Graduate admission test for EE4 - Control stream

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Table 1: Marks

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20

Instructions:

1. *Time duration* : **2 hours** *Max marks:* **75**
2. *Marks for each question along with the part marking is given at the right side of each question*
3. *Read the entire question paper to choose questions that you are most familiar with*
4. *Attempt maximum number of questions*

1. Consider the following sets of linear equations: **(2+2)**

$$Ax_1 = \begin{bmatrix}1\\0\\0\end{bmatrix},\ Ax_2 = \begin{bmatrix}0\\1\\0\end{bmatrix},\ Ax_3 = \begin{bmatrix}0\\0\\1\end{bmatrix}$$

   (a) On what conditions on A are the solutions x_1, x_2 and x_3 unique?

   (b) Let $X \in \mathbb{R}^{3\times 3}$ be a matrix whose columns are x_1, x_2 and x_3. What can you say about X with respect to A?

2. Check if the following statements are true or false. If true give reasons/proof and if false give a counter example. **(2+2)**

   (a) If A and B are invertible, then $A + B$ is also invertible.

   (b) Determinant of $(AB - BA)$ is 0

3. $A \in \mathbb{R}^{3\times 3}$ has 3 eigenvalues 0,1,2. **(1+1+2)**

   (a) What is the determinant of A?

   (b) What is the rank of A?

   (c) What are the eigenvalues of $(A+I)^{-1}$ and A^2?

4. A random variable X has a probability density function $f(x)$ as given below **(3)**

$$f(x) = \begin{cases} a + bx & 0 \le x \le 1 \\ 0 & \text{other wise} \end{cases}$$

   If the mean of X is $\frac{2}{3}$, then find the variance of X.

5. A function $f(x, y)$ is defined as $f(x, y) = x + y$. Find the maximum value of $f(x, y)$ such that $x^2 + y^2 = 1$. **(2)**

6. A function $f(x)$ is defined as $f(x) = -2x^3 + 3x^2$. Find the greatest value of $f(x)$ for $x \in (-\frac{1}{2}, 3)$. **(2)**

7. The Newton Raphson's iterative formula to find the roots of $f(x) = 0$ is given as $2x_{n+1} = \frac{(x_n^2+2)}{(x_n+1)}$. Find $f(x)$. **(2)**

8. Find the coefficient of $(x-2)^4$ in the Taylor series expansion of $f(x) = 3\log x - \log x^2$ about $x = 2$.

(2)

9. Consider the following second order systems with their specifications. For each system find the location of poles and state the nature of their step response. (overdamped, critically damped etc.)

(2+2)

(a) Transfer function : $\frac{10(s+7)}{(s+10)(s+20)}$

(b) Settling time: 7 seconds, Peak time: 2 seconds

10. Consider the circuit shown in the Figure 1. **(2+2+1)**

(a) Find the transfer function of the above system.

(b) Find a state space representation for the same.

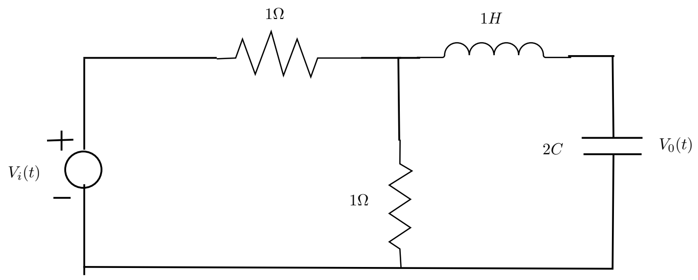

Figure 1: RLC circuit

(c) Is the state space representation unique?

11. Consider the following system: **(2+3)**

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & -1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

(a) Analyze the stability of the system

(b) Design a state feedback controller to stabilize the system

12. Linearize the following non-linear system around its equilibrium point (0,0) and analyze its stability.

(2+2)

$$\dot{x}_1 = x_2(1 - x_1^2)$$
$$\dot{x}_2 = -(x_1 + x_2)(1 - x_1^2)$$

13. Check if the following (see Figure 2) can be a root locus plot. Justify. **(2+2)**

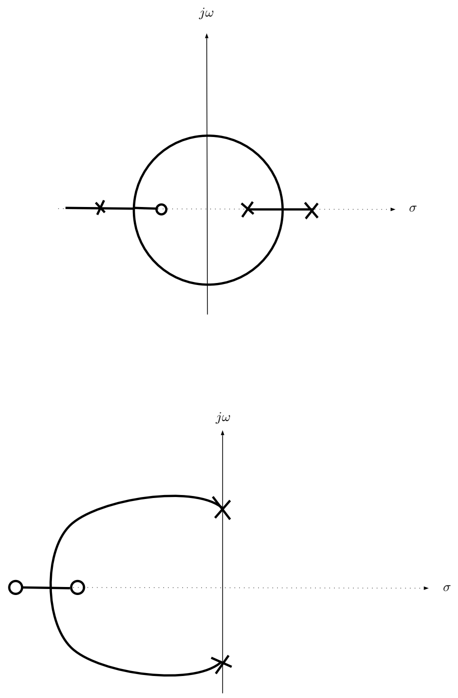

Figure 2: Root locus plot

14. Consider the following linear system in state space representation. **(1+1+1+1)**

$$\dot{x} = \begin{bmatrix} -1 & 0 \\ -1 & a \end{bmatrix} x + \begin{bmatrix} 1 \\ 2 \end{bmatrix} u$$
$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} x$$

Provide a value of a for the following cases:

(a) system is controllable
(b) system is not controllable
(c) system is observable
(d) system is not observable

15. Consider a minimum-phase (no zero in RHS plane) system whose asymptotic Bode Magnitude plot is shown in Figure 3. Determine the transfer function $G(s)$ of the system. **(4)**

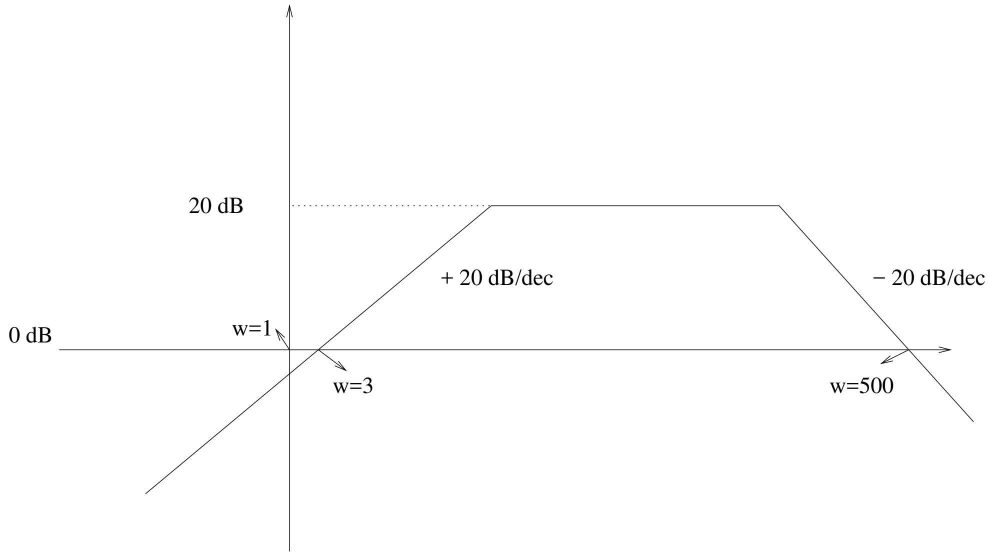

Figure 3: Bode magnitude plot

16. The signal flow graph of a system is shown below: **(3+1+1)**

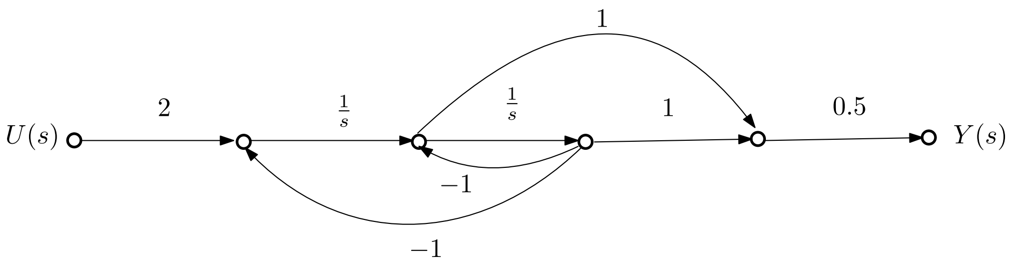

Figure 4: Signal flow graph

Provide the state variable representation of the system. Verify if the system is (a) controllable and (b) observable.

17. Solve the following differential equation: **(5)**

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + x = 5e^{-2t} + t$$

$$x(0) = 4, \frac{dx}{dt}(0) = 1$$

18. Match the unit-step responses, plotted in Figures 5, 6, 7 and 8 with their respective transfer-functions **(1+1+1+1)**

$$\begin{aligned} G_1(s) &= \frac{(50\pi)^2}{(s^2 + 2 \times 0.1 \times 50\pi s + (50\pi)^2)} \\ G_2(s) &= \frac{(50\pi)^2}{(s^2 + 2 \times 0.3 \times 50\pi s + (50\pi)^2)} \\ G_3(s) &= \frac{(50\pi)^2(s - 100)}{-100(s^2 + 100\pi s + (50\pi)^2)} \\ G_4(s) &= \frac{2 \times (50\pi)^2}{(s + (50\pi)^2)}. \end{aligned}$$

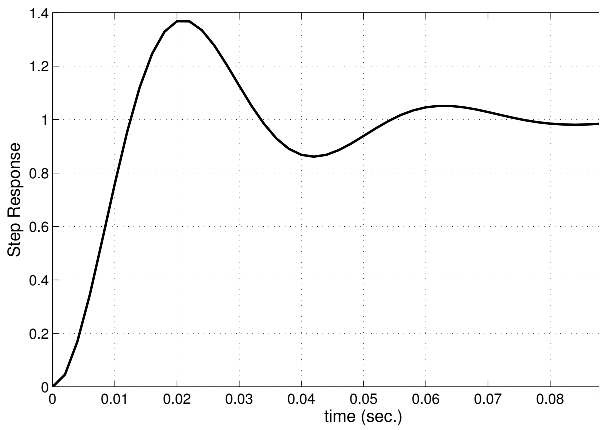

Figure 5: Unit-step response

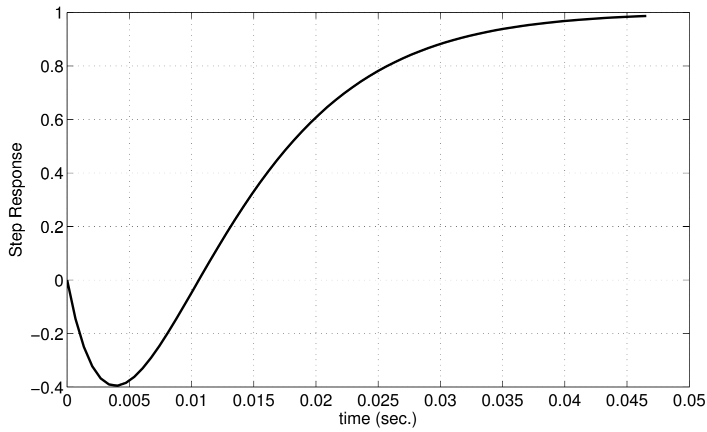

Figure 6: Unit-step response

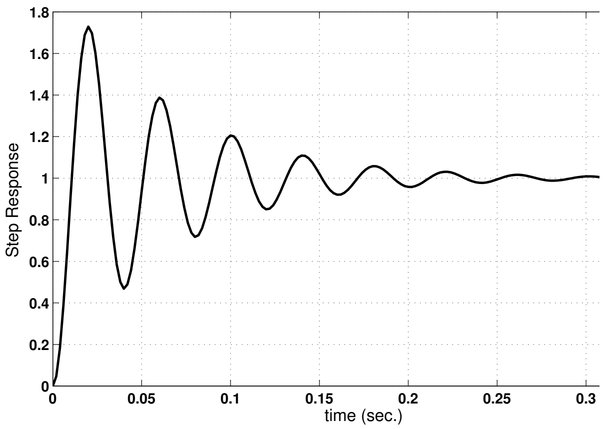

Figure 7: Unit-step response

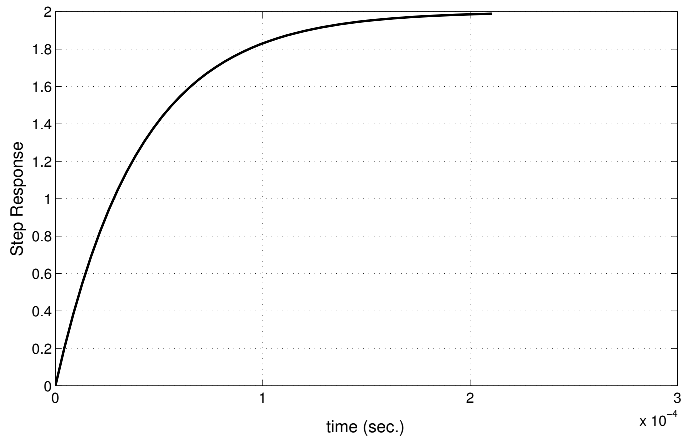

Figure 8: Unit-step response

19. Certain small scale ICs contain a single two-input gate. The ICs are manufactured in three varieties: NAND, OR and EXCLUSIVE-OR as indicated by a printed label on the IC's package. By mistake, a batch of all the three varieties is manufactured without their labels. Devise an efficient test that a technician can apply to any IC from this batch to determine which gate type it contains. **(4)**

20. It is proposed to determine if a point P given by (x,y) lies on a line L joining (x1, y1) and (x2, y2) by substituting the coordinates of P into the equation for L. Suppose all the coordinates are 8-bit two's complement numbers, the digital logic elements required for the task would be

    (a) 4 8-bit subtractors and 1 comparator.

    (b) 4 8-bit subtractors, 2 8-bit multipliers, 1 16-bit subtractor and 1 comparator.

    (c) 7 8-bit subtractors, 1 16-bit subtractor and 1 8-bit multiplier.

    (d) None of the above.

    Choose one of (a), (b), (c), (d) and establish your solution. **(4)**