Lecture 30

Note Title 3/27/2008

$$G(D) = \left[g^{(0)}(D) \;\; g^{(1)}(D) \; \cdots\cdots \; g^{(m-1)}(D) \right]$$

rate - $1/m$ convolutional encoder.

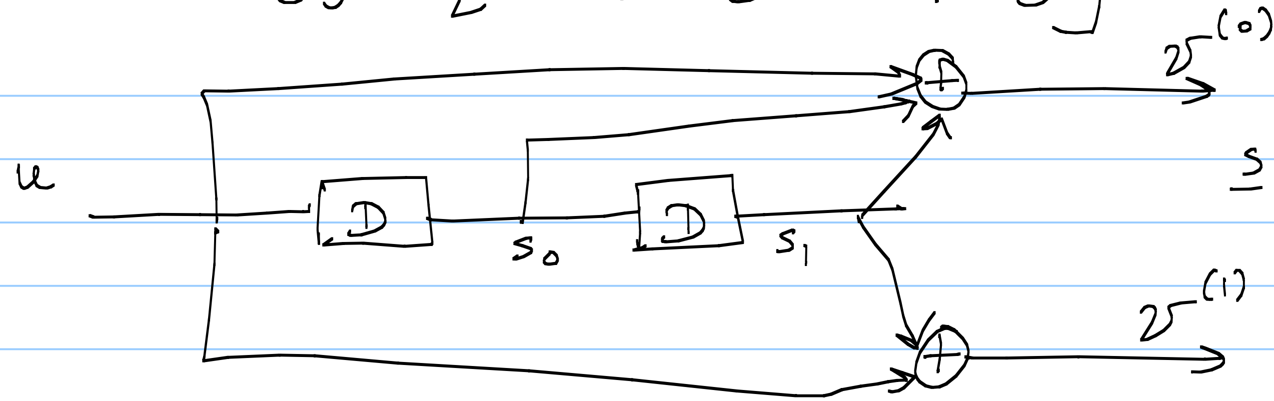
→ No. of states = 2^(no. of memory elements) (μ)

→ Trellis representation

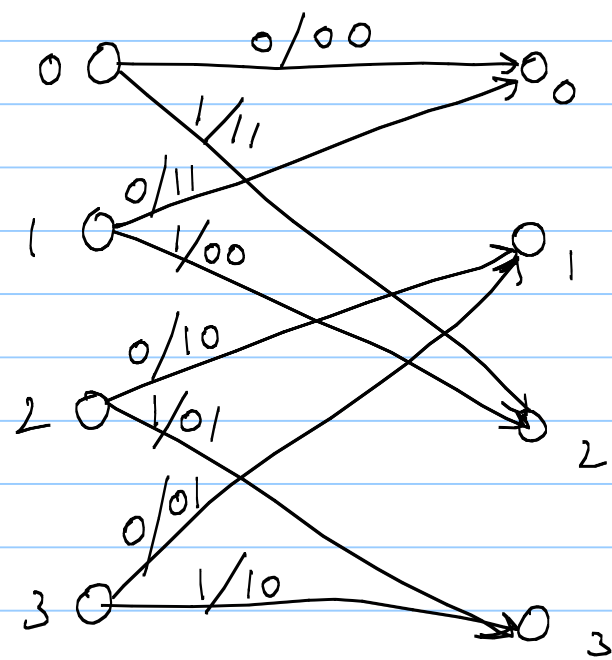
- → paths ⟺ codewords
  - sequence of branches
- → termination.
- → k-bit message ⟶ $(k+\mu)m$-bit codeword.

Ex:

$$G(D) = [\,1 + D + D^2 \quad 1 + D^2\,]$$



$\underline{s} = [s_0\ s_1]$



Viterbi Decoder:

- rate - $1/m$ convolutional encoder
- Memory $= \mu$

$$\underline{u} = [u_1 \quad u_2 \quad \cdots \quad u_k]$$

(rate-1/2) $$\underline{v} = [v_1^{(0)} v_1^{(1)} \quad v_2^{(0)} v_2^{(1)} \quad \cdots \quad v_k^{(0)} v_k^{(1)} \quad v_{k+1}^{(0)} v_{k+1}^{(1)} \quad \cdots \quad v_{k+\mu}^{(0)} v_{k+\mu}^{(1)}]$$

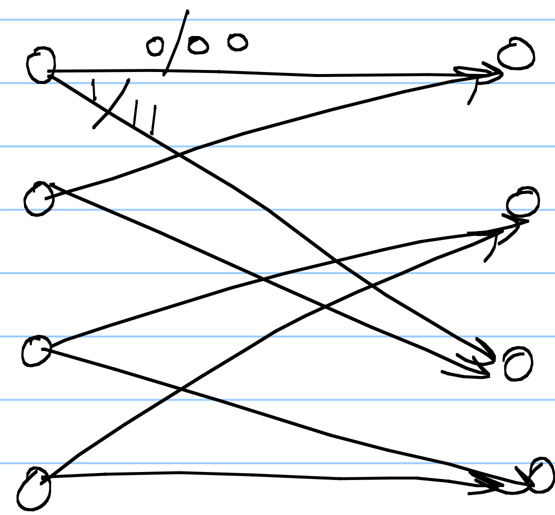
$\downarrow$ BPSK over AWGN

$$\underline{r} = [\underbrace{r_1^{(0)} r_1^{(1)}}_{\text{Stage } 1} \quad \underbrace{r_2^{(0)} r_2^{(1)}}_{\text{Stage } 2} \quad \cdots \quad \underbrace{r_k^{(0)} r_k^{(1)}}_{\text{Stage } k} \quad r_{k+1}^{(0)} r_{k+1}^{(1)} \quad \cdots \quad \underbrace{r_{k+\mu}^{(0)} r_{k+\mu}^{(1)}}_{\text{Stage } k+\mu}]$$

ML rule: $$\arg \min_{\underline{s} \,\in\, \text{Codeword symbols}} \|\underline{r} - \underline{s}\|^2$$

$$\|\underline{r}-\underline{s}\|^2 = \sum_{i=1}^{k+\mu} \Big(\underbrace{(r_i^{(0)} - s_i^{(0)})^2 + (r_i^{(1)} - s_i^{(1)})^2} \Big)$$

$$\longrightarrow \underline{s} = \left[s_1^{(0)}\ s_1^{(1)}\ \ s_2^{(0)}\ s_2^{(1)}\ \ \cdots\cdots\ s_{k+\mu}^{(0)}\ \ s_{k+\mu}^{(1)} \right]$$



Stage i

$r_i^{(0)},\ r_i^{(1)}$

Branch: particular value for $s_i^{(0)}, s_i^{(1)}$

Branch Metric:

$$(r_i^{(0)} - s_i^{(0)})^2 + (r_i^{(1)} - s_i^{(1)})^2$$

Path metric:

$$\text{path} = \{\text{Branch } 1, \text{Branch } 2, \ldots, \text{Branch } a\}$$

$$\text{path metric} = \sum_{i=1}^{a} \text{Branch metric (Branch } i)$$

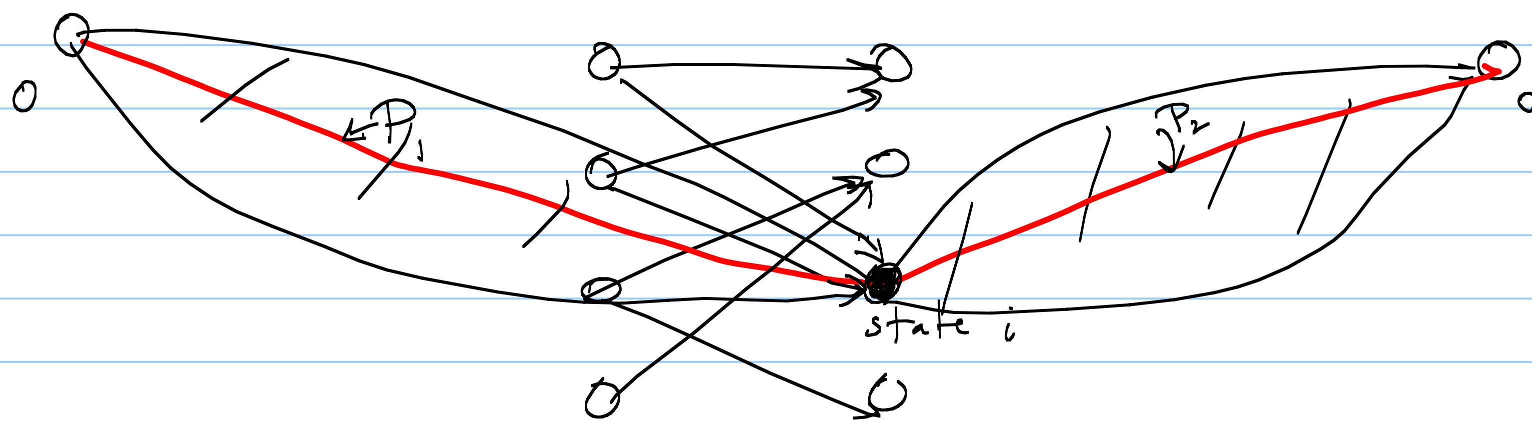
$$\text{path} = \{\text{state } 1, \text{state } 2, \ldots, \text{state } a+1\}$$

$$\text{path metric} = \sum_{i=1}^{a} \text{Branch metric}\{\text{state } i \rightarrow \text{State}_{i+1}\}$$

$$\|\underline{r} - \underline{s}\|^2 = \text{path metric of path corresponding to } \underline{s}$$

ML rule: $\underset{\text{paths}}{\arg\min}\,(\text{path metric})$

<u>Argument</u> :



Stage 1 - - - - Stage i — — — . Stage k+μ

$$\underset{\underline{\text{paths through State } i}}{\arg\min} \text{ path metric } = [P_1 \ P_2]$$

$\arg\min$ path metric $= P_1$

(paths from

State 0 at stage 1

to

Stat i after stage i)

$\arg\min$ path metric $= P_2$

(paths from

State i after stage i

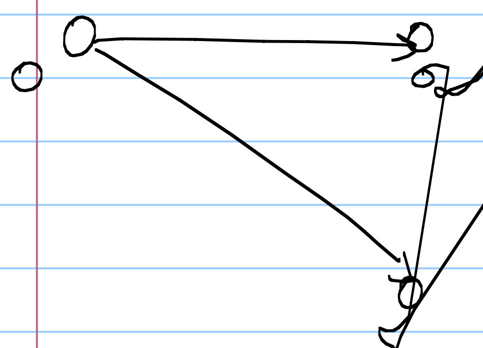
to

State 0 after stage k+m)

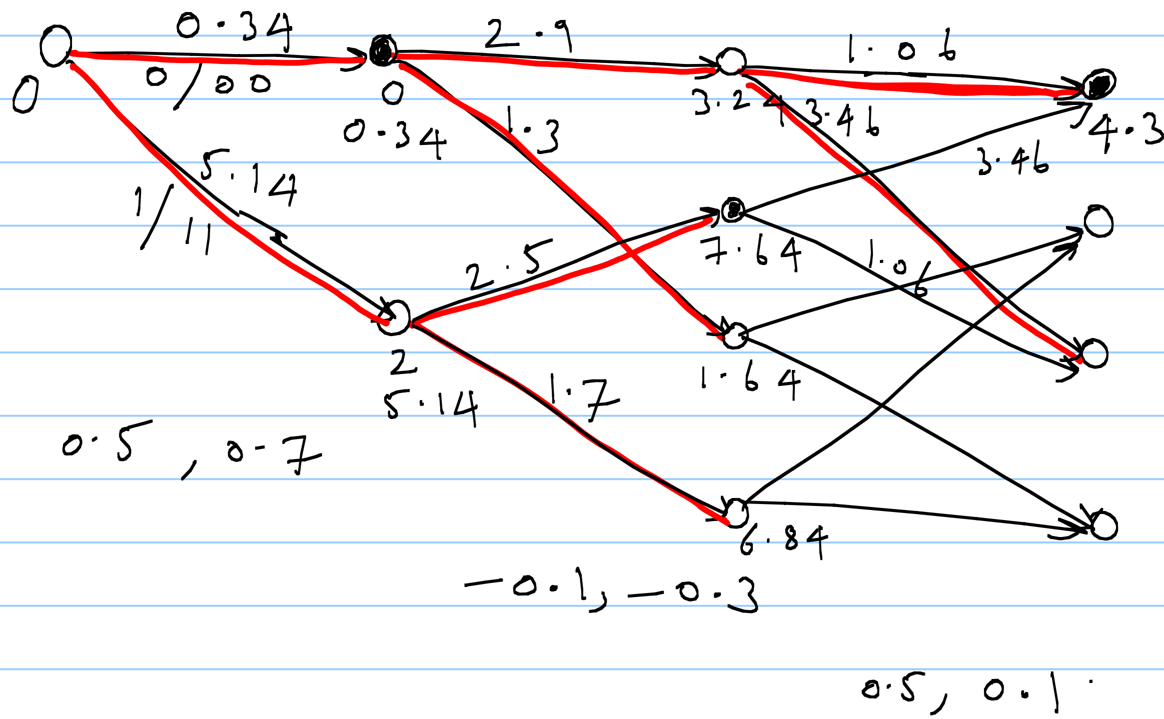
Viterbi algorithm:

Ex: 4-state code.

$\underline{r} = [0.5 \quad 0.7, \quad -0.1 \quad -0.5, \quad . \quad . \quad . \quad . \quad]$



Ex:


0.34
0/00
0
0
2.9
0.34
1.06
3.24
3.46
1.3
4.3
5.14
3.46
1/11
7.64
2.5
1.06
2
5.14
1.64
1.7
0.5, 0.7
6.84
-0.1, -0.3
0.5, 0.1