EE611 Problem Set 1

1. The following four waveforms are used for signaling in a digital communication system: ($rect(t) = 1$ for $0 \le t < 1$ and zero elsewhere)

$$\begin{aligned} s_0(t) &= rect(t) + rect(t-2) \\ s_1(t) &= rect(t-1) + rect(t-3) \\ s_2(t) &= rect(t-1) + rect(t-2) \\ s_3(t) &= rect(t-1) - rect(t-3) \end{aligned}$$

   (i) Determine an orthonormal basis and the corresponding constellation in two ways:

      (a) by using Gram-Schmidt Orthogonalisation starting with $s_0(t)$ and going in sequence, and

      (b) by inspection of the waveforms without any computations.

   (ii) Verify that one constellation can be obtained from the other simply by rotation.

   (iii) What is the distance of each point $\underline{s}_i$ from the origin? Is it the same in both constellations? Why? How do the distances between $\underline{s}_0$, $\underline{s}_1$, $\underline{s}_2$ and $\underline{s}_3$ compare in either case?

2. *(Wozencraft & Jacobs pp. 269-273:)* What is the minimum number of orthonormal basis functions required to represent the four signal waveforms in Figure 1?

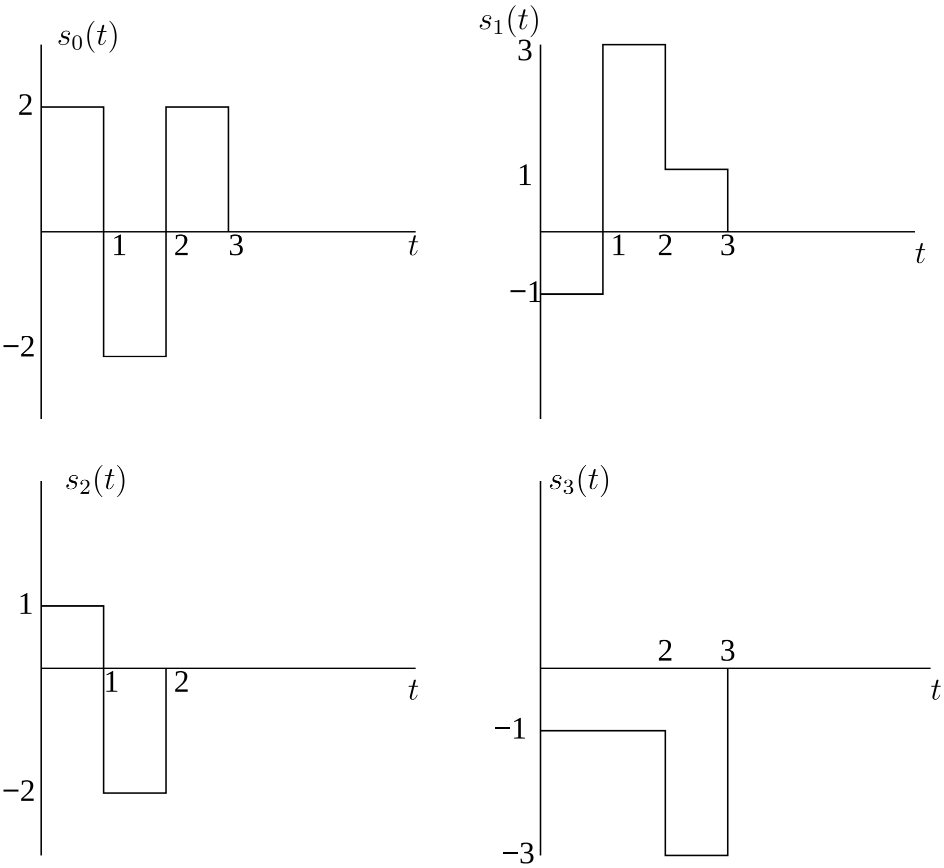

Figure 1:

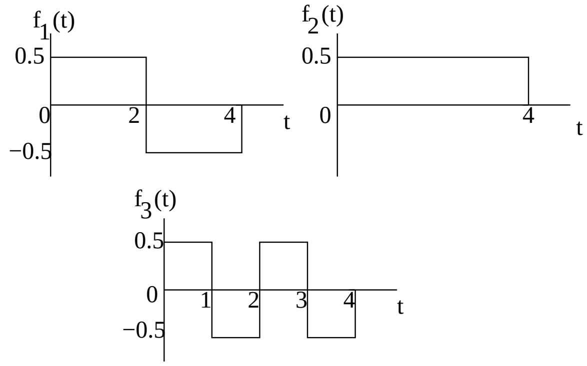

Figure 2:

3. Consider the three waveforms $f_n(t)$ shown in Figure 2.

   (a) Show that these waveforms are orthonormal.

   (b) Express the waveform $x(t)$ as a weighted linear combination of $f_n(t)$, $n = 1, 2, 3$, if

$$x(t) = \begin{cases} 3 & (1 \leq t < 2) \\ -2 & (2 \leq t < 3) \\ 1 & (3 \leq t < 4) \\ 0 & \text{else} \end{cases}$$

   and determine the weighting coefficients.

4. A binary communication system uses the two waveforms shown in Figure 3 for signalling. Sketch a signal constellation representation of the signals.

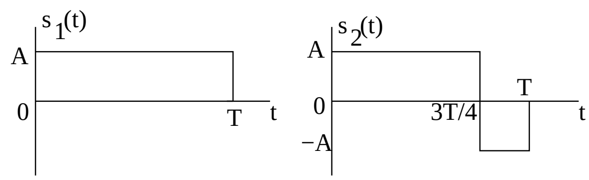

Figure 3:

5. Sketch the decision region for the optimal receiver that minimizes probability of error when the signal constellation in Figure 4 is transmitted over an AWGN channel. Assume that the symbols are equally likely.

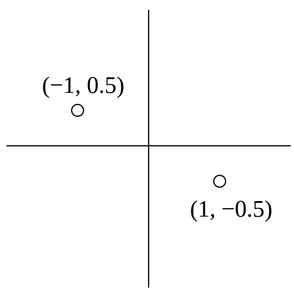

Figure 4: