Special Manpower Development Program in VLSI

IEP : RF Integrated Circuits

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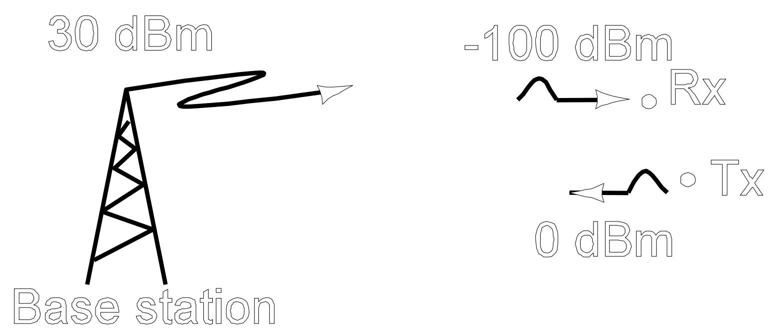
Introduction to RFICs

- RF problem –
  - Received signal is *very small*, in presence of interferers



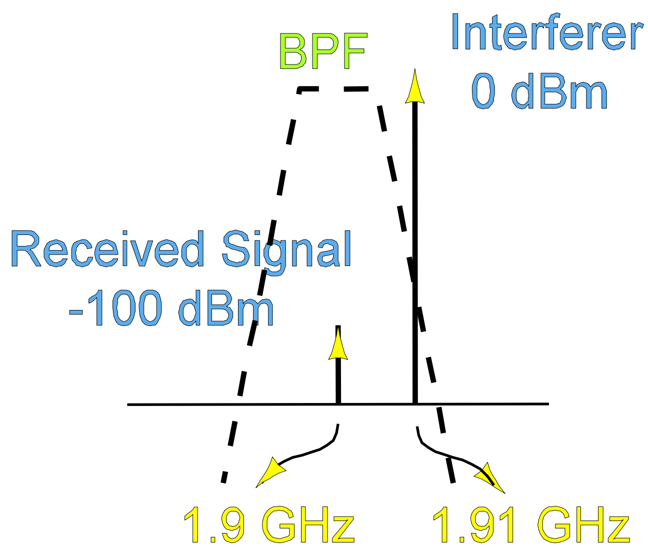
- Power measured in '**dBm**'

dB → log scale, since the range of power involved is huge.
m → relative to 1 mW

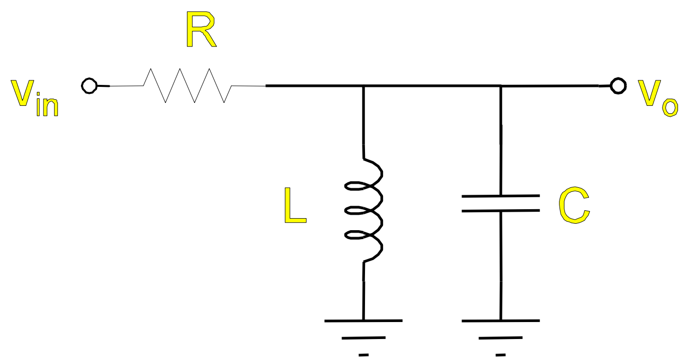
$$\text{Power in dBm} = 10\log_{10}\left(\frac{P}{1\,\text{mW}}\right)$$

0 dBm = 1 mW; -100 dBm = 0.1 pW !!!!

Simple RF receiver



Use a filter to knock off the interferer



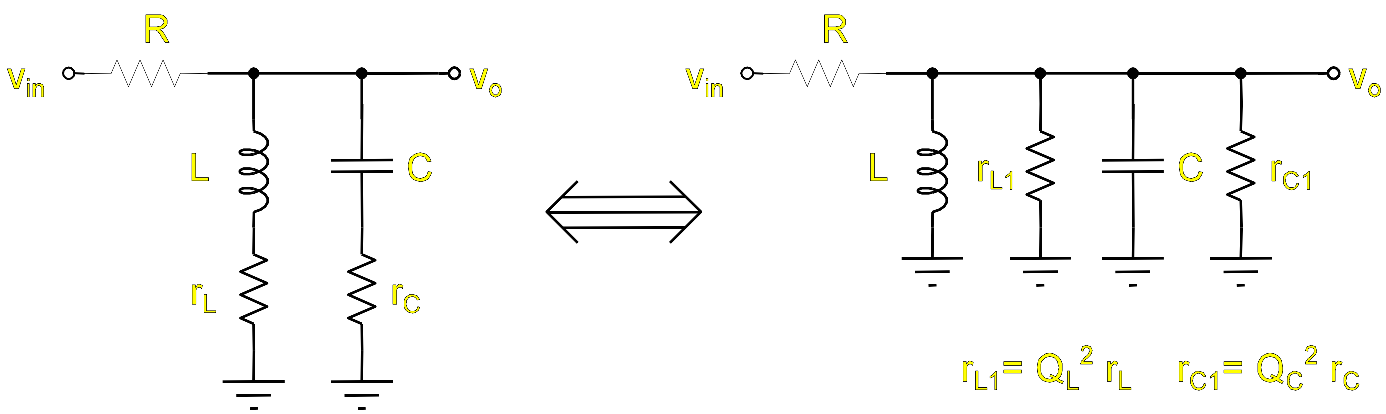
$$\omega_o = \frac{1}{\sqrt{LC}}; \quad Q = \frac{R}{\sqrt{L/C}}$$

- What will be the Q required to reject the interferer ?

  The Q required to get 3 dB rejection at 1.91 GHz (10 MHz from 1.9 GHz)

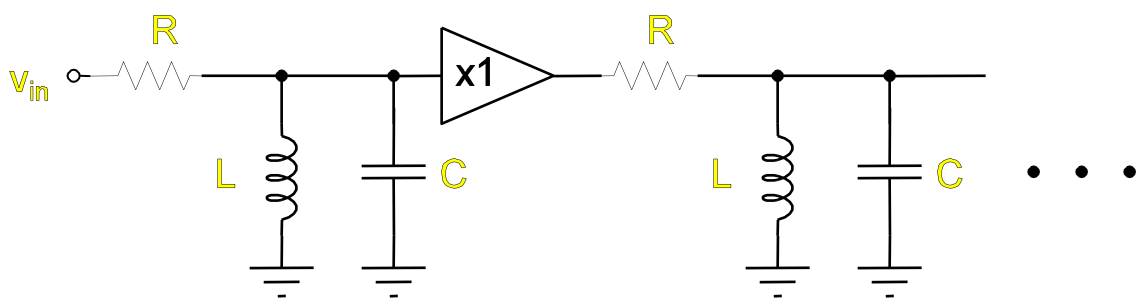
  = f_o/BW = 1.9 GHz / 20 MHz = 85

- The Q of the filter is limited by the Q of passive components



$$\frac{1}{Q_{eq}} = \frac{1}{R} + \frac{1}{Q_L} + \frac{1}{Q_C}$$

- Very High Q passive components are impossible to realize !
- Cascading the filters, another way to realize sharp filters

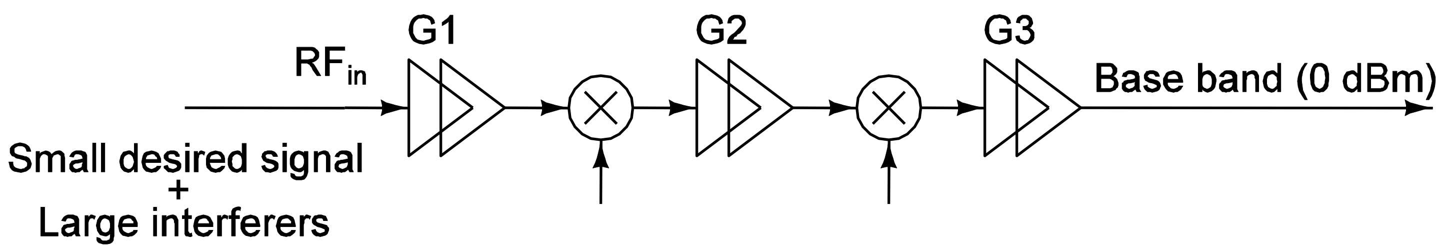


Limitations:

- Precise matching of L, C are required
  - Else, results in broad bandwidth
- Shift in all L's or C's => picking up interferer but not the desired signal
- High gain (of the order of 10^5) is impractical at RF.
  - Parasitic coupling between i/p and o/p of the amplifier leads to stability problem

- Therefore the gain is distributed at several frequencies.
  - breaks the loop due to parasitic coupling

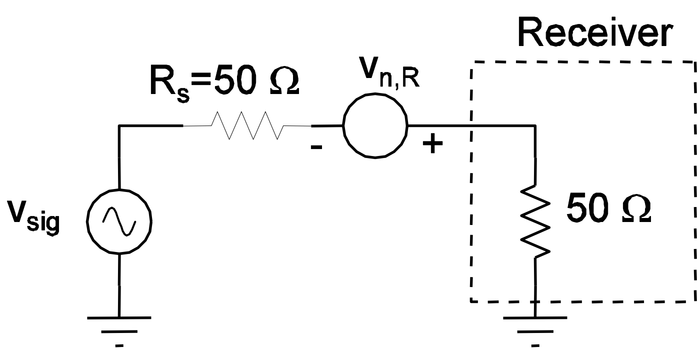
Example :



Issues in RF Receivers

1. Noise
   - Frequency translation and amplification needs devices (both active and passive)
   - These devices add noise
   - Even with zero input signal, output has noise (due to internal noise sources

Consider the model of an antenna,



v_{sig} : signal received by the antenna,

R_s : Impedance of the antenna

$V_{n,R}$: Equivalent voltage noise of the antenna

Signal power at the input of the receiver = $\dfrac{\overline{v_{sig}^2}}{4\,R_S}$ watts

Avg. noise power dissipated at the input of the receiver = $\dfrac{\overline{v_{n,R}^2}}{4\,R_S} = \dfrac{(4KTR_S)B}{4R_S}$ watts

$4KTR_S$: Voltage noise power spectral density of R_S

B : Band width

Noise power at the input (in dBm) = 10 *log* ($4KTR_S$) + 10 *log* (B)

= -174 dBm/Hz + 10 *log* (B)

For a channel with bandwidth of 200 kHz, the noise power = -121 dBm

If the received signal power = -100 dBm, then

the SNR @ input = 21 dBm

=> Receiver output SNR cannot be greater than 21 dBm (for this case)

- ***Sensitivity*** :

  Minimum i/p signal power required for a specified SNR @ o/p.

  Sensitivity > SNR_{out} (in dB) + 10 *log* (B) – 174 dBm

- ***Noise factor*** :

$$\text{Noise Factor} = \frac{\text{SNR @ i/p}}{\text{SNR @ o/p}}$$

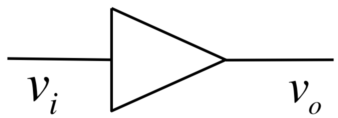
  Noise Figure (NF) = 10 *log* (Noise Factor)

  - Noise factor is the measure of SNR degradation when the signal passes through the system.
  - For a noise less receiver, SNR @ i/p = SNR @ o/p

    i.e. Noise factor = 1
  - For a noisy receiver Noise factor > 1

2. Large signal issues

- Amplifiers are non-linear in reality.



$$v_o = a_o + a_1 v_i + a_2 v_i^2 + a_3 v_i^3 + \cdots$$

$$(A\cos\omega_1 t + A\cos\omega_2 t)$$

$$v_o = (a_o + a_2 A^2) + (a_1 A + \frac{9}{4} a_3 A^3)(\cos\omega_1 t + \cos\omega_2 t)$$

$$+ \frac{a_2 A^2}{2}(\cos 2\omega_1 t + \cos 2\omega_2 t) + \frac{a_3 A^3}{4}(\cos 3\omega_1 t + \cos 3\omega_2 t)$$

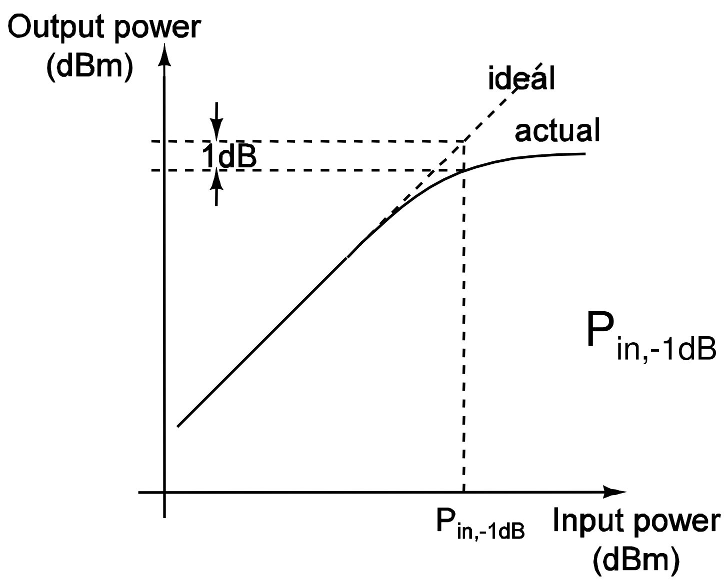
$$+ a_2 A^2[\cos(\omega_1 - \omega_2)t + \cos(\omega_1 + \omega_2)t]$$

$$+ \frac{3}{4} a_3 A^3[\cos(2\omega_1 - \omega_2)t + \cos(2\omega_2 - \omega_1)t]$$

$$+ \frac{3}{4} a_3 A^3[\cos(2\omega_1 + \omega_2)t + \cos(2\omega_2 + \omega_1)t]$$

- Amplitude of the fundamental component at the output of the amplifier is, $(a_1 A + \frac{9}{4} a_3 A^3)$

- Typically a_3 is negative
- Therefore, amplitude of the fundamental at the output decreases with the increasing input signal strength

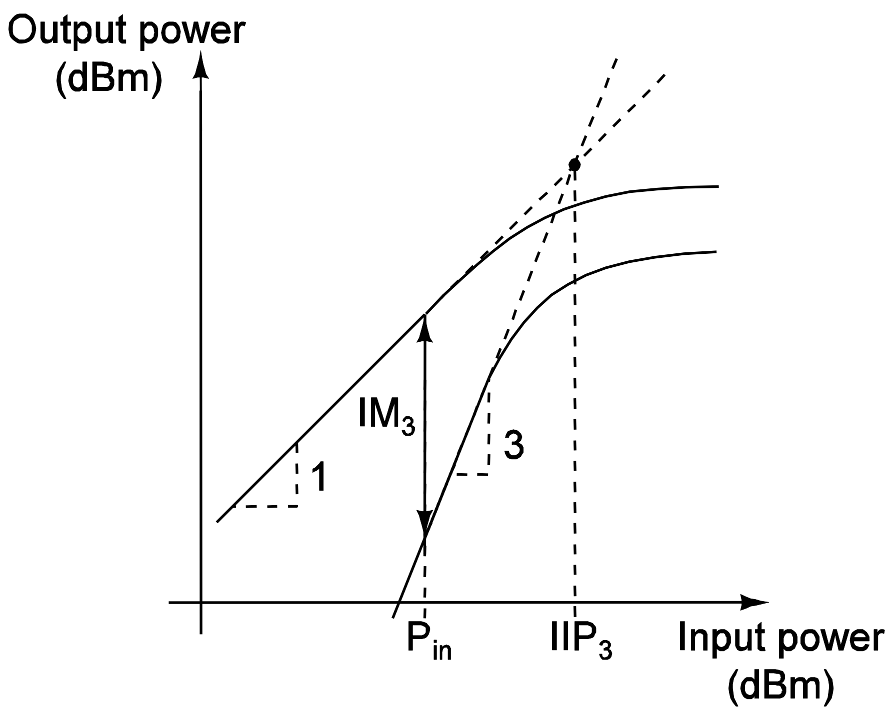


$P_{in,-1dB}$: 1 dB compression point

Intermodulation

- Output components of frequency $(2\omega_1 - \omega_2)$ and $(2\omega_2 - \omega_1)$ are third order intermodulation terms.

- Third order intermodulation = $IM_3 = \dfrac{\frac{3}{4} a_3 A^3}{a_1 A} = \dfrac{3}{4}\dfrac{a_3 A^2}{a_1}$



IIP_3 : Third order Input Intercept Point

$(IIP_3 - P_{in})\,3 = IM_3 + (IIP3 - Pin)$

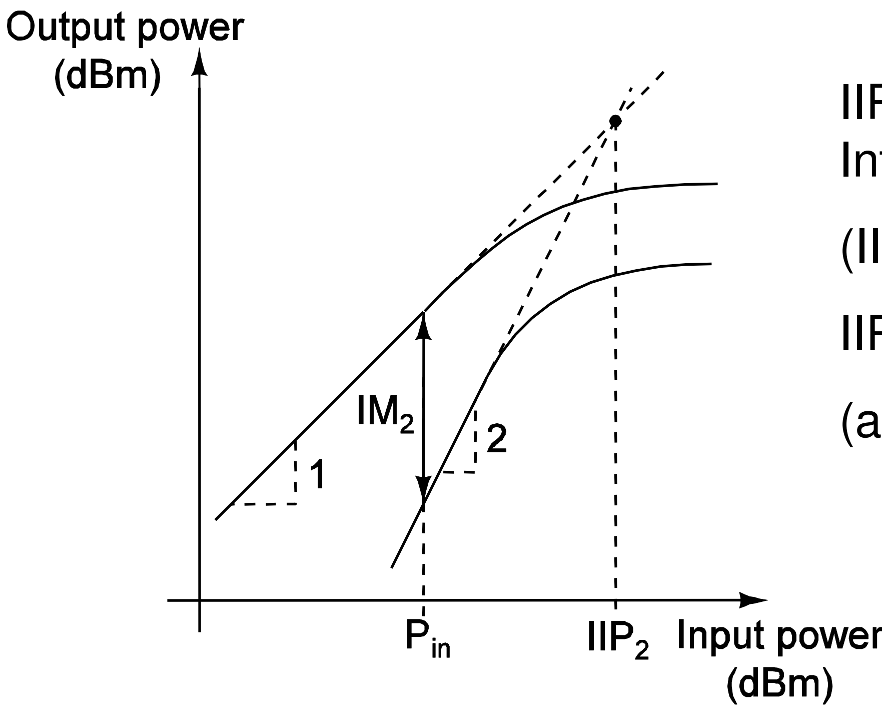
$IIP_3 = P_{in} + IM_3/2$

(all quantities in dB)

Intermodulation

- Output components of frequency $(\omega_1 - \omega_2)$ and $(\omega_1 + \omega_2)$ are second order intermodulation terms.

- Second order intermodulation = $IM_2 = \dfrac{a_2 A^2}{a_1 A} = \dfrac{a_2 A}{a_1}$



IIP_2 : Second order Input Intercept Point

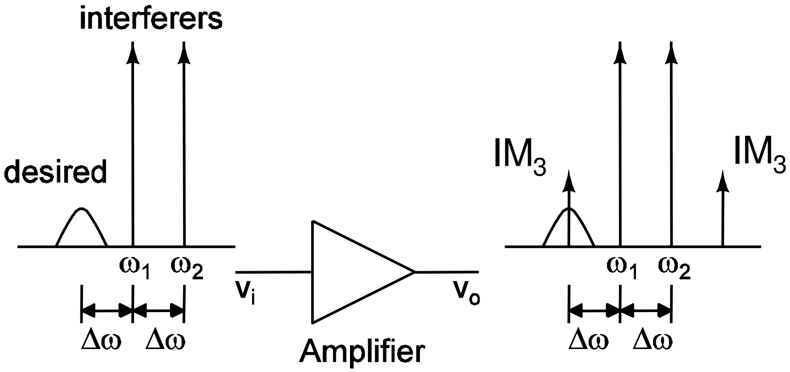
$(IIP_3 - P_{in})\ 3 = IM_3 + (IIP3 - Pin)$

$IIP_3 = P_{in} + IM_3/2$

(all quantities in dB)

Problems with large signals:

Intermodulation-



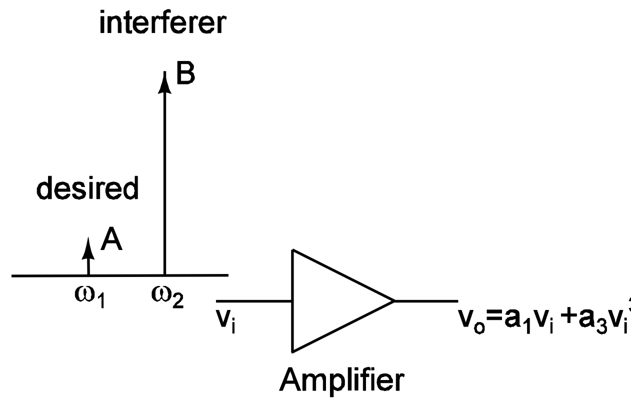
Blocking-

Amplitude of o/p fundamental

$$=(a_1 A+\frac{3}{2}a_3 AB^2)$$

Gain for the fundamental

$$=(a_1+\frac{3}{2}a_3 B^2)$$

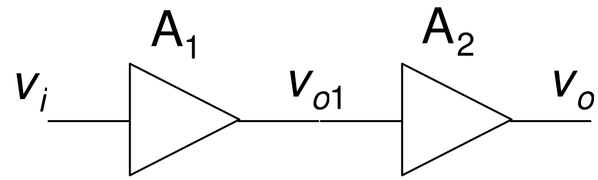


- Gain of desired channel reduced in the presence of large interferer (a_3 is typically negative)
- If the interferer is modulated, 'B' changes with time => *Cross Modulation*

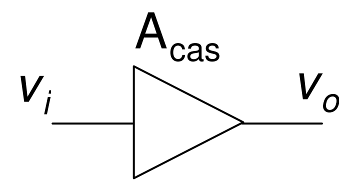
Distortion in cascaded amplifiers

Let the non-linearity of the amplifiers A_1 and A_2 be given as

$$v_{o1} = a_1 v_i + a_3 v_i^{\,3} \;\; ; \;\; v_o = b_1 v_{o1} + b_3 v_{o1}^{\,3}$$



$$\Rightarrow \quad v_o \approx (a_1 b_1) v_i + (b_1 a_3 + b_3 a_1^{\,3}) v_{o1}^{\,3}$$



For the amplifier A_1, $$A_{IIP3,1} = \sqrt{\frac{4}{3}\frac{a_1}{a_3}}$$

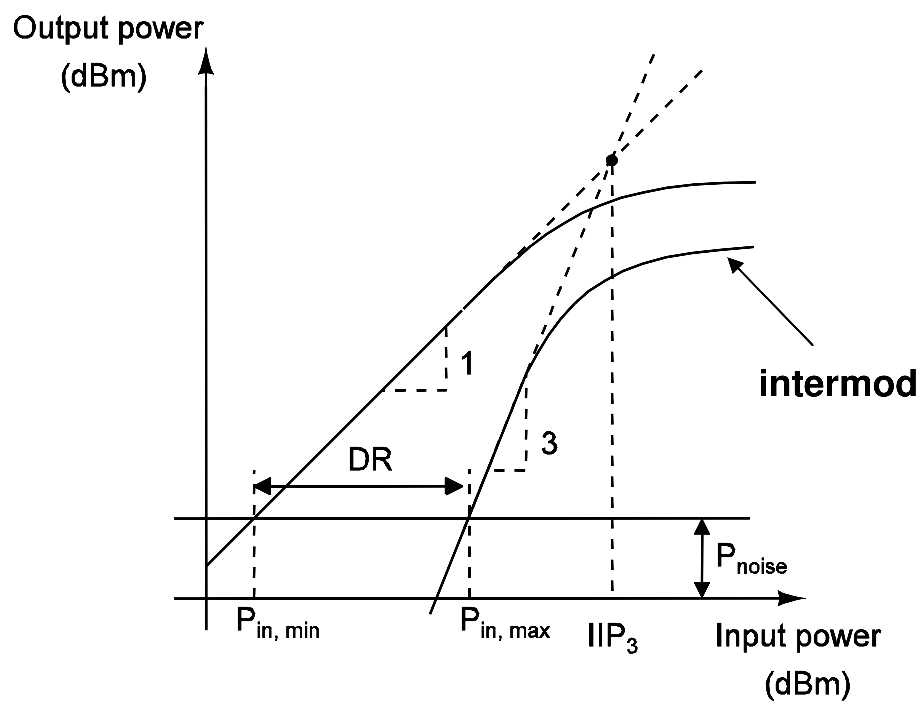
For the amplifier A_2, $$A_{IIP3,2} = \sqrt{\frac{4}{3}\frac{b_1}{b_3}}$$

Then for the equivalent amplifier A_{cas}, $$A_{IIP3,cas} = \sqrt{\frac{4}{3}\frac{b_1 a_1}{b_1 a_3 + b_3 a_1^{\,3}}}$$

Re-arranging the above equation, it can be shown that,

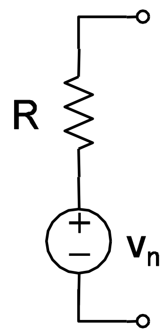
$$\frac{1}{A^2_{IIP3,cas}} = \frac{1}{A^2_{IIP3,1}} + \frac{a_1^{\,2}}{A^2_{IIP3,2}}$$

Dynamic Range (DR)-



Noise in Electronic Circuits

1. Resistor



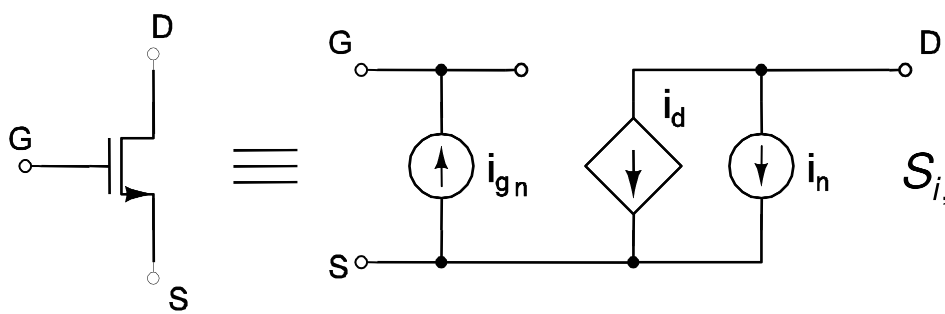
v_n : Equivalent noise voltage of resistor R. It is assumed to be Gaussian white.

$\overline{v_n} = 0$; $\overline{v_n^2} = 4KTR\Delta f$ assuming the resistor is in equilibrium with the surrounding

Noise power spectral density = $S_v(f) = 4KTR \ \ \mathrm{V}^2/\mathrm{Hz}$

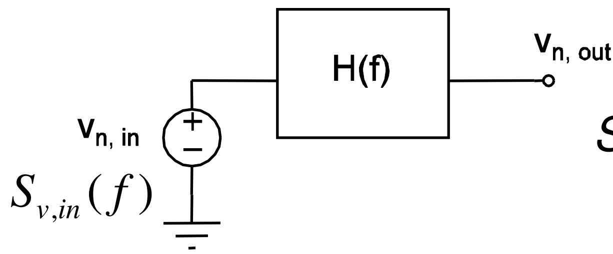
2. L & C : No noise

3. MOSFET



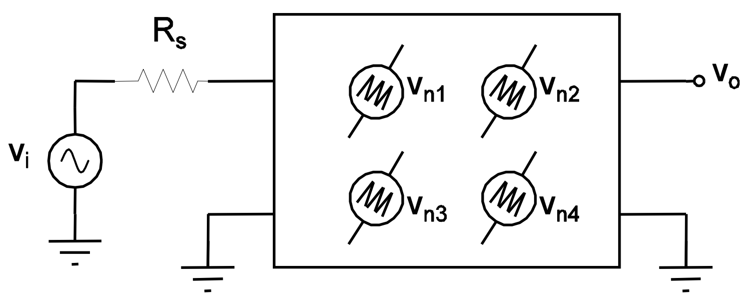
$$S_{i,n}(f) = \frac{8}{3} KT\eta g_m \ \ \mathrm{V}^2/\mathrm{Hz}$$

4. Noise through a linear filter



$$S_{v,out}(f) = S_{v,in}(f)\left|H(f)\right|^2$$

5. Noise in networks



Output noise voltage =

$$v_{n,o} = v_{n1}H_1(f) + v_{n2}H_2(f) + v_{n3}H_3(f) + \cdots$$

where, $H_i\ (f)$ is the transfer function from i^{th} noise source to the output

Output noise power spectral density =

$$S_{v,o}(f) = S_{v,n1}(f)\left|H_1(f)\right|^2 + S_{v,n2}(f)\left|H_2(f)\right|^2 + S_{v,n3}(f)\left|H_3(f)\right|^2 + \cdots$$

The transfer function H_i *(f)* can be written in terms of R_s as $H_i(f) = \frac{A_i R_s + B_i}{C R_s + D}$

$$\therefore \quad v_{n,o} = \sum_{k=1}^{N} v_{nk} \frac{A_k R_s + B_i}{C R_s + D}$$

If $v_{n,in\text{-}eq}$ is the equivalent input referred noise, then $v_{n,o} = v_{n,in-eq} \frac{A_{eq} R_s + B_{eq}}{C R_s + D}$

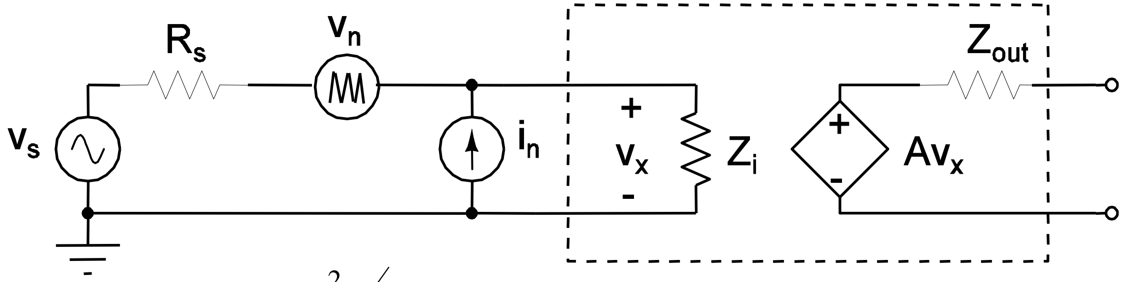
As $R_s \to \infty, v_{n,o} \to 0$ and also $v_{n,o} = v_{n,in-eq} \frac{A_{eq}}{C} \Rightarrow A_{eq} = 0$

$$\therefore \quad v_{n,o} = v_{n,in-eq} \frac{B_{eq}}{C R_s + D} = \sum_{k=1}^{N} v_{nk} \frac{A_k R_s + B_i}{C R_s + D}$$

$$\Rightarrow \qquad v_{n,in-eq} = R_s \left(\sum_{k=1}^{N} v_{nk} \frac{A_k}{B_{eq}} \right) + \left(\sum_{k=1}^{N} v_{nk} \frac{B_k}{B_{eq}} \right)$$

$$v_{n,in-eq} = R_s I_n + V_n$$

Noise factor of amplifier



Available SNR @ i/p $= \dfrac{v_s^2 / 4R_s}{4KTR_s / 4R_s} = \dfrac{v_s^2}{4KTR_s}$ taking 1 Hz bandwidth

Available signal power at the output $= v_s^2 \left| \dfrac{Z_i A}{Z_i + R_s} \right|^2 \dfrac{1}{4\Re e[Z_{out}]}$

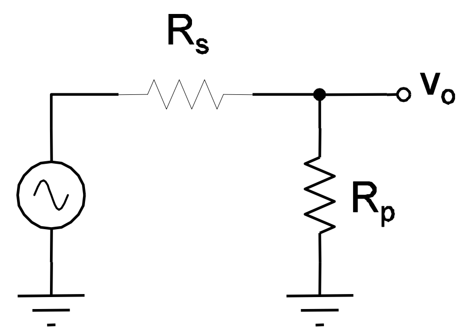
Available noise power at the output $= \left[4KTR_s + \overline{\left(v_n + i_n R_s\right)^2} \right] \left| \dfrac{Z_i A}{Z_i + R_s} \right|^2 \dfrac{1}{4\Re e[Z_{out}]}$

Available SNR @ o/p $= \dfrac{v_s^2}{4KTR_s + \overline{\left(v_n + i_n R_s\right)^2}}$

$$\text{Noise Factor} = \frac{\text{Available SNR @ i/p}}{\text{Available SNR @ o/p}} = 1 + \frac{\overline{\left(v_n + i_n R_s\right)^2}}{4KTR_s}$$

$$\overline{(v_n + i_n R_s)^2} = \overline{v_n^{\,2}} + R_s^{\,2}\overline{i_n^{\,2}} + 2R_s\overline{v_n i_n}$$

Noise factor of a simple potential divider



Taking 1 Hz bandwidth,

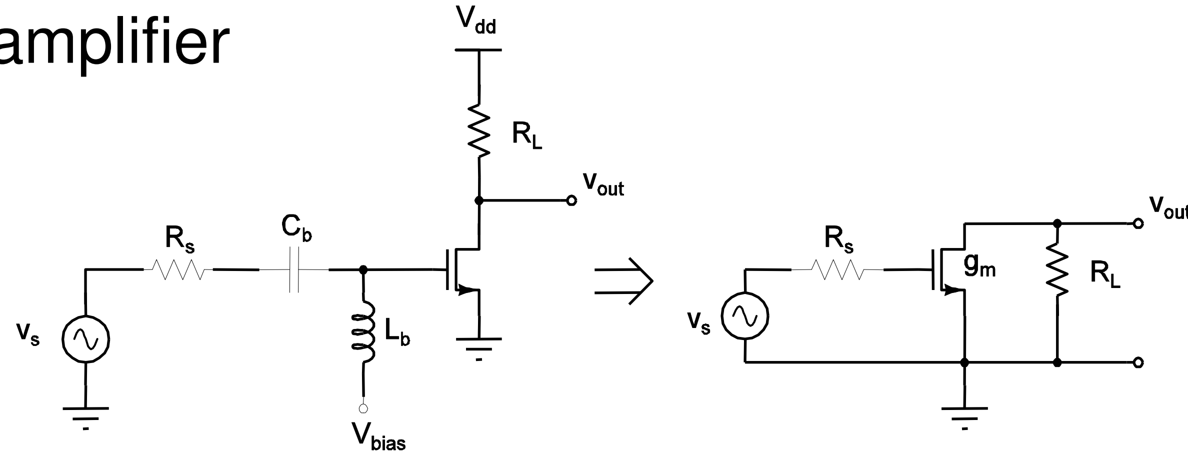
Available SNR @ i/p $= \dfrac{v_s^{\,2}/4R_s}{4KTR_s/4R_s} = \dfrac{v_s^{\,2}}{4KTR_s}$

Available SNR @ o/p $= \dfrac{v_s^{\,2}\left(\dfrac{R_p}{R_p+R_s}\right)^2}{4KT\dfrac{R_sR_p}{R_p+R_s}} = \dfrac{v_s^{\,2}}{4KT}\dfrac{R_p}{R_p+R_s}$

$$\text{Noise Factor} = \frac{\text{Available SNR @ i/p}}{\text{Available SNR @ o/p}} = 1 + \frac{R_s}{R_p}$$

For best noise figure, $Rp = 0$; for Maximum power transfer, $Rp = Rs$

Noise factor of a CS amplifier



Available SNR @ i/p $= \dfrac{v_s^2}{4KTR_s}$

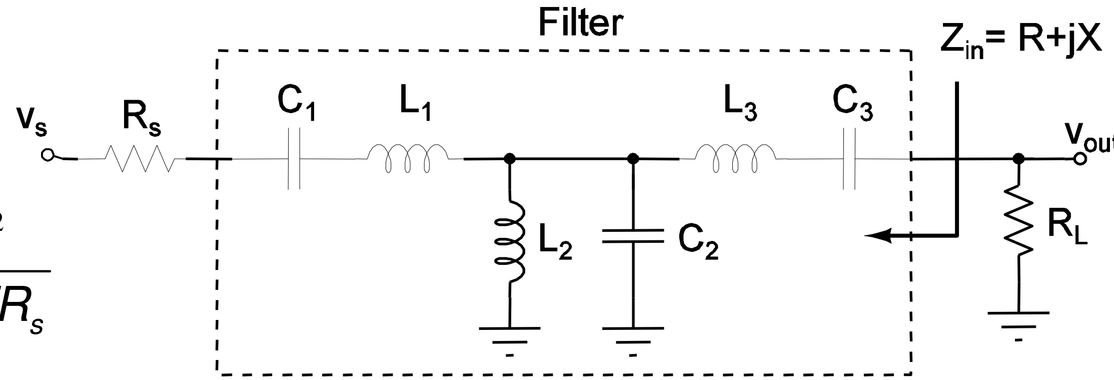
Available signal power at the output $= \dfrac{v_s^2 (g_m R_L)^2}{4R_L}$

Available noise power at the output $= \left[4KTR_s (g_m R_L)^2 + \dfrac{8}{3} KT g_m R_L^2 + 4KTR_L\right] \dfrac{1}{4R_L}$

Available SNR @ o/p $= \dfrac{v_s^2}{4KTR_s + \dfrac{8}{3}\dfrac{KT}{g_m} + 4KT\dfrac{1}{g_m^2 R_L^2}}$

$$\text{Noise Factor} = \frac{\text{Available SNR @ i/p}}{\text{Available SNR @ o/p}} = 1 + \frac{2}{3}\frac{1}{g_m R_s} + \frac{1}{g_m^2 R_s R_L}$$

Noise factor of filters



Available SNR @ i/p $= \dfrac{v_s^2}{4KTR_s}$

Available SNR @ o/p $= \dfrac{v_{out,th}^2}{4KT\Re e[Z_{in}]}$

For a <u>*loss less*</u> filter, at f = $f_{c,}$ $v_{out,th} = v_s$ and $Re[Z_{in}] = R_s$

$\Rightarrow$ Available SNR @ o/p $= \dfrac{v_s^2}{4KTR_s}$

$\Rightarrow$ Noise factor = 1

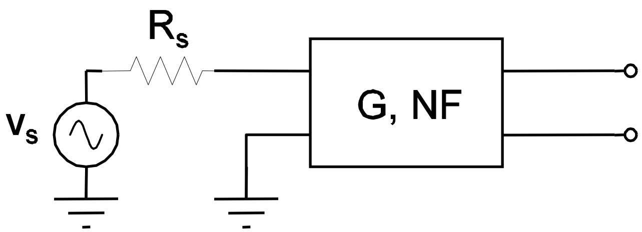
For a <u>*lossy*</u> filter,

Available SNR @ o/p $= \dfrac{v_s^2 G}{4KTR_s}$ G: the power gain of the filter

Therefore, Noise factor = 1/G = L L: Loss

Noise factor of cascaded amplifiers

- Consider an amplifier having power gain ‘G’ and noise factor ‘NF’



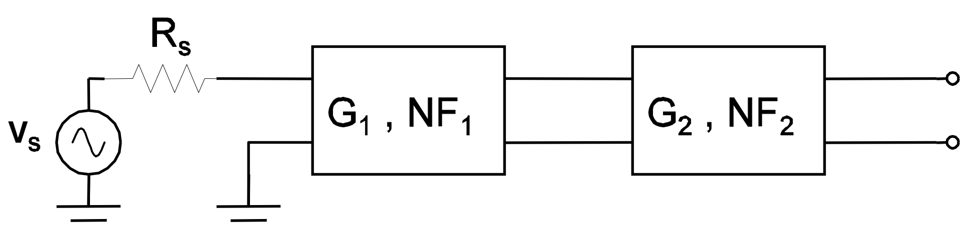
$$NF = \frac{\dfrac{\overline{v_s^2}}{4KTR_s}}{\dfrac{\overline{v_s^2}G/4R_s}{\text{Available noise power @ o/p}}}$$

$\therefore$ Available noise power @ o/p $= NF\ KTG$

But, KTG = Available noise power @ o/p due to R_s alone

$\therefore$ $KTG\,(NF - 1)$ = Available noise power @ o/p due to the amplifier alone

- Now, consider two amplifiers connected in cascade



Available signal power @ o/p $= \dfrac{\overline{v_s^2} G_1 G_2}{4R_s}$

Available noise power @ o/p $= KT(G_1 G_2) \; + \; [KTG_1(NF_1 - 1)]\, G_2 \; + \; KTG_2(NF_2 - 1)$

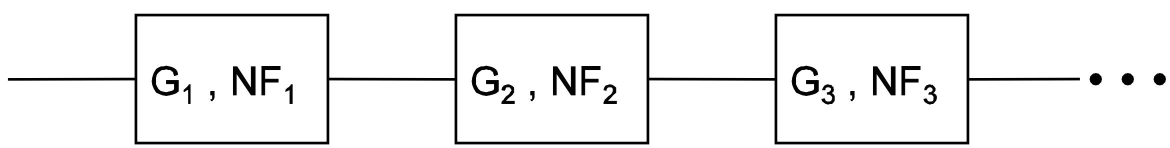
Available SNR @ i/p $= \dfrac{\overline{v_s^2}}{4KTR_s}$

Available SNR @ o/p $= \dfrac{\dfrac{\overline{v_s^2} G_1 G_2}{4KTR_s}}{KT(G_1 G_2) \; + \; [KTG_1(NF_1 - 1)]\, G_2 \; + \; KTG_2(NF_2 - 1)}$

Therefore, the total Noise factor (NF) can be written as,

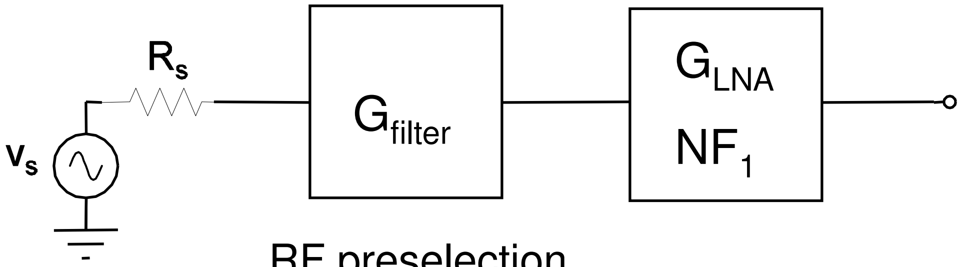
$$NF_{total} = NF_1 + \frac{(NF_2 - 1)}{G_1}$$

Generalizing the above result for any number of cascade,



$$\boxed{NF_{total} = NF_1 + \frac{(NF_2 - 1)}{G_1} + \frac{(NF_3 - 1)}{G_1 G_2} + \cdots}$$ ***Friis Formula***

Example



RF preselection filter

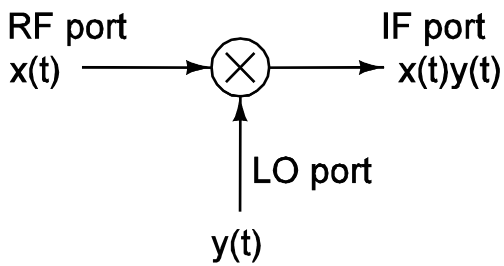
$$NF_{total} = NF_{filter} + \frac{(NF_1 - 1)}{G_{filter}}$$

$$= \frac{1}{G_{filter}} + \frac{(NF_1 - 1)}{G_{filter}}$$

$$= \frac{NF_1}{G_{filter}} = NF_1 \, L_{filter}$$

In ‘dB’ the Noise figure can be written as $NF_{total}\big|_{dB} = NF_1\big|_{dB} + L_{filter}\big|_{dB}$

Mixer and Mixer Noise

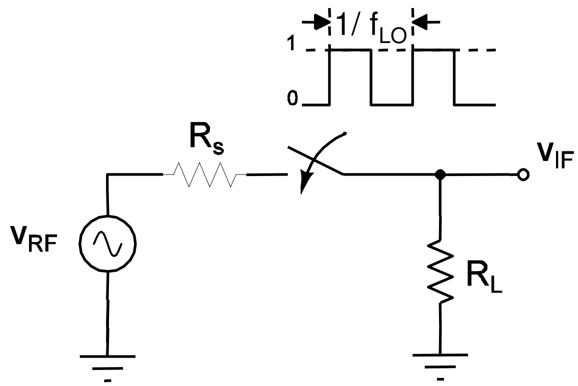


$$x(t) = A_{RF}\cos\omega_{RF}t,$$
$$y(t) = A_{LO}\cos\omega_{LO}t$$

$$x(t)y(t) = \frac{A_{RF}A_{LO}}{2}\left[\underbrace{\cos(\omega_{RF}-\omega_{LO})\,t}_{\text{o/p of down conversion mixer}} - \underbrace{\cos(\omega_{RF}+\omega_{LO})\,t}_{\text{Rejected by filter}}\right]$$

- In general, $v_{IF}(t) = f(x(t))\,g(y(t))$
  The functions 'f' and 'g' are non-linear
- Voltage gain of the mixer
  $$= \frac{\text{signal amplitude @ IF port}}{\text{signal amplitude @ RF port}} = \text{Conversion gain}$$
- Available power gain
- Noise factor $$= \frac{\text{available power @ IF port}}{\text{available power @ RF port}} = \frac{\text{available SNR @ RF port}}{\text{available SNR @ IF port}}$$

Example 1



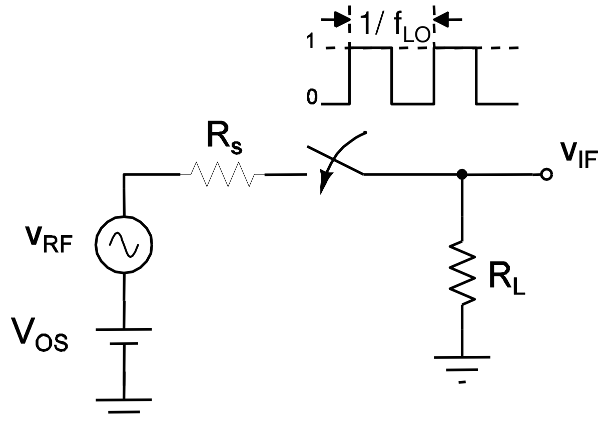
$$v_{RF}(t) = A_{RF}\cos\omega_{RF}t$$

$$v_{IF} = A_{RF}\cos(\omega_{RF}\ t)\frac{R_L}{R_L+R_s}\left[\frac{1}{2} + \frac{1}{2}\frac{4}{\pi}\cos(\omega_{LO}\ t)+\frac{1}{2}\frac{4}{3\pi}\cos(3\omega_{LO}\ t) + \cdots\right]$$

$$= \frac{A_{RF}}{2}\frac{R_L}{R_L+R_s}\frac{2}{\pi}\cos(\omega_{IF}t) + \cdots$$

Conversion gain $= \dfrac{R_L}{R_L+R_s}\dfrac{1}{\pi}$

Example 2



Unbalanced Mixer

$$v_{IF} = \frac{R_L}{R_L + R_s}\left[V_{OS} + A_{RF}\cos(\omega_{RF}\ t)\right]\left[\frac{1}{2} + \frac{1}{2}\frac{4}{\pi}\cos(\omega_{LO}\ t) + \cdots\right]$$

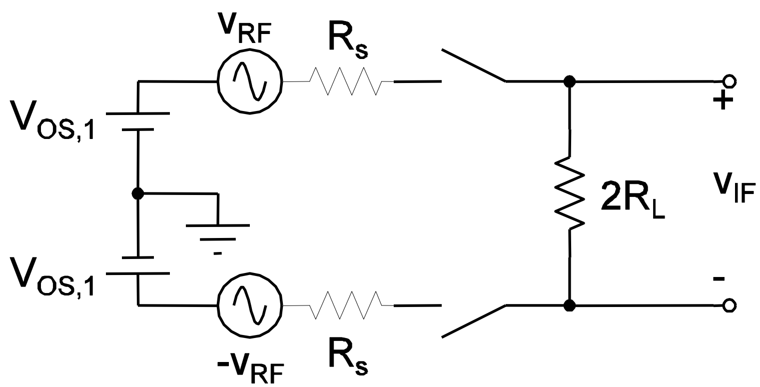
$$= \underbrace{\frac{A_{RF}}{2}\frac{R_L}{R_L + R_s}\cos(\omega_{RF} t)}_{\text{RF feed through}} + \frac{V_{OS}}{2} + \underbrace{\frac{V_{OS}}{2}\frac{2}{\pi}\cos(\omega_{LO} t)}_{\text{LO feed through}} + \frac{A_{RF}}{2}\frac{R_L}{R_L + R_s}\frac{2}{\pi}\cos(\omega_{IF} t) + \cdots$$

RF feed through => Noise, low frequency components

LO feed through => Desensitizes subsequent amplifiers ($A_{LO} >> A_{RF}$)

Improvement

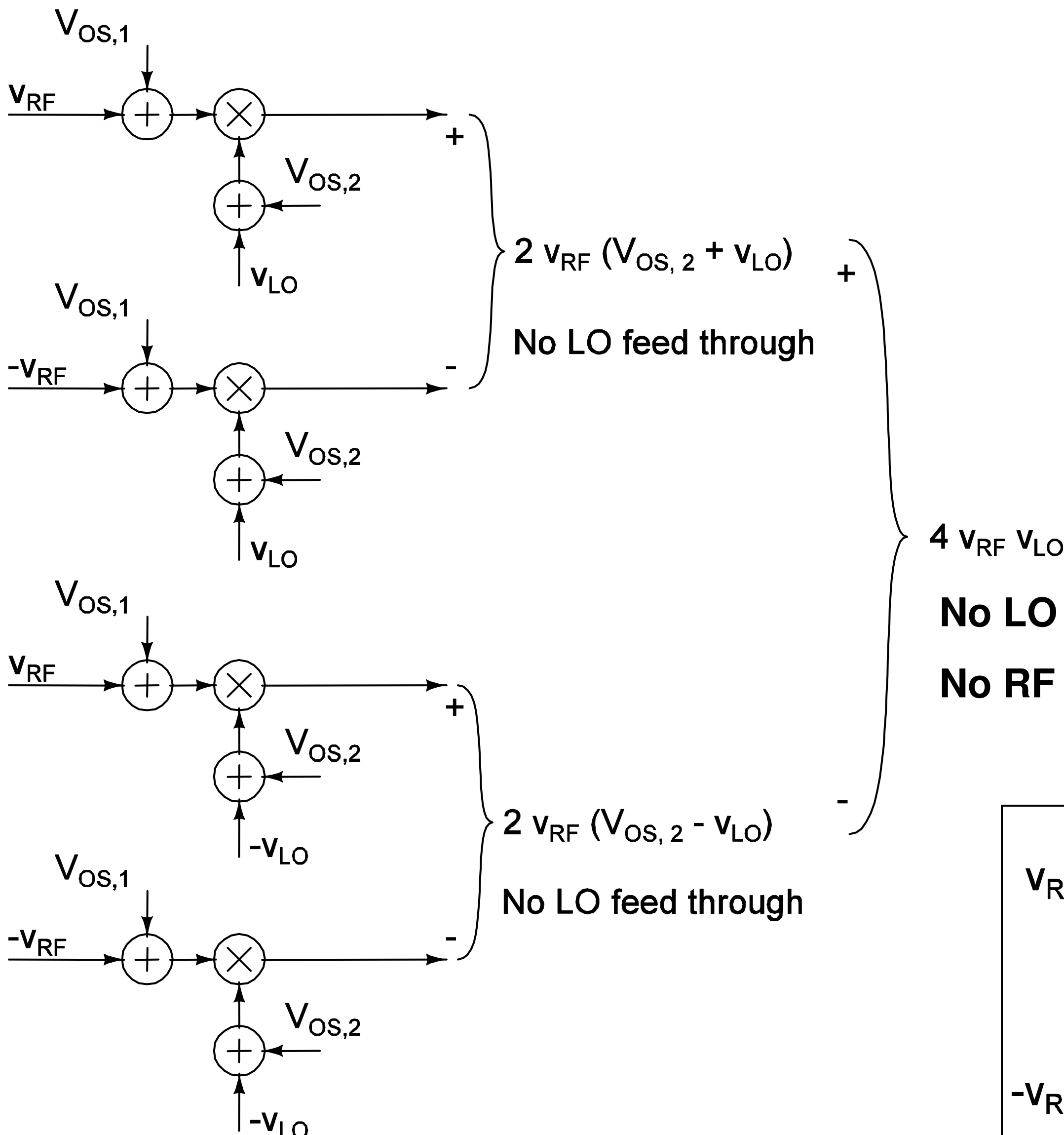
1. Single balanced mixer



$$v_{IF} = 2A_{RF}\frac{R_L}{R_L + R_s}\cos(\omega_{RF}t)\left[\frac{V_{OS}}{2} + \frac{1}{2} + \frac{2}{\pi}\cos(\omega_{LO}t) + \cdots\right]$$

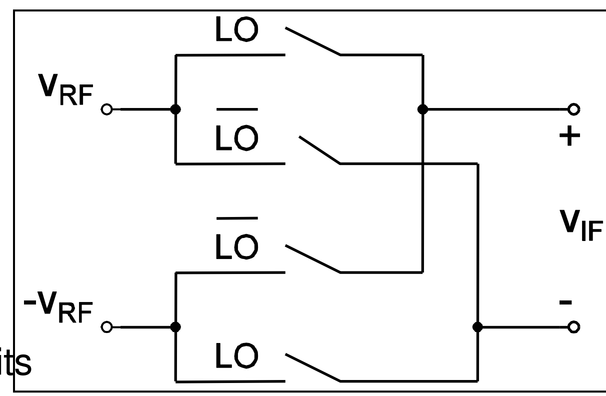
- No LO feed through

2. Double balanced mixer



No LO feed through

No RF feed through



Noise in Mixers

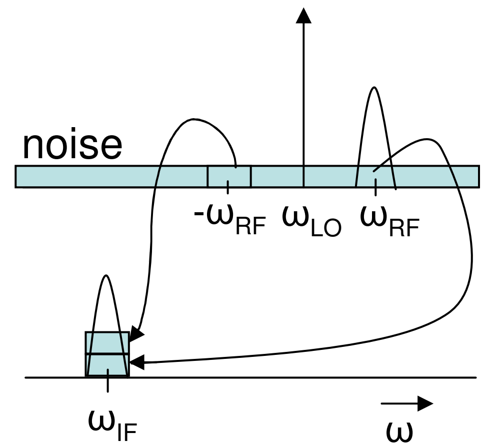
Consider an ideal mixer-

SNR @ input of mixer $= \dfrac{\overline{v_s^2}}{4KTR_s}$

1. Single side band signal -

➢ Thermal noise of both the signal band and the image band are translated to IF.

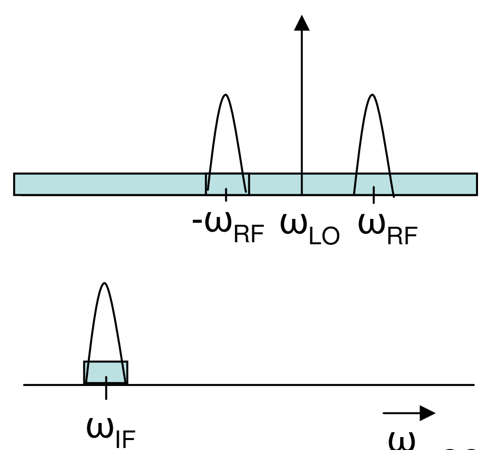
➢ SNR @ output is half the SNR @ input
=> Noise figure = 3 dB



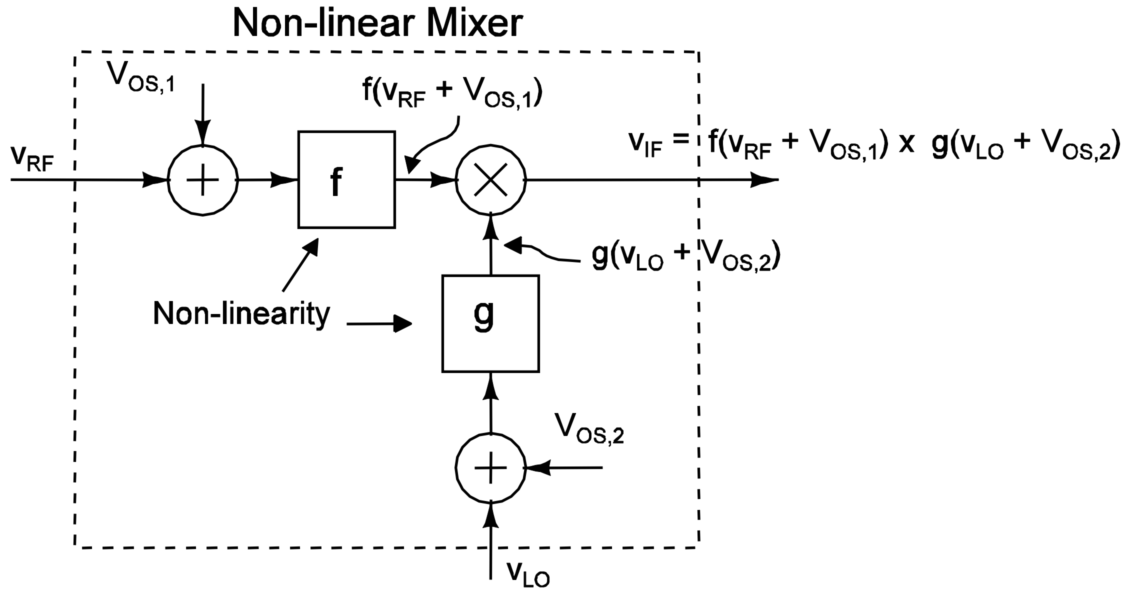
2. Double side band signal -

➢ SNR @ output is equal to the SNR @ input
=> Noise figure = 0 dB

➢ DSB Noise figure is 3 dB smaller than SSB Noise figure



Spurious responses in Mixers



- $f(v_{RF} + V_{OS,1})$ will have the frequency mf_{RF} *, m=0,1, 2, . . .*
- $g(v_{LO} + V_{OS,2})$ will have the frequency nf_{LO} *, n=0,1, 2, . . .*
- $f_{LO} - f_D = f_{IF}$ => High side LO ; $f_{RF} - f_D = f_{IF}$ => Low side LO ;

  f_D *: desired signal*
- For every combination of m and n, there is some frequency $f_{RF} = f_S$ that satisfies, $|mf_S \pm nf_{LO}| = f_{IF}$. Then f_S is the *'Spurious Signal'*
- m=0 => LO feed through ; n=0 => RF feed through

- Possible combinations –

$$mf_S + nf_{LO} = f_{IF}$$
$$mf_S - nf_{LO} = f_{IF}$$
$$mf_S - nf_{LO} = -f_{IF}$$

- Consider High side LO,

$$f_{LO} - f_D = f_{IF} \Rightarrow f_{LO} = f_D + f_{IF}$$

$$|mf_S \pm nf_{LO}| = f_{IF}$$
$$|mf_S + n(f_D + f_{IF})| = f_{IF}$$

Rearranging,

$$mf_S = -n(f_D + f_{IF}) + f_{IF}$$
$$\text{and} \quad mf_S = n(f_D + f_{IF}) \pm f_{IF}$$

Normalizing w.r.t. f_{IF},

$$S = \frac{-n+1}{m} + \frac{n}{m}D$$
$$\text{and} \quad S = \frac{n \pm 1}{m} \pm \frac{n}{m}D$$
$$S = \frac{f_S}{f_{IF}} \quad \& \quad D = \frac{f_D}{f_{IF}}$$

- m=1 and n=1 is the desired mode
- Plot of S vs D for given m and n is called 'Spur chart'

Example-

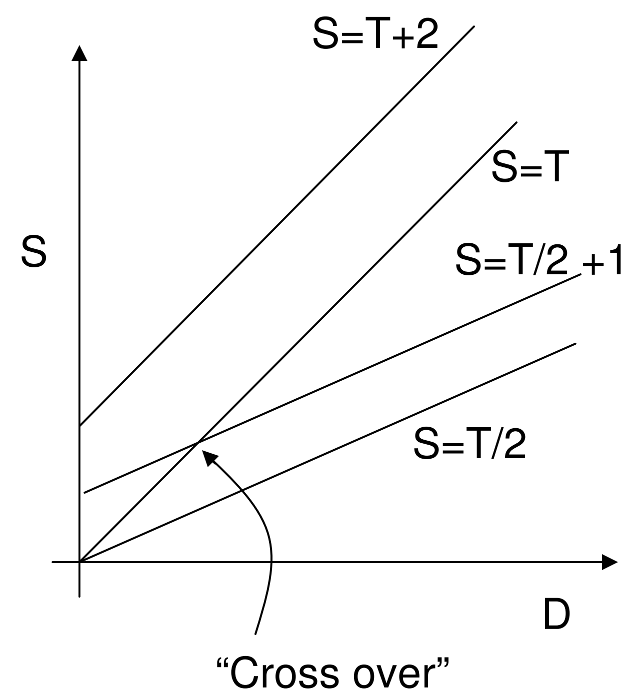
1. m=2, n=1 $\Rightarrow \boldsymbol{S = \frac{n \pm 1}{m} \pm \frac{n}{m} D} \Rightarrow \boldsymbol{S = \frac{D}{2} + \frac{1 \pm 1}{2}}$

2. m=2, n=2 $\Rightarrow \boldsymbol{S = \frac{n \pm 1}{m} \pm \frac{n}{m} D}$

$$\Rightarrow \boldsymbol{S = \frac{D}{2} + \frac{1}{2} \;\; or \;\; S = \frac{D}{2} + \frac{3}{2}}$$

Second harmonic of RF beating with second harmonic of LO demodulating to IF

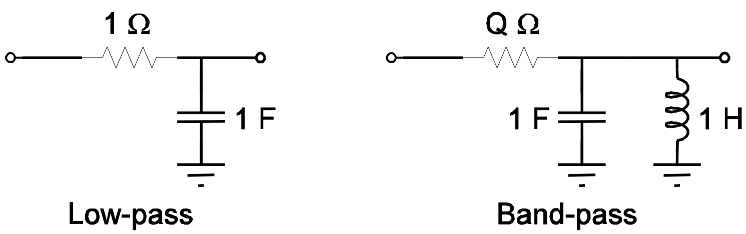
$$\boldsymbol{S = \frac{D}{2} + \frac{1}{2} \Rightarrow \text{Half IF problem}}$$

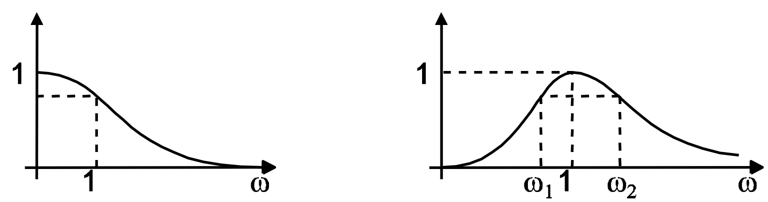


Filters

- Need a Linear filter (to avoid inter-modulation)
- Therefore passive filters are used in general
  - LC filters
  - Crystal filters
  - Ceramic filters

Frequency transformation (from Low pass to band-pass)





$$H(s) = \frac{1}{1 + Q\left(s + \frac{1}{s}\right)}$$

$$H(s) = \frac{1}{1 + s}$$

Transformation from low pass to band pass $\Rightarrow s \to Q\left(s + \frac{1}{s}\right)$

Let ω_{LP} be the cut-off frequency of the low pass filter. Then, for band pass filter,

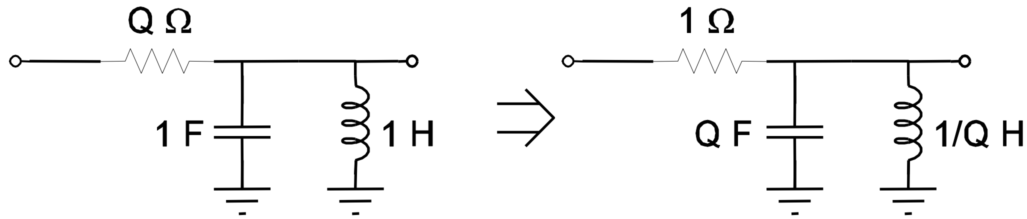
$$j\omega_{LP} = Q\left(j\omega + \frac{1}{j\omega}\right)$$

$$\Rightarrow \omega = \frac{\omega_{LP}}{2Q} \pm \sqrt{\frac{\omega_{LP}^{2}}{4Q^2} + 1}$$

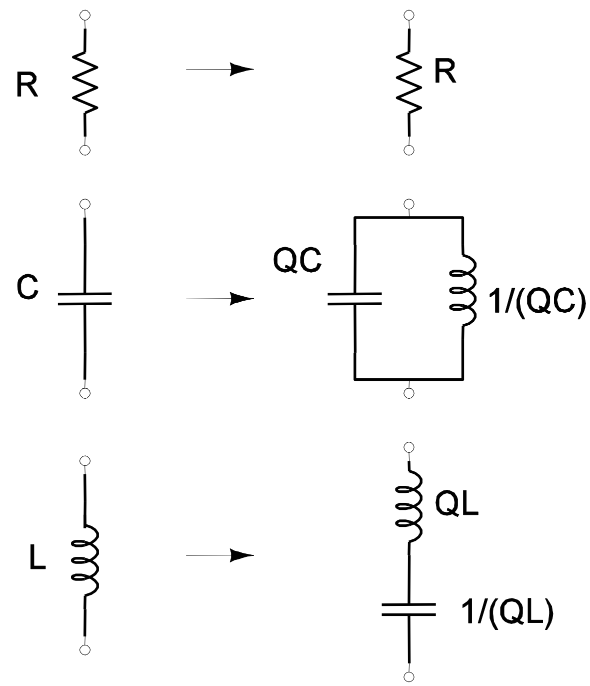
$$\Rightarrow \omega_1 = \frac{\omega_{LP}}{2Q} - \sqrt{\frac{\omega_{LP}^{2}}{4Q^2} + 1} \;\; \& \;\; \omega_2 = \frac{\omega_{LP}}{2Q} + \sqrt{\frac{\omega_{LP}^{2}}{4Q^2} + 1}$$

Center frequency $= \omega_o = \dfrac{1}{\sqrt{\omega_1 \omega_2}}$

Quality factor $= Q = \dfrac{\omega_o}{\omega_2 - \omega_1}$



$$s \rightarrow Q\left(s+\frac{1}{s}\right)$$



Example:

A 4th order low-pass Butterworth band-pass has 3 dB frequencies 100 MHz and 121 MHz. Find the equivalent frequency of the low-pass prototype corresponding to 121 MHz. -

$$f_o = \frac{1}{\sqrt{100 \times 121}} = 110 \ \text{MHz} \ ; \quad Q = \frac{\omega_o}{\omega_2 - \omega_1} = \frac{110}{21} = 5.25$$

$$\textbf{Equivalent frequency of the low pass prototype} = \mathbf{Q}\left(\frac{\mathbf{f}}{\mathbf{f_o}} - \frac{\mathbf{f_o}}{\mathbf{f}}\right) = \mathbf{5.25}\left(\frac{\mathbf{121}}{\mathbf{110}} - \frac{\mathbf{110}}{\mathbf{121}}\right)$$

$$= \mathbf{1\,Hz}$$

Local Oscillators

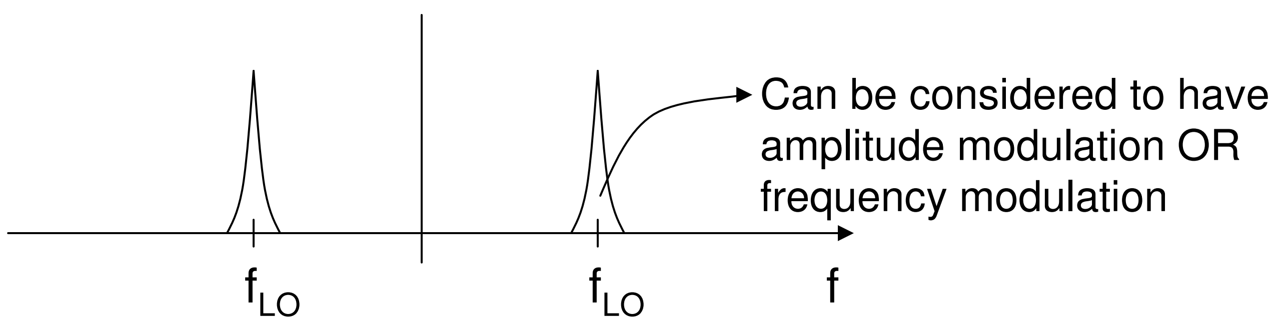
- Ideally Local oscillator should give a sinusoidal signal of constant amplitude and phase

  i.e. $v_{LO} = A\cos(\omega_{LO}t + \phi)$

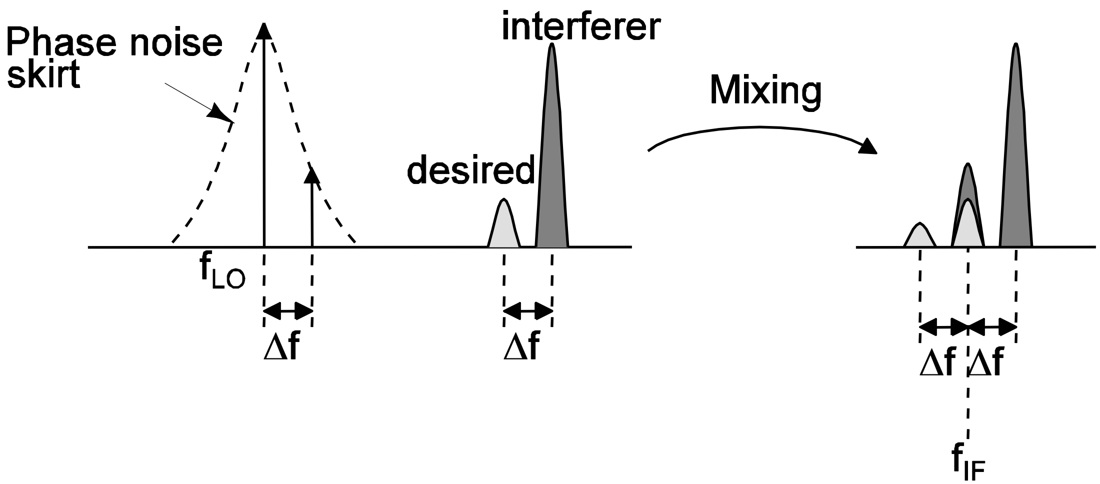
- In reality both the amplitude and phase will be function of time

  i.e. $v_{LO} = A(t)\cos(\omega_{LO}t + \phi(t))$

- *A(t)* : Amplitude noise & $\phi(t)$: Phase noise
- The spectrum LO will be like-



- Amplitude noise is not of concern. A Zero-Cross Detector can be used to switch the switch in the mixer.
- Phase noise is of concern because, zero crossing is the function of time, resulting in ‘timing jitter’.
- The timing jitter may cause the interferer to leak into the IF after mixing.

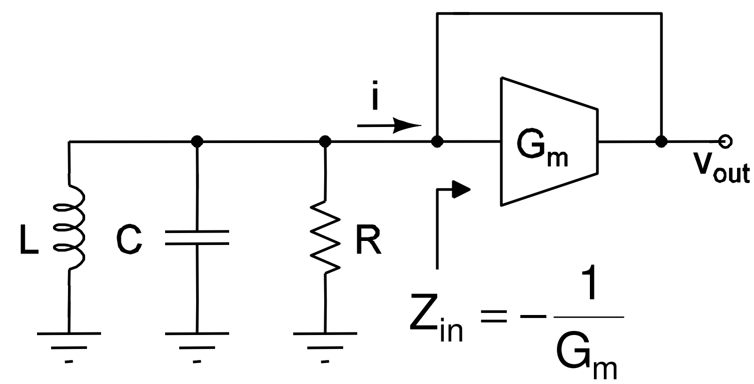


Example: Let the desired channel and interferer strengths be -98 dBm and

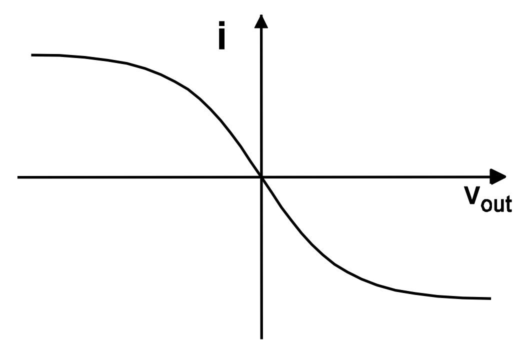
-23 dBm respectively, separated by Δf = 200 kHz. Calculate the reduction in

LO skirt at Δf = 200 kHz so as to have an SNR of 6 dB (Ans: 98-23+6 = 81 dB)

An oscillator schematic is shown below.



- R is the equivalent resistance of the tank circuit formed by L & C
- The transconductor G_m connected in unity feed back has an input impedance
$$Z_{in} = -\frac{1}{G_m}$$
- For oscillation, $G_m \geq \frac{1}{R}$



- The amplitude of oscillation is limited by the non-linearity of the transconductor

Let us assume the transconductor has a non-linearity of the form,

$$i = -\left(G_o v_{out} - G_1 v_{out}^3\right)$$

Let $v_{out} = A\sin(\omega t)$

$$\Rightarrow i = -\left(G_o A\sin(\omega t) - G_1 A^3 \sin^3(\omega t)\right)$$

$$= -\left(G_o A\sin(\omega t) - G_1 A^3 \frac{3\sin(\omega t) - \sin(3\omega t)}{4}\right)$$

$$= -A\sin(\omega t)\left[G_o - \frac{3}{4}G_1 A^2\right] - \frac{G_1 A^3}{4}\sin(3\omega t)$$

- 3rd harmonic current => 3rd harmonic in v_{out} , but will have less amplitude compared to fundamental due to the tank selectivity
- The amplitude will be stable when, $\left|\frac{i}{v}\right| = \frac{1}{R}$

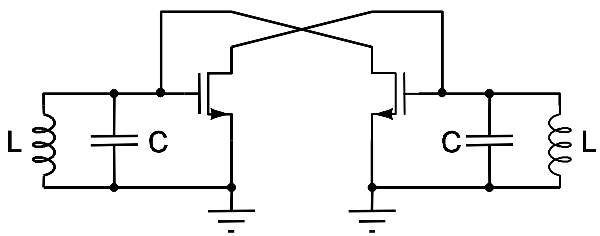
i.e. $$\frac{i}{A\sin(\omega t)} = \left[G_o - \frac{3}{4}G_1 A^2\right] = \frac{1}{R}$$

$$\Rightarrow \qquad A = \sqrt{\frac{G_o - 1/R}{3/4\, G_1}}$$

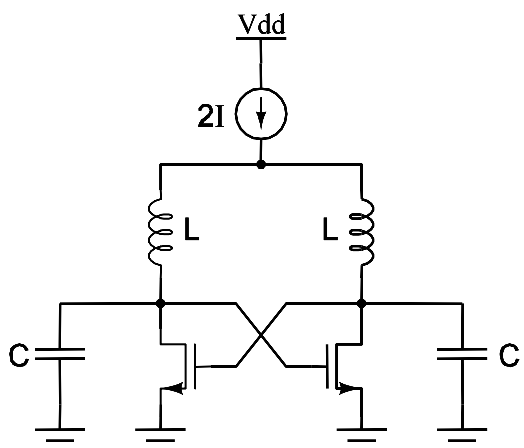
- The differential equation governing the KCL at the output node is

$$\frac{1}{L}v_{out}+\frac{1}{R}\frac{dv_{out}}{dt}+C\frac{d^2v_{out}}{dt^2}-\left(G_O\frac{dv_{out}}{dt}-3G_1v_{out}^2\right)=0 \rightarrow \text{ Vander Pol equation}$$

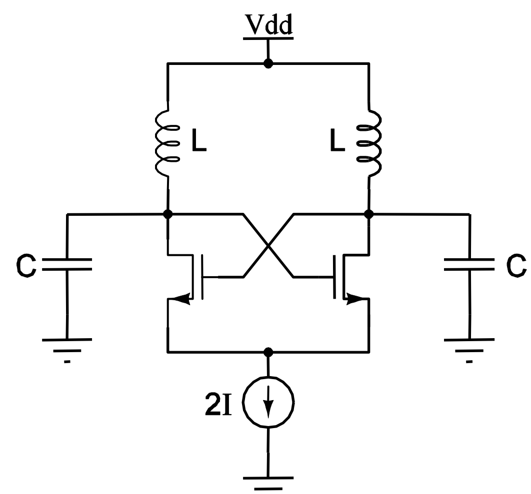
- Fully differential circuit implementation-



Differential signal picture



Fully differential schematic

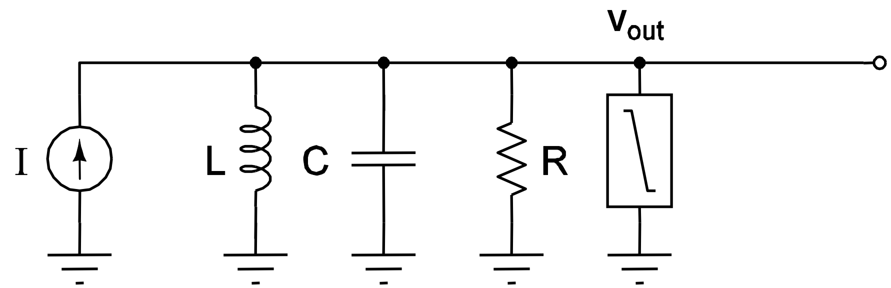


Alternative schematic

Phase noise models

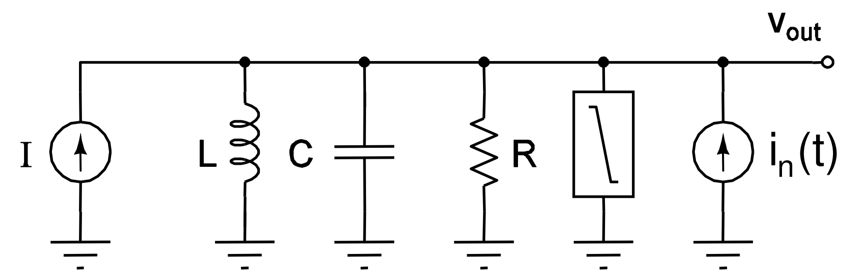
1. Leeson's model

If $v_{out} = Af(t)$, where *f(t)* is a solution, then $f(t+t_o)$ is also a solution (but depends on the initial condition).



- A charge dumped on tank circuit, causes an amplitude shift and phase shift in the voltage across it.
- If the dumping of charge is due to the noise, the phase shift in the oscillator will be noise dependent
- i.e. noise in the phase => 'Phase Noise'

Assuming linear circuit and the active device is noise less,



$$S_{i_n}(f) = \frac{4KT}{R}$$

In steady state, the impedance at v_{out} near oscillation frequency ω_o is

$$Z(j\omega) = \frac{j\omega L}{1-\omega^2 LC} = \frac{j\omega L}{1-\left(\omega/\omega_o\right)^2}$$

Therefore, the voltage noise spectral density at frequency (ω_o +$\Delta\omega$) is

$$S_{v_{n,o}}(f) = \frac{4KT}{R}\left|Z(j\omega_o + j\Delta\omega)\right|^2$$

$$\approx \frac{4KT}{R}\frac{L}{C}\left(\frac{\omega_o}{2\Delta\omega}\right)^2$$

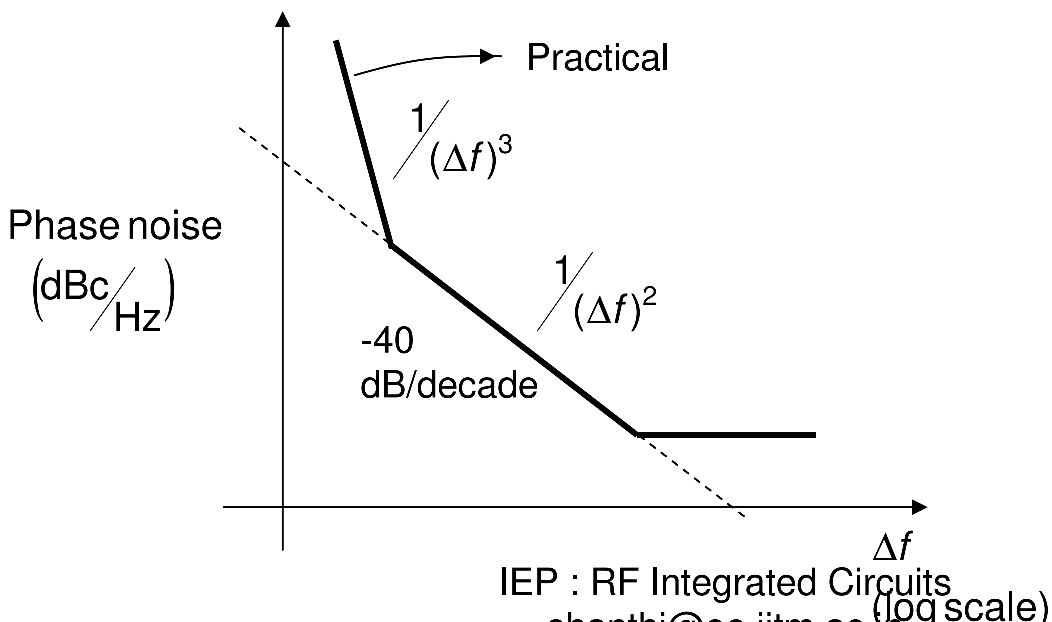
This contains both the Amplitude and Phase noise.

Assuming Phase noise is half of the total noise,

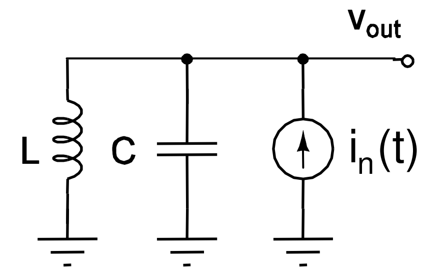
$$\text{Phase noise} \approx \frac{2KT}{R}\frac{L}{C}\left(\frac{\omega_o}{2\Delta\omega}\right)^2$$

$$\approx \frac{KTR}{2}\left(\frac{\omega_o}{Q\Delta\omega}\right)^2$$

$$\frac{\text{Phase noise @ }(\omega_o+\Delta\omega)}{\text{Carrier power @ }\omega_o} \approx \frac{1}{A_{rms}^2}\frac{4KTR}{2}\left(\frac{\omega_o}{Q\Delta\omega}\right)^2 \quad \text{dBc/Hz}$$



1. Hajimiri's phase noise model



- For an LC oscillator, if a charge is dumped, the oscillation will settle to a new amplitude and phase. The change is dependent on the time at which the charge is dumped.

- The plot of time instance of charge dumping verses the phase change will be a sinusoid of a amplitude = $\Delta q / CV_o$

  where, Δq is the charge and V_o is the amplitude of oscillation prior to charge dump.

$$i.e.\quad \Delta\phi(t) = -\frac{\Delta q}{CV_o}\sin(\omega_o\tau)u(t-\tau)$$

$$\frac{\Delta\phi(t)}{\Delta q}CV_o = -\sin(\omega_o\tau)u(t-\tau)$$

$$\left(\frac{\Delta\phi(t)}{\Delta q}\right)Q_{\max} = -\Gamma(\tau)u(t-\tau)$$

$$\left(\frac{\Delta\phi(t)}{\Delta q}\right): \text{unit impulse response} \quad \& \; \Gamma(\tau): \text{Impulse Sensitivity Function (ISF)}$$

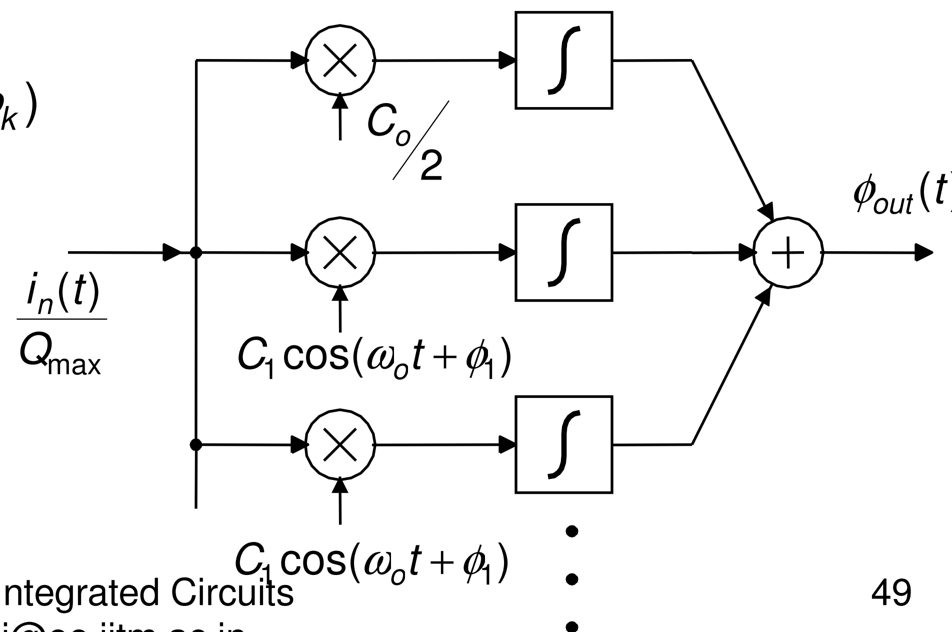
➢ The total phase noise due the charge dumped by the noise source $i_n(t)$ on the LC tank is,

$$\text{Total phase noise} = \int_0^{\infty} i_n(\tau)\left(\frac{1}{Q_{max}}\right)\Gamma(\tau)\, u(t-\tau)\, d\tau$$

$$= \int_0^{t} i_n(\tau)\left(\frac{1}{Q_{max}}\right)\Gamma(\tau)\, d\tau$$

➢ $\Gamma(\tau)$ is periodic with period $\dfrac{2\pi}{\omega_o}$

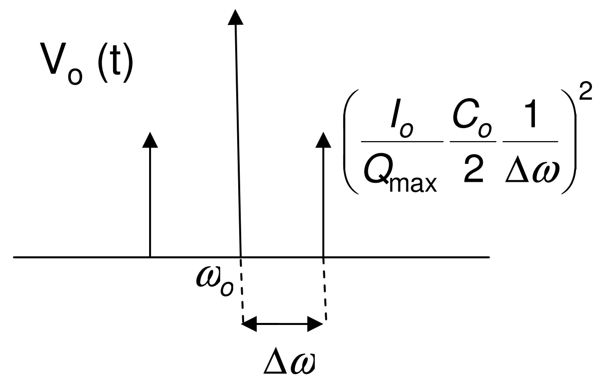
$$\Rightarrow \Gamma(\tau) = \frac{C_o}{2} + \sum_{k=1}^{\infty} C_k \cos(k\omega_o\tau + \phi_k)$$



- The oscillator o/p is of the form $v_o(t) = \cos(\omega_o t + \phi(t))$
- For very small $\phi(t)$, $v_o(t) = \cos(\omega_o t) - \phi(t)\sin(\omega_o t)$
- Therefore, low frequency component of $\phi(t)$ is of interest, as it beats with $\sin(\omega_o t)$ and the resulting frequency will be very close to ω_o

If, $\dfrac{i_n(t)}{Q_{max}} = \dfrac{I_o \sin(\Delta\omega t)}{Q_{max}}$

Then, $\phi(t) \approx \dfrac{I_o}{Q_{max}} \dfrac{C_o}{2} \dfrac{1}{\Delta\omega} \cos(\Delta\omega t)$ $\longrightarrow$



- Generalizing,

If, $i_n(t) = I_o \sin((k\omega_o + \Delta\omega)t)$

Then, $\phi(t) \approx \dfrac{I_o}{Q_{max}} \dfrac{C_k}{2} \dfrac{1}{\Delta\omega} \cos((k\omega_o + \Delta\omega)t)$

- Assuming $i_n(t)$, is white noise with noise spectral density = $\dfrac{\overline{i_n^2}(t)}{\Delta f}$

- Noise power at the oscillator output, at a frequency offset of Δω is

$$= 10\log_{10}\left[\frac{\overline{i_n^2}/\Delta f}{(4Q_{\max})^2}\times\frac{1}{(\Delta\omega)^2}\times\sum_{k=2}^{\infty}C_k^{\,2}\right] \quad \text{dBc}/\text{Hz}$$

- Considering 1/f noise, NSD of $i_n(t)$ is $= \dfrac{\overline{i_n^{\,2}(t)}}{\Delta f}\left(\dfrac{\omega_{1/f}}{\Delta\omega}\right)$

The noise power at the oscillator output, at a frequency offset of Δω is

$$= 10\log_{10}\left[\frac{\left(\overline{i_n^2}/\Delta f\right)\omega_{1/f}}{(4Q_{\max})^2}\times\frac{1}{(\Delta\omega)^3}\times C_o^{\,2}\right] \quad \text{dBc}/\text{Hz}$$

$\because$ for very low frequency $\Delta\omega$, C_o is only dominant

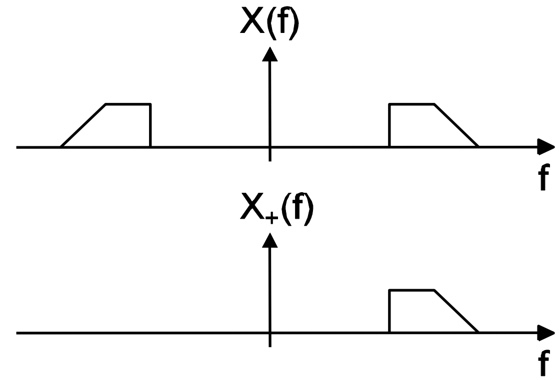
Image Rejection

- Two ways to reject the filter
  1. Using Image reject filter

     High Q filters are required – difficult to achieve
  2. Cancelling the image
- Latter of the two is preferred
- Image rejection techniques
  1. Hartley image reject mixer
  2. Weaver image reject mixer
  3. Direct conversion

Hilbert transform

X_+(f) can be obtained from X(f) as,



$$X_+(f) = X(f)\frac{1}{2}\left[1+\mathrm{sgn}(f)\right] \quad ; \quad \mathrm{sgn}(f) \text{ is complex}$$

$$X_+(f) = X(f)\frac{1}{2}\left[1+ j(-j\,\mathrm{sgn}(f))\right]$$

$-j\,\mathrm{sgn}(f)$ is real and known as Hilber filter

$$X_+(f) = \frac{1}{2}X(f) + j\frac{1}{2}X(f)(-j\,\mathrm{sgn}(f))]$$

$$= \frac{1}{2}X(f) + j\frac{1}{2}\hat{X}(f) \qquad \hat{X}(f) \text{ is the Hilbert transform of X(f)}$$

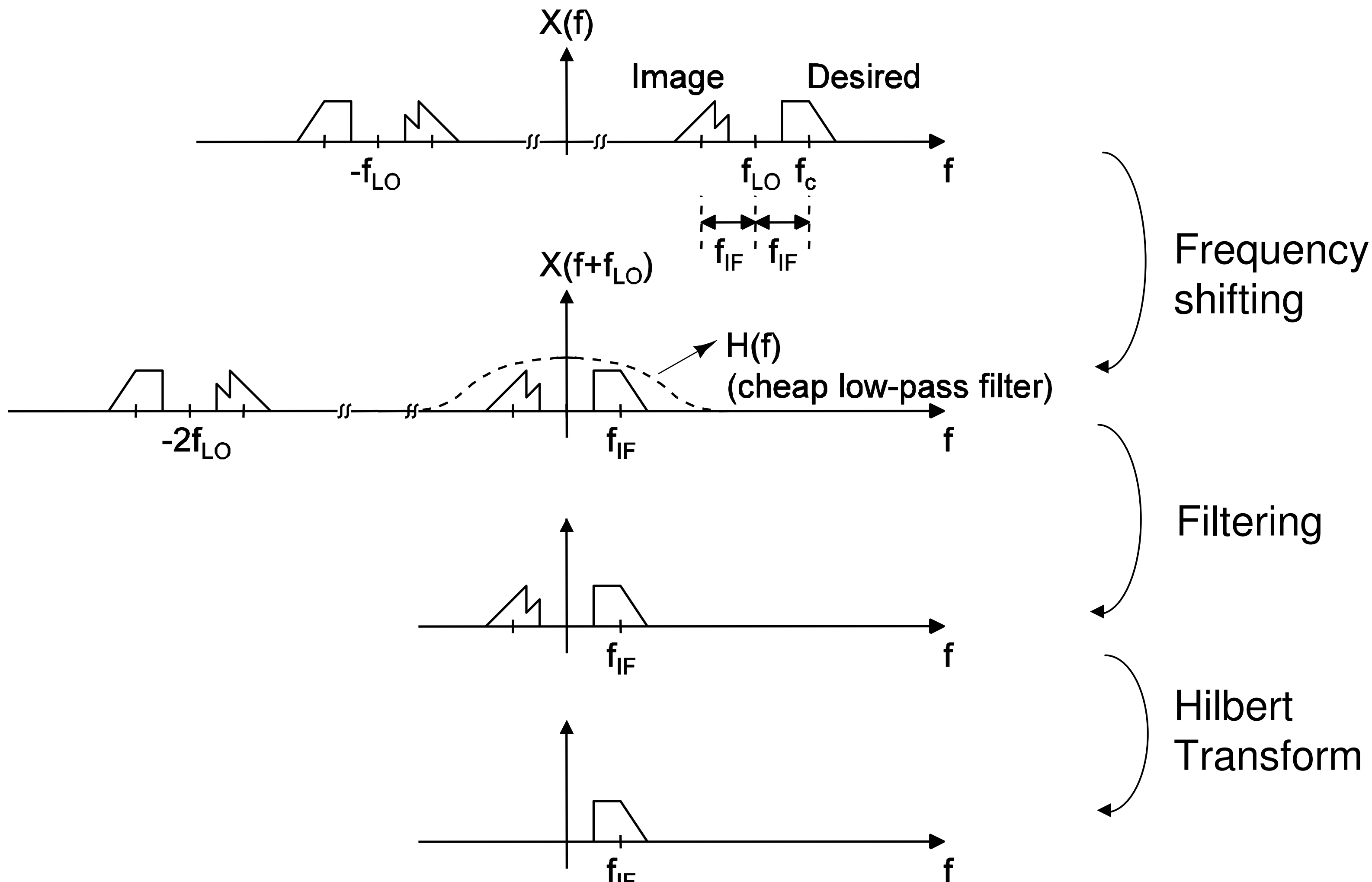
$$\Rightarrow x_+(t) = \frac{1}{2}\left[x(t) + j\hat{x}(t)\right] \quad \rightarrow \text{Analytic signal}$$

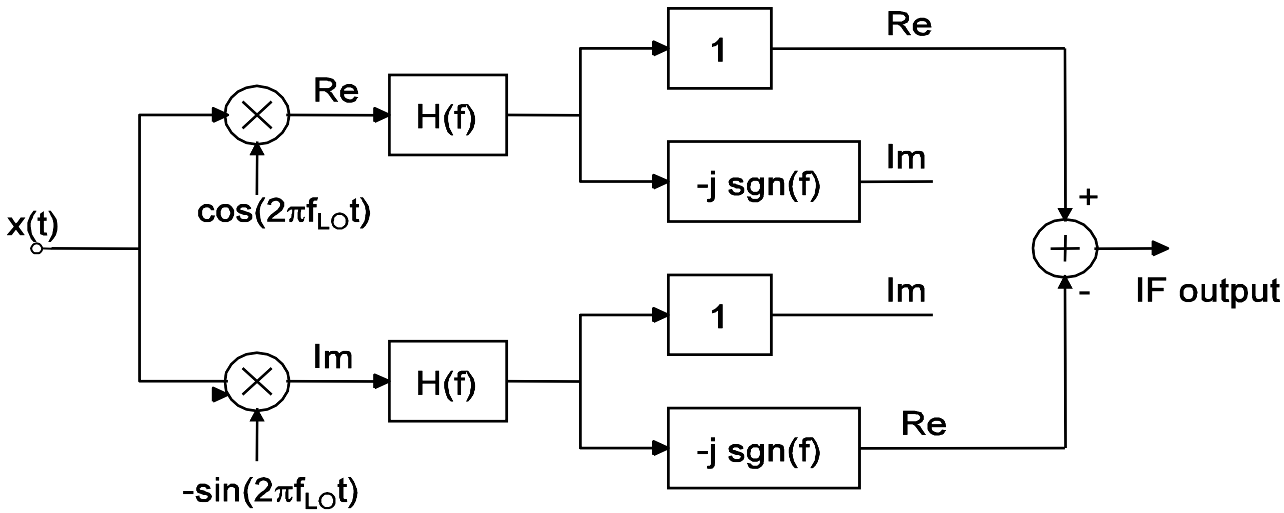
To get back x(t) from x_+(t),

$$\Rightarrow x(t) = 2\,\Re\mathrm{e}\left[x_+(t)\right]$$

1. Hartley Image Reject Mixer

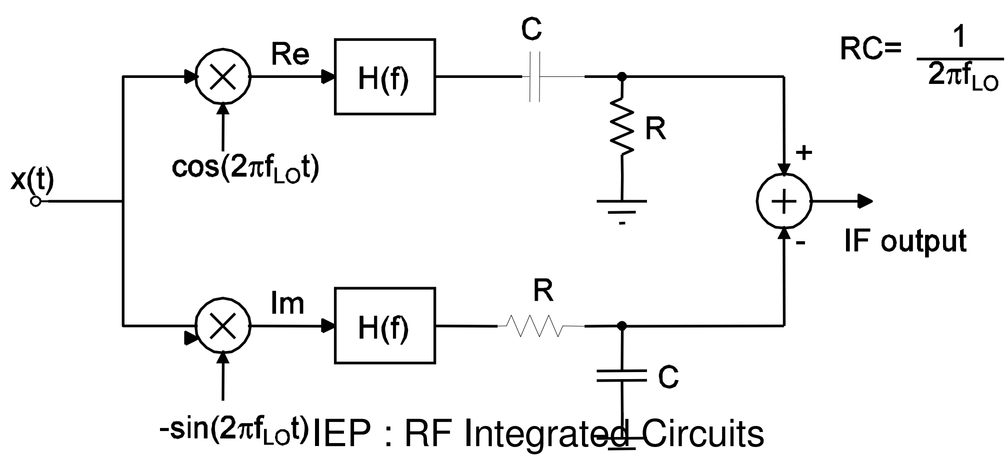
- The steps followed in rejecting the image is shown in the figure below





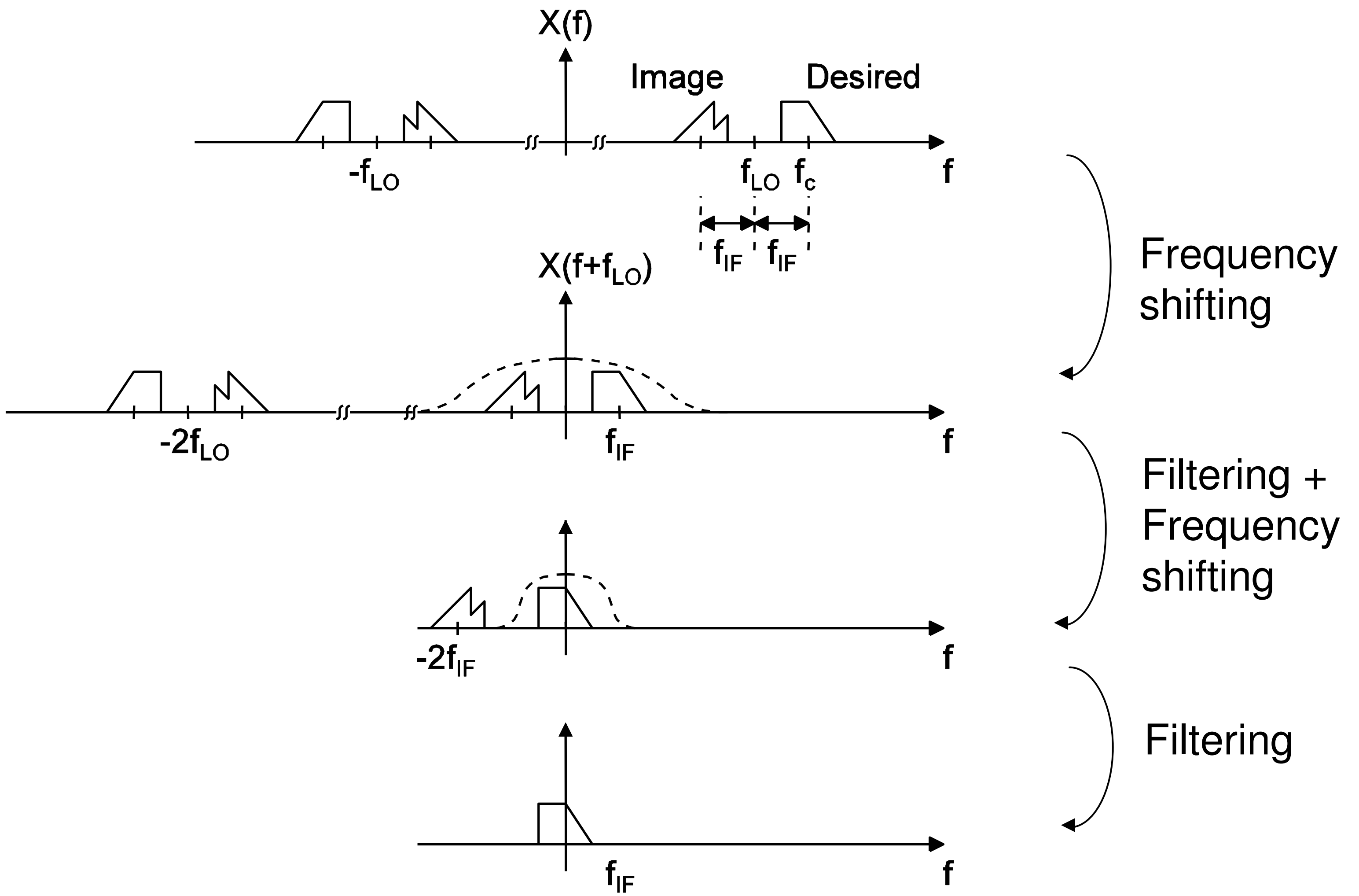
Hartley image reject mixer

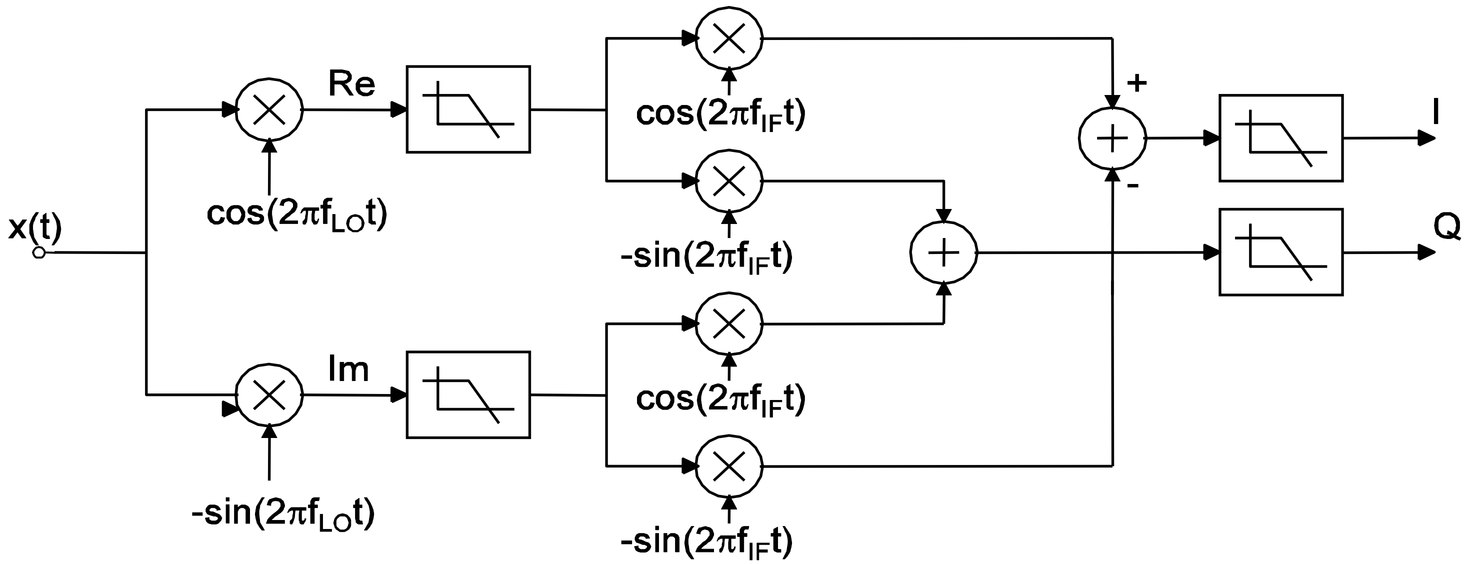
- If the RF signal is very narrow band, (-sgn(f)) block need to provide 90º phase-lag only at f_{IF}. The Hartley image reject mixer can be simplified as-



2. Weaver Image Reject Mixer

- The steps followed in rejecting the image is shown below

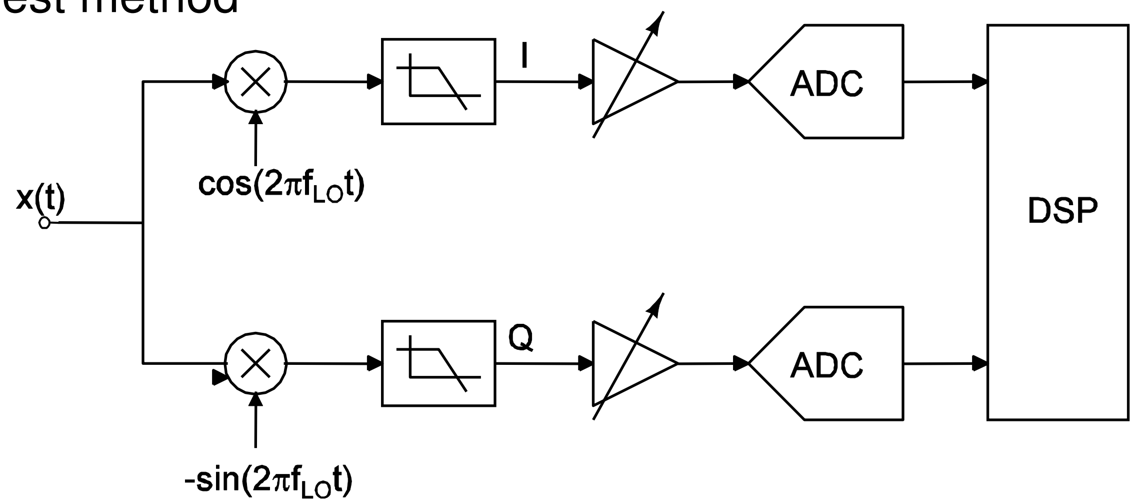




Weaver image reject filter

3. Direct conversion (Zero IF) mixer

- One step conversion from RF band to base band
- Simplest method



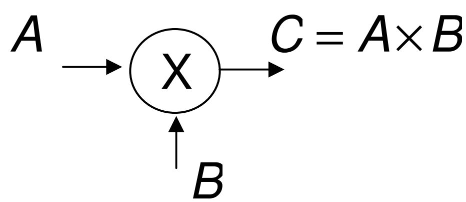
Limitations

a) Predominant component of I & Q are 'DC'

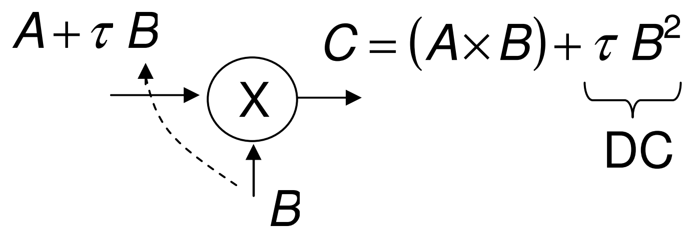
- Filter DC off-sets will contaminate the information
- 1/f noise => slow varying DC
- 2nd order intermodulation (out-of band IIP_2)
- 3rd order intermodulation

b) Mixer

- Finite isolation between the RF port and LO port, causes a ‘DC’ component to appear at the mixer output



Ideal mixer



Practical mixer

c) LNA

- Finite reverse isolation of LNA makes the LNA non-unilateral.
- The LO component leaked to the input of LNA will reflect back, causing a ‘DC’ component at mixer output