CORDIC - Basic Algorithm and Enhancements

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The CORDIC algorithm provides an iterative method of performing vector rotations by arbitrary angles using only shifts and adds.

**Vector rotation transform:** For rotating in a Cartesian plane by angle $\phi$.

\[
\begin{align*}
    x' &= x \cos \phi - y \sin \phi \\
    y' &= y \cos \phi + x \sin \phi
\end{align*}
\]
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\begin{align*}
x' &= x \cos \phi - y \sin \phi \\
y' &= y \cos \phi + x \sin \phi
\end{align*}
\]

OR

\[
\begin{align*}
x' &= \cos \phi [x - y \tan \phi] \\
y' &= \cos \phi [y + x \tan \phi]
\end{align*}
\]
If rotation angles are selected such that \(\tan \phi = \pm 2^{-i}\), then

\[
X_{i+1} = K_i (X_i - Y_i \cdot d_i \cdot 2^{-i})
\]
\[
Y_{i+1} = K_i (Y_i + X_i \cdot d_i \cdot 2^{-i})
\]
\[
Z_{i+1} = Z_i - d_i \cdot \tan^{-1}(2^{-i})
\]

where

\[
K_i = \cos(\tan^{-1}(2^{-i})) = \frac{1}{\sqrt{1 + 2^{-2i}}}
\]  \hspace{1cm} (1)

The scale factor \(K_i\) can be accumulated and the vector is scaled by

\[
A_n = \prod_{n} \sqrt{1 + 2^{-2i}}
\]
Polar \((R, \theta)\) to Rectangular \((X, Y)\) transformation:

\[
\begin{align*}
+X \quad +Y \\
\theta = 65^\circ
\end{align*}
\]

Rotation by 65°
Polar \((R, \theta)\) to Rectangular \((X,Y)\) transformation:

Initialize

\[
\begin{align*}
X_0 &= R \\
Y_0 &= 0 \\
Z_0 &= \theta
\end{align*}
\]
Polar \((R, \theta)\) to Rectangular \((X,Y)\) transformation:

Rotate by \(\tan^{-1}(2^0) = 45^\circ\)

\[
\begin{align*}
X_1 &= X_0 - Y_0 \\
Y_1 &= Y_0 + X_0 \\
Z_1 &= Z_0 - \tan^{-1}(2^0)
\end{align*}
\]
Polar \((R, \theta)\) to Rectangular \((X, Y)\) transformation:

Rotate by
\[
\tan^{-1}(2^{-1}) = 26.5^\circ
\]

\[
X_2 = X_1 - \frac{Y_1}{2}
\]
\[
Y_2 = Y_1 + \frac{X_1}{2}
\]
\[
Z_2 = Z_1 - \tan^{-1}(2^{-1})
\]
Polar \((R, \theta)\) to Rectangular \((X,Y)\) transformation:

Rotate by \(\tan^{-1}(2^{-2}) = 14^\circ\)

\[
X_3 = X_2 + \frac{Y_2}{4}
\]

\[
Y_3 = Y_2 - \frac{X_2}{4}
\]

\[
Z_3 = Z_2 + \tan^{-1}(2^{-2})
\]
Basics of CORDIC

Goal
Enhancement
References

Example
Conventional CORDIC architecture

Diagram of CORDIC architecture

- **MUX**
- **Register**
- **Barrel Shifter**
- **Adder/Subtractor**
- **Scalar**
- **ROM**
- **Counter**
- **Comparator**
- **Subtractor**
- **Mode Select**

Flow of operations:
1. **Input**: \(X_0, X_0[MSB]\)
2. **Mode Selection**
3. **Operation**: 
   - **Basic CORDIC**
   - **Counter**
   - **ROM**
4. **Output**: \(X_{out}\), \(Y_{out}\), \(Z_{out}\)

Key Components:
- **MUX** for selection
- **Register** for storage
- **Barrel Shifter** for shifting
- **Adder/Subtractor** for arithmetic operations
- **Scalar** for scaling
- **ROM** for storing lookup values
- **Counter** for iterative processing
- **Comparator** for decision-making
Reduce area consumption without affecting the performance in terms of **accuracy** and **number of iterations**.
- ROM: The size of the ROM is $2^\lceil \log_2(\text{no. of iterations}) \rceil$.
- Barrel shifters.
- Range is limited to $|Z| \leq 99^\circ$. Multiplexers (both at input and output) are required to extend the range.
1. Completely eliminates barrel-shifters.

2. Represents all the angles in $[-180^\circ, 180^\circ]$ using combinations of two signed elementary angles, $\tan^{-1}2^{-1}$ and $\tan^{-1}2^{-3}$.

OR

\[
Z = k_0 \cdot \tan^{-1}(2^{-1}) + k_1 \tan^{-1}(2^{-3})
\]
Either

\[ X = K_1 \cdot (X - (-1)^{sgn(k_0)} \cdot Y \cdot 2^{-1}) \]
\[ Y = K_1 \cdot (Y + (-1)^{sgn(k_0)} \cdot X \cdot 2^{-1}) \]

Or

\[ X = X - (-1)^{sgn(k_1)} \cdot Y \cdot 2^{-3} \]
\[ Y = Y + (-1)^{sgn(k_1)} \cdot X \cdot 2^{-3} \]
Either

\[ X = K_1 \cdot (X - (-1)^{sgn(k_0)} \cdot Y \cdot 2^{-1}) \]
\[ Y = K_1 \cdot (Y + (-1)^{sgn(k_0)} \cdot X \cdot 2^{-1}) \]

Or

\[ X = X - (-1)^{sgn(k_1)} \cdot Y \cdot 2^{-3} \]
\[ Y = Y + (-1)^{sgn(k_1)} \cdot X \cdot 2^{-3} \]
max(|k_0| + |k_1|) = 13

for

170° = 4 \cdot \tan^{-1}(2^{-1}) + 9 \cdot \tan^{-1}(2^{-3})

and

177° = 8 \cdot \tan^{-1}(2^{-1}) - 5 \cdot \tan^{-1}(2^{-3})

Found using C program.
\[
\text{max}(|k_0| + |k_1|) = 13
\]

for
\[
170^\circ = 4 \cdot \tan^{-1}(2^{-1}) + 9 \cdot \tan^{-1}(2^{-3})
\]

and
\[
177^\circ = 8 \cdot \tan^{-1}(2^{-1}) - 5 \cdot \tan^{-1}(2^{-3})
\]

Found using C program.
Basics of CORDIC 

Goal 

Enhancement 

References 

Features 

Iterative equations 

Convergence 

Architecture 

1

MUX 

0

oscaler 

360 X 10

ROM 

1

MUX 

0

1

MUX 

0oscaler 

state 

machine 

sub 

1

MUX 

0

sel 

sel 

sel 

+

+/−

sub 

+

+/−

accumulator 

accumulator 

10

sel

sel

4 4

m stop−bit 

4 0389 5

Z_{in} 

\text{sgn}(k_0) 

\text{sgn}(k_1) 

X 

\frac{X}{2} 

\frac{X}{8} 

\frac{Y}{2} 

Y 

Y_i 

1

MUX 

0 

M_0 

\text{MUX} 

1

0

\text{MUX} 

1

M_1 

\text{MUX} 

1

M_2 

\text{MUX} 

1

M_3 

\text{MUX} 

1

M_4 

\text{sgn}(k_1) 

|k_1| 

\text{sgn}(k_0) 

|k_0|

