Multistage Relaying Using Interference Networks

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Abstract

Wireless networks with multiple nodes that relay information from a source to a destination are expected to be deployed in many applications. In this work, we consider multihopping decode and forward (MDF) relaying protocols for multistage half-duplex relay networks with no direct link between the source and destination nodes. Each state of the half-duplex network is considered as an interference network. Receivers in each state employ interference processing/cancellation; however, no cooperation across relay nodes is assumed for encoding or decoding. The scheduling of interference network states is optimized to maximize the rate for a given realization of channel gains. For arbitrary networks with two node-disjoint paths between source and destination, we analytically characterize strong and weak interference channel-gain regimes, and show an explicit two-state schedule that approaches the cheap relay cutset bound in these regimes. Numerical evaluation in example networks illustrate the capacity-approaching performance of MDF protocols and the effectiveness of interference processing. Our results suggest that multistage half-duplex relaying with practical constraints on cooperation and finite SNR is comparable to point-to-point links and full-duplex relay networks, if there are multiple node-disjoint paths from source to destination and if suitable coding is employed in the interference network states.

I. Introduction

One of the key technologies in next generation wireless communication systems for achieving high throughput and providing better coverage is *relaying*. Relaying has attracted a high level of recent research interest with several papers focusing on various aspects of communicating using relays with different constraints and assumptions. In this work, we are concerned with the

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capacity of multistage relaying from one source to one destination through an arbitrary network of half duplex relays.

While the proposed protocols could be applied to arbitrary multistage half-duplex networks, an example network that we consider in detail for ease of explanation and clarity is the two stage relay network shown in Fig. 1. In this 6-node network, the source node $S = 1$ intends to communicate with the sink node $D = 6$ through 4 relay nodes $\{R_1 = 2, R_2 = 3, R_3 = 4, R_4 = 5\}$ connected as shown. The channel gains (α, β, γ, δ) are shown next to the corresponding edges. For simplicity, some of the gains are assumed to be identical. For a multistage half-duplex relay network such as the one in Fig. 1, we study coding methods and protocols needed to achieve the best possible rate from source to destination for different ranges of the channel gains.

There are two aspects to multistage relaying when relays are connected in an arbitrary fashion: (1) scheduling transmissions and receptions by nodes, and (2) coding and decoding methods employed by nodes during transmissions and receptions, respectively. One strategy for scheduling is to avoid interference altogether. However, the maximum data rate under Interference Avoidance (IA) is limited, because the source is transmitting only for a fraction of the total time. To improve upon IA, more states of the network with the source in transmit mode need to be considered. The scheduling task is to determine those states that are crucial for obtaining higher rates.

When multiple nodes transmit, interference network states are created in the network based on the connectivity. For illustration, two interference network states are shown in Fig. 2 for the network of Fig. 1. In one state, S, R_1, and R_3 are transmitters and, in the other state, S, R_2, and R_4 are transmitters. Note that, in both the states shown in Fig. 1, the source node is a transmitter and the destination node is a receiver.

The capacity region and the corresponding optimal coding strategy are not known for intereference networks, and we study different coding strategies. We fix that the receivers decode by successive interference cancellation. At the transmitter, we consider four different strategies of increasing complexity. The first strategy is common broadcast, where the transmitter sends at a rate that can be decoded by its weakest receiver. The common broadcast strategy is just as complex as point-to-point transmission. The second strategy is superposition coding, where the transmitter orders the receivers according to their channel strength and sends additional information to more capable receivers. Superposition coding, though more complex than common broadcast, will be seen to be important for approaching capacity when the channel gains out of a transmitter vary over a wide range. For both these strategies, no cooperation is needed for encoding. The transmitting nodes only need to know the relative channel strengths of their receivers. The third coding strategy is dirty paper coding (DPC) at the source node and common broadcast at the other transmitters. Since the source is the originator of all messages, we assume that the source node knows the message to be sent by relay nodes in interference network states. For instance, in the state S_1 shown in Fig. 1, the source node is assumed to know the codeword transmitted by the relay nodes R_1 and R_3. Under this assumption, the source node employs DPC to cancel the interference from the relays. As shown in Sec. VI, DPC at the source node proves to be crucial for approaching capacity in certain regimes of channel gains. However, some additional cooperation is needed for DPC, as the source needs to know the channel gains between relays as well. The fourth coding strategy is dirty paper coding (DPC) at the source node and superposition coding at the other transmitters.

For each of the four coding strategies, suitable rate regions are determined for each state (or interference network). The overall rate achievable from the source to the destination is computed using an optimization over the time-sharing of the rate regions for each state, subject to additional flow constraints that ensure compatibility of the rate vectors used for individual states.

For an arbitrary network with two node-disjoint paths from source to destination, we study the MDF protocols over a simple two-state schedule chosen using the source-destination paths. In this setting, we analytically compute weak and strong interference channel gain regimes, where capacity is approached by some of the MDF protocols. To further study the protocols, numerical evaluation is performed on two specific networks, where we show that the cut-set bound is approached for several regimes of channel gains.

To place our work better, we review a sample of the relevant prior literature. The relay channel is a classic setting, introduced in [1], and studied extensively [2]–[4]. One result of particular interest is the cut-set bound for half-duplex relay networks operating by time-sharing over a finite number of states [5]. This “cheap relay” bound has been used by several authors as an outer bound for achievable rates. Recent and past studies of the relay channel can be classified based on the following considerations: (1) number of stages and topology of the relays, (2) duplex (half or full) constraints assumed for the relays, (3) cooperation assumed between nodes, and (4) the analysis method - Diversity Multiplexing Gain Tradeoff (DMT) or capacity computations. The half-duplex diamond network with two relays has been studied in [6]–[9]. The *multi-hopping* *decode and forward* (MDF) protocol, proposed in [6] and extended in [7], achieves rates close to the cheap relay cut-set bound. Wang *et al* [8] consider a modified diamond network with an additional link between the relays and propose a coding strategy using Dirty Paper Coding (DPC), which is shown to approach the cut-set bound. More protocols for general half-duplex wireless relay networks have been studied in [10], [11]. For an arbitrary number of relays in a general topology, capacity approximations have been established in [12] under the full-duplex and full-cooperation assumptions. The optimal DMT for arbitrary relay networks with full-duplex and half-duplex nodes have been determined in [13] and [14], respectively. In relation to the above, in our work, we propose and study multi-hopping decode and forward (MDF) protocols in the following setting: (1) A *general topology* of relays, (2) *Half-duplex nodes*, (3) *No receive cooperation*, (4) *Finite SNRs*.

The rest of this article is organised as follows. The model and operation of the relay network are described in Section II. The cutset outer bound is described in Section III followed by a description of the MDF protocols in Section IV. The achievable rate for a simple schedule is analyzed in Section V in the strong and weak interference regimes. Numerical results are presented in Section VI, and concluding remarks are made in Section VII.

II. Model

We represent a wireless network with m nodes as an undirected graph $G = (V, E)$, where the vertex set $V = \{1, 2, \ldots, m\}$ represents the wireless nodes. An edge $(i, j) \in E$ indicates that Node i and Node j are connected by an additive white Gaussian noise (AWGN) channel with constant gain denoted as h_{ij}. Further, $(i, j) \in E$ implies that Node j is connected to Node i with a channel gain $h_{ji} = h_{ij}$.

Each node is subject to an average power constraint P and a noise variance σ^2. In addition, a half-duplex constraint is imposed on the nodes so that they can either transmit, receive, or be idle at any given time. Therefore, in this work, an m-node half-duplex wireless network can be in $M \leq \mathscr{M} = 3^m$ states that are denoted S_1, S_2, $\cdots$, S_M. In such a network, we are interested in maximizing the communication rate R from an arbitrary source $S \in V$ to an arbitrary sink $D \in V$. Nodes in $V \setminus \{S, D\}$ act as relays. Information flow from source to destination happens by a time-sharing of the states S_k, $1 \leq k \leq M$, and may reach the destination in multiple hops depending on the connectivity of the graph. Hence, the specific problem considered in this work can be termed *multihop, half-duplex relaying* in an arbitrary wireless network.

The total transmission time is normalized to one time unit, and state S_k is active for a λ_k fraction of the time (λ_k could be zero) with $\sum_{k=1}^{M} \lambda_k = 1$. As in [6], [7], we assume that the state sequence and the time-sharing parameters are known to all nodes before transmission. Let $I_k = \{i \in V : \text{Node } i \text{ is a transmitter in State } S_k\}$ be the set of active transmitters in State S_k, and let $J_k = \{i \in V : \text{Node } i \text{ is a receiver in State } S_k\}$ be the set of active receivers in State S_k. When state S_k is active, simultaneous transmissions from nodes in I_k can interfere at one or more of the receivers in J_k depending on the connectivity of the nodes in I_k and J_k. Thus, each state $S_k = (I_k, J_k)$ is an *interference network* [15] or *hyperedge* with I_k and J_k as the two disjoint vertex sets. We use the terms interference network, hyperedge and state interchangeably. The choice of a specific coding and decoding strategy for each state $S_k = (I_k, J_k)$ determines possible operating rate vectors in an achievable rate region for that state. Since the capacity region and optimal coding scheme are not known for general interference networks, we consider four suboptimal strategies for each state based on different broadcast and interference processing techniques. In all these strategies, we impose the constraint that the receivers J_k cannot cooperate in decoding. Similarly, the nodes in I_k are assumed to encode their messages independently; however, in two schemes, the source is assumed to know the messages transmitted by the relays.

III. Cut-set Bound

A cut-set upper bound for half-duplex relay networks operating by time-sharing over a finite number of states has been derived in [5]. This bound is presented here, briefly.

Let $X^{(i)}$ and $Y^{(i)}$ be the transmitted and received variables at node i when it is in transmit and receive states, respectively. The maximum achievable information rate R between source S and destination D in a half-duplex network is bounded as

$$R \leq \sup_{\lambda_k} \min_{\Omega} \sum_{k=1}^{\mathscr{M}} \lambda_k I(X_{(k)}^{\Omega}; Y_{(k)}^{\Omega^c} | X_{(k)}^{\Omega^c}), \tag{1}$$

for some joint distributions $\{p(x^{(1)}, x^{(2)}, \cdots, x^{(m)}|k)\}$, $1 \leq k \leq \mathscr{M}$, where the supremum is over all $\lambda_k \geq 0$ such that $\sum_{k=1}^{\mathscr{M}} \lambda_k = 1$, and the minimization is over all Ω such that $S \in \Omega$, $D \in \Omega^c$, $X_{(k)}^{\Omega} = \{X^{(i)} : i \in \Omega \cap I_k\}$, $Y_{(k)}^{\Omega^c} = \{Y^{(i)} : i \in \Omega^c \cap J_k\}$, $X_{(k)}^{\Omega^c} = \{X^{(i)} : i \in \Omega^c \cap I_k\}$. The above upper bound can be computed by solving a linear program [7]. The mutual information $I(X_{(k)}^{\Omega}; Y_{(k)}^{\Omega^c} | X_{(k)}^{\Omega^c})$ is computed exactly using known sum rate capacity results [16] when the choice of Ω and k results in multiple access or broadcast channels. When the sum rate capacity is not known exactly (e.g. for interference channels), the multiple-input multiple-output (MIMO) sum capacity is used as an upper bound.

IV. Multihop Half-duplex Relaying Strategies

In this section, we present the four MDF strategies that we study in the context of a general relay network with half-duplex nodes. In all these strategies, the network operates by time-sharing between the states, where each state is an interference network in general. The strategies differ in the encoding scheme in each state. The decoder at each receiver employs successive interference cancellation (SIC).

A. Common Broadcast (CB) Scheme

In state $S_k = (I_k, J_k)$, each transmitter $i \in I_k$ sends a common message at rate R_i^k to the set of all its receivers denoted Γ_-^i. Each receiver $j \in J_k$ must decode the messages from the set Γ_+^j (say) of all the transmitters connected to j. The decoding constraints at each receiver for achievability are the constraints for the multiple access channel corresponding to the SIC receiver. Therefore, the achievable rate region for each state S_k is defined by the constraints:

$$\sum_{i \in A} R_i^k \leq \frac{1}{2} \log \left(1 + \frac{\sum_{i \in A} h_{ij}^2 P}{\sigma^2} \right), \tag{2}$$

for all $A \subseteq \Gamma_+^j$ and for all $j \in J_k$. When each transmitter is connected to all receivers, i.e., $\Gamma_-^i = J_k$ for each $i \in I_k$, the above region is the same as the compound multiple access rate region in [17].

B. Superposition Coding (SC) Scheme

In this scheme, in state S_k, each transmitter $i \in I_k$ sends d_-^i independent messages to its receivers in Γ_-^i using superposition coding. For simplicity of notation, we assume that the d_-^i receivers in Γ_-^i are arranged in descending order of channel magnitude from transmitter i, and $\Gamma_-^i[p:q]$ denotes the set of elements of Γ_-^i starting from the p^{th} element to the q^{th} element. Let the j^{th} codeword transmitted from transmitter i be $\mathbf{x}_{ij}$. Let the power used for this codeword be $P_j = \alpha_{ij} P$ and R_{ij}^k be the rate. Therefore, the transmitter i transmits a superposition of codewords given by $\mathbf{x}_i = \sum_{j \in \Gamma_-^i} \mathbf{x}_{ij}$.

The received word at receiver j is

$$\mathbf{y}_j = \sum_{i \in \Gamma_+^j} h_{ij} \sum_{l \in \Gamma_-^i} \mathbf{x}_{il} + \mathbf{w}_j,$$

where $\mathbf{w}_j$ is the additive white Gaussian noise at receiver j. Each receiver j decodes the codewords intended for itself and all other *weaker* receivers. Let receiver j be the l_i^{th} receiver in Γ_-^i. The codewords with indices l_i to d_-^i are decoded at the j^{th} receiver. The codewords of the weaker receivers $\Gamma_-^i[l_i+1 : d_-^i]$ are canceled in the SIC receiver. Therefore, only the codewords to the stronger receivers $\Gamma_-^i[1 : l_i - 1]$ will interfere. The received word can be written as

$$\mathbf{y}_j = \underbrace{\sum_{i \in \Gamma_+^j} \sum_{l \in \Gamma_-^i[1:l_i-1]} h_{ij}\mathbf{x}_{il}}_{\text{interference codewords}} + \underbrace{\sum_{i \in \Gamma_+^j} h_{ij}\mathbf{x}_{ij} + \sum_{i \in \Gamma_+^j} \sum_{l \in \Gamma_-^i[l_i+1:d_-^i]} h_{ij}\mathbf{x}_{il}}_{\text{decoded codewords}} + \mathbf{w}_j.$$

Therefore, the achievable rate region for each state S_k is defined by the following constraints:

$$R_{ij}^k \leq \frac{1}{2}\log\left(1 + \frac{h_{ij}^2 \alpha_{ij} P}{\sigma^2 + \sum_{l \in \Gamma_-^i[1:l_i-1]} h_{ij}^2 \alpha_{il} P}\right), \quad \forall i \in I_k, \tag{3}$$

$$\sum_{j \in \Gamma_-^i} \alpha_{ij} \leq 1, \quad \forall i \in I_k, \tag{4}$$

$$\sum_{(p,q) \in A} R_{pq}^k \leq \frac{1}{2}\log\left(1 + \frac{\sum_{(p,q)\in A} h_{pj}^2 \alpha_{pq} P}{\sigma^2 + \sum_{i \in \Gamma_+^j} \sum_{l \in \Gamma_-^i[1:l_i-1]} h_{il}^2 \alpha_{il} P}\right), \tag{5}$$

$\forall A \subseteq Q_j = \{(p,q) : p \in \Gamma_+^j, q \in \Gamma_-^p[l_i : d_-^i]\}$ and $\forall j \in J_k$.

Using superposition coding allows each transmitter to send messages to a subset of its receivers. This *receiver selection* ability allows better spatial reuse.

C. Dirty Paper Coding (DPC) - CB Scheme

In the DPC-CB scheme, the source is assumed to know the messages transmitted by all the relays since all messages originate from the source. Therefore, when $S \in I_k$, Dirty Paper Coding (DPC) is used by the source to cancel interference to its receiver caused by simultaneous transmissions from relay nodes. Other transmitters in I_k transmit common messages similar to the CB scheme. The receiver r to which the source is sending its DPC-coded message at rate

R_s^k is not affected by interference from other relays and will decode only this message. The other receivers must decode all the messages from all the transmitters (except the source) that are connected to it. For example, in the state S_1 shown in Fig. 1, S transmits a DPC-coded message to R_2 using its prior knowledge of the messages transmitted by R_1 and R_3 (and the corresponding channel gains). Receiver R_4 decodes the common messages transmitted by R_1 and R_3, and receiver D decodes the common message transmitted by R_3. In general, for the above DPC-CB scheme, the achievable rate region for state S_k is given by the following constraints:

$$R_s^k \leq \frac{1}{2}\log\left(1+\frac{h_{sr}^2 P}{\sigma^2}\right), \tag{6}$$

$$\sum_{i\in A} R_i^k \leq \frac{1}{2}\log\left(1+\frac{\sum_{i\in A} h_{ij}^2 P}{\sigma^2}\right), \begin{pmatrix}\forall A \subseteq \Gamma_+^j \\ \forall j \in J_k \setminus r\end{pmatrix}. \tag{7}$$

D. Dirty Paper Coding (DPC) - SC Scheme

In the DPC-SC scheme, the source uses DPC as in the DPC-CB scheme. Other transmitters transmit messages as in the SC scheme. For the DPC-SC scheme, the achievable rate region for state S_k is given by the following constraints: Equation (6) and

$$R_{ij}^k \leq \frac{1}{2}\log\left(1+\frac{h_{ij}^2\alpha_{ij}P}{\sigma^2+\sum_{l\in\Gamma_-^i[1:l_i-1]} h_{ij}^2\alpha_{il}P}\right), \quad \forall i \in I_k \setminus S, \tag{8}$$

$$\sum_{j\in\Gamma_-^i} \alpha_{ij} \leq 1, \quad , \alpha_{ir}=0, \quad \forall i \in I_k \setminus S, \tag{9}$$

$$\sum_{(p,q)\in A} R_{pq}^k \leq \frac{1}{2}\log\left(1+\frac{\sum_{(p,q)\in A} h_{pj}^2\alpha_{pq}P}{\sigma^2+\sum_{i\in\Gamma_+^j}\sum_{l\in\Gamma_-^i[1:l_i-1]} h_{il}^2\alpha_{il}P}\right), \tag{10}$$

$\forall A \subseteq L_j = \{(p,q) : p \in \Gamma_+^j, q \in \Gamma_-^p[l_i : d_-^i] \setminus r\}$ and $\forall j \in J_k \setminus r$.

E. Flow Constraints and Optimization

Now, we present a constrained flow problem to compute the achievable rate from source S to destination D in the multistage relay network and the corresponding time-sharing between the states. Let x_{ij}^k denote the information flow rate from node i to node j in state S_k towards the sink.

Let z_{ij}^k be the maximum information flow through link (i,j) in state S_k. Let z_i^k denote the total information flow out of node i in state S_k. In the CB and DPC-CB schemes, each transmitter i in a state sends only one message. Since any receiver j can get information only from this message a single flow variable x_{ij}^k is sufficient. However, when SC is used, a receiver j can get information from transmitter i through all messages that it can decode. Therefore, flow variables corresponding to each transmitted message are required. Let $x_{ij,s}^k$ denote the information flow rate from node i to node j via the s^{th} transmitted message by node i in state S_k. In this case:

$$\sum_{s=l_i}^{d_-^i} x_{ij,s}^k = x_{ij}^k, \forall j \in \Gamma_-^i, i \in I_k. \tag{11}$$

The optimization problem can now be stated as:

$$\max_{\{x_{ij}^k\},\{x_{ij,s}^k\},\{\lambda_k\}} R, \text{ subject to:} \tag{12}$$

- Flow constraints: For all $i \in V$, we have

$$\sum_{\{k:i\in I_k\}} \sum_{j\in\Gamma_-^i} x_{ij}^k - \sum_{\{k:i\in J_k\}} \sum_{j\in\Gamma_+^i} x_{ji}^k = \begin{cases} R & \text{if } i = S \\ -R & \text{if } i = D \\ 0 & \text{else} \end{cases}.$$

- Scheduling constraints: $\sum_k \lambda_k \leq 1$ and $\lambda_k \geq 0\ \forall k$.
- Rate region constraints: The achievable rate region constraints for each state depend on the encoding and decoding scheme used. The rate constraints for each of the three proposed schemes for each state S_k are as follows:

  1) CB scheme:

$$\sum_{j\in\Gamma_-^i} x_{ij}^k \leq z_i^k, \quad \forall i \in I_k, \tag{13}$$

$$\sum_{i\in A} z_i^k \leq \lambda_k(\text{RHS of (2)}), \begin{pmatrix} \forall A \subseteq \Gamma_+^j \\ \forall j \in J_k \end{pmatrix}, \tag{14}$$

  where RHS of (2) is the right hand side of (2).

  2) SC scheme: Equations (4), (11) and:

$$\sum_{b\in\Gamma_-^i[1:l_i]} x_{ib,l_i}^k \leq z_{ij}^k, \quad \forall j \in \Gamma_-^i, i \in I_k, \tag{15}$$

$$z_{ij}^k \leq \lambda_k(\text{RHS of (3)}), \forall i \in I_k, \tag{16}$$

$$\sum_{(p,q)\in A} z_{pq}^k \leq \lambda_k(\text{RHS of (5)}), \tag{17}$$

for all $A \subseteq Q_j$ and for all $j \in J_k$.

3) DPC-CB scheme:

$$\sum_{j \in \Gamma_-^i} x_{ij}^k \leq z_i^k, \quad \forall i \in I_k, \tag{18}$$

$$z_s^k \leq \lambda_k(\text{RHS of (6)}), \tag{19}$$

$$\sum_{i \in A} z_i^k \leq \lambda_k(\text{RHS of (7)}), \tag{20}$$

for all $A \subseteq \Gamma_+^j$ and for all $j \in J_k \setminus r$.

4) DPC-SC scheme: Equations (8), (19), (11) and:

$$\sum_{b \in \Gamma_-^i[1:l_i]} x_{ib,l_i}^k \leq z_{ij}^k, \quad \forall j \in \Gamma_-^i, i \in I_k \setminus S, \tag{21}$$

$$z_{ij}^k \leq \lambda_k(\text{RHS of (8)}), \forall i \in I_k \setminus S, \tag{22}$$

$$\sum_{(p,q) \in A} z_{pq}^k \leq \lambda_k(\text{RHS of (10)}), \tag{23}$$

for all $A \subseteq L_j$ and for all $j \in J_k \setminus r$.

For the CB and DPC-CB schemes, the above optimization problem is a linear program. However, for the SC and DPC-SC schemes, it is not a linear program since the power sharing variables α_{ij}'s are also optimized. Numerical solutions for the SC and DPC-SC schemes may be computed using generic constrained optimization routines such as the function *fmincon* in MATLAB.

F. Example

As an example, we provide the constraints for the State S_1 in Fig. 2 when the SC scheme is used. These correspond to equation (15). Assume the channel gains are such that: $\alpha \leq \delta \leq \gamma$. In State S_1, source node S transmits only one message, relay node R_1 transmits two messages, and relay node R_3 transmits three messages. The first message from R_1 can be decoded by R_4 and the second message can be decoded by both R_2 and R_4. Similarly, The first message from R_3 can be decoded by R_2, the second message can be decoded by both R_2 and R_4, and the third message can be decoded by R_2, R_4, and D. Therefore, the constraints are as follows:

- At transmitter S: $x_{13,1}^k \leq z_{13}^k$
- At transmitter R_1: $x_{25,1}^k \leq z_{25}^k, \quad x_{25,2}^k + x_{23,2}^k \leq z_{23}^k$
- At transmitter R_3: $x_{43,1}^k \leq z_{43}^k, \quad x_{43,2}^k + x_{45,2}^k \leq z_{45}^k$, and $x_{43,3}^k + x_{45,3}^k + x_{46,3}^k \leq z_{46}^k$

In the CB scheme, transmitter i sends out a common codeword to all its receivers Γ^i_-. The explicit constraints are $x^k_{13} \leq z^k_1$ at S, $x^k_{25} + x^k_{23} \leq z^k_2$ at R_1, and $x^k_{43} + x^k_{45} + x^k_{46} \leq z^k_4$ at R_3.

V. Achievable Rate and Capacity

In general, computing the rate achieved by the above relaying schemes requires optimization over all possible schedules. In this section, we fix a simple schedule and compute the rate achieved by it. We show that the cut-set bound can be approached with this simple schedule over certain networks.

Definition 1 (Two-path, equal-source-gain networks): Consider a half-duplex relay network represented by a graph $G = (V, E)$ connecting a source $S \in V$ and destination $D \in V$ with channel gains h_{ij}, $(i, j) \in E$ and maximum transmit power P per node. Such a network is said to be a two-path network if there are at least two node-disjoint paths connecting S and D. The network is an equal-source-gain network if the source gains $h_{Sj} = \alpha$ (a constant) $\forall (S, j) \in E$.

In general, the information rate in a relay network is upper-bounded by the maximum information transfer across any cut. In an equal-source-gain network, the information rate R from S to D is upper bounded by the source cut as

$$R \leq I(X^S; Y^{(V \setminus S)} | X^{(V \setminus S)}) \leq \frac{1}{2} \log(1 + \alpha^2 P),$$

when there is no receiver cooperation. For two-path, equal-source-gain networks, we will show an explicit schedule (using interference network states) that achieves an information rate equal to the source cut under some conditions on the channel gains h_{ij}.

Two-path schedule: The specific schedule works for any two-path network, and is constructed using two node-disjoint paths $P_1 = \{S, n_{11}, n_{12}, \cdots, n_{1,l_1}, n_{1,l_1+1} = D\}$ and $P_2 = \{S, n_{21}, n_{22}, \cdots, n_{2,l_2}, n_{2,l_2+1} = D\}$ of lengths $l_1 + 1$ and $l_2 + 1$ edges ($l_1 \geq l_2$, without loss of generality), respectively. If there are multiple pairs of paths, any one can be chosen as the specific pair P_1 and P_2. However, to reduce interfering links within a path, we select P_1 and P_2 to be the two shortest paths (i.e. for any other pair of node-disjoint paths P'_1 and P'_2 in the network with respective lengths $l'_1 + 1$ and $l'_2 + 1$ ($l'_1 \geq l'_2$), we have $l_1 \leq l'_1$ and $l_2 \leq l'_2$). Using P_1 and P_2, two interference network states $S_1 = (I_1, J_1)$ and $S_2 = (I_2, J_2)$ are constructed as follows.

$$I_1 = \{S, n_{12}, n_{14}, \ldots, n_{1,2\lfloor l_1/2 \rfloor}, n_{21}, n_{23}, \ldots, n_{2,2\lceil l_2/2 \rceil - 1}\},$$

$$J_1 = \{n_{11}, n_{13}, \ldots, n_{1,2\lfloor l_1/2 \rfloor + 1}, n_{22}, n_{24}, \ldots, n_{2,2\lceil l_2/2 \rceil}\},$$

$$
\begin{aligned}
I_2 &= \{S, n_{11}, n_{13}, \ldots, n_{1,2\lceil l_1/2 \rceil -1}, n_{22}, n_{24}, \ldots\ n_{2,2\lfloor l_2/2 \rfloor}\},\\
J_2 &= \{n_{21}, n_{12}, n_{14}, \ldots, n_{1,2\lceil l_1/2 \rceil}, n_{23}, \ldots, n_{1,2\lfloor l_1/2 \rfloor +1}\}.
\end{aligned}
$$

Observe that: (1) The source S is a transmitter in both states. The destination D is never a transmitter. (2) The destination D is a receiver in both states if $l_1 = l_2 \mod 2$. If $l_1 \neq l_2 \mod 2$, $D \in J_1$ when l_1 is even (l_2 odd), and $D \in J_2$ if l_1 is odd (l_2 even). The states are illustrated in Fig. 3 for two scenarios. The interfering links between the two paths are not shown in the figure for simplicity. Note that in Fig. 3(b) D is present as a receiver only in S_2 connected to two nodes from one path each.

In general, the two states S_1 and S_2 are interference networks, which are time-shared for normalized time periods of τ_1 and $\tau_2 = 1 - \tau_1$, respectively, in the final schedule. Transmissions on the edges in the paths P_1 and P_2 are processed as intended messages at the receivers and they carry a nonzero information flow. All other edges have zero information flow and are processed as interference at the receivers. Therefore, the flow of information is along the two paths P_1 and P_2 from S to D. Let $C_1 = 1/2 \log(1 + |h_{S,n_{11}}|^2 P)$ and $C_2 = 1/2 \log(1 + |h_{S,n_{21}}|^2 P)$ be the capacities of the two edges out of the source into paths P_1 and P_2, respectively.

Strong interference condition in the CB scheme: We suppose that the source S transmits at rates $R_1 \leq C_1$ and $R_2 \leq C_2$ in states S_1 and S_2 to nodes n_{11} and n_{21}, respectively. Further, information received by a node at rate R_1 when state S_1 is operational is forwarded in state S_2 by the same node at rate R_2. For flow conservation, we require that $R_1\tau_1 = R_2\tau_2$. Using $\tau_1 + \tau_2 = 1$, we have $\tau_1 = R_2/(R_1 + R_2)$ and $\tau_2 = R_1/(R_1 + R_2)$.

In summary, each node in I_t transmits at a common rate R_t for a time period τ_t in state S_t for $t = 1, 2$. Hence, the relaying scheme considered is the CB scheme. The question that remains to be addressed is the condition for successful decoding by receivers in each state. In each state, a receiving node sees a Gaussian MAC channel with different channel gains and a transmit power constraint P. If the receiving node is not the destination D, exactly one of these links carries information at a rate R_1 or R_2. If the receiving node is D, two of these links might carry information at rate R_1 or R_2 depending on the parity of l_1 and l_2.

Lemma 1: Consider a Gaussian Multiple Access Channel (GMAC) with K transmitters with a power constraint P, channel gains h_i, $1 \leq i \leq K$ and normalized unit noise variance at the receiver. For $R = \frac{1}{2}\log(1 + |g|^2 P)$, the length-$K$ rate vector $(R, R, \ldots, R)$ is achievable, if the channel gains satisfy either (a) $|h_i| \geq \sqrt{\frac{(1+|g|^2P)^K-1}{KP}}$, or (b) $h_1 = |g|$, $|h_i| \geq \sqrt{\frac{(1+|g|^2P)^K-(1+|g|^2P)}{(K-1)P}}$ for $2 \leq i \leq K$.

Proof: The sum-rate constraint imposed by the Gaussian-MAC results in the following requirement on the channel gains h_i:

$$KR = \frac{K}{2}\log(1+|g|^2P) \leq \frac{1}{2}\log(1+(|h_1|^2+|h_2|^2+\cdots+|h_K|^2)P).$$

The above condition is satisfied if

$$\frac{K}{2}\log(1+|g|^2P) \leq \frac{1}{2}\log(1+Kh_{\text{min}}^2P) \leq \frac{1}{2}\log(1+(|h_1|^2+|h_2|^2+\cdots+|h_K|^2)P),$$

where $h_{\text{min}} = \min\{|h_1|, |h_2|, \cdots, |h_K|\}$. Hence, we need $|h_i| \geq |h_{\text{min}}| \geq \sqrt{\frac{(1+|g|^2P)^K-1}{KP}}$. Since the sum-rate constraint is satisfied, all other MAC constraints are satisfied for the rate vector $(R, R, \ldots, R)$. This proves the conditions in part (a). For part (b), we use the sum rate constraint $\frac{K}{2}\log(1+|g|^2P) \leq \frac{1}{2}\log(1+|g|^2P+(K-1){h'_{\text{min}}}^2P)$, where $h'_{\text{min}} = \min\{|h_2|, |h_3|, \cdots, |h_K|\}$ and argue similarly. ■

Consider a receiver $r \in J_t$ with neighbouring transmitters $N_t(r) = \{q \in I_t : (q, r) \in E\}$ for $t = 1, 2$, and let $d_t(r) = |N_t(r)|$. In state S_t, we have a Gaussian-MAC connecting the $d_t(r)$ transmitters in $N_t(r)$ to the receiver r. The transmissions are at a common rate R_t. Using Lemma 1, if $S \notin N_t(r)$, reception at r is successful whenever $|h_{q,r}| \geq \sqrt{\frac{(1+|h_{S,n_{t1}}|^2P)^{d_t(r)}-1}{d_t(r)P}}$ for $q \in N_t(r)$ and $t = 1, 2$. If $S \in N_t(r)$ (i.e. $r = n_{11}$ or $r = n_{21}$), we use the second sufficient condition in Lemma 1 to get $|h_{q,r}| \geq \sqrt{\frac{(1+|h_{S,n_{t1}}|^2P)^{d_t(r)}-(1+|h_{S,n_{t1}}|^2P)}{(d_t(r)-1)P}}$ for $q \in N_t(r) \setminus S$ and $t = 1, 2$.

Extending this result, all receptions in state S_t, $t = 1, 2$, will be successful if the channel gains h_{ij} for $i \in P_t \setminus S$ satisfy

$$|h_{ij}| \geq \max\left\{\sqrt{\frac{W_t^{d_t(n_{t1})}-W_t}{(d_t(n_{t1})-1)P}}, \max_{r\in J_t\setminus n_{t1},\, q\in N_t(r)} \sqrt{\frac{W_t^{d_t(r)}-1}{d_t(r)P}}\right\}, \tag{24}$$

where $W_t = 1+|h_{S,n_{t1}}|^2P$. Hence, a rate $R_1\tau_1 + R_2\tau_2 = 2R_1R_2/(R_1+R_2)$ is achieved by the two-path schedule in the CB scheme for any $R_1 \leq C_1$ and $R_2 \leq C_2$, whenever (24) is satisfied by the channel gains of the network. We state the above result as a theorem below.

Theorem 1: Consider a two-path half-duplex relay network with C_1 and $C_2 \leq C_1$ being the capacities of the edges from the source into any two node-disjoint paths. In such a network, a common broadcast relaying scheme using a two-path schedule with two states (as defined above) achieves rates up to $2C_1C_2/(C_1+C_2)$, if (24) is satisfied.

Here are a few observations about the condition (24) and the above theorem.

1) If the network in Theorem 1 is an equal-source-gain network with source gains equal to α, then a rate $C = 1/2 \log(1 + |\alpha|^2 P)$ (use $C_1 = C_2 = C$) is achievable by the common broadcast relaying scheme. Hence, under (24), the source cut-set bound is achieved in two-path, equal-source-gain networks.
2) If (24) is satisfied, then the gains on the paths P_1 and P_2 satisfy $|h_{ij}| \geq |h_{S,n_{t1}}|$ for $t = 1, 2$. Therefore, a simple one-path alternating schedule on path P_t achieves a rate of $C_t/2$. Also, for $P << 1$, (24) approximates to $|h_{q,r}| \geq |h_{S,n_{t1}}|$.
3) By the above, a rate of $\max\{C_1/2, 2C_1C_2/(C_1 + C_2)\}$ is achievable under the conditions of Theorem 1 by either one-path or two-path scheduling. We see that two-path scheduling is better than one-path, if $C_2 > C_1/3$.
4) Let $\kappa_t = \max_{r \in J_t} d_t(r)$ for $t = 1, 2$. If $(\kappa_t + 1)|h_{S,n_{t1}}|^2 P \geq 1$, (24) is satisfied whenever $|h_{q,r}| \geq \sqrt{\frac{(1+|h_{S,n_{t1}}|^2 P)^{\kappa_t} - 1}{\kappa_t P}}$.
5) Under the DPC-CB scheme with DPC at the source S, links from the source in the set $N_t(r)$ can be removed in (24). This will result in a weakening of (24) in the sense that the same rates are now achievable for a larger range of channel gains.
6) Since the SC scheme includes the CB scheme as a special case, Theorem 1 holds for the SC scheme as well. The condition (24) could presumably be weakened by a version of Lemma 1 for non-constant rate vectors.

In summary, Lemma 1 and Theorem 1 show that the cutset bound can be achieved by the CB scheme under suitable strong interference conditions on two-path, equal-source-gain networks.

Weak interference condition in the DPC-SC scheme: We now provide weak interference conditions under which the DPC-SC scheme achieves the cutset bound. We use the same two-path schedule, with P_1 and P_2 chosen to be two shortest paths. In each state S_t, a receiver $r \in J_t$ in path P_1 (say) is connected to a legitimate transmitter on the path P_1, one interferer on the same path and other possible interferers from the other path P_2. In Fig. 3, for $r = n_{12} \in J_2$, we have n_{11} as the legitimate transmitter and n_{13} as the interferer on the same path. Since the paths are shortest, there cannot be other interferers from the same path.

In general, for the MAC at receiver r in the path P_t, we have two links on the data flow path (one of them interfering) and other $d_t(r) - 2$ possible interfering edges connecting the two paths. Suppose that r does MAC-decoding for the two links on the data flow path and treats all inter-path interference as noise. Then, at r not connected to the source or destination, we have a two-user Gaussian-MAC with effective normalized receiver noise variance $\leq 1+(d_t(r)-2)h_{\max}^2P$, where $h_{\max} = \max\{|h_{ij}| : i \in P_1, j \in P_2, (i,j) \in E, i \neq S, j \neq D\}$ is the maximum gain on any inter-path edge. For $(r,D) \in P_t$, the destination is the next node on the path; since the destination is never a transmitter, we have a single-user Gaussian channel with effective noise $\leq 1 + (d_t(r)-1)h_{\max}^2P$. Note that we assume Gaussian codebooks at all transmitters. A rate vector $[R\ R]$ for $R = 1/2\log(1+|g|^2P)$ is achievable on the two-user MAC for r such that $(r,D) \notin P_t$ whenever

$$2R = \log(1+|g|^2P) < 1/2\log\left(1+\frac{2h_{\min}^2P}{1+(d_t(r)-2)h_{\max}^2P}\right),$$

where $h_{\min} = \min\{|h_{ij}| : (i,j) \in P_1 \text{ or } P_2, i \neq S\}$ is the minimum gain on any edge (not originating from the source) in the chosen paths. The above condition reduces to

$$h_{\max} \leq \min_{r\in J_1\cup J_2, r\neq n_{t1}, (r,D)\notin P_t} \frac{c}{\sqrt{(d_t(r)-2)P}}, \text{ with } 1+c^2 = \frac{h_{\min}^2}{|g|^2+|g|^4P/2} > 1. \tag{25}$$

For a receiver r such that $(r,D) \in P_t$, a rate $R < 1/2\log\left(1+\frac{h_{\min}^2P}{1+(d_t(r)-1)h_{\max}^2P}\right)$ is achievable. The above condition reduces to

$$h_{\max} \leq \min_{r\in J_1\cup J_2, r\neq n_{t1}, (r,D)\in P_t} \frac{c'}{\sqrt{(d_t(r)-1)P}}, \text{ with } 1+c'^2 = \frac{h_{\min}^2}{|g|^2} > 1. \tag{26}$$

Clearly, for (25) and (26) to be valid, we need $h_{\min} > |g|$. Since DPC-SC is employed, (25) and (26) need not be satisfied for receivers r connected to the source i.e. $r \neq n_{11}$ or $r \neq n_{21}$. For these receivers, DPC at the source eliminates all interference. Hence, for DPC-SC in a two-path equal-source gain network with source-gain α, we can set $g = \alpha$ and achieve rate $R = 1/2\log(1+\alpha^2P)$, which equals the source cut, whenever h_{ij} satisfy (25) and (26). Essentially, (25) and (26) provide a lower bound on the gains of data-carrying edges on the chosen paths ($h_{\min}$), while imposing an upper bound on the gains of interfering inter-path edges ($h_{\max}$).

VI. Numerical Results

We evaluate and compare the rate achieved by the MDF schemes: (1) CB, (2) SC, (3) DPC-CB, and (4) DPC-SC for two different network topologies and channel realizations. The cheap relay cut-set upper bound for half-duplex relay networks and the rate achieved by the IA scheme are also evaluated. The rate achieved by each scheme is obtained by solving the optimization problem in (12) with appropriate rate region constraints. The strong and weak interference regimes are illustrated when appropriate.

Since the diamond network has been studied in detail in [7], [8], we skip details and simply mention that the proposed MDF protocols recover similar results for the diamond network. Note that the protocols and optimization methods of Section IV work for all relay networks with arbitrary topology. We have chosen two simple networks for illustration purposes.

A. Two stage relay network

Consider the network shown in Fig. 1. For evaluating the cut-set bound, all the $2^2 \cdot 3^4 = 324$ states were considered (The source is never in receive state and the destination is never in transmit state.). For this network, the interference avoidance states are the states with a single transmitting node. For the proposed MDF protocols, interference network states with two transmitters ($\binom{5}{2} = 10$ states) and some states with three transmitters (5 out of $\binom{5}{3} = 10$ states) are used along with the IA states. Two of the states with three transmitters are shown in Fig. 2 for illustration.

In Fig. 4, we set $\alpha = \gamma = 1$ and vary $\beta = \delta$. The cut-set bound, determined by the source cut, is 1 for all β. As seen in the figure, there is a significant gap in performance between the IA scheme and the cut-set bound. The proposed MDF schemes perform significantly better than the IA scheme and achieve the cut-set bound for certain channels. **For large** $\beta(= \delta)$**:** (1) All four MDF schemes achieve capacity of 1 by equal time-sharing of states $S_3 = (\{S, R_2, R_3\}, \{R_1, R_4, D\})$ and $S_4 = (\{S, R_1, R_4\}, \{R_2, R_3, D\})$. The receivers in both these states see strong interference, which can be canceled at the receiver. For instance, in state S_3, the receiver R_1 can decode the source's message in the presence of strong interference from R_2 and R_3. (2) According to the condition for strong interference derived in Section V, the CB scheme achieves the cut-set bound for $\beta > 5.01$ dB, and the DPC-CB scheme achieves the cut-set bound for $\beta > 1.99$ dB. The analytical bounds clearly agree with the numerical results obtained by solving the optimization problem. **For small** β**:** (1) Common broadcast at the relays is limited by a weak receiver with close-to-zero capacity. DPC-CB is better, but still limited by common broadcast to weak receivers. (2) Superposition coding, which enables different rates to receivers, proves to be better at low values of β. For SC, states S_1 and S_2 (shown in fig. 1) are used, and the rate is limited by the interference at relays R_1 and R_2. When $\beta = 1$ (0 dB), DPC-CB is better as SC becomes identical to CB for identical channel gains. (3) The DPC-SC scheme performs the best and approaches the cut-set bound as $\beta(=\delta) \to 0$. i.e., when interference becomes weak.

In Fig. 5, we set $\alpha = \beta = \delta = 1$ and vary γ. The cut-set bound, determined by the source cut, is 1 for all γ. **For large γ:** (1) From the analysis of the strong interference regime in Section V, the DPC-CB and DPC-SC schemes achieve the cut-set bound for $\gamma > 3.01$ dB. DPC-SC and DPC-CB achieve capacity by time-sharing the states S_1 and S_2. The interference at relays R_1 and R_2 are canceled using DPC, while the interference at R_3 and R_4 are overcome because the gains of the $R_2 \to R_3$ and $R_1 \to R_4$ links increase with γ. (2) The same states are used for the SC scheme as well. However, interference at R_1 and R_2 are overcome only for $\gamma \to \infty$. (3) As $\gamma \to \infty$, the CB scheme also approaches the cut-set bound by time-sharing between the states $(\{S, R_4\}, \{R_1, D\})$ and $(\{R_1\}, \{R_4\})$. However, it performs worse than the SC scheme for a fixed γ. **For small γ:** (1) All MDF schemes approach rates lower than the cut-set bound. DPC-SC and DPC-CB achieve rates of 0.76 and 0.7, while the SC and CB achieve rates of 0.67 and 0.55 respectively. Since $\beta = \delta = 1$, interference is never weak in this case. (2) For both DPC-CB and CB schemes, states S_3 and S_4 are used. While the interference at R_3 and R_4 limits the DPC-CB scheme, the CB scheme is limited by the interference at relays R_1 and R_2.

In Fig. 6, we fix $\alpha = 1$, $\beta = 1.25$ and vary $\gamma = \delta$ to illustrate both the strong and weak interference regimes. The two interference network states chosen according to the simple schedule in Section V have been used here to achieve the cut-set bound. The DPC-SC scheme achieves the cut-set bound for $\gamma(=\delta) > 2.68$ dB and for $\gamma < -3.63$ dB, as per the bounds in Section V. The DPC-CB scheme achieves the cut-set bound only for $\gamma(=\delta) > 2.68$ dB.

B. Rectangular grid network

In the 4×3 rectangular grid network shown in Fig. 7, we use a path-based heuristic to limit the number of possible states in the MDF protocols. We first select three non-overlapping paths from the source node $S = 1$ to the destination node $D = 11$, since multiple flow paths with interference processing is effective. The paths chosen are $S \to 4 \to 7 \to D$, $S \to 5 \to 8 \to D$ and $S \to 6 \to 9 \to D$. Using the nodes on these paths, three states chosen for scheduling are $(\{S, 6, 8\}, \{4, 9, D\})$, $(\{S, 4, 9\}, \{5, 7, D\})$, and $(\{S, 5, 7\}, \{6, 8, D\})$. Note that the source node is a transmitter and the destination node is a receiver in all three chosen states. Also, the other two transmitters are chosen to be at different distances from the source. With this choice of states, we have a two-stage relay network with six relay nodes $\{4, 5, 6, 7, 8, 9\}$ aiding communications from the source to the destination. For the proposed MDF schemes, we also use 5 important IA states ($T_1 = (\{S, 3, 6\}, \{2, 9, 10, 11\})$, $T_2 = (\{S, 4, 9\}, \{3, 7, 11, 12\})$, $T_3 = (\{2, 3, 11\}, \{4, 6, 10, 12\})$, $T_4 = (\{S, 6\}, \{4, 8\})$, $T_5 = (\{S, 4\}, \{6, 8\})$) in addition to the 3 states chosen above.

In Fig. 8, the gains β and γ are set to 1, and the gain α is varied. The cut-set bound is calculated using all possible states and the IA scheme uses all interference avoidance states. **For large α:** (1) The CB and SC schemes are limited by the interference at relays 4 and 5 even for large α. (2) The DPC-CB and DPC-SC schemes achieve a rate of 1 for $\alpha > 3.01$ dB. Note that since the gains of the paths chosen depend only on β and γ, a maximum rate of 1 can only be achieved. However, increasing α does change the interference. The cut-set bound is finite even if α is large since there is a cut separating nodes 1-6 from nodes 7-12 that is determined only by β and γ. **For small α:** (1) The DPC-CB and CB schemes are limited by the common broadcast constraint at the relays. (2) While SC scheme can perform better, it is still limited by interference at relays 4 and 5 compared to the cut-set bound. (3) The DPC-SC scheme performs the best and approaches the cut-set bound as $\alpha \to 0$.

In summary, in larger networks, the choice of schedule is important. We have used a path-based heuristic and relied on interference-processing for approaching the cut-set bound.

Multicast Communication: In Fig. 9, we present the performance of the proposed MDF relaying schemes for multicast communication. We include the possibility of network coding for multicast communication and modify the flow constraints in the optimization in (12) for multicast as in [18]. For illustration, we consider the DPC-CB scheme with a multicast session over the 4×3 grid network. The source node is S=1 and the sinks are nodes 10, 11 and 12.

We select the two paths connecting source and each sink. The paths chosen for sink 10 are $S \to 4 \to 7 \to 10$, $S \to 5 \to 8 \to 10$ and the paths chosen for sink 11 are $S \to 4 \to 7 \to 11$, $S \to 6 \to 9 \to 11$. Similarily, the paths chosen for sink 12 are $S \to 5 \to 8 \to 12$, $S \to 6 \to 9 \to 12$. Based on the path-based heuristic schedule, we select the following two interference-processing states: $S_1 = (\{S, 4, 6, 8, 11\}, \{5, 7, 9, 10, 12\})$, $S_2 = (\{S, 5, 7, 9\}, \{4, 6, 8, 10, 11, 12\})$. Along with those, we use six interference avoidance states: $T_1, T_2, T_3, T_4, T_5, T_6 = (8, \{10, 11, 12\})$. With these eight states, the DPC-CB scheme always achieves better multicast throughput than the interference avoidance scheme which is evident in Fig. 9. Thus, the proposed MDF relaying schemes achieve significant improvement for multicast communication as well.

C. Discussion

Single hop vs Multihop: For the network in Fig. 1, the cut-set bound is upper bounded by $C_{pp} = \frac{1}{2}\log(1 + \alpha^2 P/\sigma^2)$, which can be interpreted as the capacity of a point-to-point link with power constraint P and channel gain α. Using the protocols in this work, we have shown that rates up to C_{pp} are achievable by multistage half-duplex relaying in the network of Fig. 1 for certain ranges of the channel gains α, β, γ, and δ. A necessary condition for achieving the point-to-point capacity under the half-duplex constraint is that the source needs to be in transmit mode at all times. From our work, it appears that continuous transmission by the source and information transfer through the half-duplex relays is possible as long as there are two or more non-overlapping paths from the source to the destination (which is true in Figs. 1 and 7). Further, coding in interference networks created by multiple transmitters and receivers of the relay network is crucial for enabling the information flow.
Full-duplex Relaying vs Half-duplex Relaying: The second comparison is with full-duplex relays. The achievable rate even with full duplex relays is bounded by the sum rate across the source-broadcast cut, which is equal to C_{pp}, for the network in Fig. 1. Once again, we observe that two non-overlapping paths through the relays and interference-network coding enable a half-duplex relay network to achieve the full-duplex cut-set bound for certain ranges of channel gains.

VII. Conclusions

In this work, we have studied multihopping decode and forward (MDF) protocols over a relay network under practical assumptions such as half-duplex nodes, no cooperation among relay nodes and finite SNR. The states of the network (nodes either transmit, receive or remain idle) are seen as interference networks that support a certain rate region. Information flow from source to destination is optimized over the time-sharing of the interference network states.

Through analytical derivations, we show that MDF protocols used with a simple two-state schedule can approach the cutset bound under strong and weak interference regimes of channel gains. Use of multiple source-destination paths and dirty paper coding (DPC) at the source appear to be important tools for approaching the cutset bound in half-duplex relay networks.

Numerical studies have been performed on some example networks to illustrate the results. As expected, processing interference provides useful gains over interference avoidance in all scenarios. Even in larger networks, heuristic scheduling methods using states formed from multiple non-overlapping paths prove to be useful for approaching the cutset bound.

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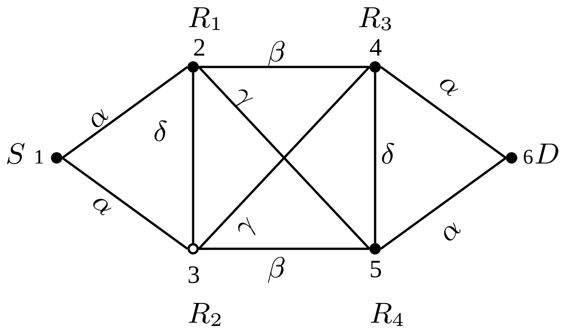

Fig. 1. Two stage relay network.

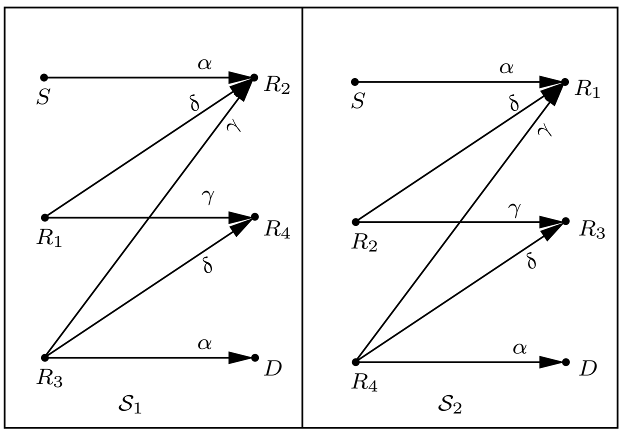

Fig. 2. Interference network states for the two stage relay network.

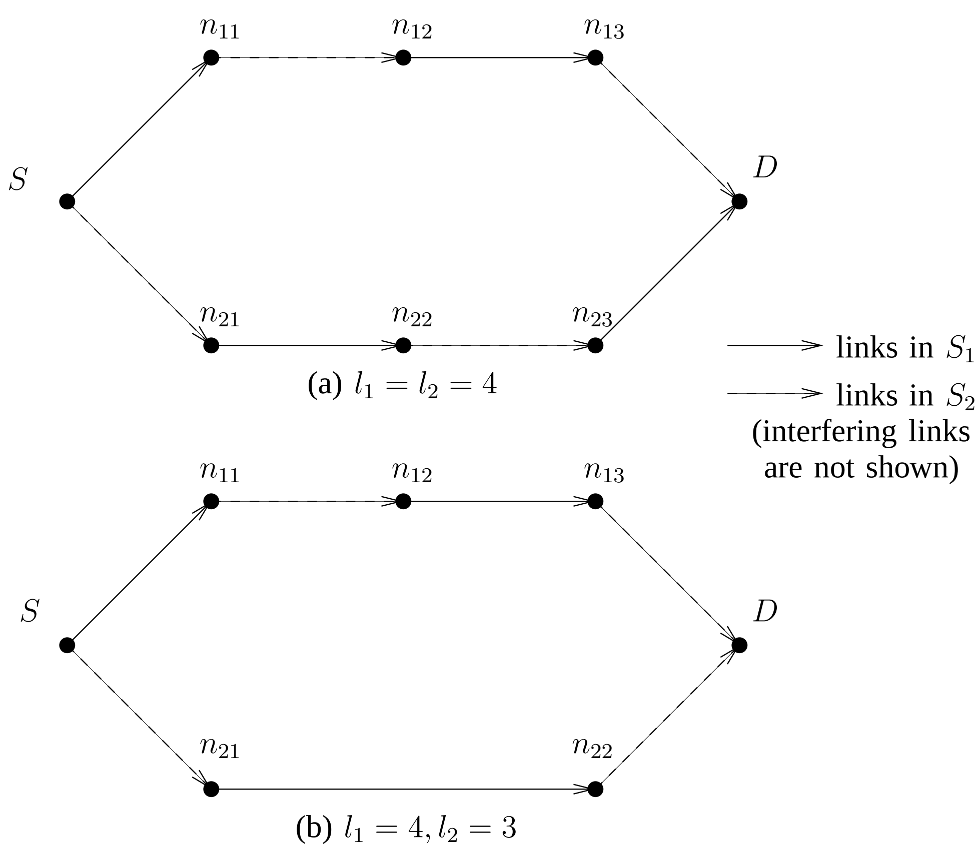

Fig. 3. Illustration of schedule in two-path networks.

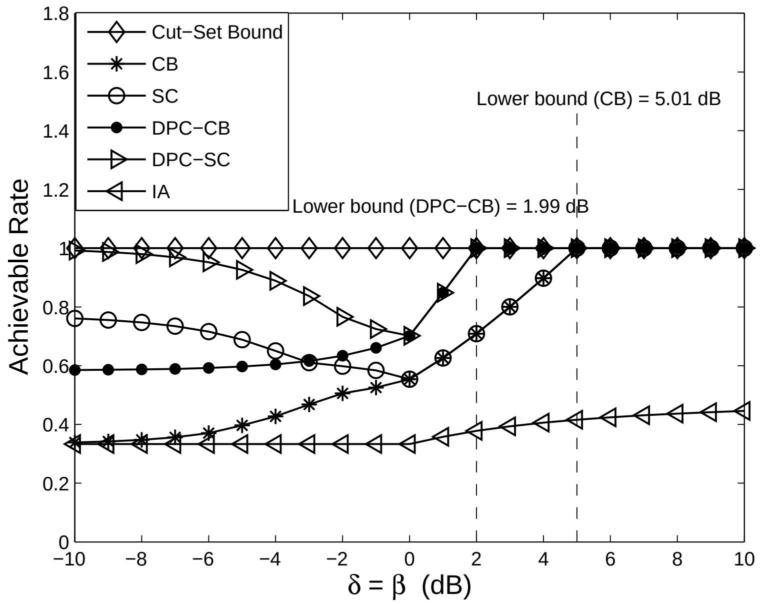

Fig. 4. Performance in two stage relay network, $\alpha = 1, \gamma = 1$, vary $\beta = \delta$.

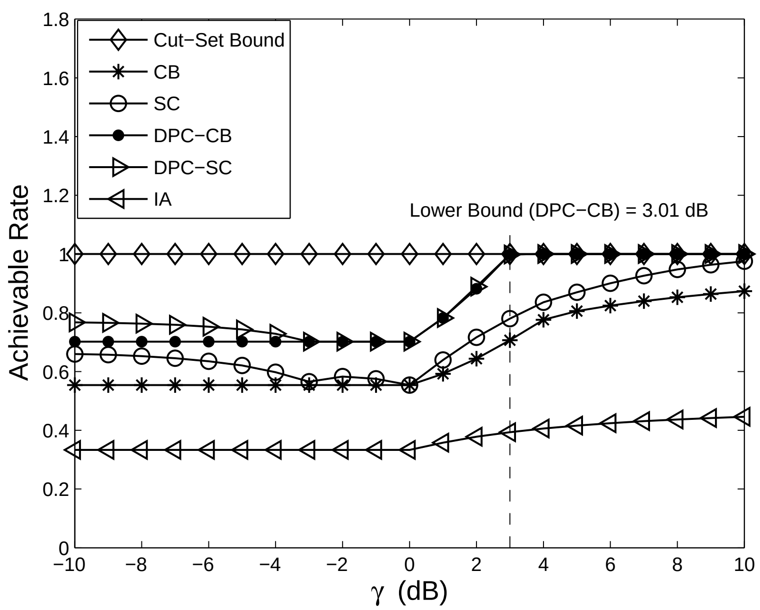

Fig. 5. Performance in two stage relay network, $\alpha = 1, \beta = 1, \delta = 1$, vary γ.

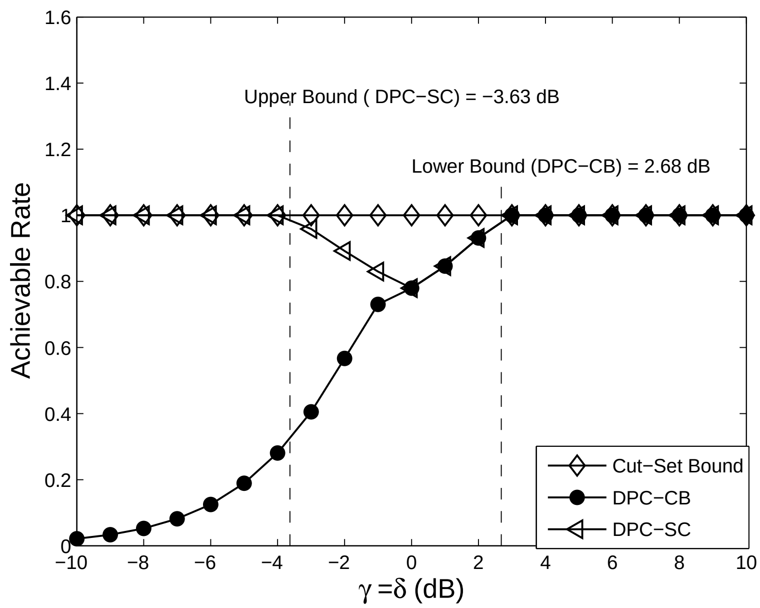

Fig. 6. Performance in two stage relay network, $\alpha = 1, \beta = 1.25$, vary $\gamma = \delta$.

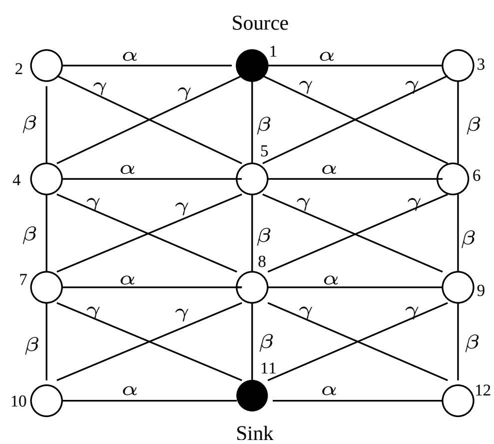

Fig. 7. 4×3 Grid Network.

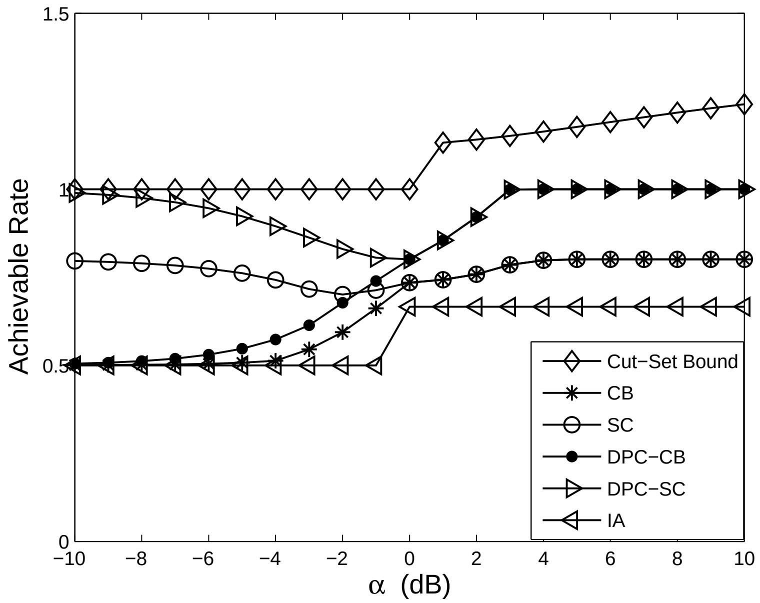

Fig. 8. Performance in Grid Network, $\beta = 1, \gamma = 1$, vary α.

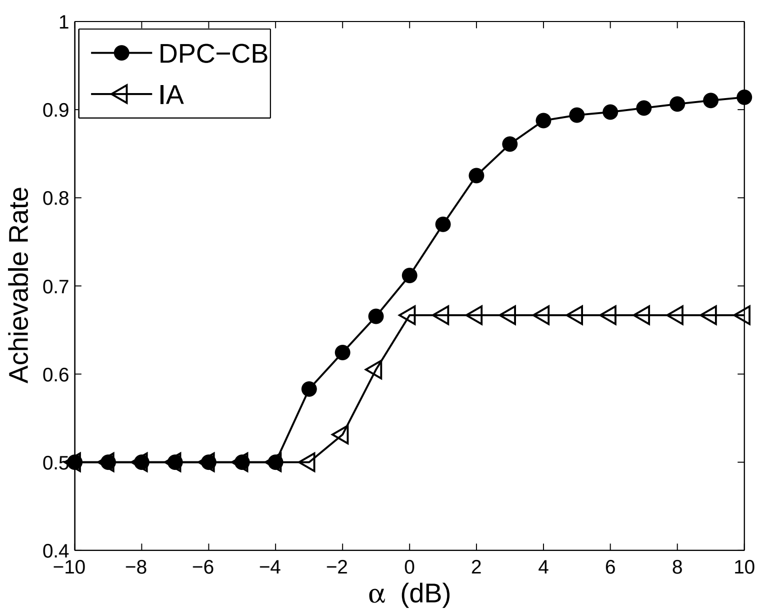

Fig. 9. Multicast throughput in Grid Network, $\beta = 1, \gamma = 1$, vary α.