Lecture 34

Note Title 4/9/2008

→ branch : (s_1, s_2) of stage i

(γ) Branch metric at stage i $\quad p\left((s_1, s_2) \mid \underline{r}_i \right)$

$\downarrow$

$\underline{s}_i$ (symbols)

$\downarrow$

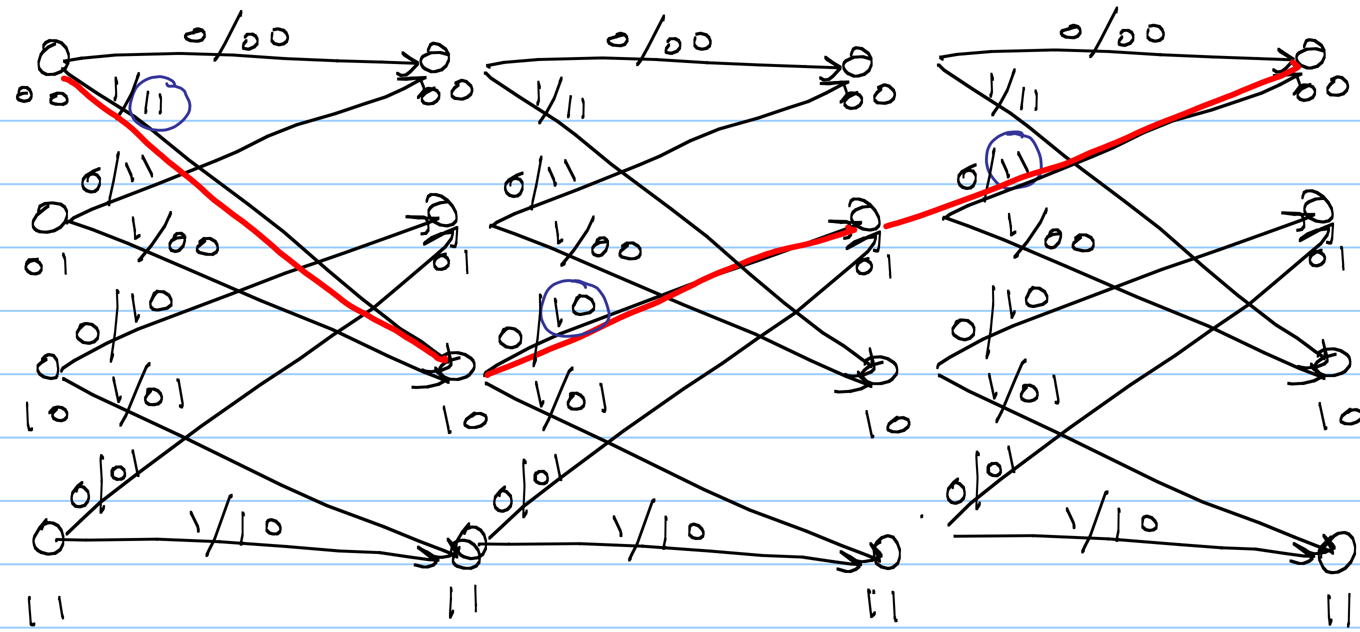
u_i (input bit)

$\downarrow$

a priori probabilities are used here.

Free distance & Error events:

$$G(D) = \begin{bmatrix} 1+D+D^2 & 1+D^2 \end{bmatrix}$$



Error event: "most likely" erroneous paths

Free distance: wt. of "most likely" error event
(d_{free})

$d_{free} = 5$ (in above trellis)

Moderately high SNRs: Errors will be in <u>bursts</u>

Ex: Catastrophic encoders

$$G(D) = \begin{bmatrix} (1+D)D & (1+D)(1+D) \end{bmatrix}$$

$$u(D) = \frac{1}{1+D} \qquad wt(\underline{u}) = \infty$$

$$v(D) = \begin{bmatrix} D & 1+D \end{bmatrix} \qquad wt(\underline{v}) = 3$$

Trellises for block codes

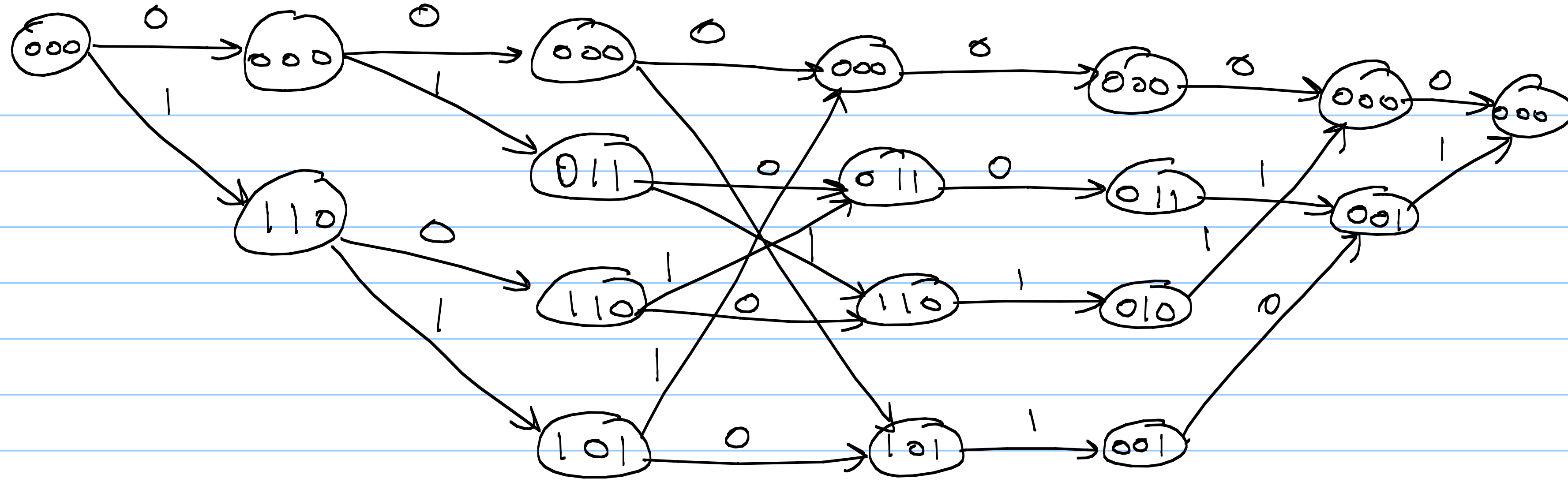
Ex: (6, 3, 3) code

$$H = \begin{array}{c} \begin{matrix} m_1 & m_2 & m_3 & p_1 & p_2 & p_3 \end{matrix} \\ \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

$$H \underline{c}^T = 0$$

$$\begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \\ m_3 \\ p_1 \\ p_2 \\ p_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$m_1 \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + m_2 \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} + m_3 \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + p_1 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + p_2 \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + p_3 \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

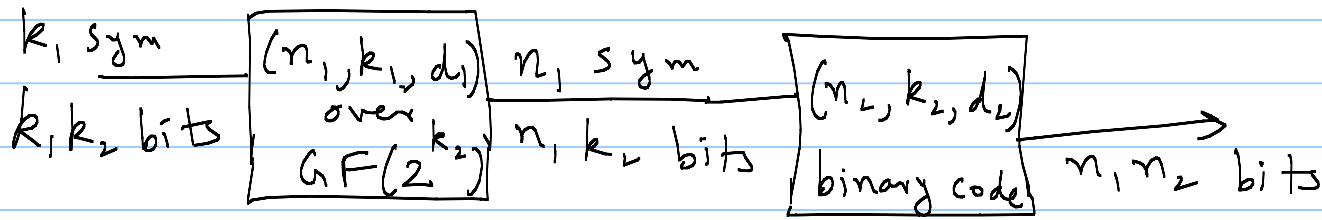


Trellis for (6, 3, 3) code.

→ changes with PC matrix H.

Concatenation:

I



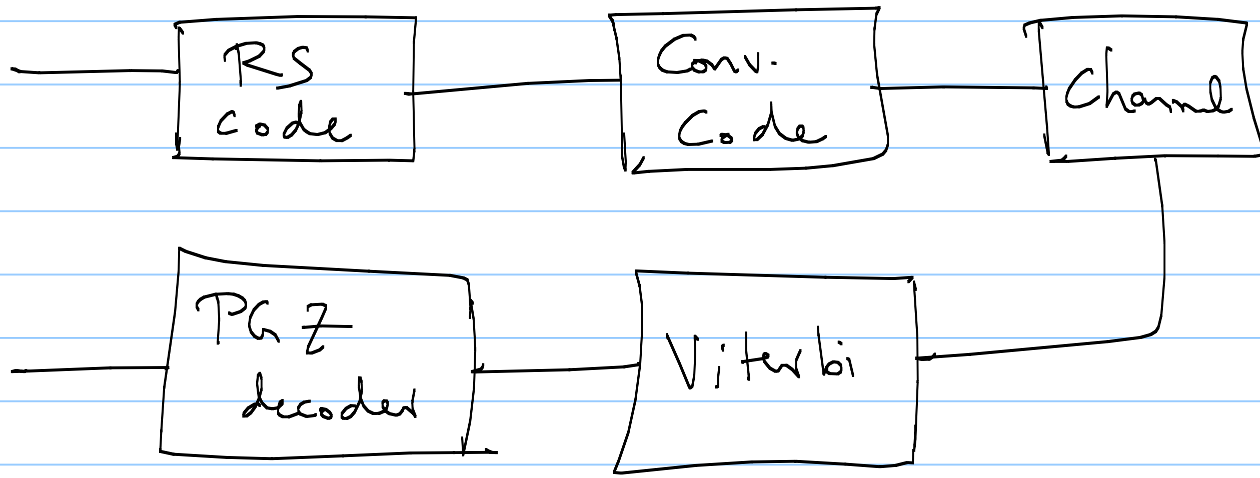
Overall: $(n_1 n_2, k_1 k_2)$ linear binary code.

$$(n_1 n_2, k_1 k_2, \geq d_1 d_2) \text{ code}$$

$\longrightarrow$ good codes are possible by above method.

$\longrightarrow$ decoding remains a problem.
(up to $\frac{d_1 d_2 - 1}{2}$)

II Reed-Solomon-Viterbi (RSV)



III Turbo Product Code

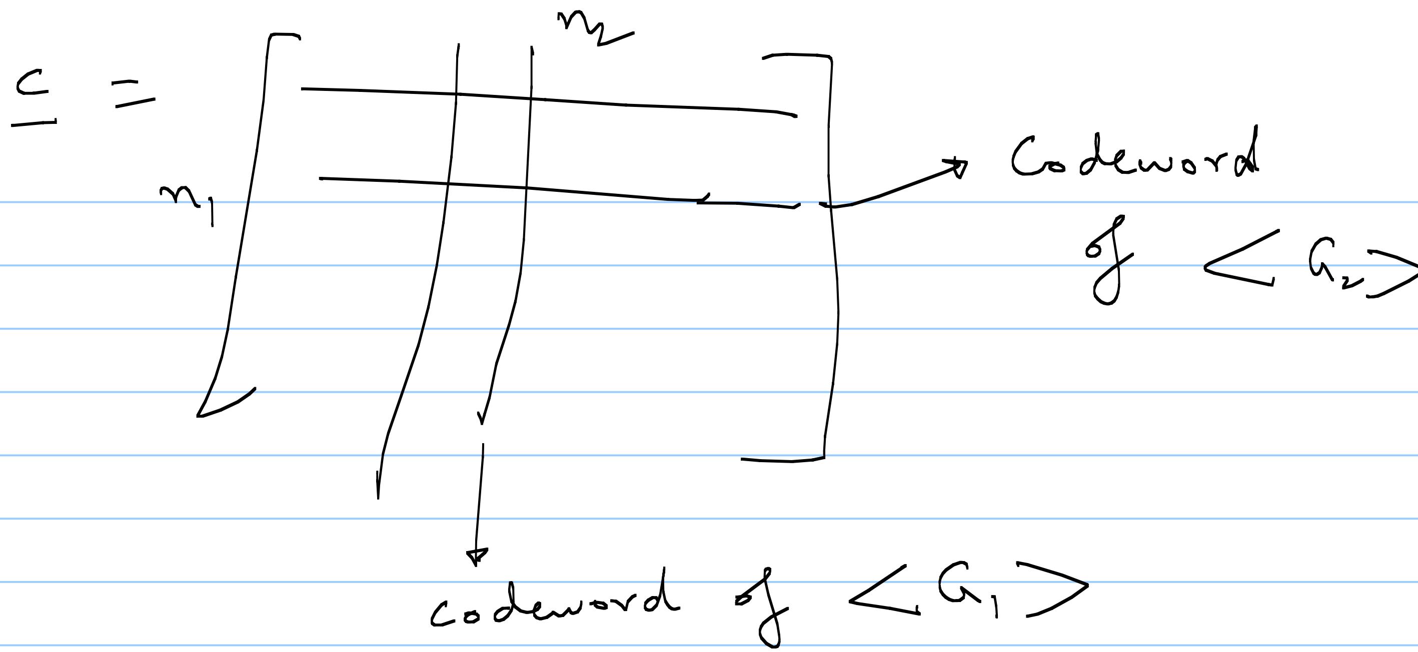
$d_1 \leftarrow G_1 : k_1 \times n_1$ matrix

$d_2 \leftarrow G_2 : k_2 \times n_2$ matrix

$\underline{m}$

$k_1 \times k_2$

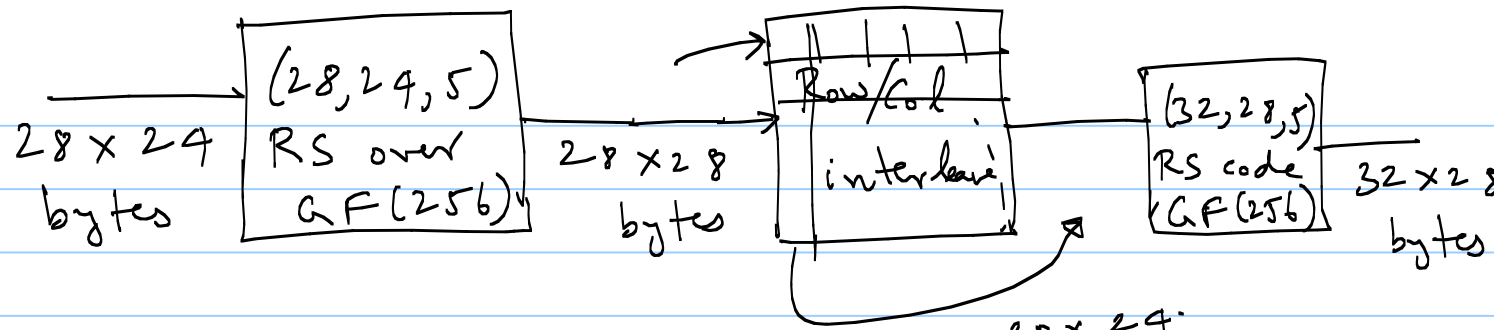
$$\underset{n_1 \times n_2}{\underline{c}} = G_1^T \, \underline{m} \, G_2$$



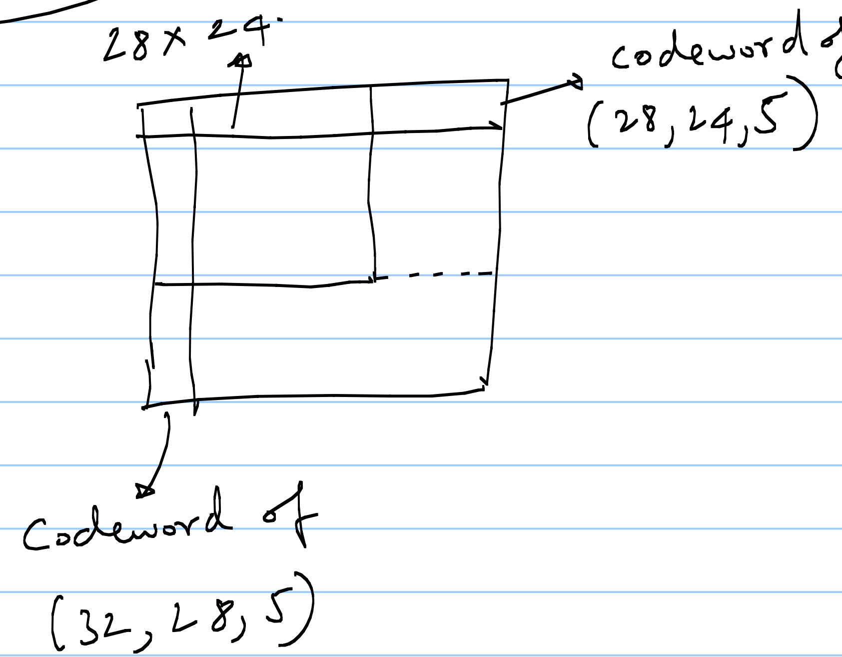
Overall code: $(n_1 n_2, k_1 k_2, \geq d_1 d_2)$

C_1 & C_2 : extended Hamming Codes $(64, 57, 4)$

IV



→ early CD standards



Coded Modulation: