Assignment on Cyclic Codes

EE512: Error Control Coding

Questions marked (Q) or (F) are questions from previous quizzes or final exams, respectively.

1. What is the ideal describing the cyclic code $\{0000, 0101, 1010, 1111\}$?
2. Describe the smallest cyclic code containing the vector 0011010.
3. Show that in an (n, k) cyclic code any k consecutive bits can be taken to be the message bits.
4. Consider the $n = 7$ binary cyclic code generated by $g(x) = 1 + x + x^3$.
   (a) Find all codewords of the code.
   (b) The allzero codeword $c(x) = 0$ is obtained uniquely by multiplying $g(x)$ by $m(x) = 0$ in $\mathrm{GF}(2)[x]$. Find all $f(x) \in \mathrm{GF}(2)[x]/(x^7 + 1)$ such that $f(x)g(x) = 0$ in $\mathrm{GF}(2)[x]/(x^7 + 1)$.
5. A binary cyclic code of length 15 has generator polynomial $g(x) = (x^4+x+1)(x^4+x^3+x^2+x+1)$. Give a generator matrix and parity-check matrix for the code. Find the generator matrix for the dual of the code.
6. Find the dimension and generator polynomial for every binary cyclic code of length 15, 17, 21, 31, 51, 73, 85.
7. Let C be the $n = 3$ cyclic code over GF(4)= $\{0, 1, \alpha, \alpha^2\}$ ($\alpha^3 = 1$, $\alpha^2 = 1 + \alpha$) generated by $g(x) = x + \alpha$.
   (a) Find all codewords of C. Find the minimum distance of C.
   (b) Find the check polynomial of C. Find the generator polynomial of $C^{\perp}$.
   (c) Find all cyclic subcodes of C i.e. cyclic codes that are contained in C.
8. Let two length-n cyclic codes C_1 and C_2 be generated by $g_1(x)$ and $g_2(x)$ respectively.
   (a) Show that $C_1 \subseteq C_2$ iff $g_2(x)|g_1(x)$.
   (b) State the condition $g_2(x)|g_1(x)$ in terms of the zeros of C_1 and C_2.
   (c) Find a necessary and sufficient condition for a cyclic code C to be self-orthogonal, i.e. $C \subseteq C^{\perp}$, in terms of the zeros of C.
9. Consider the linear block code with generator matrix
   (a)
$$G = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix}.$$
   (b)
$$G = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 \end{bmatrix}.$$
   (c)
$$G = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix}.$$

In which of the above cases is the code cyclic? Can you think of a method other than enumeration of all codewords to answer this question?

10. Show that $g(x) = 1 + x^2 + x^4 + x^6 + x^7 + x^{10}$ is a generator polynomial for a (21,11) cyclic code. Find the check polynomial of this code.

11. Let $g(x)$ be the generator polynomial of a binary cyclic code of length n.

    (a) Show that if $(x + 1)$ is a factor of $g(x)$, the code contains no odd-weight codewords.

    (b) If n is odd and $(x+1)$ is not a factor of $g(x)$, show that the code contains the all-1s codeword.

12. Consider a binary $[n, k, d]$ cyclic code C with generator polynomial $g(x)$. Show that $g^*(x) = x^{n-k}g(x^{-1})$ is also a generator polynomial of a cyclic code C^*. What is the minimum distance of C^*?

13. List all polynomials of the ideal $C =< 1 + x + x^2 + x^4 >$ in the ring $\mathrm{GF}(2)[x]/(x^5 + 1)$. Find the generator polynomial of C. Note that C can be generated by more than one polynomial as an ideal, but only one among them will be the generator polynomial.

14. Let the generator and check polynomials of a cyclic code be $g(x)$ and $h(x)$, respectively. Find the generator and check polynomials of the cyclic codes $< g(x) >^{\perp}$, $< h(x) >$, and $< h(x) >^{\perp}$.

15. (Q) C is a $(15, 11)$ binary cyclic code. The dual code $C^{\perp}$ does not contain any codewords of odd weight.

    (a) Show that C contains the vector [111111111111111].

    (b) Given that [001111111111000] belongs to C, find the generator polynomial $g(x)$ for C.

    (c) Find the minimum distance of C. Find a nonzero codeword of least weight.