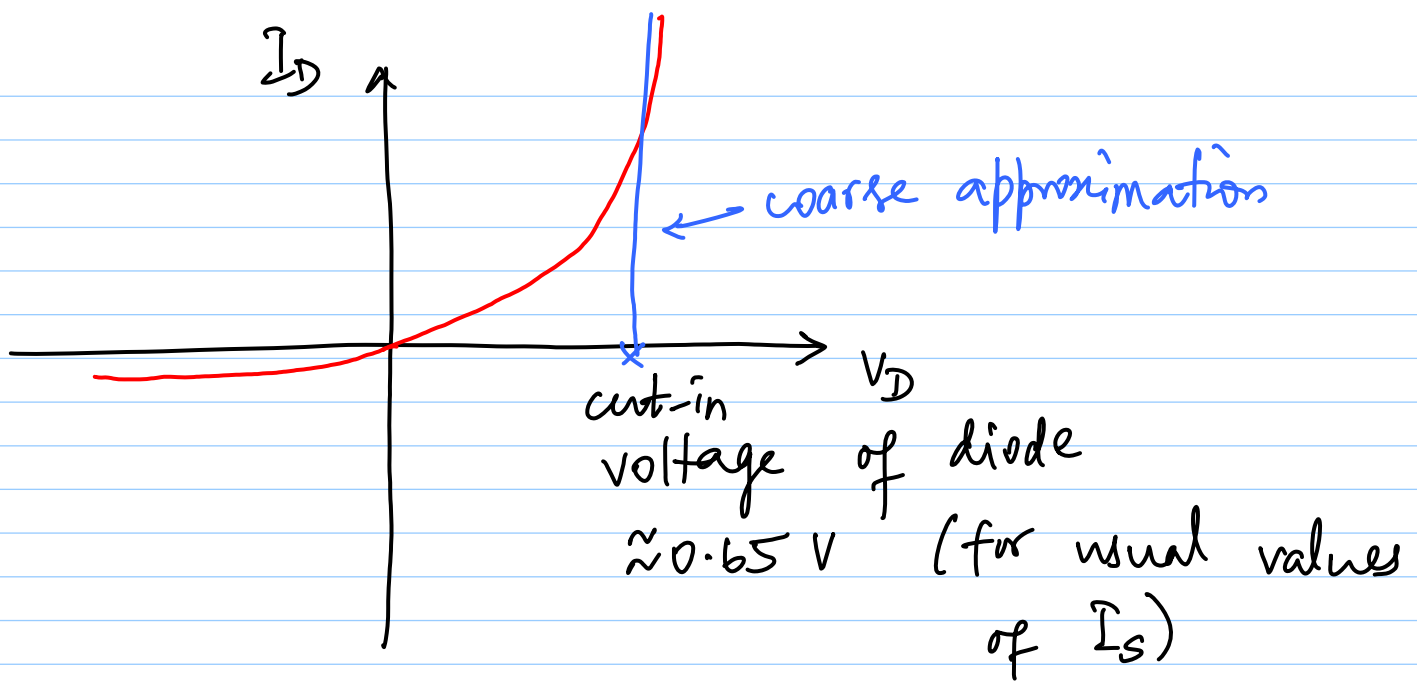
$$= I_D R_S + V_t \ln \left(1 + \frac{I_D}{I_S} \right)$$

to find out V_D, I_D

several methods

1) zeroth order method

— use exponential characteristic to approximate V_D

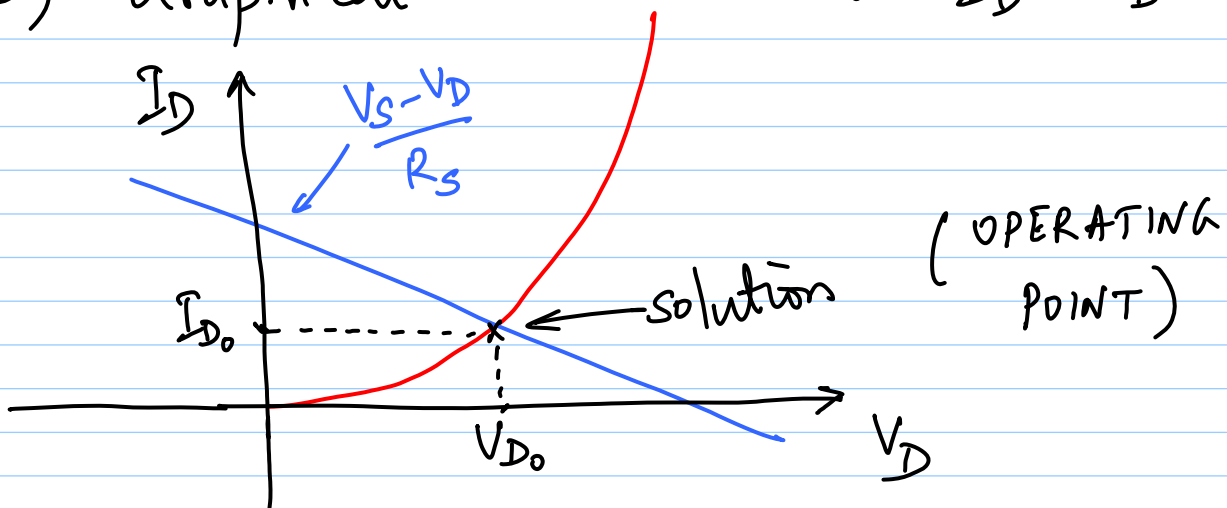


voltage drop across a fwd biased diode (V_D) $\approx 0.65 \text{ V}$

$$\Rightarrow I_D \approx \frac{V_S - 0.65}{R_S}$$

2) Numerical solution (iteration)

3) Graphical solution on $I_D - V_D$ plot



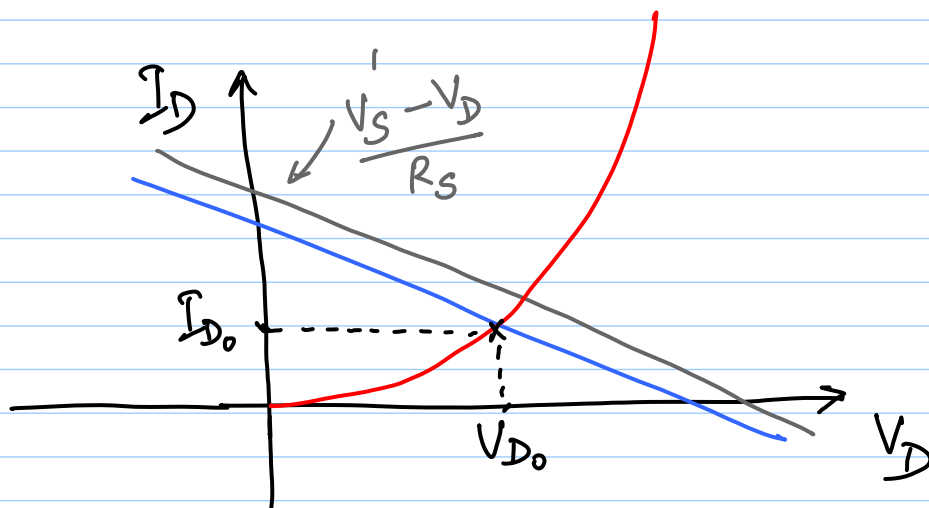
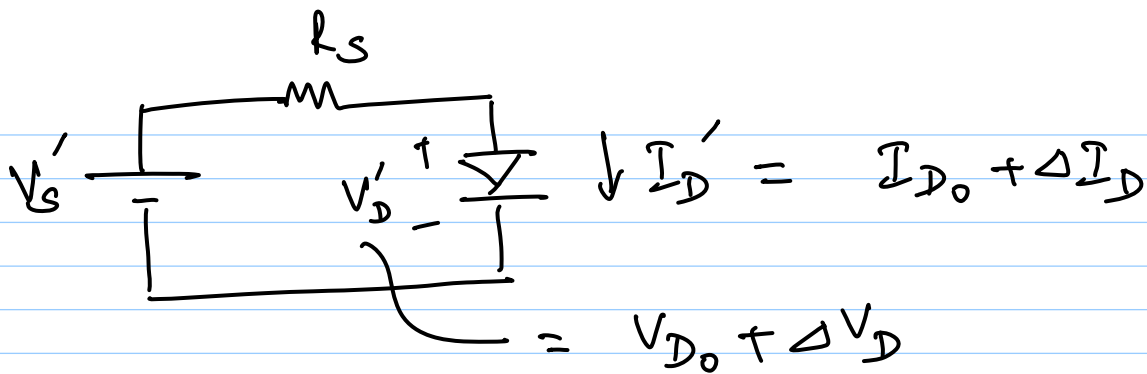
$$I_D = I_S \exp\left(\frac{V_D}{V_t}\right) \quad \leftarrow \text{diode eq.}$$

$$I_D = \frac{V_S - V_D}{R_S} \quad \leftarrow \text{KVL eq.}$$

Solution is at intersection of the two curves.

If V_S (or R_S) changes, can we find the new solution easily?

e.g. $V'_S = V_S + \Delta V_S$



Orig. Sol.:

$$\frac{V_S - V_{D_0}}{R_S} = I_{D_0} = I_S \left\{ \exp\left(\frac{V_{D_0}}{V_t}\right) - 1 \right\}$$

$$\frac{V'_S - V'_D}{R_S} = I'_D = I_S \left\{ \exp\left(\frac{V'_D}{V_t}\right) - 1 \right\}$$

$$\begin{aligned} \frac{(V_S + \Delta V_S) - (V_{D_0} + \Delta V_D)}{R_S} &= I_{D_0} + \Delta I_D \\ &= I_S \left\{ \exp\left(\frac{V_{D_0} + \Delta V_D}{V_t}\right) - 1 \right\} \end{aligned}$$

$$\frac{V_S - V_{D_0}}{R_S} + \frac{\Delta V_S - \Delta V_D}{R_S} = I_{D_0} + \Delta I_D$$

= Taylor series expansion
of diode equation
around V_{D_0}

for $y = f(x)$ around x_0

$$y = \sum_{n=0}^{\infty} \frac{f^{(n)}(x_0)}{n!} \cdot (x - x_0)^n$$

$$= \underline{f(x_0)} + f'(x_0) \cdot (x - x_0)$$

operating point $\rightarrow$

$$+ \frac{f''(x_0)}{2} \cdot (x - x_0)^2 + \dots$$

$$\begin{aligned}
 & I_S \left\{ \exp \left(\frac{V_{D0} + \Delta V_D}{V_t} \right) - 1 \right\} \\
 &= I_S \left\{ \exp \left(\frac{V_{D0}}{V_t} \right) - 1 \right\} + \frac{I_S}{V_t} \exp \left(\frac{V_{D0}}{V_t} \right) \cdot (\Delta V_D) \\
 &\quad + \dots \rightarrow \Delta V_D^2, \Delta V_D^3 \text{ etc.}
 \end{aligned}$$

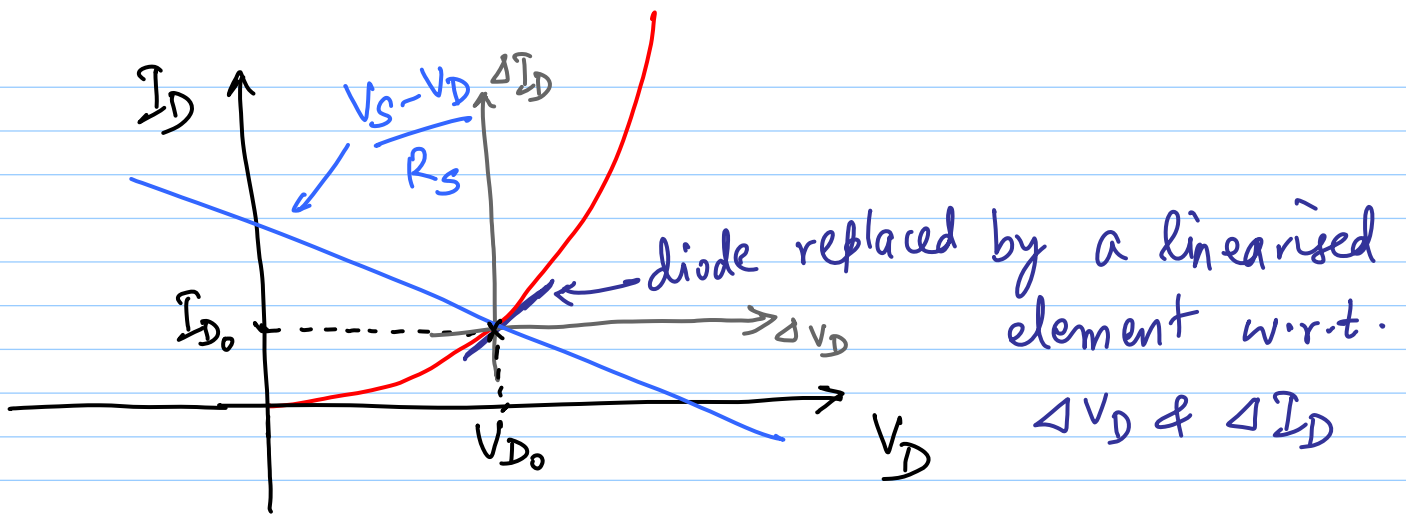
* for "small" values of ΔV_D , you can neglect $\Delta V_D^2, \Delta V_D^3$ etc.

$$\begin{aligned}
 & \frac{V_S - V_{D0}}{R_S} + \frac{\Delta V_S - \Delta V_D}{R_S} = I_{D0} + \Delta I_D \\
 &= I_S \left\{ \exp \left(\frac{V_{D0}}{V_t} \right) - 1 \right\} + \frac{I_S}{V_t} \exp \left(\frac{V_{D0}}{V_t} \right) \cdot (\Delta V_D)
 \end{aligned}$$

or terms cancel out

$$\begin{aligned}
 \frac{\Delta V_S - \Delta V_D}{R_S} &= \Delta I_D \\
 &= \frac{I_S}{V_t} \exp \left(\frac{V_{D0}}{V_t} \right) \cdot (\Delta V_D)
 \end{aligned}$$

* Linear in incremental quantities
 $\Delta V_S, \Delta V_D, \Delta I_D$



slope @ OP = conductance