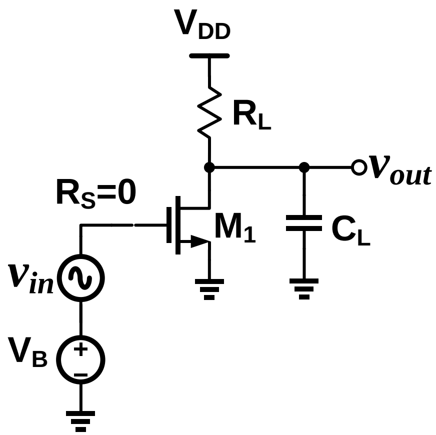
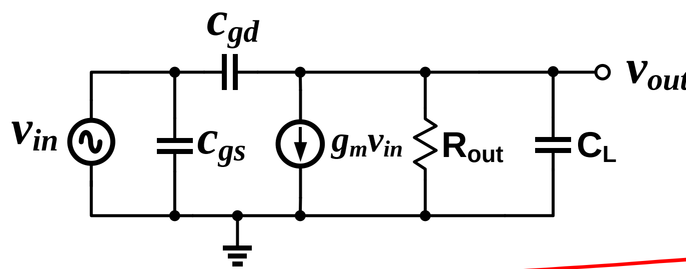
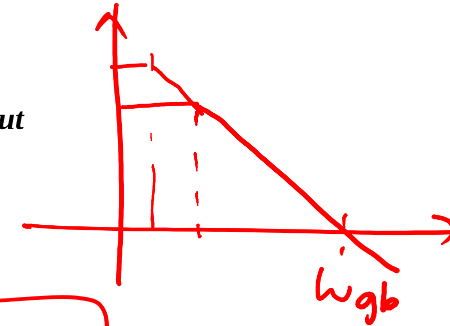
Gain-Bandwidth (GBW) Product







$$A_v(s) = -g_m R_{out} \frac{1 - sC_{gd}/g_m}{1 + s(C_{gd} + C_L)R_{out}}$$

$$\text{DC gain } A_0 = g_m R_{out}$$

$$\text{Bandwidth (BW) } \omega_{-3dB} = \omega_p = \frac{1}{(C_{gd} + C_L)R_{out}}$$

$$\text{Gain} \times \text{Bandwidth(GBW product)} = \frac{g_m}{C_{gd} + C_L} = \omega_{ugb}$$

Need: Large DC gain and large GBW product

Technology Limitation

$$\text{DC gain } A_0 = g_m(R_L \parallel r_{ds})$$

$$\text{Max DC gain } A_{0_{max}} = g_m r_{ds} = \frac{2}{\lambda V_{ov}}$$

$$\omega_{ugb} = \frac{\mu_n C_{ox}(W/L)V_{ov}}{C_{gd} + C_L}$$

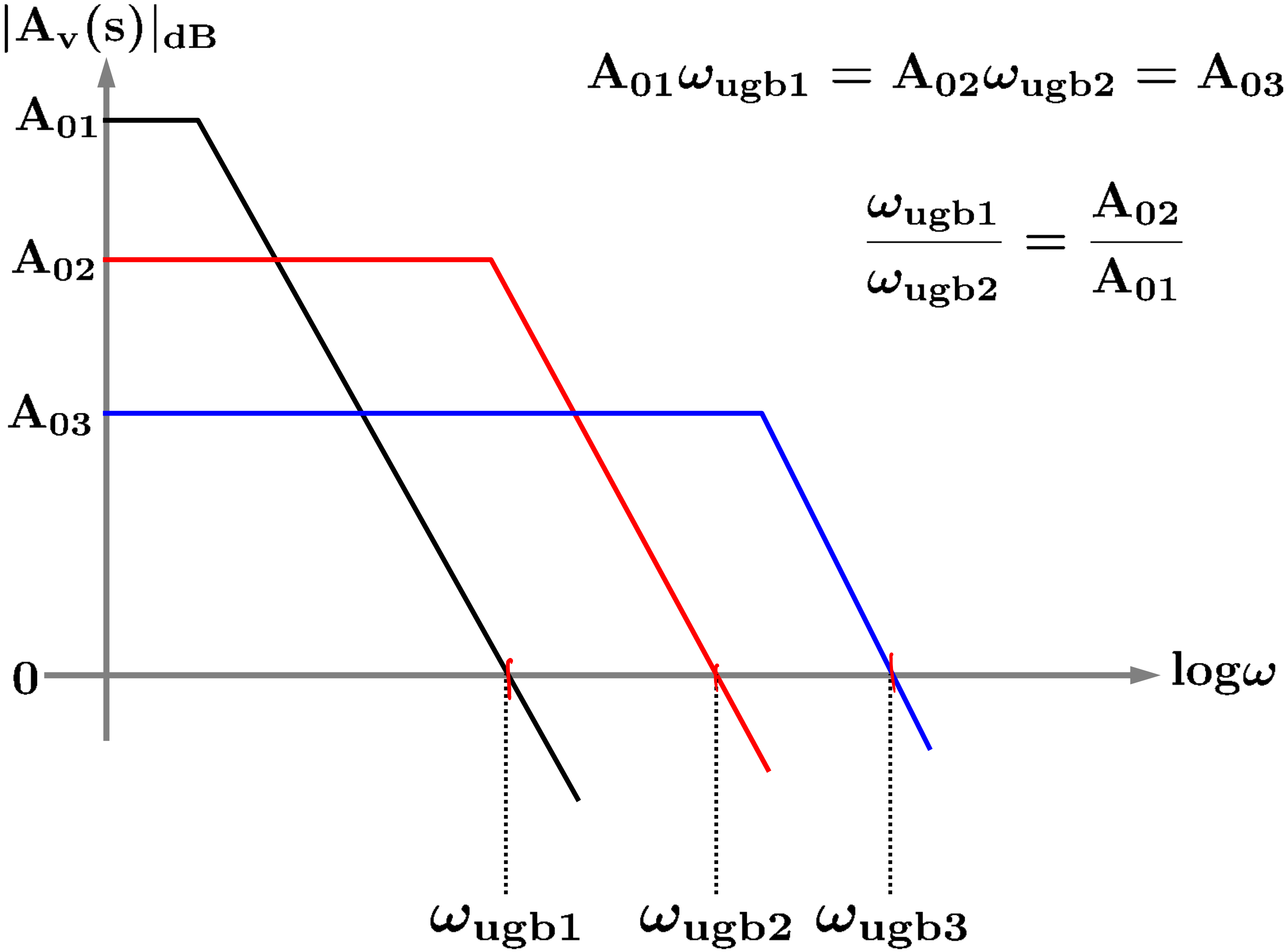
$$= \frac{g_m}{C_{gd} + C_L}$$

$$A_0 \cdot \omega_{ugb} = \frac{2\mu_n C_{ox}(W/L)}{\lambda(C_{gd} + C_L)}$$

$$C_{gd} = k_c W \qquad C_L = 0$$

$$A_0 \cdot \omega_{ugb}|_{max} = \frac{2\mu_n C_{ox}}{\lambda k_c L} = \text{constant}$$

Graphical Illustration

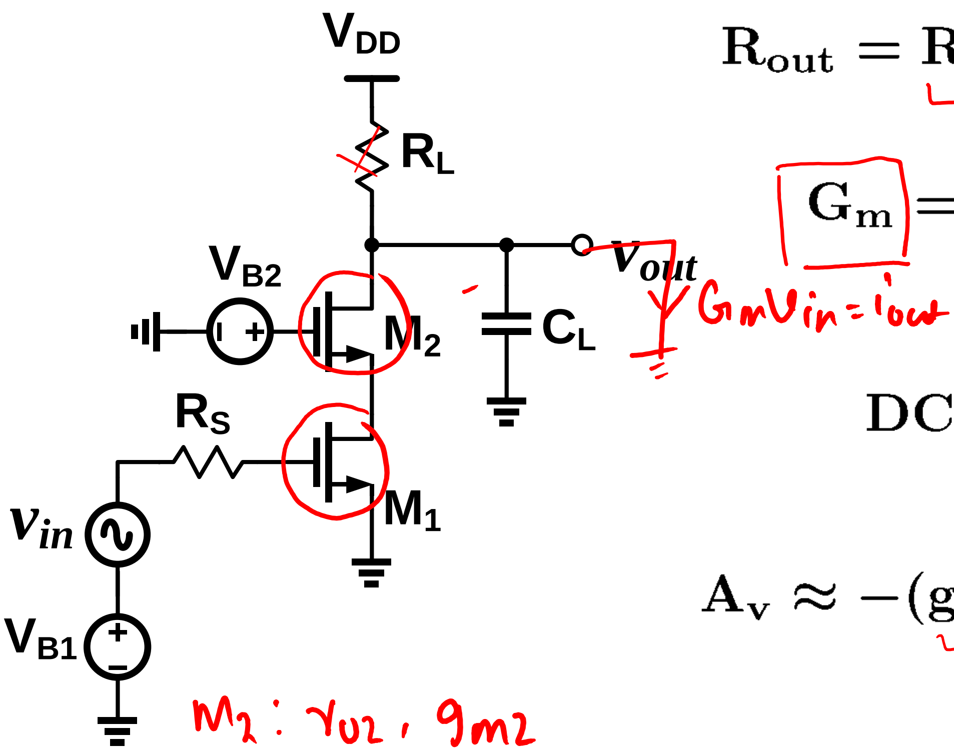


$$A_{01}\omega_{ugb1} = A_{02}\omega_{ugb2} = A_{03}\omega_{ugb3}$$

$$\frac{\omega_{ugb1}}{\omega_{ugb2}} = \frac{A_{02}}{A_{01}}$$

Cascoded CS (CCS) Amplifier

$$\text{Common source} + \text{Common gate} = \text{Cascode}$$



M_2 : r_{o2}, g_{m2}

M_1 : r_{o1}, g_{m1}

$$R_{out} = R_L \,\|\, (r_{o2} + r_{o1} + g_{m2} r_{o2} r_{o1})$$

$$G_m = g_{m1} \frac{r_{o1}}{r_{o1} + 1/g'_{m2}} \approx g_{m1}$$

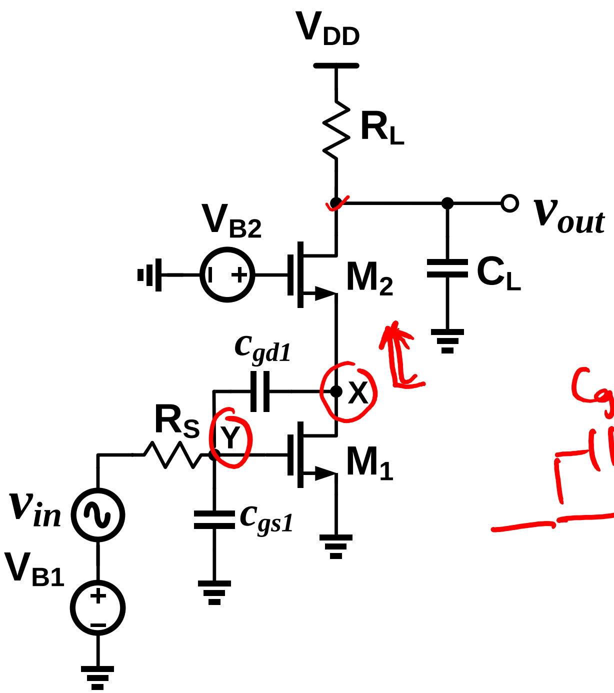
$$\text{DC gain } A_v = -G_m R_{out}$$

$$A_v \approx -(g_{m1} r_{o1})(g_{m2} r_{o2}) \text{ for large } R_L$$

$$A_v \propto (g_m r_o)^2$$

Gain is boosted by $g_m r_o$

Bandwidth Extension



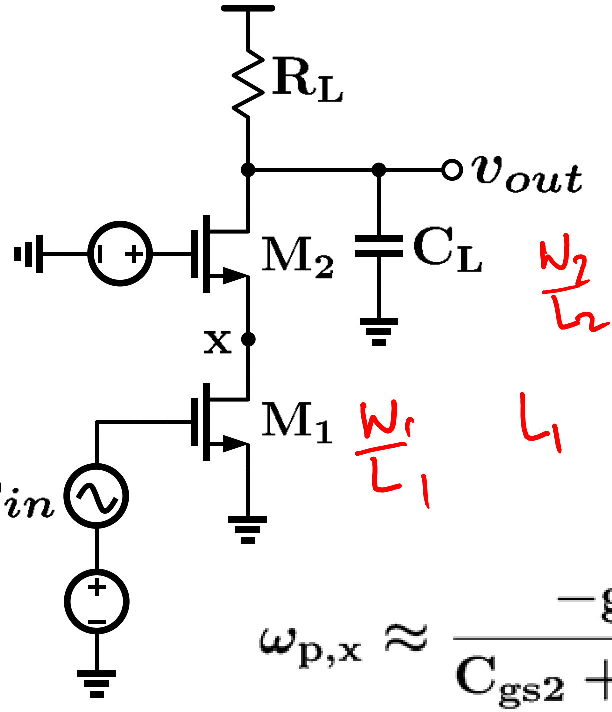
$$g'_{m2} = g_{m2} + g_{mb2}$$

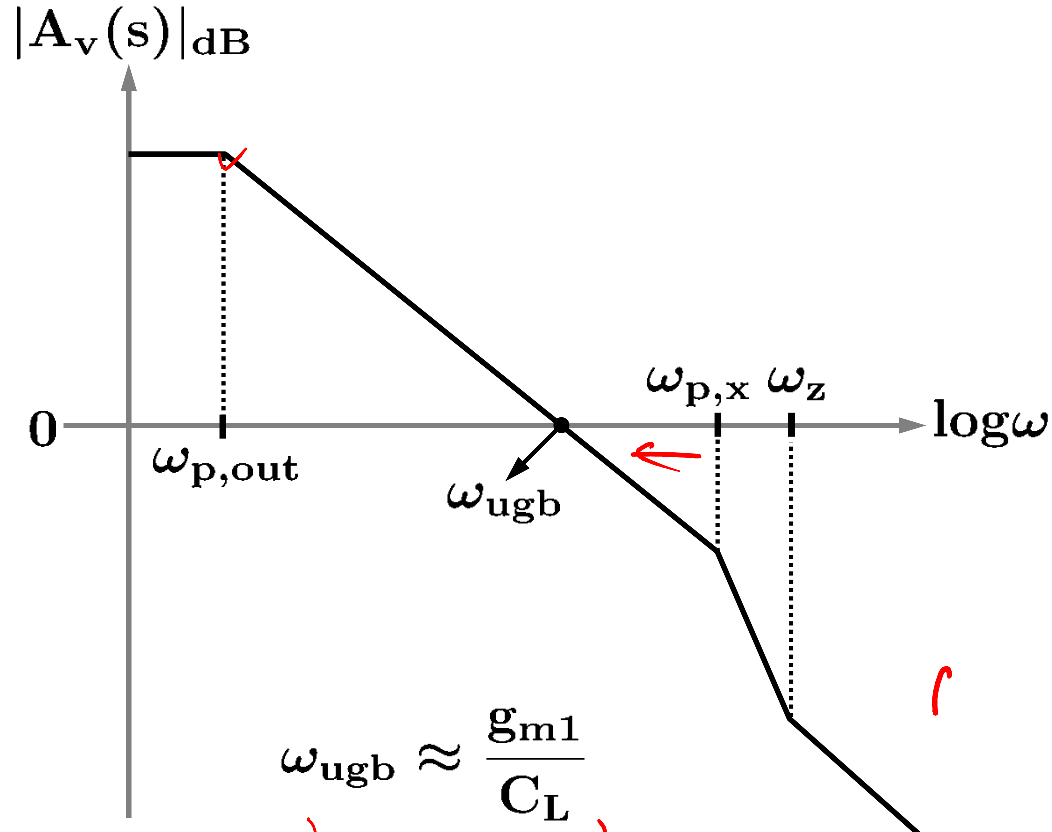
$$\omega_{in} \approx \frac{1}{R_S\left[C_{gs1} + \left(1 + \frac{g_{m1}}{g'_{m2}}\right) C_{gd1}\right]}$$

$$\frac{g_{m1}}{g'_{m2}} \leq 1$$

- Cascode transistor reduces the gain from node Y to node X → suppresses miller effect
- Negligible impact of R_L on this benefit

CCS Amplifier Frequency Response



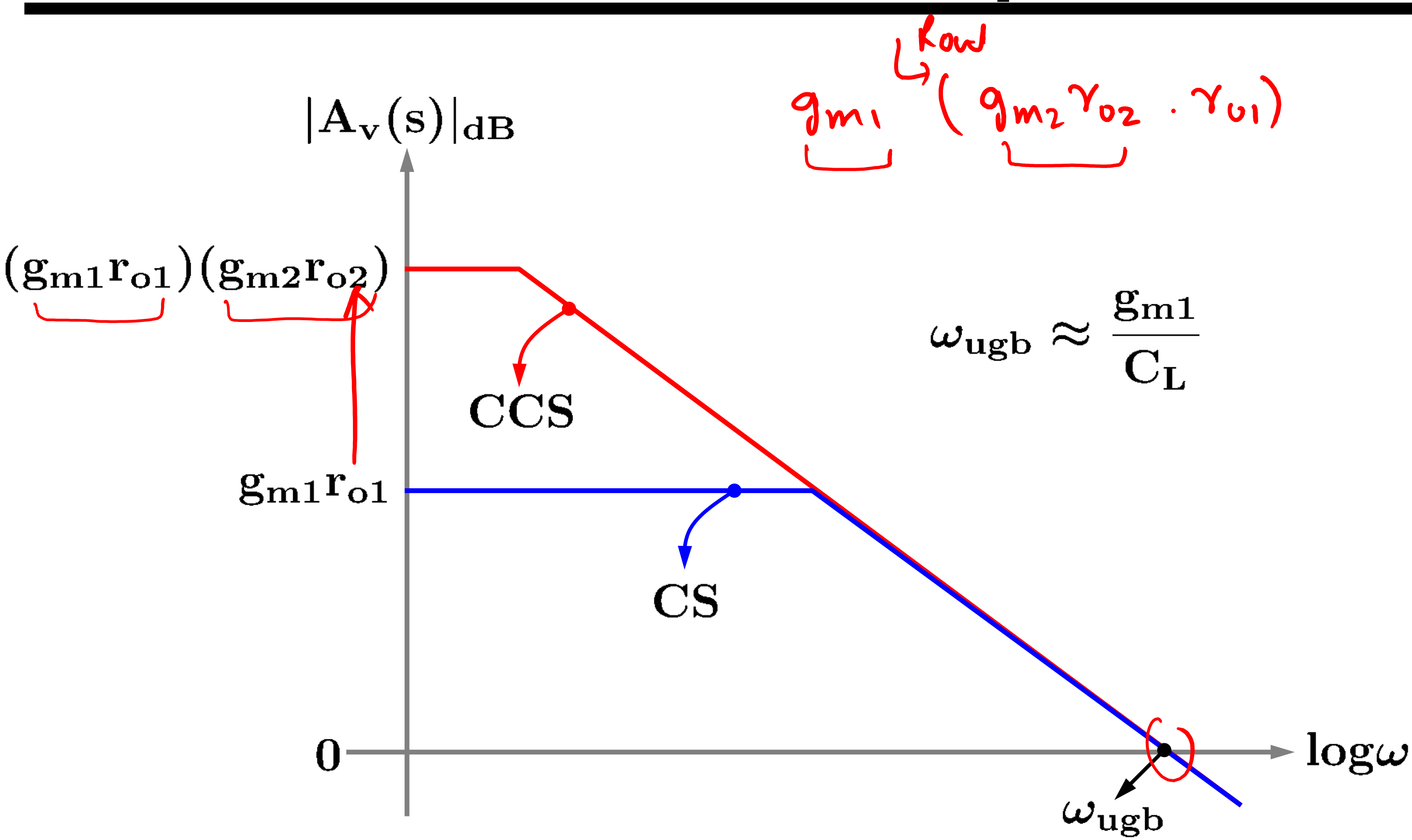


$$\omega_{p,x} \approx \frac{-g'_{m2}}{C_{gs2} + C_{gd1}}$$

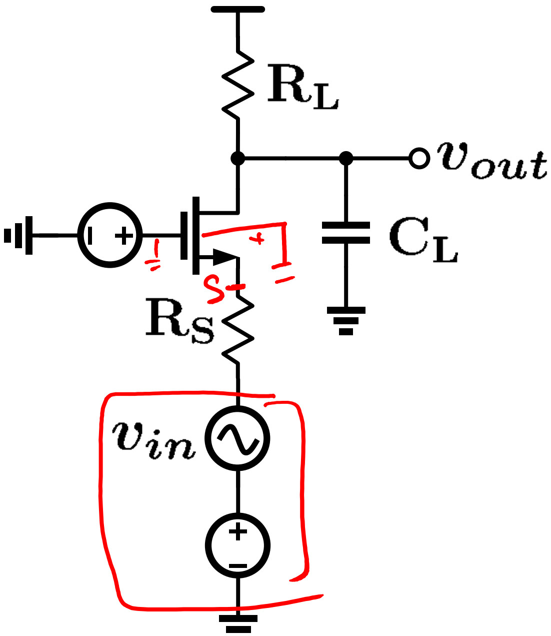
$$\omega_{ugb} \approx \frac{g_{m1}}{C_L}$$

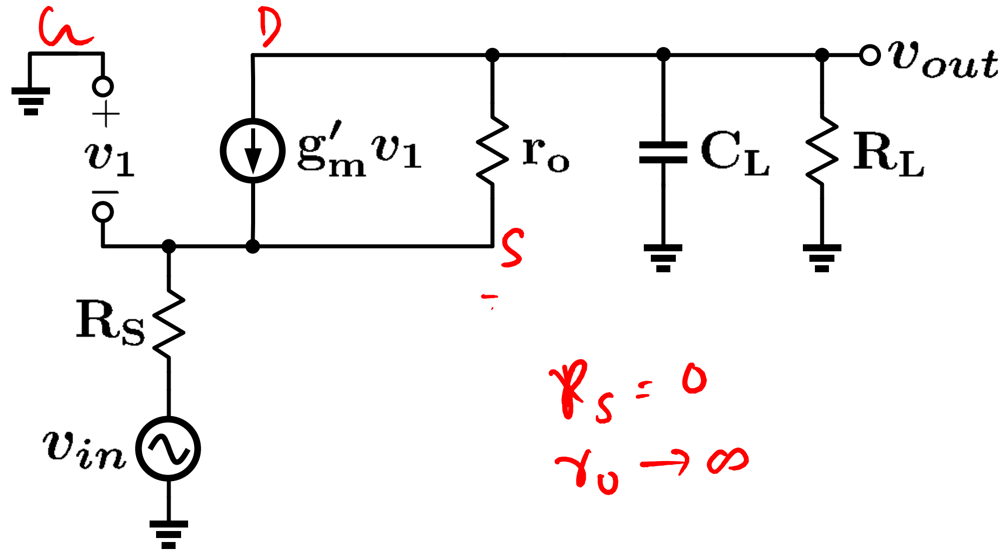
$$\omega_{p,out} \approx \frac{-1}{R_{out} C_L} \qquad \omega_z \approx \frac{g_{m1}}{C_{gd1}}$$

CS Vs. CCS Amplifier



Common-Gate Amplifier





$R_S = 0$

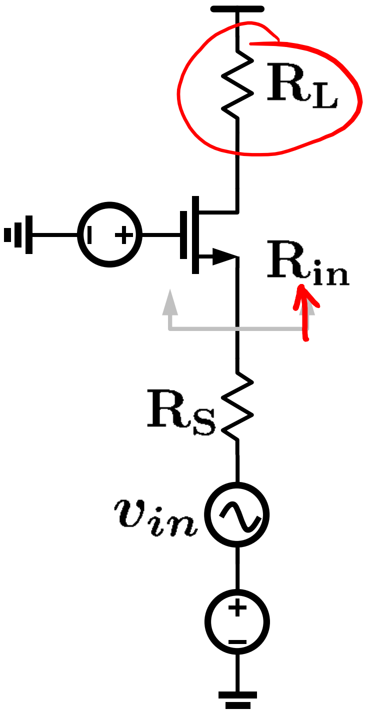
$r_o \to \infty$

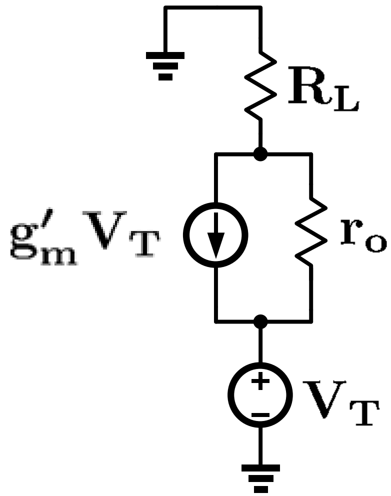
$$g'_m = g_m + g_{mb}$$

$V_{out} = g'_m v_{in} \cdot R_L$

$$\frac{v_{out}(s)}{v_{in}(s)} = \frac{1 + g'_m r_o}{r_o + R_S + g'_m r_o R_S + R_L} R_L$$

CG Amplifier Input Impedance





$$R_L I_T + r_o \left(I_T - g'_m V_T\right) = V_T$$

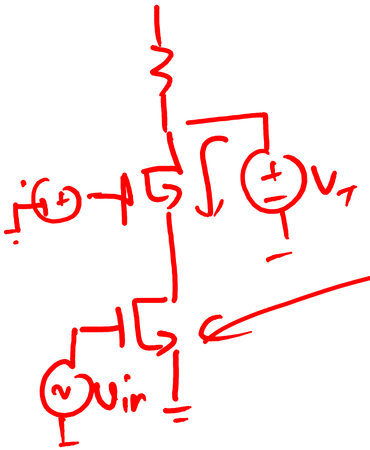
$$R_{in} = \frac{V_T}{I_T} = \frac{R_L + r_o}{1 + g'_m r_o}$$

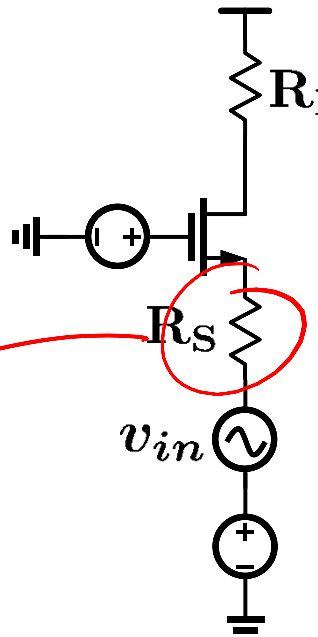
$$R_{in} \approx \frac{1}{g'_m}\left(1 + \frac{R_L}{r_o}\right) \text{ if } g'_m r_o \gg 1$$

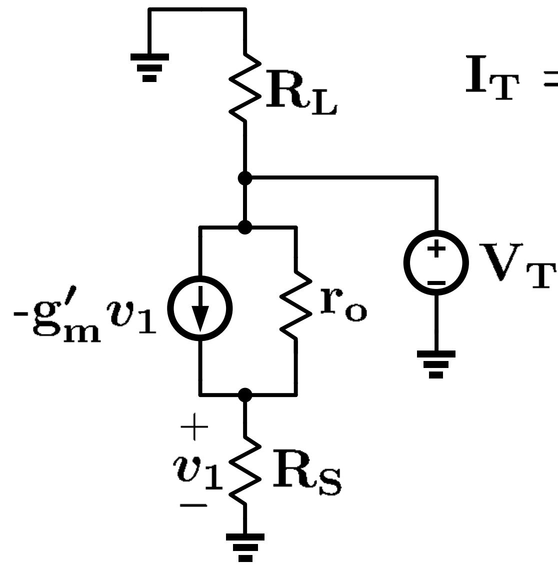
$$R_{in} \approx \frac{1}{g'_m} \text{ if } R_L \ll r_o \text{ and } g'_m r_o \gg 1$$

$$R_{in} \approx \frac{R_L}{g'_m r_o} \text{ if } R_L \gg r_o \text{ and } g'_m r_o \gg 1$$

CG Amplifier Output Impedance







$$I_T = I'_T + I_{R_L}$$

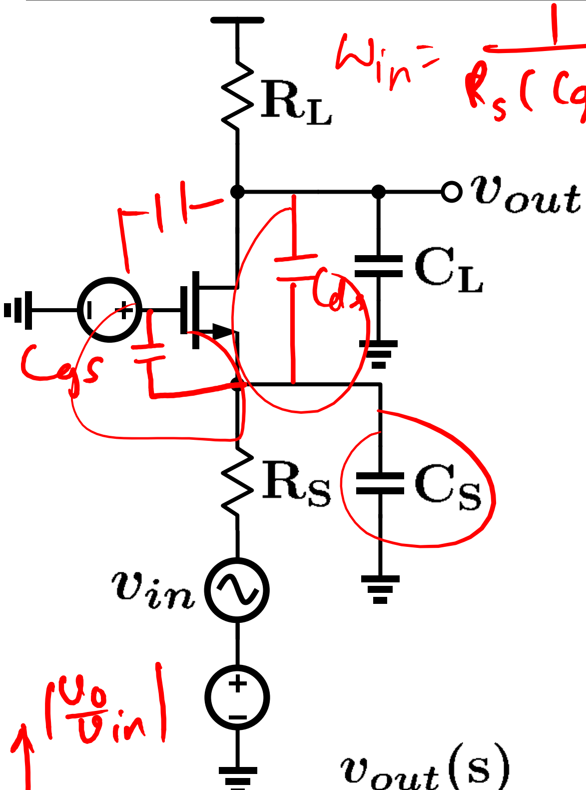
$$V_T = r_o\left(I'_T + g'_m R_S I'_T\right) + I'_T R_S$$

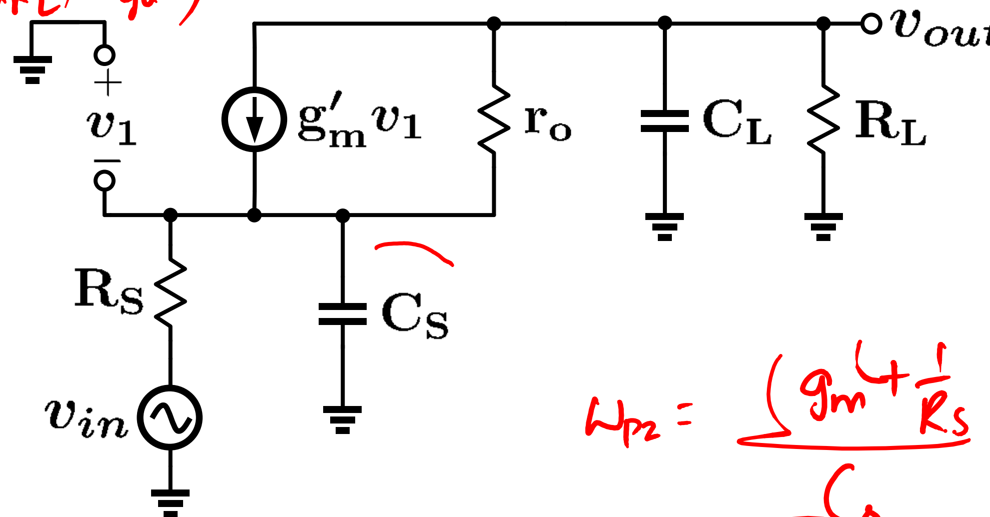
$$R_{out} = \frac{V_T}{I_T} = R_L \parallel \left(r_o + R_S + g'_m r_o R_S\right)$$

$$R_{out} \approx R_L \parallel r_o\left(1 + g'_m R_S\right)$$

CG Amplifier Frequency Response

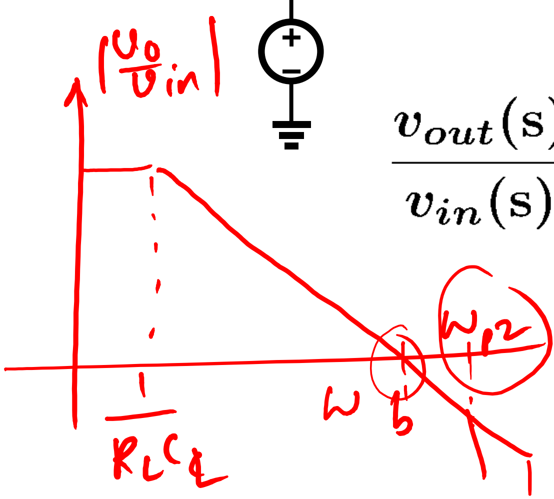
$$\omega_{in} = \frac{1}{R_s\left(C_{gs} + (1+g_m R_L)\,C_{gd}\right)}$$





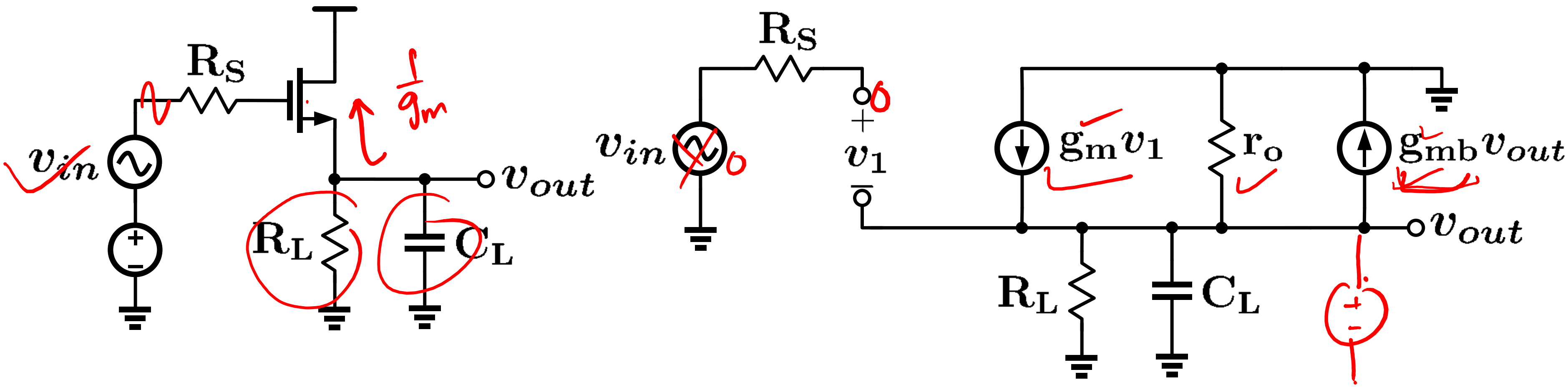
$$\omega_{p2} = \frac{\left(g_m + \frac{1}{R_S}\right)}{C_S} \approx \frac{1}{R_S C_S}$$

if $r_o = \infty$



$$\frac{v_{out}(s)}{v_{in}(s)} = \frac{g'_m R_L}{1 + g'_m R_S}\,\frac{1}{\left(1 + \dfrac{C_S}{\left(g'_m + \dfrac{1}{R_S}\right)} s\right)\left(1 + R_L C_L s\right)}$$

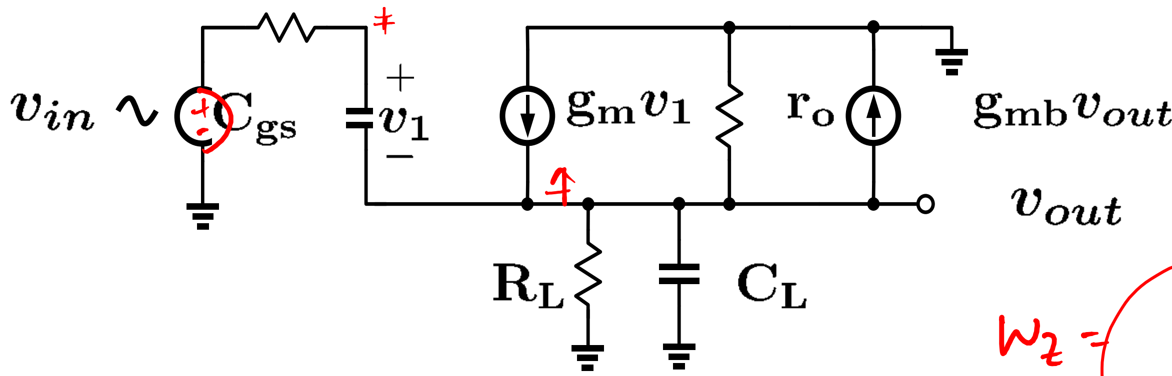
Source Follower



$$R'_L = r_o \parallel \frac{1}{g_{mb}} \parallel R_L$$

$$\frac{v_{out}(s)}{v_{in}(s)} = \frac{R'_L}{R'_L + \frac{1}{g_m}} = \frac{g_m}{gm + \frac{1}{R'_L}} = \frac{g_m R'_L}{1 + g_m R'_L}$$

Source Follower Frequency Response



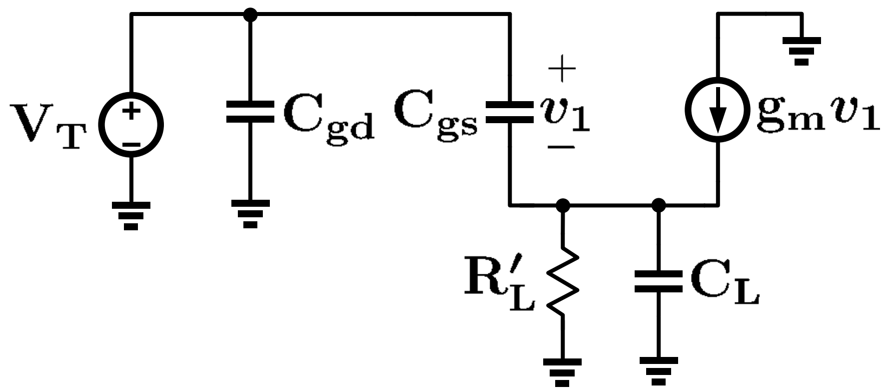
if $R_S = 0$ and $C_{gd} = 0$

$$\frac{v_{out}(s)}{v_{in}(s)} = \frac{g_m}{g_m + \frac{1}{R_L'}} \frac{1 + sC_{gs}/g_m}{\left(1 + \frac{s(C_{gs} + C_L)}{g_m + \frac{1}{R_L'}}\right)}$$

$$\omega_z = \frac{g_m}{C_{gs}}$$

$$\omega_p = \frac{g_m + \frac{1}{R_L'}}{C_{gs} + C_L} \approx \frac{g_m}{C_{gs} + C_L} = \frac{g_m}{C_{gs}}$$

Source Follower Input Impedance



$$I_T = I'_T + I_{C_{gd}}$$

$$V_T = \frac{I'_T}{sC_{gs}} + \left(I'_T + \frac{g_m I_T}{C_{gs}s}\right)\left(R'_L \,||\, \frac{1}{C_L s}\right)$$

$$Z_{in}(s) = \left[\frac{1}{C_{gs}s} + \left(1 + \frac{g_m}{C_{gs}s}\right)\left(\frac{R'_L}{1 + R'_L C_L s}\right)\right] \,||\, \frac{1}{C_{gd}s}$$

Source Follower Input Capacitance

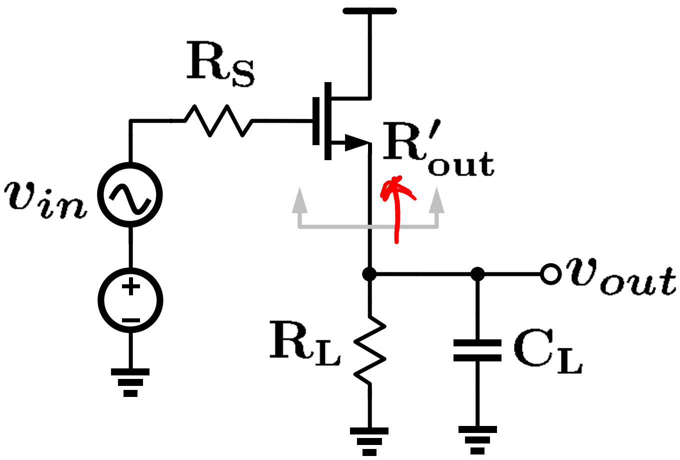
$$\mathrm{for}\ \left|R_L' C_L s\right| \ll 1$$

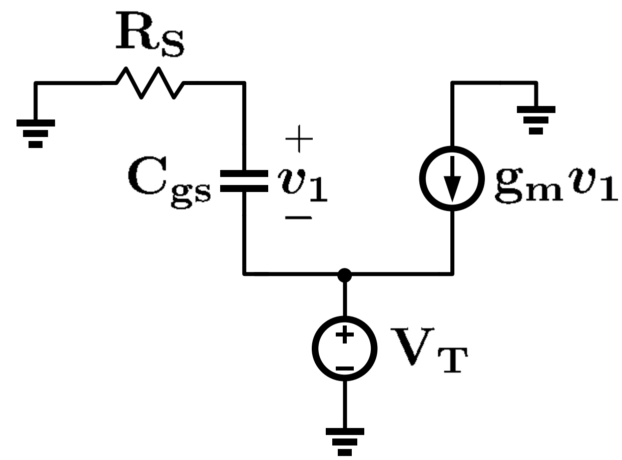
$$Z_{in}(s) \approx \left[\frac{1}{C_{gs}s} + (1 + \frac{g_m}{C_{gs}s})R_L'\right] \| \frac{1}{C_{gd}s}$$

$$C_{in}(s) \approx C_{gd} + C_{gs}(\frac{g_{mb}}{g_m + g_{mb}})$$

A very small input capacitance until moderate frequencies

Source Follower Output Impedance





$$v_1 C_{gs} s + g_m v_1 = -I_T$$

$$v_1 C_{gs} R_S s + v_1 = -V_T$$

$$Z_{out}(s) = \frac{R_S C_{gs} s + 1}{g_m + C_{gs} s} \,||\, \frac{1}{sC_L} \,||\, R'_L$$

A closer look at SF Output Impedance

$$Z_{out}(s) \approx \frac{1 + R_S C_{gs} s}{g_m + C_{gs} s}$$