Lecture 29: Colpitts Oscillator; Quadrature Signal Generation

Colpitts Oscillator

\[ w_0 = \frac{1}{\sqrt{L - \frac{C_1 C_2}{C_1 + C_2}}} \]

Hartley Oscillator

Colpitts Oscillator

\[ V_a = V_0 \cos w_0 t \]

Assume large amplitude

\[ V_s \]

Assume \( w_0 \) = high freq.

Assume \( C_1 \) is short @ \( w_0 \)

\[ i_d = I_{bias} \left\{ = \frac{1}{T} \int_{0}^{T} i_d(t) dt \right\} \]
M1 conducts only when \( V_a \) is large (\( \approx V_o \)).

- When \( V_a \ll V_o \), M1 cuts off, Ibias discharges \( C_1 \).

* \( i_a \) = periodic pulses of current @ \( \omega_0 \).

Fourier series:

\[
i_d(t) = I_0 + \sum_{n=1}^{\infty} I_n \cos(n\omega_0 t)
\]

\( t=0 \) reference = peak of \( V_o \cos \omega_0 t \)

\( I_{dc}(C_1) = 0 \Rightarrow I_0 = I_{bias} \)

\( \Rightarrow \) fundamental component of \( i_d \) is

\[
\hat{I}_1 = \frac{2}{\pi} \int_0^T i_d(t) \cos \omega_0 t \, dt
\]

\[
\hat{I}_1 = 2I_{bias}
\]

- If \( i_d \) flows through a tank tuned to \( \omega_0 \), only \( \hat{I}_1 \) creates a voltage

\[
G_m \approx \frac{\hat{I}_1}{V_0} = \frac{2I_{bias}}{V_0}
\]

\( \leq \) applicable to any device

Recall that

\[
g_m \text{ (long channel)} = \frac{2I_{bias}}{V_{DSAT}} \text{ constant (due to } C_1) \]
Short channel:

\[ I_D = \frac{\mu n C_0}{2} \left( \frac{W}{L} \right) (V_{GS} - V_T) \cdot L \cdot \varepsilon_c \text{ (voltage saturation)} \]

\[ \Rightarrow g_m = \frac{\partial I_D}{\partial V_{GS}} = \frac{I_{bias}}{V_{DSAT}} \]

\[ \Rightarrow \text{in general,} \]

\[ \frac{I_{bias}}{V_{DSAT}} \leq g_m \leq \frac{2I_{bias}}{V_{DSAT}} \]

\[ \Rightarrow \]

\[ \frac{V_{DSAT}}{V_0} \leq \frac{g_m}{g_m} \leq \frac{2V_{DSAT}}{V_0} \]

\[ \text{Resistive of } \frac{1}{g_m} \]
assume \( Y_{gm} \) does not load
the cap divider
(i.e. \( Q \gg 1 \))

\[
V_1 = \frac{C_2}{C_1 + C_2} \cdot V_{out} = \frac{1}{n} V_{out}
\]

\[
n = \frac{C_1 + C_2}{C_2} = 1 + \frac{C_1}{C_2} \quad \text{<--- equivalent ratio}
\]

equivalent net:

overall oscillator net becomes:

\[
C_{eq} = \frac{C_1 C_2}{C_1 + C_2}
\]

resonance: set \( \omega_0 = \frac{1}{\sqrt{L \cdot C_{eq}}} \)

\[
V_{out} = 2 I_{bias} \cdot R_p \parallel \frac{n^2}{G_m}
\]

\[
= 2 I_{bias} \cdot \frac{R_p \cdot n^2/G_m}{R_p + n^2/G_m}
\]
\[
= \frac{2I_{\text{bias}} R_p}{1 + \frac{R_p G_m}{n^2}} \\
\Rightarrow V_{\text{out}} \left[ 1 + \frac{R_p G_m}{n^2} \right] = 2I_{\text{bias}} R_p \\
V_{\text{out}} \left[ 1 + \frac{R_p}{n^2} \cdot \frac{2I_{\text{bias}}}{V_{\text{out}}/n} \right] = 2I_{\text{bias}} R_p \\
\Rightarrow V_{\text{out}} + \frac{2I_{\text{bias}} R_p}{n} = 2I_{\text{bias}} R_p \\
V_{\text{out}} = 2I_{\text{bias}} R_p \left( 1 - \frac{1}{n} \right)
\]
* For startup, use small-signal \( f_m \)

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**Quadrature Signal Generation**

1) 

\[
\begin{align*}
W_0 & \quad R \quad C \quad -45^\circ \\
C & \quad R \quad +45^\circ
\end{align*}
\]

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**Polyphase Filter**

\[
W_0 \quad 180^\circ \quad 180^\circ \\
\text{Buffer} \quad 180^\circ \quad \text{Buffers}
\]

* Buffers consume extra power
* R,C mismatches affect phase error
2) \[ 2 \omega_0 \rightarrow \square \rightarrow 90^\circ \]

* Basically a synchronous counter (that counts to 2)
* Power consumption @ high frequencies ↑

3) \text{Quadrature VCOs}

* Couple 2 identical oscillators in quadrature

* A - inputs (coupling)
* B - oscillator output
a) parallel coupling

* Exercise: Analyze the circuit with waveforms
  → assume outputs @ 0° & 180° phase and see what happens (both cases will result in no oscillations - i.e. any 0°/180° components will die out)

b) series coupling
(a) & (b): extra parasitic cap on tank (↓ tuning range)
(b): quadrature coupling devices have to be much larger than unscoupled devices (to increase headroom for CC devices)
(c) **Harmonic injection**
- Single-ended outputs
  - Even harmonics exist in voltages
- Common-mode nodes
  - Common-mode paths and/or currents

- Harmonics have specific phase relationship with fundamental freq.

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\[ \text{single-ended output - even harmonics (voltage)} \]
\[ \text{even harmonics (voltage)} \]
\[ \downarrow \text{even harmonic (current)} \]
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(i) C.S. coupling

(ii) Capacitive coupling
(iii) Transformer coupling

(iv) Backgate coupling
Other features of the VCO

1) Tuning & tuning range

\[
\frac{f_2 - f_1}{f_0}
\]

Varactor is a 2-terminal device

\[
\text{low-Q node}
\]

\[
\text{high-Q node}
\]

\[
\text{losses}
\]

\[
\text{parametric up}
\]

\[
\text{Tuning range} = \frac{f_2 - f_1}{f_0}
\]
2) Centre frequency \( f_c \) = \( \frac{f_1 + f_2}{2} \)

3) \( V CO \) gain
\[
K_{VCO} = \frac{f_2 - f_1}{\Delta V_C} \quad \{ \text{typically MHz/V} \}
\]

4) Power consumption

5) Supply pushing/rejection

- Noise on supply can show up at \( V CO \) output (around \( V CO \) freq)
- As phase noise or spurs

6) Phase noise