Assignment on Reed-Muller Codes

EE512: Error Control Coding

Questions marked (Q) or (F) are questions from previous quizzes or final exams, respectively.

1. Determine the dimension and minimum distance for Reed-Muller (RM) codes of length 16, 32, 64 and 128. Indicate the dual of each code, and point out the self-dual codes.

2. Fill out the following table for Reed-Muller codes.

length n								
m	2	3	4	5	6	7	8	9
distance d	dimension							
1								
2								
4								
8								
16								
32								
64								
128								
256								
512								

3. Find generator and parity check matrices for RM$(2,4)$.

4. Let $C_1 = \text{RM}(1,3)$ and let C_2 be the (8,1) repetition code. Find the length, dimension and minimum distance of the code C obtained by the performing the $[u|u+v]$ construction with C_1 and C_2. Provide a generator matrix for C.

5. Let $f(v_1, v_2, \ldots, v_m)$ be an arbitrary Boolean function. Show that $v_m + f(v_1, v_2, \ldots, v_m)$ takes the values 0 and 1 equally often.

6. Show that

$$\text{RM}(r+1,m) = \{u+v : u \in \text{RM}(r,m), v = \mathbf{0} \text{ or a polynomial in } v_1, \ldots, v_m \text{ of degree exactly } r+1\}.$$

7. For a code of length n, show that the number of errors that can be corrected by 1-step majority-logic decoding is at most $\dfrac{n-1}{2(d'-1)}$, where d' is the minimum distance of the dual code.

8. For a code of length n, show that the number of errors that can be corrected by L-step majority-logic decoding is at most $\dfrac{n}{d'} - \dfrac{1}{2}$, where d' is the minimum distance of the dual code.

9. Prove the following: if each coordinate of a linear code has J parity checks orthogonal on it, then the minimum distance is at least $J+1$.

10. Devise majority-logic decoders for the (7,4) Hamming code and the (15,7) double-error-correcting BCH code.

11. Devise a decoder for RM(r,m) that can recover up to $2^{m-r} - 1$ erasures.