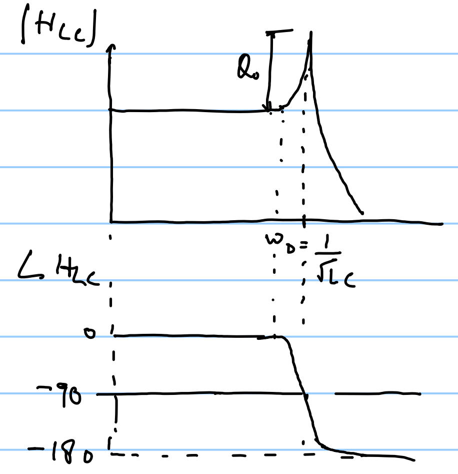
$$Q_0 = \frac{\omega_0}{\frac{R_{loss}}{L} + \frac{1}{R_{load} C}} = \frac{1/\sqrt{LC}}{\frac{R_{loss} C + L/R_{load}}{LC}}$$

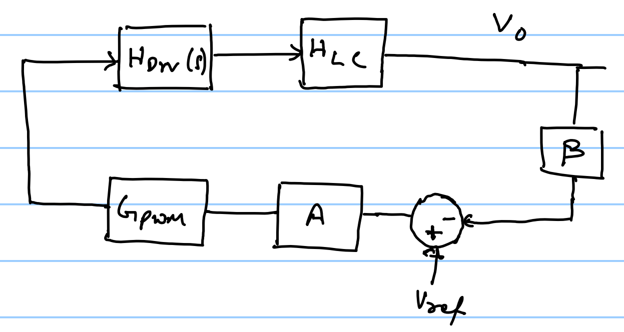
$$Q_0 = \frac{\sqrt{LC}}{R_{loss} C + L/R_{load}}$$



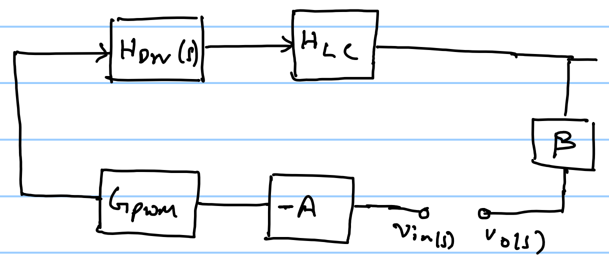
$Q_0 \Rightarrow$ max when $R_{Load} = \infty$ (no load)

$$Q_{0\text{-}max} = \frac{\sqrt{LC}}{R_{loss}C} = \frac{1}{R_{loss}}\sqrt{\frac{L}{C}}$$

Small Signal Model of a buck converter

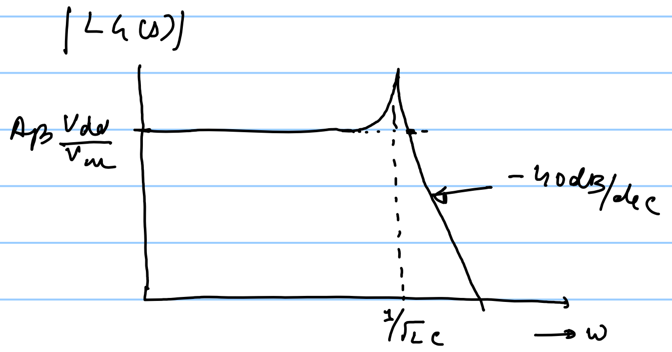


Loop Gain Analysis



$$\frac{V_o(s)}{V_{in}(s)} = LG(s) = -A\beta\, G_{pwm}\, H_{Drv}(s)\, H_{LC}(s)$$

$$= A\beta\left(\frac{1}{V_m}\right)(V_{dd})\left(\frac{1/LC}{s^2 + s\left(\frac{R_{loss}}{L} + \frac{1}{R_{load}C}\right) + \frac{1}{LC}}\right)$$



$$\left(\frac{\omega_0}{\omega_{ugb}}\right)^2 \times A\beta \frac{V_{dd}}{V_m} = 1$$

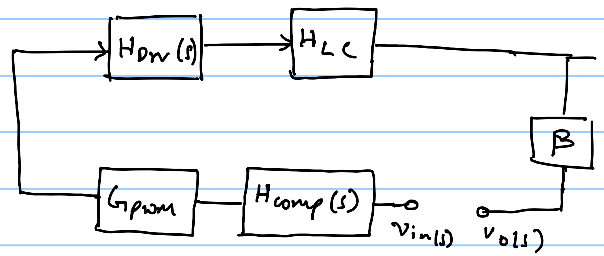
$$\omega_{ugb}^2 = \omega_0^2 A\beta \frac{V_{dd}}{V_m}$$

$$\omega_{ugb} = \omega_0 \sqrt{A\beta \frac{V_{dd}}{V_m}}$$

$$= \sqrt{\frac{A\beta V_{dd}}{V_m LC}}$$

$\omega_{ugb} >> \omega_0 \Rightarrow$ system is inherently unstable

We need to stabilize the system.



$$\frac{V_o(s)}{V_{in}(s)} = LG_{comp}(s) = \beta H_{comp}(s) G_{PWM} H_{DRV}(s) H_{LC}(s)$$

$$= \beta H_{comp}(s) \left(\frac{1}{V_m}\right) (V_{dd}) \left(\frac{1/LC}{s^2 + s\left(\frac{R_{loss}}{L} + \frac{1}{R_{load} C}\right) + \frac{1}{LC}} \right)$$

Assume

$$H_{comp}(s) = \frac{K}{s} \frac{1}{H_{LC}(s)}$$

$H_{LC}(s)$ will cancel out

$$LG_{comp}(s) = \beta \frac{V_{dd}}{V_m} \frac{K}{s} \Rightarrow \text{first order system}$$

<u>stable</u>

U.G.B. can be controlled by K

We can cancel complex poles due to LC by introducing complex zeroes in $H_{comp}(s)$ at ω_o.

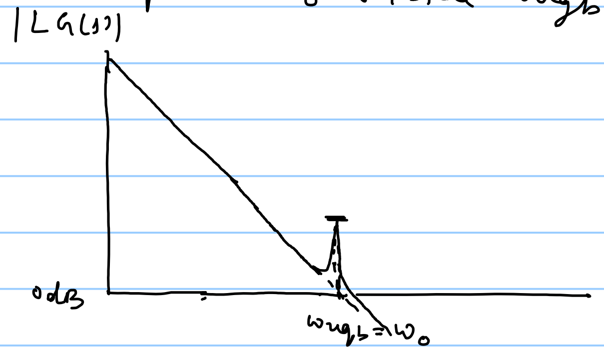
However, we can't practically implement complex zeroes with $\omega_o = \frac{1}{\sqrt{LC}}$ $\rightarrow$ Practically this stability method is not feasible.

Dominant pole compensation

we use integral or type-I compensation.

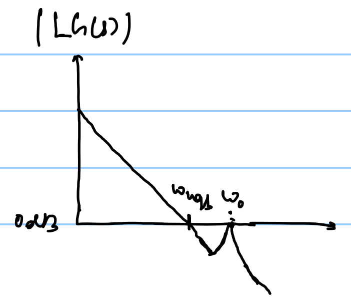
$$H_{comp}(s) = \frac{K_i}{s}$$

and push ω_0 outside ω_{UGB}



Gain at ω_{UGB} is $> 0dB$ & $PM \approx 0 \Rightarrow$ unstable.

$$\omega_{UGB} = \frac{\omega_0}{Q_0}$$



unstable, because
gain = 0dB at ω_0 (gain margin = 0dB)

$$\omega_{ugb} < \frac{\omega_o}{Q_o}$$

for $-20\,dB$ gain margin

$$\omega_{ugb} = \frac{\omega_o}{10\, Q_o}$$

$$\frac{V_o(s)}{V_{in}(s)} = LG_{comp}(s) = \beta H_{comp}(s) G_{PWM} H_{Drv}(s) H_{LC}(s)$$

$$= \beta \frac{k_i}{s} \frac{V_{dd}}{V_m} H_{LC}(s)$$

$$\beta \frac{V_{dd}}{V_m} = K_{uo}$$

$$LG_{comp}(s) = K_{uo} \frac{k_i}{s} H_{LC}(s)$$

$$K_{uo} k_i = \omega_{ugb} = \frac{\omega_o}{10\, Q_o}$$

$$k_i = \frac{\omega_o}{K_{uo}\, 10 \times Q_o} = \frac{1}{10 K_{uo}} \left(\frac{R_{loss}}{2} + \frac{1}{R_{load} C} \right)$$