Recall: \[ \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \]

In general, \[ [V] = [Z] \cdot [I] \]

Hence \[ [Y] \], \[ [H] \], \[ [G] \] can also be defined.

All of these relate \underline{voltages} and \underline{currents} of the network.

For a distributed network, we talk of \underline{incident} & \underline{reflected} waves.

\[ \Rightarrow \text{Scattering (S-) parameters} \]

\[ [V_R] = [S] \cdot [V_i] \]

\[ \begin{bmatrix} V_{R_1} \\ V_{R_2} \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} V_{i_1} \\ V_{i_2} \end{bmatrix} \]
Usually normalized w.r.t. $Z_0$:

$$a_1 = \frac{V_i_1}{\sqrt{Z_0}}; \quad a_2 = \frac{V_i_2}{\sqrt{Z_0}}$$

$$b_1 = \frac{V_r_1}{\sqrt{Z_0}}; \quad b_2 = \frac{V_r_2}{\sqrt{Z_0}}$$

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$$

$a_1^2, a_2^2, b_1^2, b_2^2 \Rightarrow$ powers of incident & reflected waves

$S_{ij}$ are usually represented in dB

i.e. $20 \log_{10} \text{(value)}$

$$S_{11} = \frac{b_1}{a_1} \bigg|_{a_2=0} = \frac{V_r_1}{V_i_1} = \Gamma_i \quad \text{(input reflection coefficient)}$$

$$S_{21} = \frac{b_2}{a_1} \bigg|_{a_2=0} = \frac{V_r_2}{V_i_1} \quad \text{(gain)}$$

$$S_{21}^2 = \frac{V_r_2^2}{V_i_1^2} = \frac{V_r_2}{V_i_1^2} = \frac{Z_0}{V_i_1^2} = \text{power gain}$$

$$S_{22} = \frac{b_2}{a_2} \bigg|_{a_1=0} = \frac{V_r_2}{V_i_2} = \Gamma_2 \quad \text{(output port reflection coefficient)}$$

$$S_{12} = \frac{b_1}{a_2} \bigg|_{a_1=0} = \frac{V_r_1}{V_i_2} \quad \text{(reverse transmission gain)}$$
Short note on units etc:

let \( x \) be any linear electrical value (e.g. \( x \) (in dB) = 20 \log (x) \text{ etc.})

power \( P \propto V^2 \) or \( I^2 \)

\[
: \quad P \left( \text{in dB} \right) = 10 \log (P)
\]

\[
P \left( \text{in dBm} \right) = 10 \log \left( \frac{P}{1 \text{mW}} \right)
\]

i.e. 0 dB reference value is 1 mW

\[
1 \text{ mW} \text{ in dBm} = 10 \log \left( \frac{1 \text{ mW}}{1 \text{ mW}} \right) = 0 \text{ dBm}
\]

\[
10 \text{ mW} \text{ in dBm} = 10 \log (10) = 10 \text{ dBm}
\]

If \( Z_0 = 50 \Omega \), 0 dBm corresponds to \( \approx 223 \text{ mV} \) rms

we can also talk about dBV etc.

\( 1 \text{ V} \leftrightarrow 0 \text{ dBV} \)

\( \text{dBc} \equiv \text{power w.r.t. carrier} \)

"C" \( \Rightarrow \) 0 dB reference is power of carrier

\( \Rightarrow \) used for noise, distortion etc.

\( ABCD \) parameters (Transmission matrix)

very useful for cascaded 2-ports

\[
\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}
\]
Consider a cascade of two 2-port networks:

\[
\begin{bmatrix}
V_1 \\
I_1
\end{bmatrix} =
\begin{bmatrix}
A & B \\
C & D
\end{bmatrix}
\begin{bmatrix}
V_2 \\
I_2
\end{bmatrix}
\]

\[
\begin{bmatrix}
V_2 \\
I_2
\end{bmatrix} =
\begin{bmatrix}
A_2 & B_2 \\
C_2 & D_2
\end{bmatrix}
\begin{bmatrix}
V_3 \\
I_3
\end{bmatrix}
\]

\[
\Rightarrow \begin{bmatrix}
V_1 \\
I_1
\end{bmatrix} =
\begin{bmatrix}
A_1 & B_1 \\
C_1 & D_1
\end{bmatrix}
\begin{bmatrix}
A_2 & B_2 \\
C_2 & D_2
\end{bmatrix}
\begin{bmatrix}
V_3 \\
I_3
\end{bmatrix}
\]

can easily relate input & output of cascade.

HW I will include the following:

1) Derive \([S]\) in terms of \([Z]\)

2) Derive \([ABCD]\) in terms of \([Z]\)

**Resonance:**

- All (narrow band) RF systems employ resonance
- Tuned bandpass amplifiers
- Impedance transformations and matching
- RF oscillators
- Series and parallel RLC categories
Paralle RLC

\[ Z(\omega) \]

Admittance \( Y(\omega) = \frac{1}{R} + j\omega C + \frac{1}{j\omega L} \)

\[ = \frac{1}{R} + j(\omega C - \frac{1}{\omega L}) \]

At resonance, \( \omega_0 C = \frac{1}{\omega_0 L} \Rightarrow \omega_0^2 = \frac{1}{LC} \Rightarrow \omega_0 = \frac{1}{\sqrt{LC}} \)

\[ f_0 = \frac{1}{2\pi\sqrt{LC}} \]

Do not confuse \( \omega \) (rad/s) & \( f \) (Hz)

\[ Y(\omega_0) = \frac{1}{R} \Rightarrow \text{purely resistive} \]

\[ \text{In} \]
convenient m-chip values:  
$|F||f| \leq 5 \text{kHz}$  

**Quality Factor (physical definition):**  
$Q = \frac{\text{Energy stored}}{\text{Average power dissipated}}$  

At $\omega = \omega_0$, $V_{\text{out}} = I_{\text{in}} \cdot R$  

Stored energy moves back and forth between $L$ & $C$  

$V_{\text{out, pk}} = I_{\text{pk}} \cdot R \Rightarrow E_{\text{stored}} = \frac{1}{2} C (I_{\text{pk}} \cdot R)^2$  

$P_{\text{ave}} = (I_{\text{rms}})^2 \cdot R = (I_{\text{pk}} \frac{\sqrt{2}}{2})^2 \cdot R = \frac{1}{2} I_{\text{pk}}^2 R$  

$\Rightarrow Q = \omega_0 \cdot \frac{E_{\text{stored}}}{P_{\text{ave}}} = \frac{\frac{1}{2} C I_{\text{pk}}^2 R^2}{\frac{\sqrt{3}}{\sqrt{3} C}} = \frac{1}{2} I_{\text{pk}}^2 R$  

$Q = \frac{R}{\sqrt{L/C}}$  

Intuitive check: at $R \to \infty$, $Q \to \infty$  

Seems correct $\Rightarrow$ no average power dissipated  

$\sqrt{\frac{C}{L}} = \text{Characteristic impedance of network}$
At resonance,

\[ |Z_L| = \omega_0 L = \frac{L}{\sqrt{LC}} = \sqrt{\frac{L}{C}} \]

\[ |Z_C| = \frac{1}{\omega_0 C} = \frac{\sqrt{LC}}{C} = \sqrt{\frac{L}{C}} \]

**Basic forms of Q for parallel RLC:**

\[ Q = \frac{R}{\sqrt{LC}} \]

\[ Q = \frac{R}{|Z_L|} = \frac{R}{\omega_0 L} \]

\[ Q = \frac{R}{|Z_C|} = \omega_0 RC \]

---

**Beware!** Branch currents at resonance.

\[ |I_L| = |I_C| = \frac{|V_{out}|}{\omega_0 L} = \frac{|I_{in}|R}{\omega_0 L} = Q \cdot |I_{in}| \]

* Very large currents can flow through L & C at resonance.

* Careful layout is required to ensure current-carrying capability (esp. for C).

* It is dangerous to think of resonance as L & C “cancelling” each other out!
The relationship:

Calculate impedance close to resonance at a frequency \( \omega = \omega_0 + \Delta \omega \)

\[
Y(\omega) = \frac{1}{R} + j \left( \frac{\omega c - \frac{1}{\omega L}}{\omega L} \right)
\]

\[
= \frac{1}{R} + \frac{j}{\omega L} \left( \frac{\omega^2 LC - 1}{\omega_0 LC} \right)
\]

\[
Y(\omega_0 + \Delta \omega) = \frac{1}{R} + \frac{j}{\omega_0 LC} \left[ \frac{(\omega_0 + \Delta \omega)^2 LC - 1}{(\omega_0 + \Delta \omega)^2 LC + 2 \omega_0 \Delta \omega LC} \right]
\]

\[
= \frac{1}{R} + \frac{j}{\omega_0 LC} \left[ \frac{\omega_0^2 LC + 2 \omega_0 \Delta \omega LC}{\omega_0^2 LC + 2 \omega_0 \Delta \omega LC} \right]
\]

\[
Y(\omega_0 + \Delta \omega) = \frac{1}{R} + \frac{j}{\omega_0 LC} \cdot \omega_0 \Delta \omega \frac{1}{C} \left[ 2 + \frac{\Delta \omega}{\omega_0} \right]
\]

\[
\approx \frac{1}{R} + j \Delta \omega c \left( 2 + \frac{\Delta \omega}{\omega_0} \right) \left( 1 - \frac{\Delta \omega}{\omega_0} \right)
\]

\[
= \frac{1}{R} + j \Delta \omega c \left[ 2 - 2 \frac{\Delta \omega}{\omega_0} + \frac{\Delta \omega}{\omega_0} - \left( \frac{\Delta \omega}{\omega_0} \right)^2 \right]
\]

\[
\approx \frac{1}{R} + j 2 \omega_0 c \quad \{\text{neglecting } \Delta \omega^2, \Delta \omega^3, \ldots\}
\]

Equivalent circuit: \( \frac{1}{R^3} = 2C \)
by symmetry, 

\[ \text{total } QW = \frac{1}{Rc} \]

\[ \frac{W_0}{BW} = \frac{Rc}{\sqrt{LC}} = \frac{R}{\sqrt{L/C}} = Q \]

Different definitions of \( Q \):

1) Fundamental physical definition:

\[ Q = \omega_0 \frac{E_{tot}}{P_{ave}} \]

also applicable to distributed systems and non-resonant systems (e.g. \( Q \) of an RC network)

2) \[ Q = \frac{\text{Im}(Z(w))}{\text{Re}(Z(w))} \]

3) \[ Q = \frac{W_0}{BW} \]

4) \[ Q = \frac{\omega_0}{2} \left| \frac{d\phi}{dw} \right| \text{ where } \phi(w) = \text{phase of open loop TF} \]

Series RLC networks

intuitively: 

low freq. 

\[ \text{high freq.} \]

\[ \text{capacitive} \]

\[ \text{inductive} \]
At resonance, reactive impedances are equal and opposite \( \Rightarrow V_L + V_C = 0 \)

\[
Z(w) = R + jwL + \frac{j}{\sqrt{LC}} = R + j\left(wL - \frac{1}{wC}\right)
\]

\[
\omega_0 = \frac{1}{\sqrt{LC}} \quad j\nu_0 = \frac{1}{2\pi\sqrt{LC}}
\]

network behaves as if purely resistive

\[\text{Vin} \xrightarrow{\omega_0} R\]

Note: HW1 will have a problem on deriving \( Z(w) \) & \( Q \) expressions for series RLC networks

---

Prove for yourself:

\[|V_L| = |V_C| = |Vin| \cdot Q\]

\( \Rightarrow \) Useful for passive voltage amplification in LNAs

Series LC is not just a short!