Lecture 14: Classical 2-port Noise Theory

\[ i = i_s \left( \frac{Y_{in}}{Y_s + Y_{in}} \right) + (i_n + e_n Y_s) \left( \frac{Y_{in}}{Y_{in} + Y_s} \right) \]

Noise Factor
- \( F = \frac{\text{total output noise power}}{\text{output noise due to input source only}} \)
- by convention, source is at a temp of 290K.
- \( F \) is a measure of degradation in SNR due to a system (degradation in SNRT \( \Rightarrow F \uparrow \))
- If system adds no noise, \( F = 1 \)

\[ \text{Equivalent circuit with noiseless 2-port} \]

\[ \text{In general, } e_n \text{ and } i_n \text{ are correlated} \]

\[ \text{OC & SC cases} \Rightarrow \text{you need both } e_n \text{ & } i_n \]

\[ \text{Actual circuit may have no physical input noise current} \]
\[ F = \frac{\bar{Is}^2 \left( \frac{\bar{Yn}}{I_c + \bar{Yn}} \right)^2 + \left| \bar{I}_n + \bar{en} \right|^2 \left( \frac{\bar{Yn}}{I_c + \bar{Yn}} \right)^2}{\bar{I}_s^2 \cdot \left( \frac{\bar{Yn}}{I_c + \bar{Yn}} \right)^2} \]

\[ = 1 + \frac{\left| \bar{I}_n + \bar{en} \right|^2}{\bar{I}_s^2} \]

Let \( \bar{I}_n = \bar{I}_c + \bar{I}_n \)

\( \bar{I}_c \) is correlated with \( \bar{en} \)

\( \Rightarrow \bar{I}_c = Y_c \cdot \bar{en}; Y_c = \) correlation admittance

\( \bar{I}_n \) is uncorrelated with \( \bar{en} \)

\[ \Rightarrow F = 1 + \left( \frac{\bar{I}_n + (Y_c + Ys)\bar{en}}{\bar{I}_s} \right)^2 \]

because \( \bar{I}_n \) is un-correlated with \( \bar{en} \)

\[ = 1 + \frac{\bar{I}_n^2 + (Y_c + Ys)^2 \bar{en}^2}{\bar{I}_s^2} \]

* Next, we define each of these 3 independent noise sources as an equivalent resistance or conductance and their associated thermal noise sources:

\[ R_N = \frac{\bar{I}_n^2}{4kT \Delta f} \]

\[ C_N = \frac{\bar{I}_n^2}{4kT \Delta f} \]

\[ G_N = \frac{\bar{I}_s}{4kT \Delta f} \]
\[ F = 1 + \frac{6u + [Y_c + Y_b]^2 \cdot RN}{G_s} \]

Now, let \( Y_c = G_c + jB_c \) and \( Y_b = G_b + jB_b \)

\[ F = 1 + \frac{6u + \left[ (G_c + G_b)^2 + (B_c + B_b)^2 \right]}{G_s} \cdot RN \]

Noise of any 2-port can be characterized by 4 parameters: \( \Sigma RN, G_u, G_c, B_c \).

Conditions that minimize \( F \) (i.e., optimum source admittance):

1. \( \frac{dF}{dB_s} = 0 \) \( \Rightarrow \) \( 2(G_c + G_b) \cdot RN = 0 \)
   \( \Rightarrow B_s = -B_c = B_{opt} \).

2. \( \frac{dF}{dG_s} = 0 \)
   \( \Rightarrow \frac{2(G_c + G_b)RN}{G_s} - \frac{G_u + \left[ (G_c + G_b)^2 + (B_c + B_b)^2 \right]}{G_s} \cdot RN = 0 \)
   \( \Rightarrow G_s^2 = \frac{G_u}{RN} + G_c^2 \)
   \( \Rightarrow G_s = \frac{\sqrt{G_u}}{\sqrt{RN}} + G_c = G_{opt} \).

**Design condition for \( F_{min} \):**

\( F_{min} \) is given by:

\[ F_{min} = 1 + \frac{G_u + \left[ (G_c + G_{opt})^2 + (B_c + G_{opt})^2 \right]}{G_{opt}} \cdot RN \]
Also, \( C_N = (G_{opt}^2 - G_c^2) \cdot R_N \)

\[
\therefore F_{min} = 1 + \frac{G_{opt}^2 R_N - G_c^2 R_N + G_c^2 R_N + 2G_cG_{opt}R_N + G_{opt}^2 R_N}{G_{opt}^2}
\]

\[
F_{min} = 1 + 2 R_N \left( G_{opt} G_c \right)
\]

\[
F_{min} = 1 + 2 R_N \left[ \sqrt{\frac{C_N}{R_N}} \cdot G_c \right]
\]

\textbf{Noise Circles}

Recall:

\[
F = 1 + \frac{C_N + \left[ (G_c + G_s)^2 + (B_c + B_s)^2 \right] R_N}{G_s}
\]

\[
G_{opt}^2 = \frac{C_N}{R_N} + G_c^2; \quad B_c = -B_{opt}
\]

\[
\begin{align*}
\text{\begin{circuitikz}
\draw (0,0) to (2,0) to (2,2) to (0,2) to (0,0);
\end{circuitikz}}
\end{align*}
\]

\[
C_N = R_N \left( G_{opt}^2 - G_c^2 \right)
\]

\[
\Rightarrow F = 1 + \frac{\left[ G_{opt}^2 - G_c^2 + (G_c + G_s)^2 \right] R_N + (B_s - B_{opt})^2 R_N}{G_s}
\]

\[
= 1 + \frac{\left[ (G_{opt}^2 - G_c^2 + G_c^2 + 2G_cG_s + G_s^2) \cdot R_N + (B_s - B_{opt})^2 R_N \right]}{G_s}
\]

\text{add & subtract } 2G_sG_{opt} \text{ to } 1^{st} \text{ term of numerator}

\[
\Rightarrow F = 1 + \frac{2(G_c + G_{opt}) \cdot G_s \cdot R_N + \left( (G_s - G_{opt})^2 + (B_s - B_{opt})^2 \right) R_N}{G_s}
\]
\[ F = F_{\text{min}} + \left[ (G_s - G_{\text{opt}})^2 + (B_s - B_{\text{opt}})^2 \right] \cdot \frac{R_N}{C_s} \]

* These are circles in the source admittance \((G_s-B_s)\) plane

* Circles of constant \(F\) in \(G_s-B_s\) plane

* Also circles on a Smith chart (mapping is a bilinear transformation)

* Conditions for \(F_{\text{min}}\) are slightly different from those for maximum power transfer!

\[ \rightarrow \text{tradeoff between max gain & min. noise} \]

* **Noise Figure** = noise factor in \(\text{dB}\)

\[ NF = 10 \log F \]

* **Noise Temperature** \(T_N\)

\[ \equiv \text{increase in temperature required of } Y_s \text { for it to account for all of the output noise at the ref. temp. } (=290K) \]

\[ F = 1 + \frac{T_N}{T_{\text{ref}}} \Rightarrow T_N = T_{\text{ref}} \cdot (F-1) \]

\[ \rightarrow \text{a 2-port that adds no noise has } T_N = 0K \]
→ Tµ is useful for cascaded amplifiers and those where F is close to 1 (i.e., NF ~ 0 dB).

→ Tµ offers a higher resolution description of noise performance.