Lecture 35: Wideband Amplifiers - II

VII Zero-peaked C-s. Amplifier

\[ Z_s(s) = R_s \parallel \frac{1}{\alpha C_s} = \frac{R_s}{1 + \alpha C_s R_s} \]

\[ G_m(s) = \frac{g_m}{1 + g_m Z_s(s)} \]

\[ Z_L(s) = \frac{R_L}{1 + \alpha C_L R_L} \]

\[ \Rightarrow G_m(s) = \frac{g_m}{1 + \frac{g_m R_s}{1 + \alpha C_s R_s}} = \frac{g_m}{1 + \frac{g_m R_s}{1 + \alpha C_s R_s}} \cdot \frac{1 + \alpha C_s R_s}{1 + \alpha C_s R_s} \]

\[ = \frac{g_m}{1 + \frac{g_m R_s}{1 + \alpha C_s R_s}} \cdot \frac{1 + \alpha C_s R_s}{1 + \alpha C_s \frac{R_s}{g_m R_s}} \]

\[ \Rightarrow G_m(s) = G_m(s) \cdot Z_L(s) \]

\[ A_v(s) = G_m(s) \cdot Z_L(s) = G_m(s) \cdot \frac{R_L}{1 + \frac{\alpha C_s}{g_m}} \cdot \frac{1 + \alpha C_s R_s}{1 + \alpha C_s \frac{R_s}{g_m}} \]
\[ (G_{m0} R_L) \left[ \frac{1 + \frac{G_m R_s}{1 + \frac{G_m}{C_L R_L}}}{1 + \frac{G_m}{C_L R_L}} \right] - \frac{1}{1 + \frac{G_m}{C_L R_L}} \]

**DC gain**

**Pole-zero cancellation**

**High freq. pole**

**Pole \( \frac{(G_m / C_L R_L)}{} \)**

**Overall response**

\[ f_T - doubling \]

\[ \omega_T = 2 \pi f_T = \frac{g_m}{C_{gs} + C_{gd}} \approx \frac{g_m}{C_{gs}} = \frac{g_m}{C_{in}} \]

(i) **Diff-pair:**

\[ i_{od} = i_1 - i_2 \]

\[ g_{m1} = g_{m1,2} \text{ unchanged} \]

\[ C_{id} = (C_{gs1}) \text{ series } (C_{gs2}) \]

\[ f_T' \approx 2 f_T \]
Remember that for a single transistor

\[ f_T = \frac{g_m}{C_{gs}} \times \sqrt{I_{bias}} \]  
(long-channel)

\[ I_{bias} \rightarrow 2I_{bias} \Rightarrow f_T \rightarrow 1.41f_T \] (best case)

But diff. pair; \( 2I_{bias} \Rightarrow f_T' = 2f_T \)

* Diff signal path may not be convenient

(i) Single-ended \( f_T \)-doubler (similar to Darlington pair)

* Interchange \( C_1 \) & \( S \) connections on one of the diff. pair devices & sum the outputs

* Both devices are biased at \( I_{bias} \)

Batties \( f_T \)-doubler (Tektronix)

* \( M_2-M_3 \) is a current mirror \( \Rightarrow \) equal \( I_{bias} \)
but $C_{gs2} \& C_{gs3}$ are in parallel

$\Rightarrow C_{as} = (C_{gs1})$ series $(C_{gs2} \parallel C_{gs3})$

$f'_T \approx (1.5) f_T$

* $f_T$ - doublers will not work well if $C_{is}$ limited by other factors

$\Rightarrow$ load cap $C_L$

$\Rightarrow$ parasitic $C_{so}$ cap

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Cascaded Amplifiers

$\begin{align*}
\text{Vin} \rightarrow D \rightarrow \cdots \rightarrow D \rightarrow \text{Vout} \\
1 & 2 & \cdots & n
\end{align*}$

* All $n$ amplifiers are identical

* Each stage has single-pole response

$A(s) = \frac{A_0}{1 + s/w_0}$

Overall cascade TF is

$H(s) = \left( \frac{A_0}{1 + s/w_0} \right)^n$
\* find -3dB BW of the cascade \( (w_{on}) \):

at \( w = w_{on} \), 

\[
|H(\omega)| = \frac{1}{\sqrt{2}} |H(0)|
\]

\[
\begin{align*}
\frac{A_0}{\sqrt{1 + \left( \frac{w_{on}}{w_o} \right)^2}} &= \frac{1}{\sqrt{2}} A_0^n \\
\Rightarrow w_{on} &= w_o \sqrt{2^n - 1}
\end{align*}
\]

**BW shrinkage**

Recall that \( A_0 w_o = w_u \) \( \Rightarrow \)

\[
\frac{w_{on}}{A_0} = \frac{w_u}{w_o} \sqrt{2^n - 1}
\]

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**a) BW shrinkage**

\* as \( n \to \infty \), \( w_{on} \to 0 \)

\* as \( n \to \infty \), DC gain \( A_{on} = A_0^n \to \infty \)

\* we want to find approximate expression for \( w_{on} \) as \( f(n) \)

\[
2^n = \exp \left\{ \ln (2^n) \right\} = \exp \left\{ \frac{1}{n} \ln 2 \right\}
\]

for large \( n \), we the first two terms in 

expansion 

\[
\exp \left\{ \frac{1}{n} \ln 2 \right\} \approx 1 \frac{1}{n} \ln 2
\]

\[
\Rightarrow w_{on} \approx w_o \sqrt{\frac{1}{n} \ln 2} \approx \frac{0.883 w_o}{\sqrt{n}}
\]
i.e. BW shrinks as $\sqrt[\nu]{n}$ for large $n$

$n \geq 4 \Rightarrow \text{error} < 5\%$

b) **Optimum gain per stage**

Given total gain $A_{\text{tot}}$, we want to find **optimal** $n$ & **maximize** BW

$$A_0 = A_{\text{tot}} \Rightarrow A_0 = A_{\text{tot}}^{\frac{1}{\nu n}}$$

$$W_{\text{on}} = \frac{W_n}{A_{\text{tot}}} \cdot \sqrt{\frac{1}{2^{\nu n} - 1}}$$

apply $\frac{dW_{\text{on}}}{dn} = 0$

After some algebra:

$$n_{\text{opt.}} = \frac{\ln 2}{\ln \left\{1 + \frac{\ln 2}{2 \ln A_{\text{tot}}}\right\}}$$

For large $A_{\text{tot}}$,

$$\ln \left\{1 + \frac{\ln 2}{2 \ln A_{\text{tot}}}\right\} \approx \frac{\ln 2}{2 \ln A_{\text{tot}}}$$

$$\ln(1 + x) \approx x \text{ for } x \ll 1$$

$$\Rightarrow n_{\text{opt.}} \approx 2 \frac{\ln A_{\text{tot}}}{\ln 2}$$
optimum gain/Stage:

\[ A_{0,\text{opt}} = (A_{\text{tot}})^{\frac{1}{\gamma_{\text{opt}}}} = \exp \left\{ \frac{1}{\gamma_{\text{opt}}} \ln A_{\text{tot}} \right\} \]

\[ \approx e^{\frac{1}{2}} \]

\[ A_{0,\text{opt}} = \sqrt{e} \]

optimum BW:

\[ \omega_{n,\text{opt}} = \frac{W_n}{A_{\text{tot}}^{\frac{1}{\gamma_{\text{opt}}}}} \sqrt{\frac{1}{2} \gamma_{\text{opt}} - 1} \]

\[ \approx \frac{W_n}{\sqrt{e}} \left[ \exp \left\{ \frac{1}{\gamma_{\text{opt}}} \ln 2 \right\} - 1 \right]^{\frac{1}{2}} \]

\[ \omega_{n,\text{opt}} \approx W_n \sqrt{\frac{\ln 2}{2e \ln A_{\text{tot}}}} \]

\[ \omega_{n,\text{opt}} \approx \frac{0.357 W_n}{\sqrt{\ln A_{\text{tot}}}} \]

In other words,

* \( BW \times \sqrt{\ln a} = \text{constant} \)

* If \( A_{\text{tot}} \to A_{\text{tot}} \times 100 \), \( BW \to < BW \times 2 \)

* Overall amp does not have constant \( G \times BW \) product (obviously, be cause \( G \times BW \) is constant only for single-pole systems)
* Gain product for this cascaded amp

\[ G = A_{\text{tot}} \cdot W_0 \]

\[ = A_{\text{tot}} \cdot \frac{0.357W_0}{\sqrt{\ln(A_{\text{tot}})}} \Rightarrow \text{increases without bound} \]

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**Gain-bandwidth-delay Tradeoff**

* Delay is less important in systems with 1-way comm. (e.g. TV, optical fibre comm.)
* Coupling between A & BW is weak for higher order cascaded systems
* If delay can be arbitrary, what A or B can be achieved?

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* Recall: \( BW \propto \frac{1}{\text{rise time}} \)

* Imagine an amp that stores energy in input step for a long time, then dumps it suddenly into an output \( \Rightarrow \) very fast rise time \( \Rightarrow \) high BW

**Distributed Amplifier** (Travelling Wave Amp.)

![Distributed Amplifier Diagram](image-url)
* inputs to transistors supplied by tapped delay line
* output fed into another tapped delay line
* assume input has a voltage step.
  → after a delay through each line, it appears at transistor inputs
  → each transistor generates current equal to \( g_m \times \text{input step} \)
  → currents of all transistors sum coherently in time, if delays of input & output line are matched

* at each point of the tapped delay line,
  \[ \text{Zin} = \frac{Z_0}{2} \]
  
* overall gain \( Av = \frac{n \cdot g_m \cdot Z_0}{2} \) for \( n \) stages

→ \( Av \propto n \)
→ \( Av > 0 \) if \( g_m > 0 \)
→ BW does not factor directly into tradeoff
→ assume that \( C_{in} \&\ C_{out} \) of transistors are absorbed into \( RLCG \) of TDL
→ \( C_{in} > C_{out} \) ⇒ matching between TDLs is difficult \( \{ C_{gs} > C_{db} \} \)
* Can be power-hungry
* TDLs can be replaced by lumped LC equivalents (artificial T-lines)
* Main advantage: You can achieve significant gain @ freq. close to \( f_r \)
* High gain & low NF not possible
* Area is large
* \( Z_0 \)'s for G & D T-lines need not be the same
* If T-lines are lossy, \( A_v \to 0 \) as \( N \to \infty \)

\[ \text{Vin in gate line decays exponentially} \]
\[ \Rightarrow A_v \text{ increases linearly with } n \]
\[ \Rightarrow n_{opt.} \text{ exists for a given set of TDL's and MOSFETS.} \]

**Artificial T-lines**
* Has BW limitation because of lumped LC (ideal lossless TDL has no BW limitation)
* \( T_{delay} = \sqrt{LC} \text{ per LC-section} \)
* (Ideal TDL \( T_{del.} = \sqrt{LC} \cdot z \), \( z=\text{length} \))
\[ Z_{in} = j \omega L \frac{1}{2} \left[ 1 \pm \sqrt{1 - \frac{4}{\omega^2 LC}} \right] \]

\[ \Rightarrow \text{cutoff} = \frac{2}{\sqrt{LC}} \quad \text{cutoff frequency} \]

* Choose \( LC \) for
  
  \[ \Rightarrow \text{min. loss & attenuation} \]
  
  \[ \Rightarrow \text{cutoff } > \text{required BW} \]
  
  \[ \Rightarrow \text{desired } T_{\text{delay}} \]
  
  \[ \Rightarrow \text{constant group delay (min. dispersion)} \]

* for a lossy line, constant G.D. \( \Rightarrow RC = CL \)

\[ \Rightarrow \frac{1}{\tau} = \frac{C}{L} \quad \text{equal time constants} \]