Lecture #6: RF Inductors

RF Circuits = Analog Circuits + Inductors

Std. DC process does not include inductors.

Digital ICs → NMOS, PMOS, MOS capacitors
  + resistors (rarely)

Analog ICs → (above) + capacitors (MOM or MIM)

RF → (above) + inductors

Passive inductors are indispensable!

Active inductors = higher noise,
  higher distortion,
  higher power

Take a step back — Skin Effect

DC - entire conductor cross-section is used for current flow.

AC - Faraday's Law: AC current flow establishes a magnetic field that induces an electric field whose associated currents (eddy currents) oppose the original current

* This effect is strongest at the center of the conductor (r=0) → current tends to flow in the outer portion of the conductor.

Hence the name Skin Effect
DC: \[ R_{DC} = \frac{pl}{A} \approx \frac{l}{\sigma A} \]

AC: \[ R_{AC} \approx \frac{r}{28} \text{ where } r = \text{radius of cylindrical conductor} \]

Skin depth \[ \delta = \sqrt{\frac{2}{\omega \mu_0}} = \sqrt{\frac{2}{2\pi f \cdot \mu_0}} \cdot \sqrt{\frac{1}{\sigma}} \]

\[ = \frac{1}{\sqrt{\pi \cdot (2 \times 10^9) \cdot (4\pi \times 10^{-7})}} \cdot \sqrt{\frac{1}{\sigma}} \]

\[ = 1.126 \times 10^{-2} \sqrt{\frac{1}{\sigma}} \]

Al: \[ \delta = 1.126 \times 10^{-2} \sqrt{\frac{1}{3.816 \times 10^7}} = 1.82 \mu m \]

Cu: \[ \delta = 1.126 \times 10^{-2} \sqrt{\frac{1}{5.813 \times 10^7}} = 1.48 \mu m \]

Au: \[ \delta = 1.126 \times 10^{-2} \sqrt{\frac{1}{4.078 \times 10^7}} = 1.76 \mu m \]
What are the IC options available for passive inductors?

*Bondwires*

- 25 µm diameter, low resistivity
- High Q (>20 in 1-100 kHz range)
- Thumb rule ~ 1nH/mm length
- Large variation in L value (15-20%)
due to manufacturing tolerances (these are meant for IO & VDD and, not inductance)

*Spiral Inductors*

- On-chip metal traces laid out in a spiral shape
- Metal is either Au or Au-Cu alloy, so higher series resistance ($p_{Au} \approx 250 \mu\Omega \cdot \text{m}$, $p_{Cu} \approx 25 \mu\Omega \cdot \text{m}$)
- Low Qs in std. CMOS process (Q<10 nH 1-100 kHz range)

*Only inductances <10nH are practical for IC implementation*

*Origin of inductance: time varying magnetic flux produced by current-carrying strips of metal*

*Degrees of freedom: W, S, din, dout (total area), n (number of turns)*
* Intuitive dependence on geometry:
  \[ L \propto \text{core area} \]
  \[ L \propto n \]
  \[ L_{\text{tot.}} = L + M \] (Mutual inductance)
  Self inductance

* Current in same direction \( \Rightarrow \) positive \( M \)
  - Current in adjacent turns
* Current in opposite direction \( \Rightarrow \) negative \( M \)
  - Currents in opposite sections of spiral
  - Larger inner diameter \( \Rightarrow \) larger \( Q \)
    but also larger area?
    \( \Rightarrow \) leads to more “hollow” inductors.

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High-frequency effects

![Diagram showing effects like proximity, current crowding at edges (skin effect), radiation effects, substrate injection, and lateral currents.](image)
1) Edge Effects:
- Avoid 90° routings for signal lines as much as possible
- Crowding of charge carriers around edges due to sudden 90° change in current direction
- Larger current density → lower Q
- Larger resistance

2) Substrate Losses:
- Eddy currents are induced in semiconductor substrate → magnetic losses to substrate
- Substrate losses ∝ area of inductor

Common Geometries:

Square Spiral:

Circular Spiral:
- Often not supported by process

Octagonal Spiral:
- Circular spiral gives best Q
- Largest amount of metal in least area
- No edge effects
Spiral inductor modelling:

What do we need for a physical model?

1) **Spiral inductance** - $L_S \propto \text{area, } n$

2) **Series resistance** - $R_S$

   Obvious dependence: $R \propto$ metal, thickness
   
   $R_S \propto \frac{1}{\text{length of conductor}}$
   
   (i.e. $\propto \frac{1}{\text{width}}$)

   Other factors: skin, edge & proximity effects

**Note that the above inductor is asymmetric**

* Need underpass to access both ports of inductor
* Q is often limited by underpass and via resistance
3) Input-output coupling capacitance - $C_c$
- metal layer to underpass overlap
- coupling capacitance between turns
$C_c \propto \text{area, metal-metal oxide thickness}$

4) Capacitive coupling between spiral and lossy substrate through oxide - $C_{\text{ox}}$
$C_{\text{ox}} \propto \text{area, Spiral-substrate oxide thickness}$

5) Equivalent resistance and capacitance of substrate to ground - $R_{\text{si}}, C_{\text{si}}$
$\rightarrow$ depend on doping levels etc.
(substrate resistivity)

**Lumped \( \pi \)-model** (Usually extracted from S-parameters - obtained from simulation or measurement)

![Diagram of Lumped \( \pi \)-model with labels for components: $C_c$, $L_s$, $R_s$, $C_{\text{ox}}$, $R_{\text{si}}$, $C_{\text{si}}$, and $R_{\text{eq}}$]
* Compact, simple & quite physical in nature
* Usually chosen to be symmetric (esp. if the inductor is symmetric)

* Accurate over the frequency range of interest; for a good broadband fit, Rs may need to be a function of frequency (due to skin effect etc.)