VLSI DATA CONVERSION CIRCUITS : PROBLEM SET 1

Problem 1

This problem is intended to illustrate the effect of oversampling on the required performance of an antialias filter placed before a sampler. For simplicity, assume that the anti-alias filter is an Nth order Butterworth filter, with

$$|H(f)| = \frac{1}{\sqrt{1 + \left(\frac{f}{f_{3dB}}\right)^{2N}}} \tag{1}$$

where f_{3dB} denotes the 3 dB bandwidth of the filter. The signal bandwidth is 1 MHz. It is desired that the attenuation of the anti-alias filter in the first "alias" band should be atleast 60 dB. The attenuation of signal at 1 MHz should be lesser than 0.5 dB. Consider the following cases

- The sampling rate is 4 MHz. What is the minimum order required of the anti-alias filter ? What is the bandwidth ? For the minimum order, how much can the filter bandwidth vary, while still meeting the requirements on attenuation ?

- The sampling rate is 40 MHz. What is the minimum order required of the anti-alias filter ? What is the bandwidth ? For the minimum order, how much can the filter bandwidth vary, while still meeting the requirements on attenuation ?

Problem 2

This problem is intended to understand the effects of random offset, gain mismatch and timing skew on the performance of a 2-way time-interleaved sampler. Assume that the sampling frequency of the complete sampler is f_s. The input is a 1 V sinusoid with a frequency $f_{in} = \frac{97}{1024} f_s$. To make sure you are doing the FFT right, determine the spectrum of an ideal sampler output. Draw the spectrum in all the following cases.

- The two samplers in the interleaved system have offsets of 2 mV and -5 mV respectively. How much lower in power is the tone at $f_s/2$ when compared to the input ? Do a hand calculation and confirm. Express your answer in dB.

- The two samplers in the interleaved system have gains of 0.99 and 1.01 respectively. What tones do you see in the spectrum now ? Do a hand calculation and confirm the strength of the tones. Express your answer in dB.

- Ideally, the individual samplers making up the ping-pong system sample at $2kT_s$ and $(2k+1)T_s$ respectively. In practice, there is timing skew - namely, the samplers sample at $2kT_s - t_o/2$ and $(2k+1)T_s + t_o/2$, where t_o is the "timing skew". Using the approach used in class, determine the effect of timing skew on the output spectrum for a sinewave input. Simulate for a timing offset of $T_s/100$ and confirm the results you see with hand calculations.

Problem 3

In class, we assumed that quantization "noise" can be approximated by a uniform distribution in the interval $[-\Delta/2, \Delta/2]$ where Δ is the LSB size. Further, we assumed that this noise was uncorrelated with the input signal. This problem is intended to see the range of validity of this assumption. Drive a 4-bit quantizer with a full scale sine-wave, making sure that you are using prime and integral number of input cycles. Denote the input by X and the quantized output by Xq. The *quantization error* is $Xq - X$. Divide the quantization error into 100 bins and plot a histogram. Is this uniform ? Repeat for a 6-bit, 8-bit, 10-bit and 12-bit quantizer.

Problem 4

Compute the peak Signal to Quantization Noise ratio for quantizers with resolutions of 2, 4, 6, 8, 10, 12 bits. Again, make sure that you are using a prime and integral number of input cycles, and to speed up DFT computations, choose the number of time points of the form 2^p. How do your numbers compare with the relation $\mathrm{SNR} \approx 6.02\,\mathrm{N} + 1.76\,\mathrm{dB}$?

Problem 5

A student (hereby referred to as "IT", to avoid gender bias) is testing an ideal quantizer, with N bits of resolution. IT drives the quantizer with a full scale sine-wave input, with a frequency within the Nyquist bandwidth. IT then captures 1024 output points of the quantizer, and expresses the output as a Discrete Fourier Series (DFS). IT plots the magnitudes of the DFS coefficients, (normalized to the input sine-wave amplitude), on a log scale. The resulting plot is shown in Figure 1. Assume that the "noise" floor is uniform at $-63\,\mathrm{dB}$.

- Find the frequency of the sine-wave input, in relation to the sampling frequency f_s.

- Estimate N, the resolution of the quantizer. As a reminder, make sure to CLEARLY demonstrate how you got your answer.

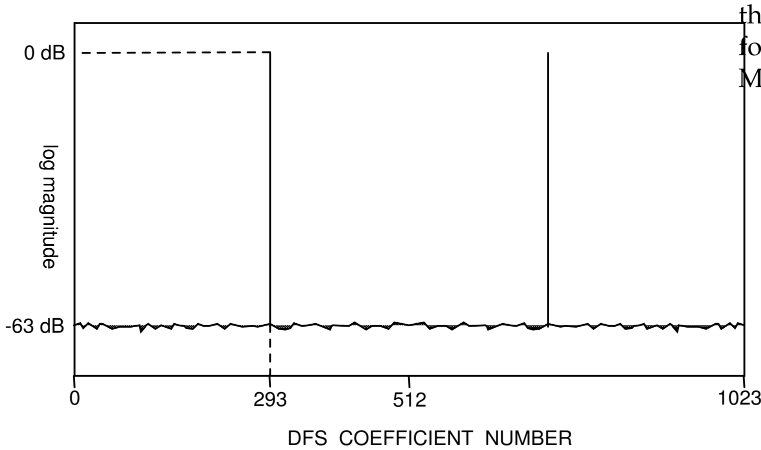

Figure 1: DFS log-magnitude plot for Problem 5.

Problem 6

For an ideal N-bit quantizer, for N ranging from 5 to 14, and excited by a full scale sine wave, find (using MATLAB) the ratio of powers of the fundamental and the largest spur at the quantizer output. This is called the Spurious Free Dynamic Range (SFDR). For uniformity, use a 2^{15} point FFT with an input at approximately $f_s/4$. Plot the SFDR (in dB) versus N. Can you explain the slope of the curve ?

Problem 7

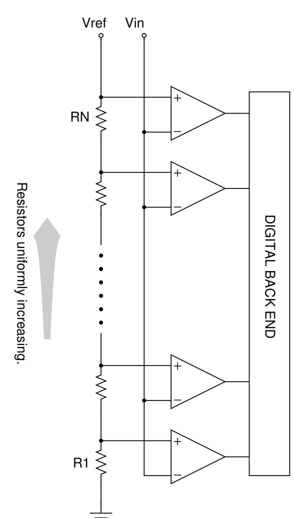

Figure 2: Flash ADC for problem 4.

A flash ADC is shown in Figure 2. Ideally, all resistors are supposed to be identical, so that the decision thresholds are all evenly spaced. On an IC, it is not possible to ensure that all resistors are equal. In this exercise, we analyze what happens to the output spectrum if there is a uniform gradient in conductivity for the material used to make the resistors. Assume an 8-bit flash ADC. Due to process imperfections, the resistors keep increasing as we progress from one end to another, so that $RN = 1.1\,R1$. Analyze and predict what happens for a full scale sinewave input. Verify your theory with MATLAB.