Phase Noise in Oscillators

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Oscillator Spectrum

- Ideally, it should be one or more delta functions
- The actual spectrum shows a non-zero width
- Close to the oscillation frequency, the PSD decreases as $(1/\Delta f)^3$ and later $(1/\Delta f)^2$
- Phase noise (dBc/Hz) is defined as

$$\mathcal{L}(\Delta f) = log\left(\frac{S_v(f_o + \Delta f)}{P_s}\right)$$

  where P_s is the total power at the fundamental frequency, $S_v(f_o + \Delta f)$ is the single sided PSD at a frequency offset Δf.
- Occurs because device noise changes the zero crossings of the signal

Linear time-invariant model

- In a high Q parallel LC oscillator,

$$Z(\omega) = \frac{j\omega L}{1 - \omega^2 LC + (j\omega L/R)}$$

$$Z(\omega_o + \Delta\omega) \approx \frac{-jR\omega_o}{2Q\Delta\omega}$$

- The output noise spectral density due to resistor noise is

$$\bar{V_n}^2 = 4kTR\left(\frac{\omega_o}{2Q\Delta\omega}\right)^2$$

Leeson's model

- For the LC oscillator

$$\mathcal{L}(\Delta\omega) = 10 log \left[\frac{2kT}{P_{sig}} \left(\frac{\omega_o}{2Q\Delta\omega} \right)^2 \right]$$

- Leeson's model includes flicker noise

$$\mathcal{L}(\Delta\omega) = 10 \log \left[\frac{2FkT}{P_{sig}} \left\{ 1 + \left(\frac{\omega_o}{2Q\Delta\omega} \right)^2 \right\} \left(1 + \frac{\Delta\omega_c}{(\Delta\omega)^3} \right) \right]$$

- F is an "effective noise figure" for the oscillator
- It is not a predictive model

Feedback oscillator

- The transfer function is given as

$$\frac{Y}{X} = \frac{H(j\omega)}{1 + H(j\omega)}$$

For $\omega = \omega_o + \Delta\omega$

$$H(j\omega) = H(j\omega_o) + \Delta\omega \frac{dH}{d\omega}$$

- At the frequency of oscillation $H(j\omega_o) = -1$
Therefore

$$|Y/X|^2 = \frac{1}{(\Delta\omega)^2 |\frac{dH}{d\omega}|^2}$$

- For white noise input, the PSD will decrease as $(1/\Delta f)^2$ and for flicker noise, it will go as $(1/\Delta f)^3$

Problems with the transfer function model

- Cannot explain how $1/f$ noise gets upconverted
- Actual phase noise is larger than predicted by the linear theory (factor "F" in Leesons model)
- Infinite power at the frequency of oscillation

Power Spectral density of noise in the phase

- The free running oscillator has amplitude stabilization, but no phase stabilization
- The output phase response to a voltage/current impulse can be modelled as a unit step. Therefore

$$\phi(t) \sim \int_{-\infty}^{\infty} u(t-\tau)v(\tau)d\tau = \int_{-\infty}^{t} n(\tau)d\tau$$

$$\Rightarrow S_{\phi}(f) \sim \frac{S_n(f)}{f^2}$$

- The output voltage is a periodic function of time. If the ideal output voltage is $cos(\omega_o t)$, the actual output $V(t)$ is

$$V(t) = cos(\omega_o t + \phi(t)) = cos(\omega_o t)cos(\phi(t)) - sin(\omega_o t)sin(\phi(t))$$

- For small $\phi(t)$, the deviation from the ideal output is

$$V_n(t) = V(t) - cos(\omega_o t) \approx -\phi(t) sin(\omega_o t)$$

- Therefore the phase noise gets upconverted to the carrier frequency
- A variation of this model is the linear periodically time varying model, where the phase response is not just a unit step, but is periodic

These models explain how the flicker noise gets upconverted. But they still predict infinite power at the frequency of oscillation. Also $\phi(t)$ is not a small perturbation - it is a non-stationary process and the variance increases with time

Nonlinear model with white noise

- A more exact nonlinear model in the presence of white noise sources gives the phase noise as

$$\mathcal{L}(f) = \frac{cf_o^2}{\pi^2 f_o^4 c^2 + (\Delta f)^2}$$

  It is a Lorentzian spectrum

- For Δf much larger than $\pi f_o^2 c$,

$$\mathcal{L}(f) = \frac{cf_o^2}{(\Delta f)^2}$$

- The PSD is finite at the oscillation frequency
- c is a parameter that relates phase noise and timing jitter

Timing jitter

- If the zero crossings occur at times t_i, a k cycle jitter $(J_k(t))$is defined as the standard deviation of $t_{i+k} - t_i$
- The period jitter is $J_1(t)$, i.e. the standard deviation of $t_{i+1} - t_i$
- The free running oscillator has accumulating jitter and is basically a Weiner process (variance increases linearly with time). It is a nonstationary process.
- It can be shown that the standard deviation of the period jitter is $\sqrt{cT}$, where T is the time period of oscillation

References

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