Lecture 36: Power Amplifiers

- Narrowband vs. Broadband
- Linear vs. Constant Envelope operation
  - AM etc.
  - PM, FM etc. (usually switching PAs)
- Tradeoffs
  - Power gain
  - Linearity
  - Output Power
  - Efficiency (drain eff. & power added eff.)

Classical PAs (linear)
- Class A, AB, B, C
- Classified based on bias conditions

\[ \text{Diagram: } \begin{align*}
\text{Input} & \quad 0 \quad \text{Bias} \quad \text{BFC} \quad i_0 \\
& \quad \text{Vin} \quad \text{Rf} \quad \text{I_d} \\
& \quad \text{Tank to filter output (tuned to } \omega_0) \\
& \quad \text{* BFC prevents DC power diss. in } R_L \\
& \quad \text{* BFL provides approximated constant current}
\end{align*} \]
* tank cir with high Q provides linear output

I Class A
* 260° conduction angle
* $V_{in\min} = V_T$

![Graph showing VDS, VDD, and VDSAT](image1)

* high linearity
* poor efficiency

\[ i_D = I_{DC} + i_{RF} \sin \omega t \]
\[ i_o = I_{DC} - i_D = -i_{RF} \sin \omega t \]
\[ V_{out} = i_o \cdot R_L = -i_{RF} R_L \sin \omega t \]
\[ V_{DS} = V_{DD} + i_o \cdot R_L = V_{DD} - i_{RF} R_L \sin \omega t \]

\[ P_{out} = i_{out} \cdot V_{out} = i_{RF}^2 R_L \sin^2 \omega t \]

![Graph showing Pout and PRF](image2)

\[ PRF = (i_{RF} \text{rms})^2 \cdot R_L = \frac{i_{RF}^2 R_L}{2} \]
\[ P_{DC} = \text{DC power from } V_{DD} \]
\[ = V_{DD} \cdot I_{DC} = V_{DD} \cdot i_{RF} \] (assume MI just turns off at lower extreme)

\[ \eta = \text{drain mirror efficiency} \]
\[ = \frac{P_{out}}{P_{Du}} = \frac{1}{2} \frac{i_{RF}^2 R_L}{i_{RF} \cdot R_D} = \frac{1}{2} \frac{i_{RF} \cdot R_L}{V_{DD}} \]

max. value of \( i_{RF} \cdot R_L = V_{DD} \) (max. swing neglecting \( V_{OSAT} \))

\[ \Rightarrow \eta = \frac{1}{2} \approx 50\% \]
practical \( \eta \approx 30 - 35\% \)

Normalised power output capability \( \equiv P_N \)
\[ P_N = \frac{P_{RF}}{V_{DSpk} \cdot i_{APk}^2} = \frac{V_{DD}^2 / 2 R_L}{(2V_{DD}) \cdot (2V_{DD}) / R_L} \]
\[ = \frac{1}{8} \quad \text{high device stress} \]

II Class B PA
+180° modulation

*Current flow only when \( V_{DS} \) is small \( \Rightarrow \text{low } P_{in} \).*
\[ i_D = i_{RF} \sin \omega t \text{ for } 0 - T/2 \]

* Tank filters out harmonics of \( i_D \), leaving a sinusoidal voltage across \( R_L \)

* Fundamental harmonic:

\[
i_D(\omega_0) = \frac{2}{T} \int_{0}^{T/2} i_{RF} \sin \omega t \cdot \sin \omega_0 t \, dt = \frac{i_{RF}}{2}
\]

\[ V_O = \frac{i_{RF} R_L}{2} \sin \omega_0 t \]

\[ V_O(\text{max}) = V_{DD} \Rightarrow \frac{i_{RF}(\text{max})}{2} = \frac{2V_{DD}}{R_L} \]

\[ P_O(\text{max}) = \frac{V_{DD}^2}{2R_L} \]

\[ i_{DC} = \frac{1}{T} \int_{0}^{T/2} \frac{2V_{DD}}{R_L} \sin \omega t \, dt = \frac{2V_{DD}}{\pi R_L} \]

\[ P_{DC} = i_{DC} \cdot V_{DD} = \frac{2V_{DD}^2}{\pi R_L} \]

\[ \eta = \frac{P_{out}}{P_{DC}} = \frac{V_{DD}^2/2R_L}{2V_{DD}^2/\pi R_L} = \frac{\pi}{4} = 78.5\% \]
\[ P_N = \frac{P_{RF}}{V_{DS(\text{max.})} I_D(\text{max.})} \]
\[ = \frac{V_{DD}^2}{2R_L} \frac{1}{2V_{DD}} = \frac{1}{8} \text{ High Stress} \]

With biasing:

\[ V_b = V_{RF} \text{ mWOT} \]

Class-A

\[ V_b > (V_T + V_{RF}) \]

Class-B

\[ V_b = V_T \]

III. Class-C PA \( \Rightarrow V_b < V_T \)

Conduction for \(< 180^\circ\)

\[ V_{DS} \]

\[ V_{DD} \]

\[ I_D \]

\[ 2\Phi = \text{conduction angle} \]

\[ \Phi = \cos^{-1} \left( -\frac{I_{DC}}{I_{RF}} \right) \]

"Bias unmen"

\[ I_{DC} = -I_{RF} \cos \Phi \text{ \{ offset unmen \}} \]

\[ \text{negative} \]
average current

\[
\bar{i}_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} (I_0 + i_{RF} \cos \phi) \, d\phi \\
= \frac{1}{\pi} \left[ \frac{2}{\pi} \pi I_0 \right] + \frac{1}{\pi} \left| \frac{i_{RF} \sin \phi}{\pi} \right|_{-\pi}^{\pi} \\
= \frac{i_{RF}}{\pi} \left( \sin \phi - \sin \phi \right)
\]

fundamental:

\[
i_{fund} = \frac{2}{\pi} \int_{0}^{\pi} i_0 \cos \omega t + dt \\
= \frac{1}{\pi} \left( 4 I_0 \sin \phi + 2i_{RF} \phi + 2i_{RF} \sin \phi \right)
\]

\[
= \frac{i_{RF}}{\pi} \left( 2 \phi - \sin 2\phi \right)
\]

max. swing \leq V_{DD}

\[
\Rightarrow V_{DD} = \frac{i_{RF}}{\pi} \frac{R_L}{\pi} \left( 2 \phi - \sin 2\phi \right)
\]

\[
\Rightarrow i_{RF} = \frac{2 \pi V_{DD}}{R_L \left[ 2 \phi - \sin 2\phi \right]}
\]

\[
i_{D, pk} = i_{RF, max} + I_{DC}
\]

\[
= 2 \pi V_{DD} \left[ 1 + \frac{\min \phi - \phi \cos \phi}{\pi} \right] \frac{1}{R_L \left( 2 \phi - \sin 2\phi \right)}
\]

\[
\Rightarrow I_{max} = \frac{2 \phi - \sin 2\phi}{4 \left( \sin \phi - \phi \cos \phi \right)}
\]
as $\phi \to 0$, $\eta \to 100\%$
but gain $\propto$ $P_{out}$ $\to 0$
* we can obtain high efficiency at
the expense of linearity, gain & $P_{out}$

**Switching PAs**

**Basic principle:** use MOSFET as a **switch**
rather than as a controlled current source
in the case of linear PAs

**Ideal switch** $\text{ON} \Rightarrow V=0$, $I>0$ $P_{out} \to \infty$
$\text{OFF} \Rightarrow V>0$, $I=0$ $P_{out}=0$

no loss in switch $\Rightarrow 100\%$ efficiency
You can show that:

**normalized power handling capability**

\[ P_N = \frac{P_{out}}{V_{DS pk} \cdot I_{D pk}} = \frac{1}{\pi} \]

\( \leq \) much lower stress than

**Linear PAs**

ideal \( \eta = 100\% \)

practical: switches must be very fast relative to \( \omega_0 \), otherwise \( \eta < 100\% \).

**IV Class-E PAs**

key ideas: switch voltage \( \approx 0 \) before current flows

\( \approx \) use higher order filter to shape the pulses

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![Class-E PA Circuit Diagram]

- **\( V_{DD} \)**
- **\( V_{DS} \approx 3.6 \times V_{DD} \)**
- **no overlap**
- **on turn-on**
- **on turn-off transient**

**\( C_1, C_2 \)**

- \( V_{out} \)
- \( V_{in} \)

**\( L \)**

**\( R_L \)**

- \( \frac{V_{DD}}{R_L} \approx 1.7 \times V_{DD} \)

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**Good (ex: BJT PA)**
Ref: Sokal & Sokal, JSSC June 1975

**Design Equations**

\[
L = \frac{Q R_L}{\omega}
\]

\[
C_1 = \frac{1}{W R_L \left( \frac{\pi^2}{4} + 1 \right) \left( \frac{\pi}{2} \right)} \approx \frac{1}{5.447 W R_L}
\]

\[
C_2 \approx C_1 \left( \frac{5.447}{Q} \right) \left( 1 + \frac{1.42}{Q - 2.08} \right)
\]

\[
P_{\text{out}} \text{ (max.)} = \frac{2}{1 + \pi^2/4} \cdot \frac{V_{DD}^2}{R_L} \approx 0.577 \cdot \frac{V_{DD}^2}{R_L}
\]

\[
P_N = \frac{P_o}{V_{DS(pk)} \cdot i_{DS(pk)}} \geq 0.098 \quad \text{high stress}
\]

Note that \( V_{DS(pk)} = 3.6 V_{DD} \), \( i_{DS(pk)} = 1.7 \frac{V_{DD}}{R_L} \) very high values.

\[
VI \quad \text{Class-F PAs}
\]

\[
V_{in} \quad 1 \quad \frac{\lambda}{4} \quad \Theta\quad 2_{o} \quad R_L \approx 2_{o}
\]

\[
BFL \quad BFC
\]
\[
\frac{\eta}{\gamma} \times \frac{z_0}{\gamma}
\]

\[
z_{in} = \frac{z_0^2}{2\eta}
\]

Here \( z_{in} = R_L @ \omega_0 \)

\( L \text{cap} C \Rightarrow Z(w) = 0 \text{ for } w \neq \omega_0 \) (short dut)

@ \( w = 2n\omega_0 \), \( R \)-line \( R = \frac{2n\gamma}{\delta} = \frac{n\gamma}{2} \)

\( \Rightarrow \text{short dut. @ Drain} \)

@ \( w = (2n+1)\omega_0 \), \( R \)-line \( R = (2n+1)\gamma \)

\( L \text{cap} C \text{ short dut } \Rightarrow \text{open dut. @ Drain} \)

\[V_{DS} \]

\[2V_{DD} \]

\[8V_{DD} \]

\[\frac{8V_{DD}}{\pi R_L} \]

\[t \]

\[i_d \]

\[\text{sinusoidal current} \]

\[\text{pulsed into drain} \]

\[V_{\text{fund.}} = \frac{4}{\pi} (V_{DD}) \]

\[P_0 = \left[ \frac{4}{\pi} \left( \frac{V_{DD}}{\sqrt{2}} \right) \right]^2 - \frac{1}{R_L} = \frac{8V_{DD}^2}{\pi^2 R_L} \]
\[ \eta_{\text{ideal}} = 100\% \]

in practice \( \eta > \eta_{\text{class-E}} \)

\[
P_N = \frac{P_0}{V_{D_Spk} \cdot I_{Dpk}} = \frac{8V_{DD}^2}{\pi^2 R_L} = \frac{1}{2\pi} \approx 0.16 \quad (\text{better than class-E})
\]

alternative topology: replace T-line with L-C

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*Note: switching PAs are constant envelope PAs

\[ V_{out} = f(V_{DD}), \text{ not } f(V_{\text{in}}) \]

Other design considerations:

1) Power-added Efficiency:

\[ \text{PAE} = \frac{P_{out} - P_{in}}{P_{DC}} \]

obviously \( \text{PAE} < \eta \)

\( \Rightarrow \) takes power gain into account

2) Stability: * gain is very important (layout)

* stability = gain trade-off
3) **Breakdown**

* Output swings up to $2V_{DD}$
* $BV$ reduces as tech. scales

→ **D B E S B diode breakdown (A)**
→ **D-S punchthrough (B)**
→ **Time-dependent dielectric breakdown (TDDDB) (C)**
→ **Gate oxide rupture (D)**

(A): Diode $BV \sim$ few $V$ (2-3x $V_{DD}$)
(B): If $V_D$ is large, depletion region extends to source, eliminating the channel

(C): Gate oxide damage due to energetic carriers - @ high fields, high energy e's create oxide traps; charge trapped here shift device $V_T$ (cumulative)

(D): Gate oxide rupture occurs @ high gate fields

4) Large - Signal impedance matching