Lecture 25

Note Title 3/12/2008

$L(x)$, $R(x)$: node-perspective

$\lambda(x)$, $\rho(x)$: edge-perspective

$$L(x) = \sum_{i=1}^{d_l} L_i x^i \qquad R(x) = \sum_{j=1}^{d_r} R_j x^j$$

Designed rate, $R = 1 - \dfrac{L'(1)}{R'(1)}$

$$\lambda(x) = \sum_{i=1}^{d_l} \lambda_i x^{i-1} \qquad \rho(x) = \sum_{j=1}^{d_r} \rho_j x^{j-1}$$

$$\longrightarrow \lambda(x) = \frac{L'(x)}{L'(1)} \qquad \rho(x) = \frac{R'(x)}{R'(1)}$$

Gallager A on irregular codes

→ running of the decoding algorithm ✓

→ all-zero codeword assumption ✓

→ Different neighbourhoods for
different bit nodes ?

→ neighbourhoods will be cycle-free
w.h.p. ✓

Idea 1: Average over all neighbourhoods
(Prob. of error)

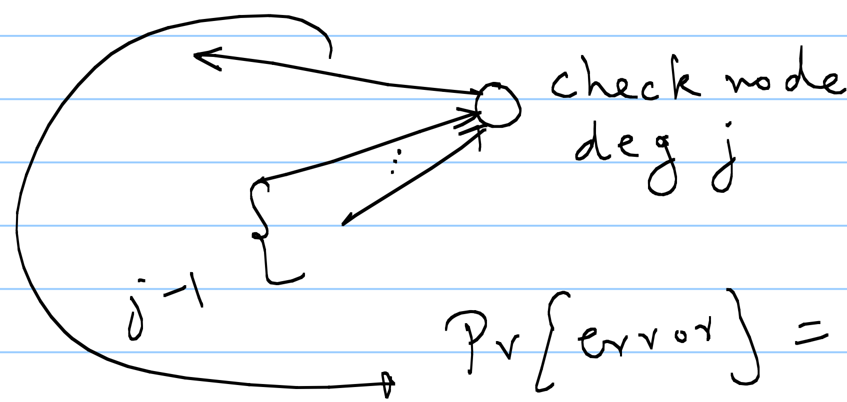
Idea 2: Concentration result.
→ Martingales

Density Evolution (irregular case, Gallager A)

Iteration 1: Step (a):

$$p^{(1)} = p$$

Step (b):



$$q^{(1)} = ?$$

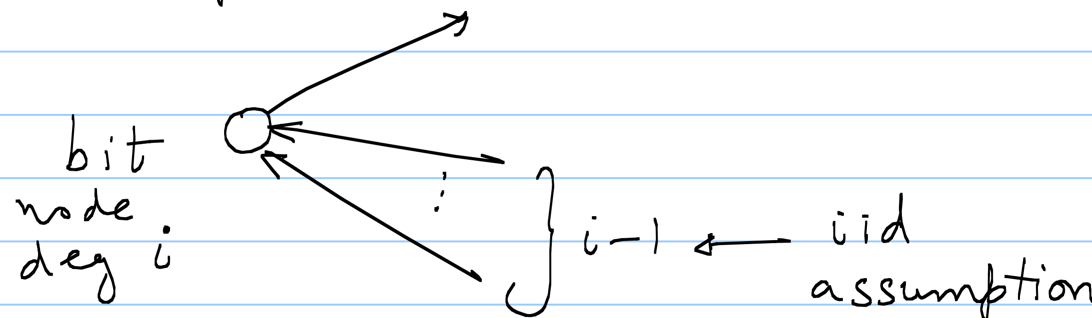
$$\Pr\{\text{error}\} = \frac{1-(1-2p)^{j-1}}{2}$$

$$\Pr\left(\begin{array}{c}\text{random edge is}\\ \text{connected to a check}\\ \text{node of deg } j\end{array}\right) = \rho_j$$

$$q^{(1)} = \sum_{j=1}^{d_r} \rho_j \left(\frac{1-(1-2p)^{j-1}}{2} \right)$$

<u>Iteration ℓ</u> : Step (a)

<u>$q^{(\ell-1)}$ is known</u>



$$\longrightarrow p^{(\ell)} = \sum_{i=1}^{d_\ell} \lambda_i \left((1-p)\left(q^{(\ell-1)}\right)^{i-1} + p\left(1-\left(1-q^{(\ell-1)}\right)^{i-1}\right) \right)$$

$$q^{(\ell)} = \sum_{j=1}^{d_r} \rho_j \left(\frac{1-\left(1-2p^{(\ell)}\right)^{j-1}}{2} \right)$$

Density Evolution Iteration $\Big\}$ $p^{(\ell)} = f_{\rho,\lambda}(p^{(\ell-1)}, p)$

Threshold: $p^* = \sup_p \{ p^{(\ell)} \to 0 \}$

$\to$ Optimization over ρ & λ is now possible.

Fix Design rate, $R = 1 - \dfrac{\sum_{j=1}^{d_r} \rho_j / j}{\sum_{i=1}^{d_\ell} \lambda_i / i}$

$\to$ right "close-to-regular" assumption

$\rho_w, \rho_{w+1} \neq 0$

$\rho_j = 0 \quad \forall j \neq w, w+1$

$\to$ try to maximize threshold.