Lecture 28

Note Title 3/19/2008

'Soft' Message-passing decoder:

— BPSK over AWGN. $\{y_1, y_2, \cdots y_n\}$: received LLRs.

Iteration 1:

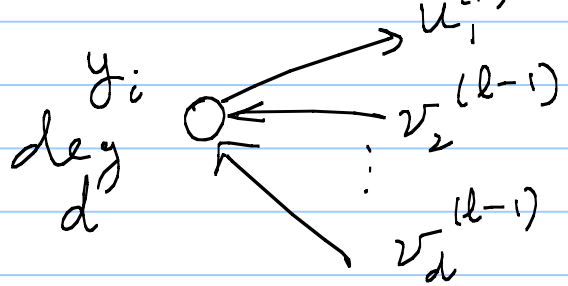
— ?

— ?

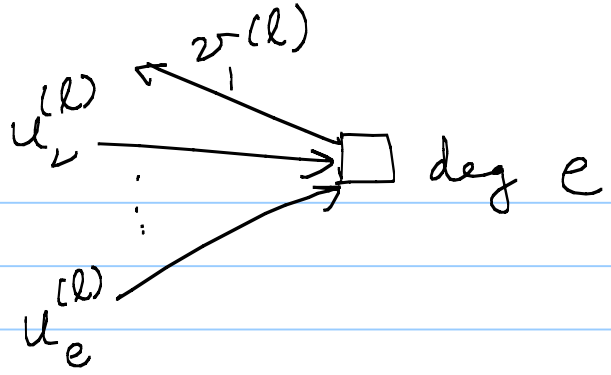
Iteration l: $l \geq 2$

$u^{(l)}$: bit-to-check message

$v^{(l)}$: check-to-bit message



$$u_1^{(l)} = y_i + v_2^{(l-1)} + \cdots + v_d^{(l-1)}$$



$$\text{mag}\left(v_1^{(\ell)}\right) = f\left(f(u_2^{(\ell)}) + f(u_3^{(\ell)}) + \cdots + f(u_e^{(\ell)})\right)$$

$$f(x) = \log \tanh \frac{|x|}{2}$$

$$\text{sign}\left(v_1^{(\ell)}\right) = \text{sign}\left(u_2^{(\ell)}\right) \cdots \text{sign}\left(u_e^{(\ell)}\right)$$

Y_i : RV denotes y_i (all-zero codeword).

$$f_{Y_i}(y_i) \sim N\left(\frac{2}{\sigma^2}, \frac{4}{\sigma^2}\right)$$

$U^{(l)}$: RV denotes $u^{(l)}$

$V^{(l)}$: " " $v^{(l)}$

degree check mag $(V^{(l)}) = f\left(f(U_1^{(l)}) + f(U_2^{(l)}) + \cdots + f(U_{e-1}^{(l)})\right)$

numerical methods

$\text{sign}(V^{(l)}) = \text{sign}(U_1^{(l)})\ \text{sign}(U_2^{(l)}) \cdots \text{sign}(U_{e-1}^{(l)})$

easily computed

$p = Pr(U^{(l)} < 0)$

$f_{V^{(l)}} : F_1\left(\bigotimes_{e-1} F(f_{U^{(l)}})\right)$

takes care of sign as well

transformation of pdf when RV is transformed by $f(x)$.

$$f_{V^{(l)}} = \sum_e p_e \, F_1 \left(\bigotimes_{e-1} F\left(f_{U^{(l)}}\right) \right)$$

$$= P\left(f_{U^{(l)}}\right)$$

deg d bit node

$$U^{(l+1)} = Y_i + V_1^{(l)} + V_2^{(l)} + \cdots + V_{d-1}^{(l)}$$

$$f_{U^{(l+1)}} = \sum_d \lambda_d \, f_{Y_i} \otimes \left(\bigotimes_{d-1} f_{V^{(l)}} \right)$$

$$= f_{Y_i} \otimes \lambda\left(f_{V^{(l)}}\right)$$

$$f_{U^{(l+1)}} = DE_{\lambda,\rho}\left(f_y, f_{U^{(l)}}\right)$$

$f_y \rightarrow N\left(\frac{2}{\sigma^2}, \frac{4}{\sigma^2}\right)$

$$\text{Prob}^{(l)}\{\text{Error}\} = \Pr\left(U^{(l)} < 0\right)$$

$$= \int_{-\infty}^{0} f_{U^{(l)}}$$

Threshold:

$$\sigma^* = \sup_{\sigma}\left\{\text{Prob}^{(l)}\{\text{Error}\} \rightarrow 0\right\}$$

σ^* → function of λ & ρ

Summary:

BSC: Gallager A decoder
- regular / irregular codes.

BPSK over AWGN: Soft message passing decoder (Belief propagation)
- Density evolution.

→ Degree distribution controls performance.

→ LDPC codes for other channels.

→ DVB-S
→ WiMax (802.16e) → protograph LDPC codes
Min sum decoder.

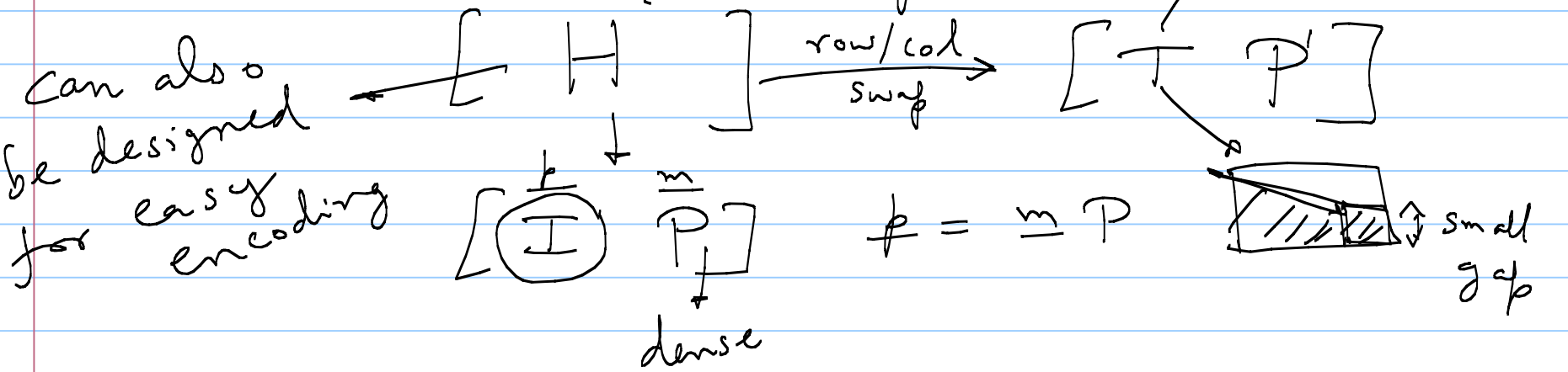
$$f\left(\underline{f(x_1)} + \underline{f(x_2)} + \cdots + \underline{f(x_d)}\right) \cong f\left(f(x_{min})\right) = x_{min}$$

$\underline{f(x_1)}$: $\log \tanh \frac{|x_1|}{2}$

→ Encoding

→ sparse

≈ upper triangular



$t = m - n$