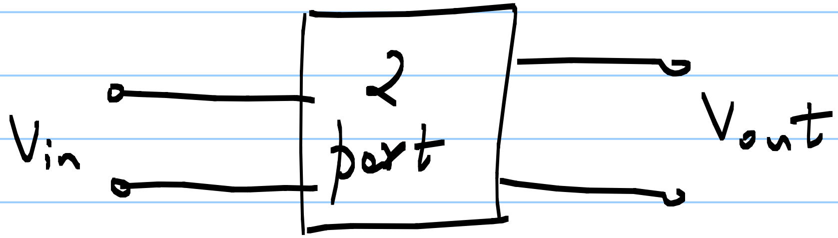
5/8/2020

Note Title

05-08-2020

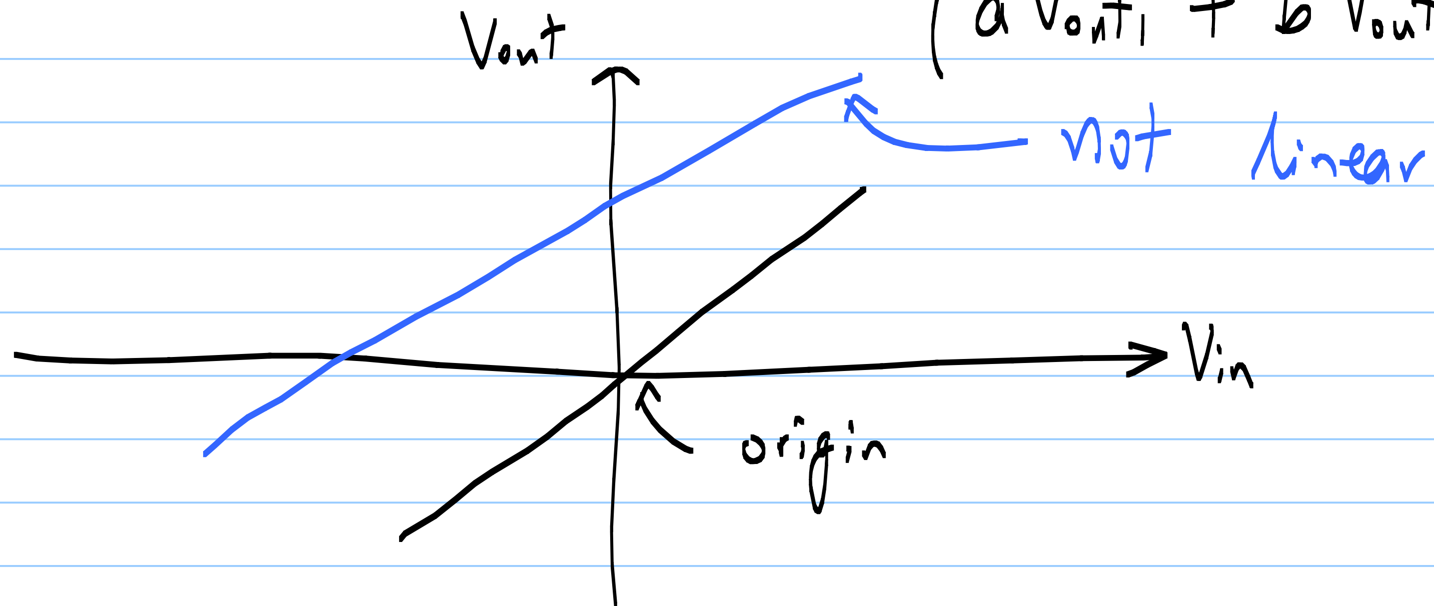
Lecture 2



Is this linear?

Based on <u>Superposition</u>

$$\left.\begin{array}{l} V_{in_1} \rightarrow V_{out_1} \\ V_{in_2} \rightarrow V_{out_2} \end{array}\right\} \quad (a V_{in_1} + b V_{in_2}) \downarrow (a V_{out_1} + b V_{out_2}) \quad \text{for all } a, b, V_{in_1}, V_{in_2}$$



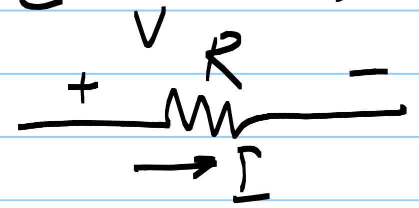
* LTI systems characterized by Impulse Response
* All practical systems are Non-linear

Linear Elements (2-terminal 1 port systems)

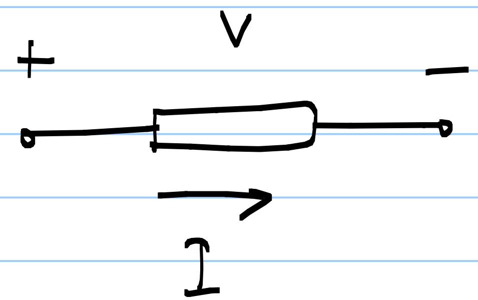
"elements"

linear elements: R, L, C → defined by V-I relationship

e.g.



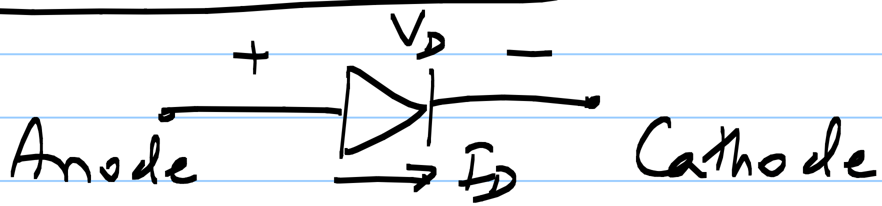
$$I = \frac{V}{R}$$



$$I = f(V)$$

Nonlinear 2-T element

Diode :



From SSD theory:

$$I_D = I_s \left[\exp\left(\frac{V_D}{V_t}\right) - 1 \right]$$

saturation current

thermal voltage

$$V_t = \frac{kT}{q} \approx 25\,mV$$
@ 300K

If $V_D > 0 \Rightarrow$ Diode is "forward biased"

$V_D < 0 \Rightarrow$ " reverse biased"

Some approximations :

1) If $V_D \gg$ few V_t,

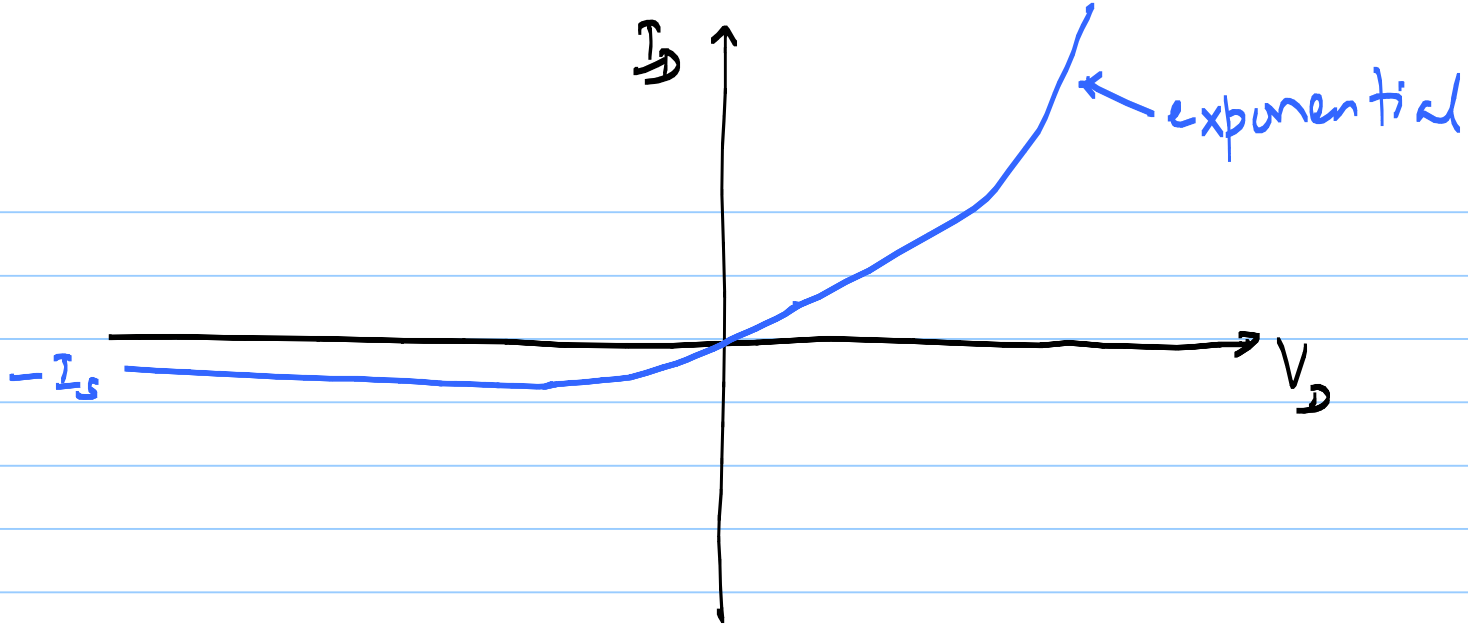
$$\exp\left(\frac{V_D}{V_t}\right) \gg 1$$

$$I_D \approx I_s \exp\left(\frac{V_D}{V_t}\right)$$

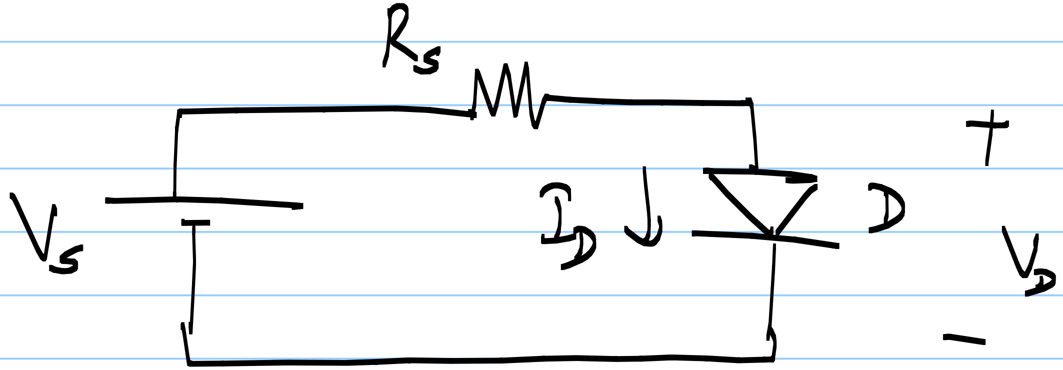
$$V_D = V_t \ln\left(\frac{I_D}{I_s}\right)$$

2) If $\frac{V_D}{V_t} << 0$, $\exp\left(\frac{V_D}{V_t}\right) << 1$

$$\Rightarrow \quad I_D \approx -I_s$$



Circuit Analysis w/ diodes



KVL :

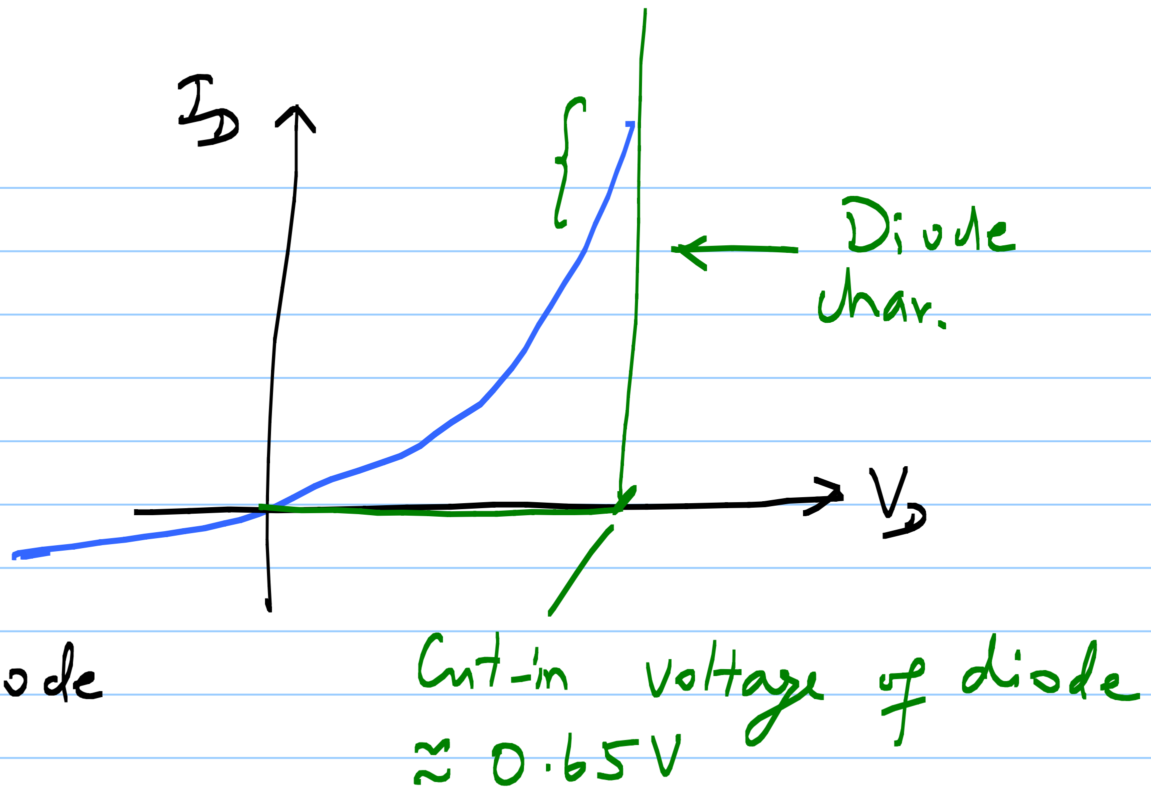
$$V_s = V_{R_s} + V_D$$

$$V_s = I_D R_s + V_D = I_D R_s + V_t \ln\left(1 + \frac{I_D}{I_s}\right)$$

Solution

1) Zeroth order:

Assume that the voltage drop across a forward biased diode $\approx 0.65V$



$$I_D \approx \frac{V_S - 0.65}{R_S}$$

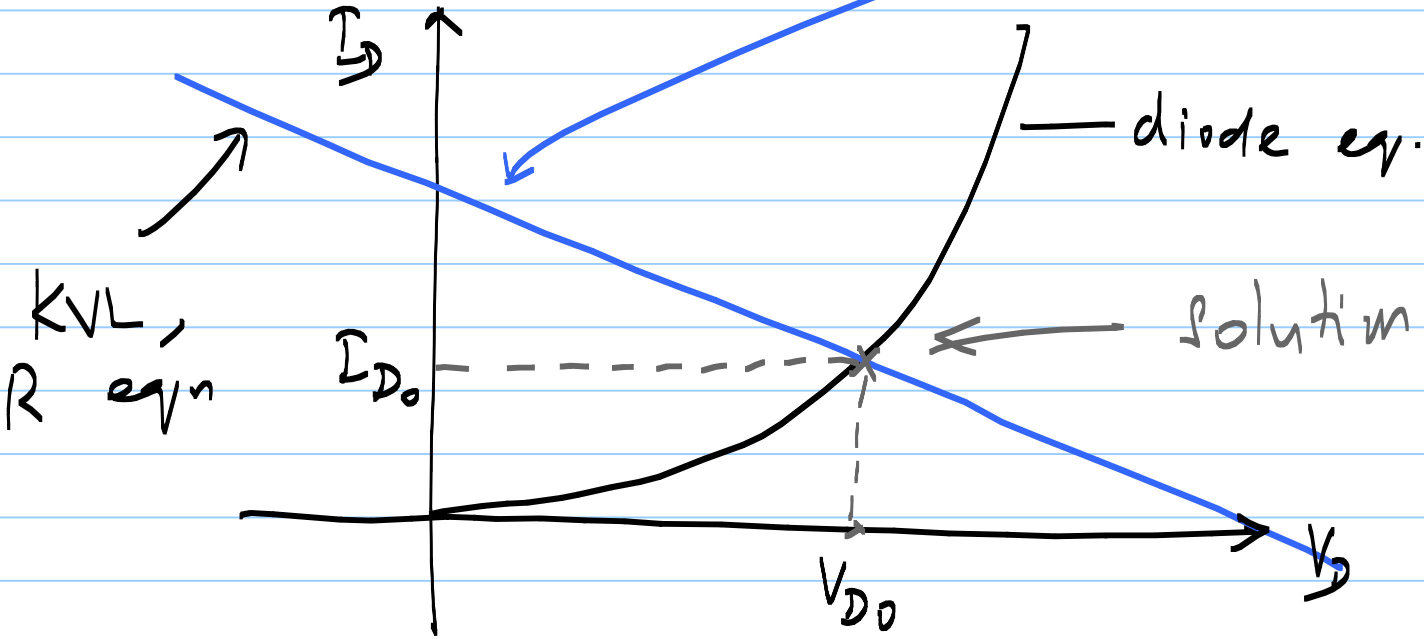
2) Exact solution: Iteration (actual $\overset{NL}{\text{characteristic}}$)

Numerical Solution

3) Graphical solution on $I_D - V_D$ plot

$I_{R_s} = I_D$ (KCL)

$$I_{R_s} = \frac{V_s - V_D}{R_s}$$



"operating (bias) point"

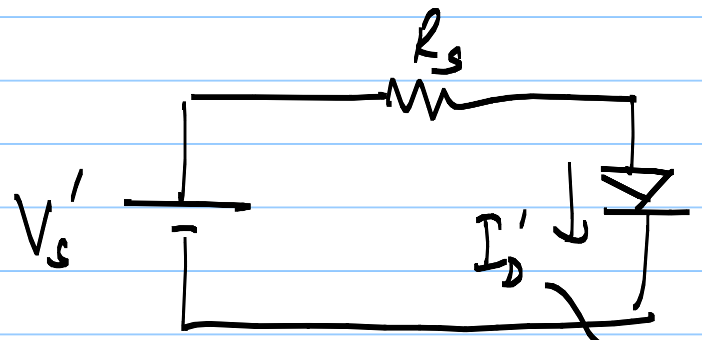
If $V_s \to V_s' = V_s + \Delta V_s$

new op. pt.?

6/8/2020

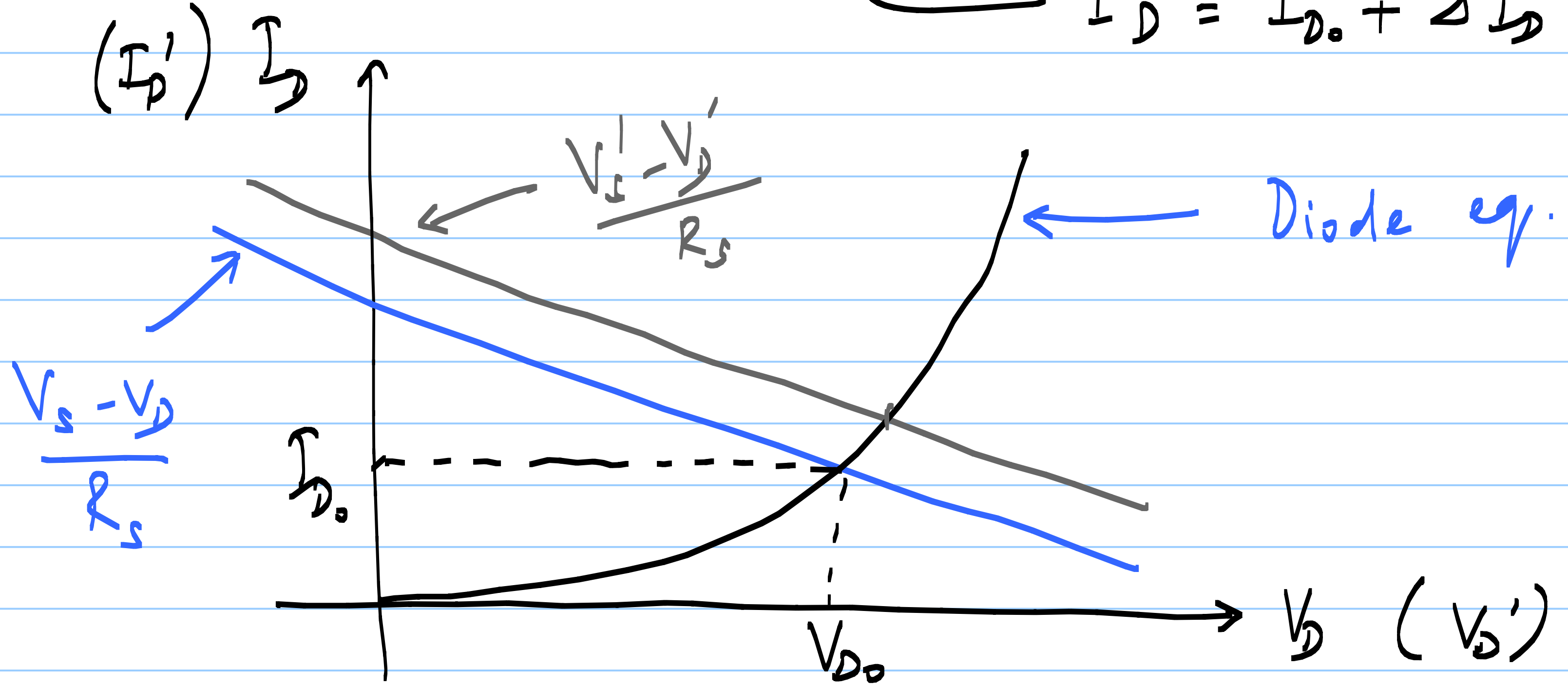
Lecture 3

* If $V_s' = V_s + \Delta V_s$ (increment), what is V_D', I_D'



$V_D' = V_{D0} + \Delta V_D$

$I_D' = I_{D0} + \Delta I_D$



Orig.

$$\frac{V_s - V_{D_0}}{R_s} = I_{D_0} = I_s \left[\exp\left(\frac{V_{D_0}}{V_t} \right) - 1 \right]$$

New

$$\frac{V_s' - V_D'}{R_s} = I_D' = I_s \left[\exp\left(\frac{V_D'}{V_t} \right) - 1 \right]$$

$$\frac{(V_s + \Delta V_s) - (V_{D_0} + \Delta V_D)}{R_s} = I_{D_0} + \Delta I_D$$

$$= I_s \left[\exp\left(\frac{V_{D_0} + \Delta V_D}{V_t} \right) - 1 \right]$$

$$\frac{V_S - V_{D0}}{R_S} + \frac{\Delta V_S - \Delta V_D}{R_S} = I_{D0} + \Delta I_D$$

$= \quad ?$ —— Taylor series expansion around (V_{D0}, I_{D0})

for $Y = f(x)$ around x_0

$$Y = \sum_{n=0}^{\infty} \frac{f^{(n)}(x_0)}{n!} \cdot (x - x_0)^n$$

$$= f(x_0) + f'(x_0) \cdot (x - x_0)$$

$$+ \left(f''(x_0)/2\right) \cdot (x - x_0)^2 + \ldots$$

oper. pt.

$$I_s\left[\exp\left(\frac{V_{D0}+\Delta V_D}{V_t}\right)-1\right] = I_s\left[\exp\left(\frac{V_{D0}}{V_t}\right)-1\right]$$

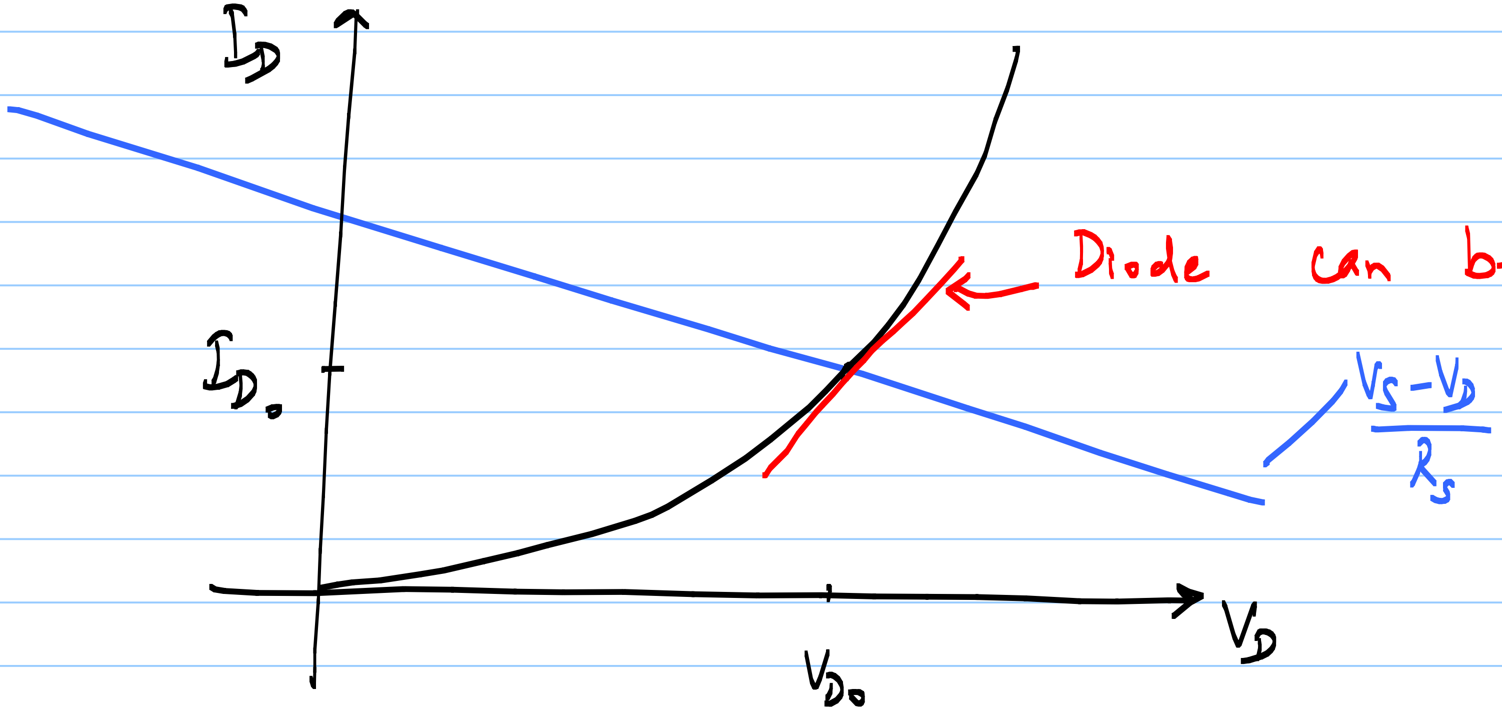
$$+\ \frac{I_s}{V_t}\exp\left(\frac{V_{D0}}{V_t}\right)\cdot(\Delta V_D)$$

$$+\ \cdots \longrightarrow \Delta V_D^{\,2},\ \Delta V_D^{\,3}\ \ldots$$

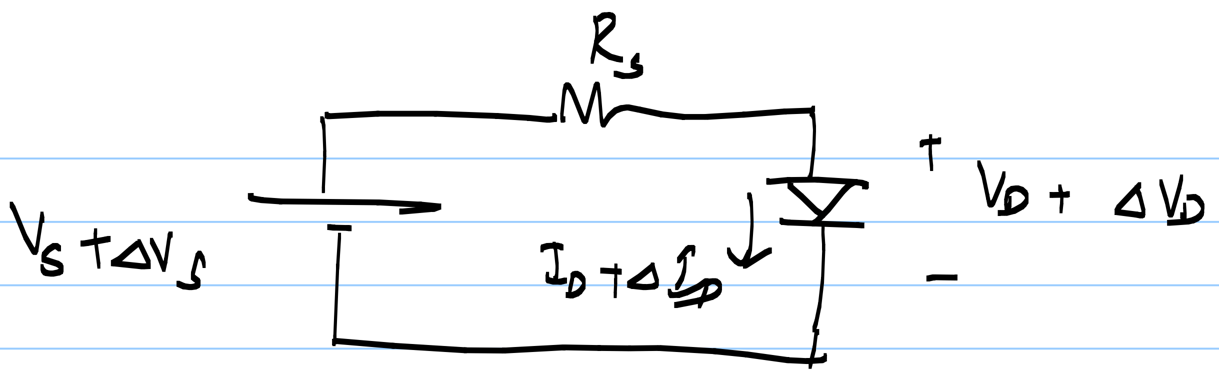
* For small increments (ΔV_D etc.), neglect $\Delta V_D^{\,2}$, $\Delta V_D^{\,3}$ ...

$$\cancel{\frac{V_s - V_{D0}}{R_s}} + \frac{\Delta V_s - \Delta V_{D0}}{R_s} = \cancel{I_{D0}} + \Delta I_D = \cancel{I_s\left[\exp\left(\frac{V_{D0}}{V_t}\right)-1\right]} + \frac{I_s}{V_t}\exp\left(\frac{V_{D0}}{V_t}\right)\cdot\Delta V_D$$

$$\underbrace{\frac{\Delta V_s - \Delta V_D}{R_s} = \Delta I_D = \frac{I_s}{V_t} \exp\left(\frac{V_{D0}}{V_t}\right) \cdot \Delta V_D}_{\text{Linear equations in } \Delta V_s,\ \Delta V_D,\ \Delta I_D}$$



Diode can be replaced by a linearized element for $\Delta V_D, \Delta I_D$



$$\Delta V_s = \Delta I_D \cdot R_s + \Delta V_D$$

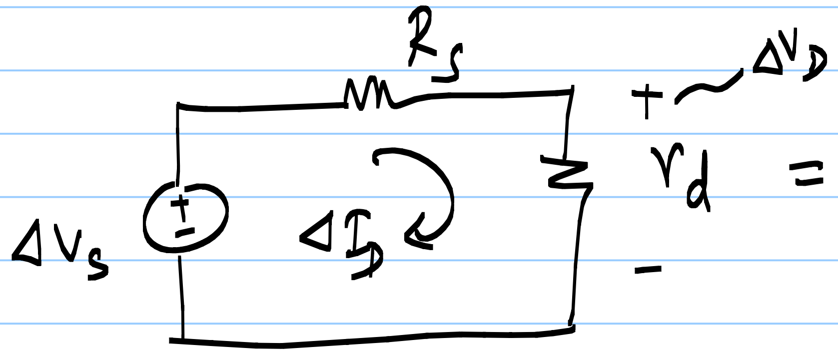
$$\Delta I_D = \underbrace{\frac{I_s}{V_t} \exp\left(\frac{V_{D0}}{V_t}\right)}_{\approx I_{D0}} \cdot \Delta V_D \approx \frac{I_{D0}}{V_t} \cdot \Delta V_D$$

$$\Delta V_D = \underbrace{\frac{V_t}{I_{D0}}}_{\text{"incremental resistance" of diode}} \cdot \Delta I_D$$

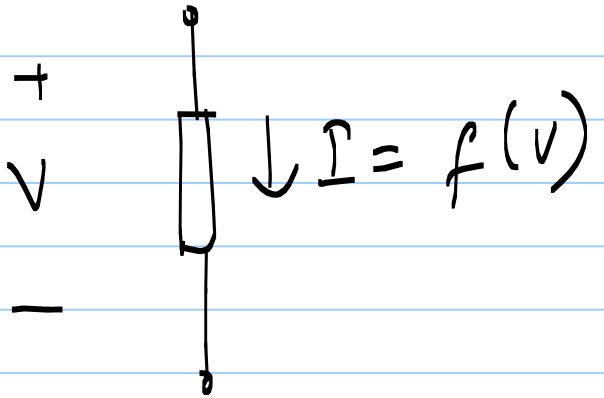
$$\Delta V_s = \Delta I_D \cdot R_s + \Delta I_D \cdot \frac{V_t}{I_{D0}}$$

* Any $I = f(V)$ can be linearized around the op. pt.
* Assume ΔV_D, ΔI_D etc are "small"

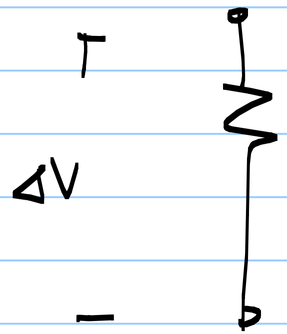
(linear) <u>Incremental eq. circuit</u> for $\Delta I_D = \dfrac{\Delta V_S}{R_S + \frac{V_t}{I_{D0}}}$



$r_d = \dfrac{V_t}{I_{D0}}$ = inc. res. of diode @ I_{D0}



$\Rightarrow$



$\Delta I = f'(V_0) \cdot \Delta V$

$r = \dfrac{1}{f'(V_0)}$

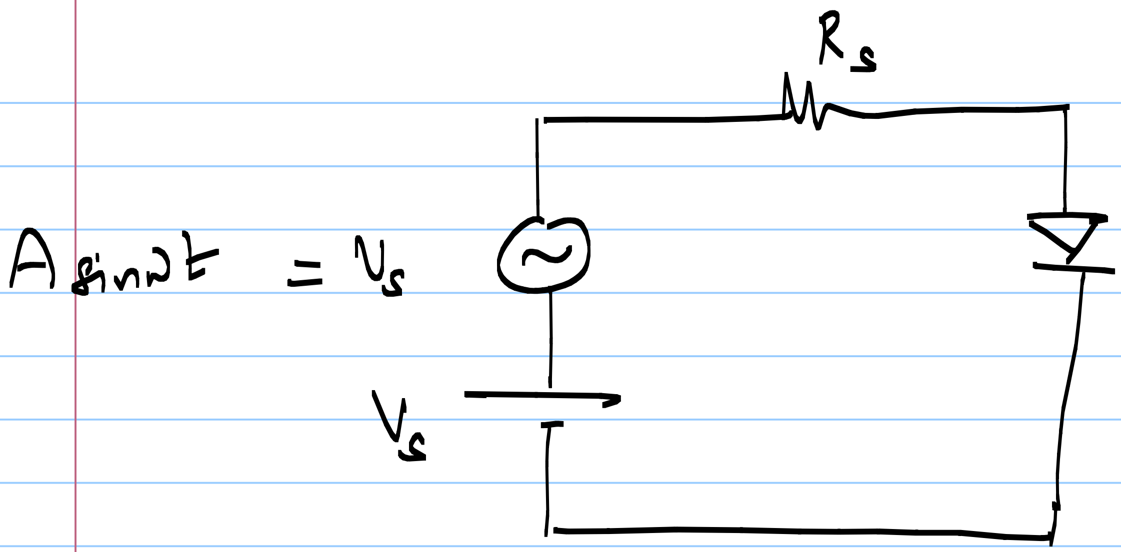
I, V, etc. $\Rightarrow$ DC

I_0, V_0 etc $\Rightarrow$ operating point,
bias point
Quiescent point

Δi, Δv $\Rightarrow$ incremental quantities

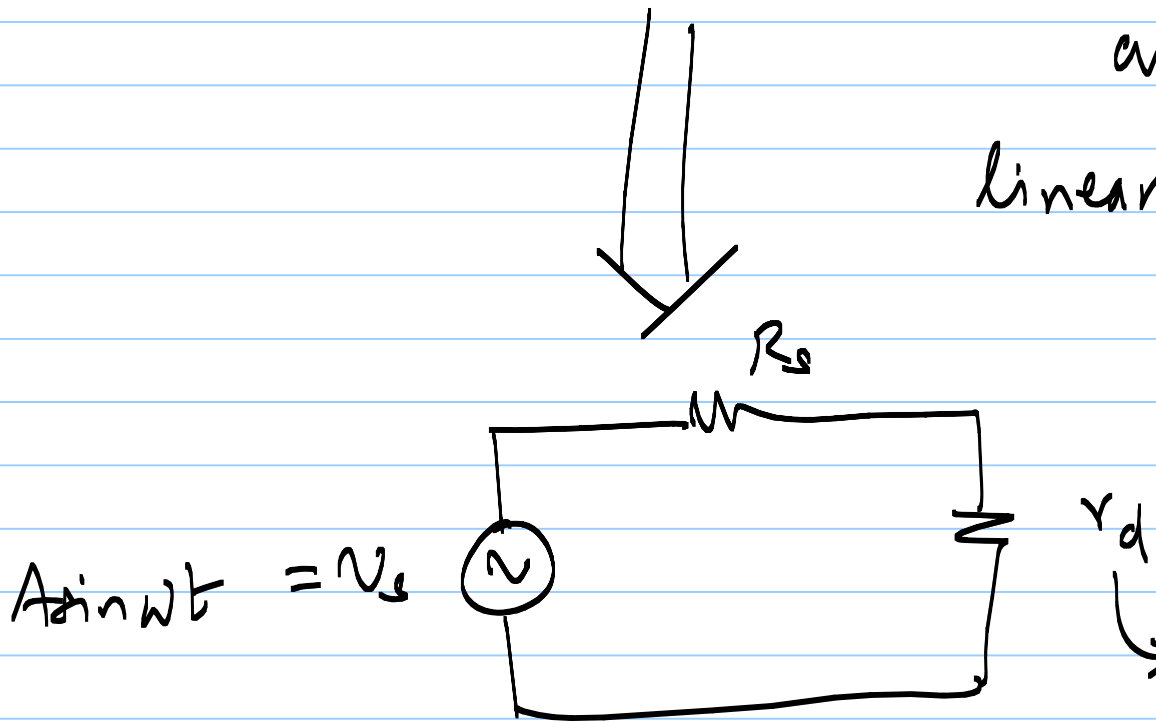
i, v $\Rightarrow$ small-signal quantities

$V_D = V_{D0} + \Delta V_D$ etc. $\Rightarrow$ total quantities



If A is very **small**:

@ every point of time, instantaneous increments are small enough that linear approx. is valid



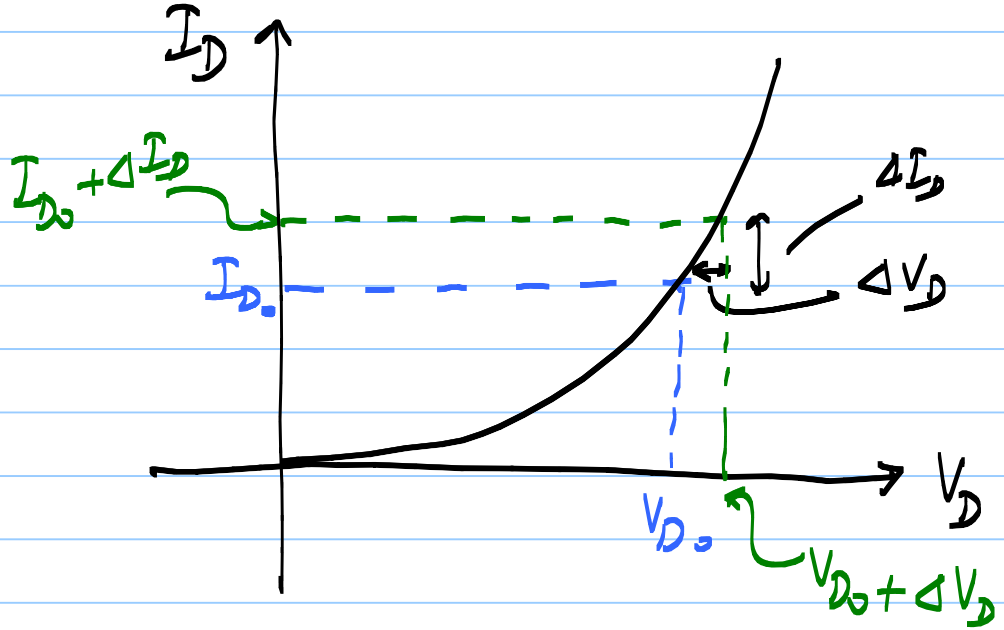
$r_d \to \dfrac{1}{f'(V_D)}$

small-signal equivalent network

For op. pt. : you have to solve the system of non-linear equations to determine I_{D0}, V_{D0} etc.

7/8/2020

Lecture 4



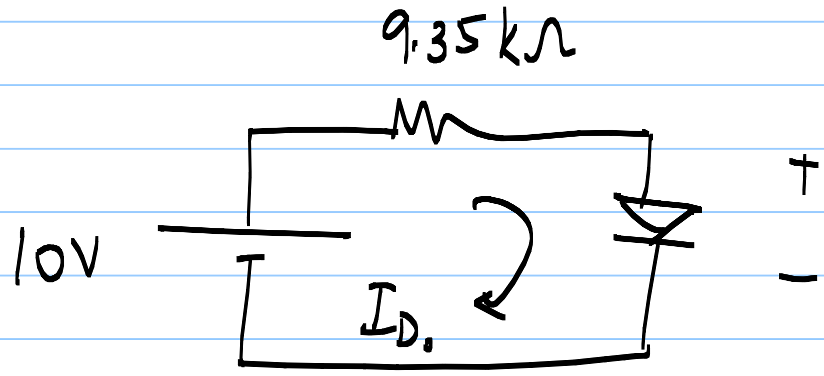
$$\frac{\Delta I_D}{\Delta V_D} = \left.\frac{dI_D}{dV_D}\right|_{(I_{D_0},\,V_{D_0})}$$

= inc. (r) conductance

dynamic (r) "

small-signal "

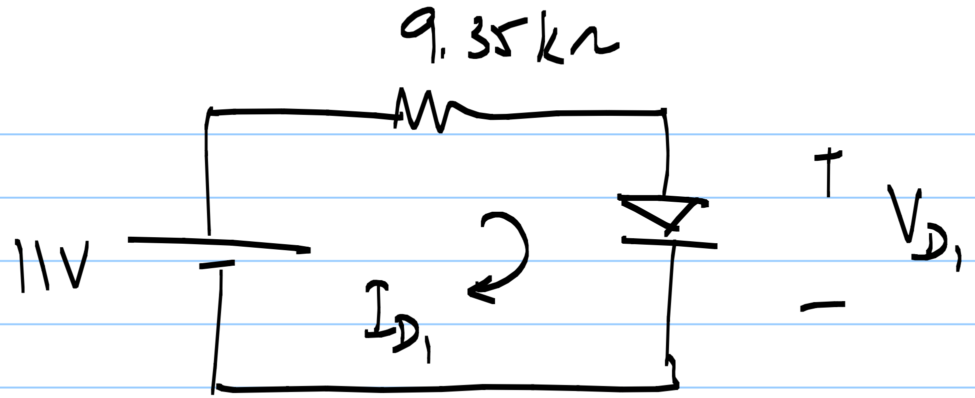
Example



$V_{D_0} \approx 0.65\,V$

$$I_{D_0} = \frac{10 - 0.65}{9.35\,k\Omega} = 1\,mA$$

change 10V → 11V

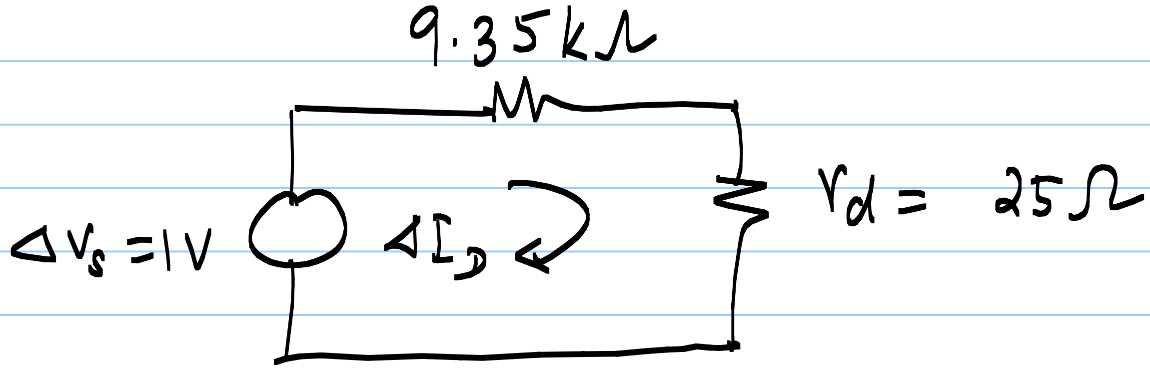


$$\Delta V_s = 1V$$

$$r_d = \frac{V_t}{I_{D_0}} = \frac{25mV}{1mA}$$

$$= 25\Omega$$

<u>Incremental Equivalent Circuit</u>



$$\Delta I_D = \frac{1\ V}{9350 + 25} \approx 106\mu A$$

$$I_{D_1} = I_{D_0} + \Delta I_D = 1.106mA$$

* If $V_s = 9V$, $I_{D_2} = ?$

$$\Delta V_s = -1V \Rightarrow \Delta I_D = -106\,\mu A$$

$$\Rightarrow I_{D_2} = 0.894\ mA$$

* If $V_s = 9.5V$, $I_{D_3} = I_{D_0} - \frac{1}{2}(106\,\mu A)$

. . .

* If $V_s = 10V + (1V)\sin\omega t$

$$I_D = 1mA + (106\,\mu A)\sin\omega t$$

<u>Next</u> : Is <u>linear approx.</u> valid?

$$\Delta V_D = \Delta I_D \cdot r_d = 106\,\mu A \times 25 \approx 2.7mV$$

$f''(V_{D_0}) = ?$

$$I_D = I_s \exp\left(\frac{V_D}{V_t}\right)$$

$$f''(V_{D_0}) = \frac{I_s}{V_t^2} \exp\left(\frac{V_{D_0}}{V_t}\right) \approx \frac{I_{D_0}}{V_t^2}$$

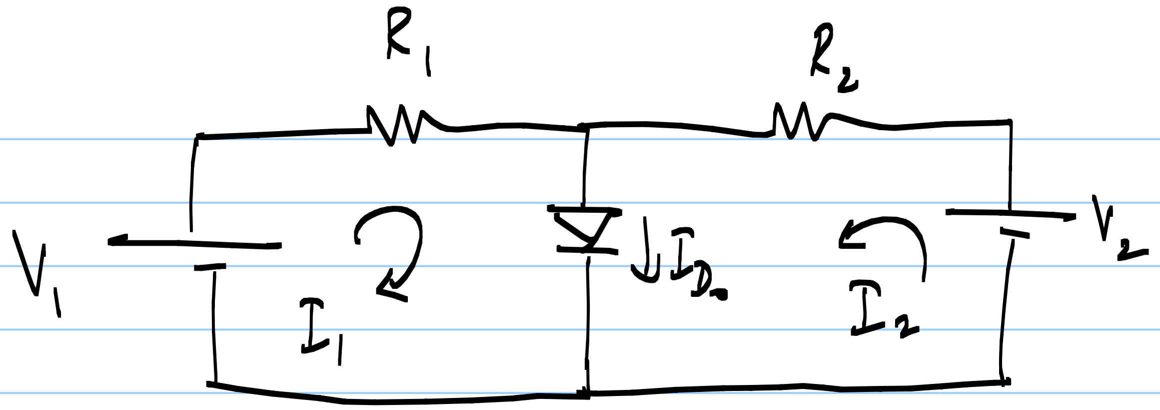
Taylor Series 3^{rd} term $= \dfrac{f''(V_{D_0})}{2} (\Delta V_D)^2$

$$= \frac{I_{D_0}}{2V_t^2} \cdot (\Delta V_D)^2$$

$$I_D = I_{D_0} + \frac{I_{D_0}}{V_t}(\Delta V_D) + \frac{I_{D_0}}{2V_t^2}(\Delta V_D)^2 + \cdots$$

compare these two

first error term is small if $\underset{2.7\,mV}{\Delta V_D} << \underset{50\,mV}{2V_t}$ valid



Assume
$V_D = 0.65$
if fwd. biased

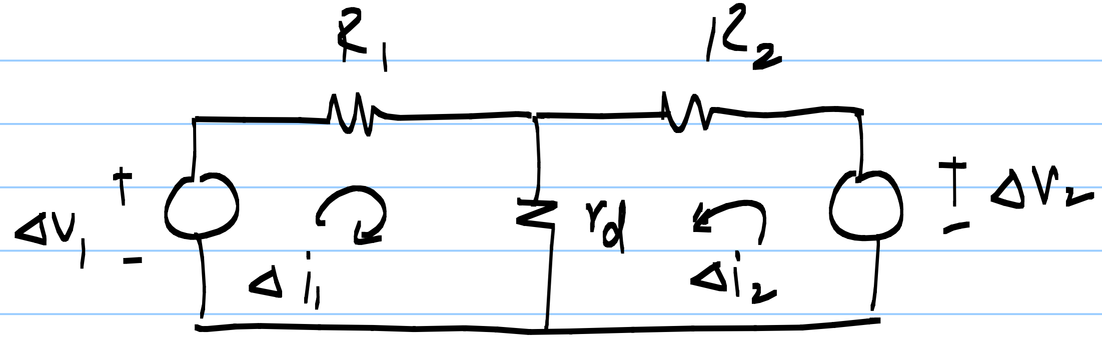
<u>Op. pt.</u> Use NL eqns.

$$I_1 = \frac{V_1 - 0.65}{R_1} \quad \& \quad I_2 = \frac{V_2 - 0.65}{R_2}$$

$$I_{D_0} = I_1 + I_2$$

$$V_1 \rightarrow V_1 + \Delta v_1 \quad \& \quad V_2 \rightarrow V_2 + \Delta V_2$$

<u>Inc. picture</u> :



$$r_d = \frac{V_t}{I_{D_0}}$$

$$\Delta i_d = \Delta i_1 + \Delta i_2$$

Incremental network

* All elements are linear
* No dc sources
* Inc. voltage across NL element

$$\Delta v_d = \Delta i_d \cdot r_d \qquad \left\{ \Delta v_d = \frac{r_d}{r_d + 9.35\,k\Omega} \cdot \Delta v_s \right\}$$

* Total voltage across diode

= Quiescent voltage + Incremental voltage

$$\{ 0.65\,V + \Delta v_d \}$$

* Can extend to networks with multiple NL elements

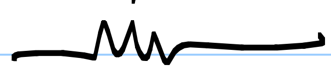
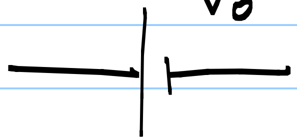
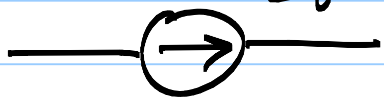
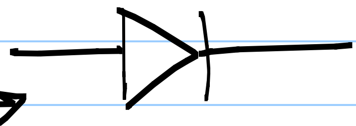
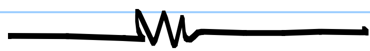
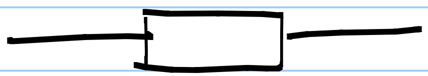
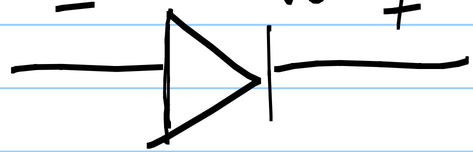
→ replace each NL element with its inc. resistance $r_i = \dfrac{1}{f_i'(v_{i0})}$

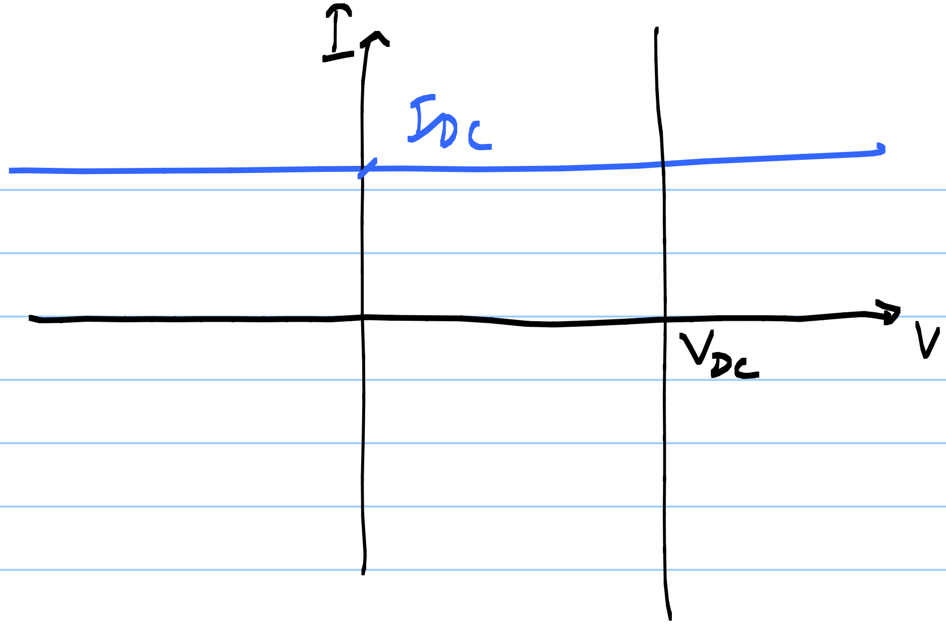
12/8/2020

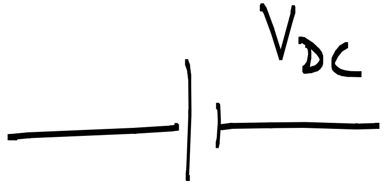
Lecture 5

1) Find operating point
   - → solve non-linear equations
   - → incremental Δvs & Δis are dependent on the Q-pt (r_is are dep. on Q-pt.)
2) Draw the incremental equivalent circuit (linear network) and solve for Δvs & Δis
3) Total Vs and Is

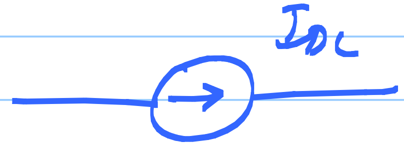
   = Quiescent V/I + Incremental V/I

Element	Incremental equivalent
 R	 R
 V_o	 short circuit
 I_o	 open circuit
 $+\ V_D\ -$, I_D	 $r_D = \dfrac{V_t}{I_D}$
 $I = f(V)$, I_o, $+\ V_o\ -$	 $r = \dfrac{1}{f'(V_o)}$
 $-\ V_o\ +$	 open circuit



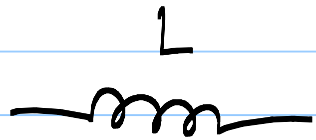


$$r_{V_{DC}} = \frac{1}{f'(V)} = 0$$

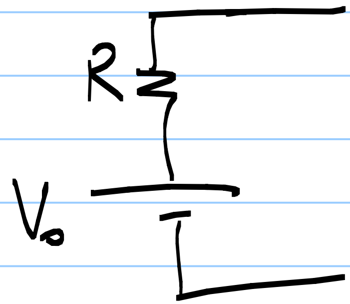


$$r_{I_{dc}} = \frac{1}{f'(V)} = \infty$$

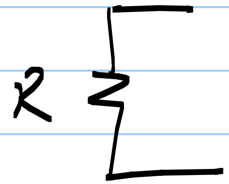
<u>HW</u> :

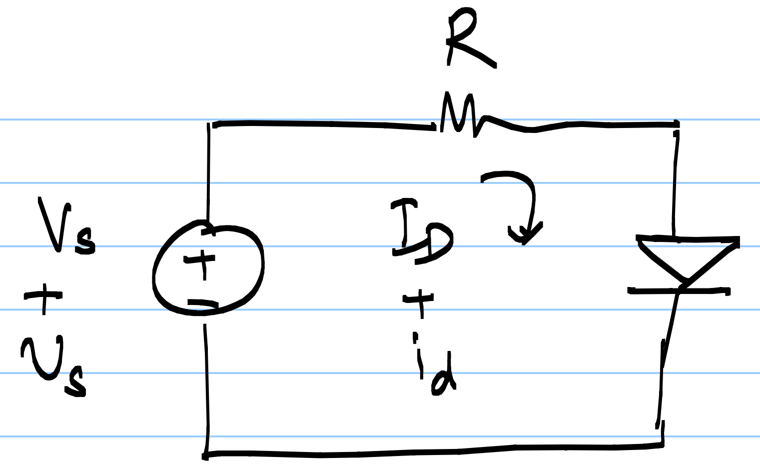


Thv. eq.



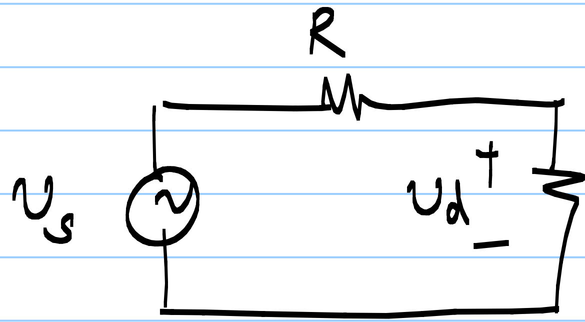
$\Rightarrow$





$+$ $V_D + v_d$ $-$

Is it possible for $v_d > v_s$?

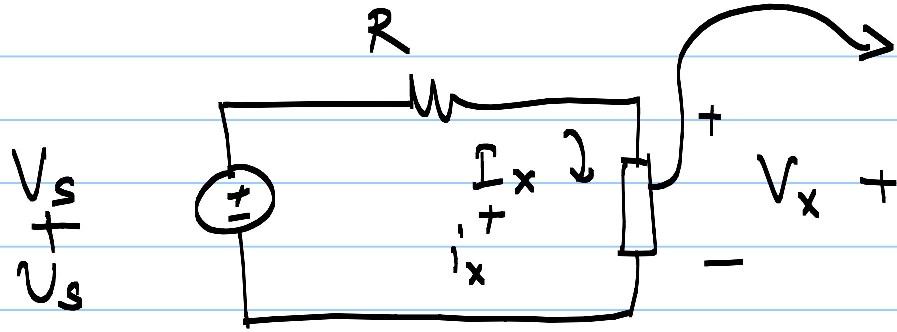


$r_d = \dfrac{V_t}{I_D}$

$$v_d = \frac{r_d}{R+r_d} \cdot v_s$$

small-signal gain $= \dfrac{v_d}{v_s}$

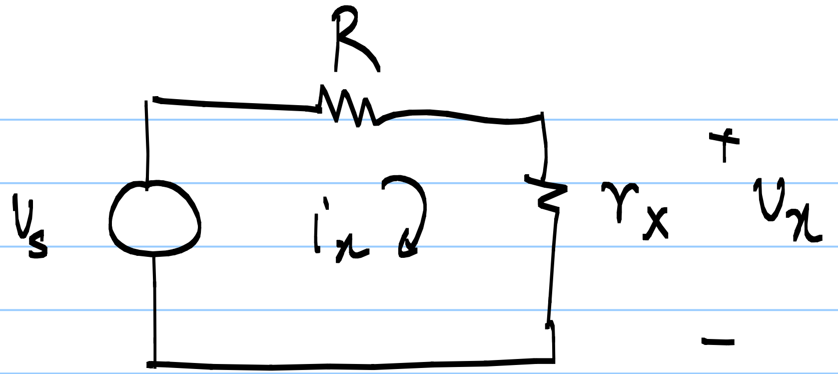
$$= \frac{r_d}{R+r_d}$$



$I = f(V)$

$+$ $V_x + v_x$ $-$

$$r_x = \frac{1}{f'(V_x)}$$

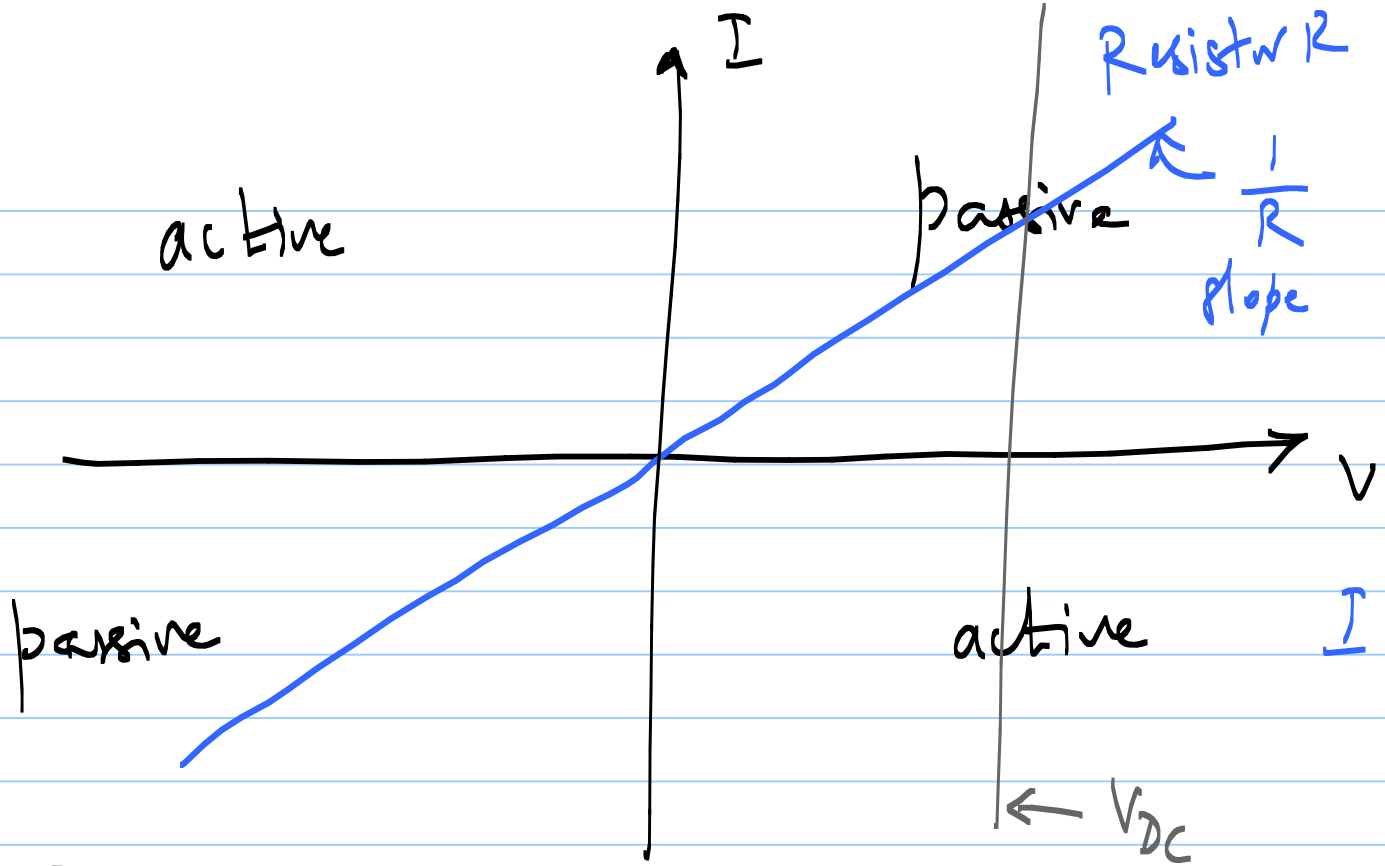


$$\frac{v_x}{v_s} = \frac{r_x}{R + r_x}$$

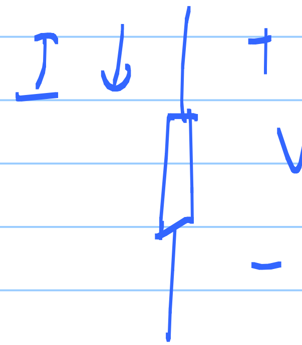
If r_x is negative, it is possible for $\left|\frac{v_x}{v_s}\right|$ to be > 1

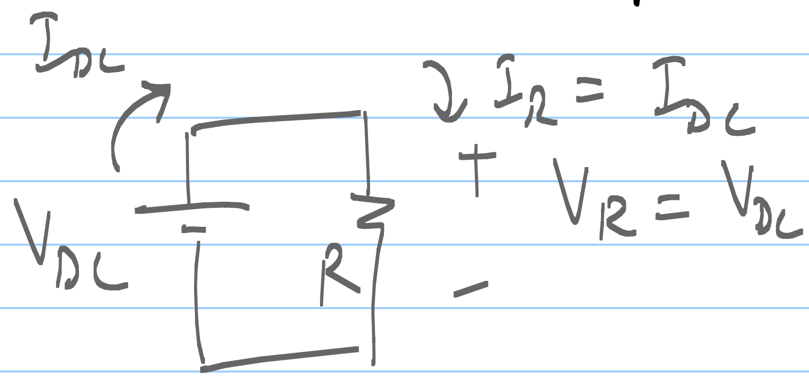
slope of $f(v)$ is negative

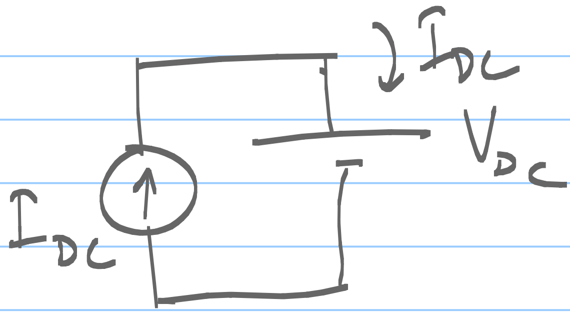
"Tunnel Diode"

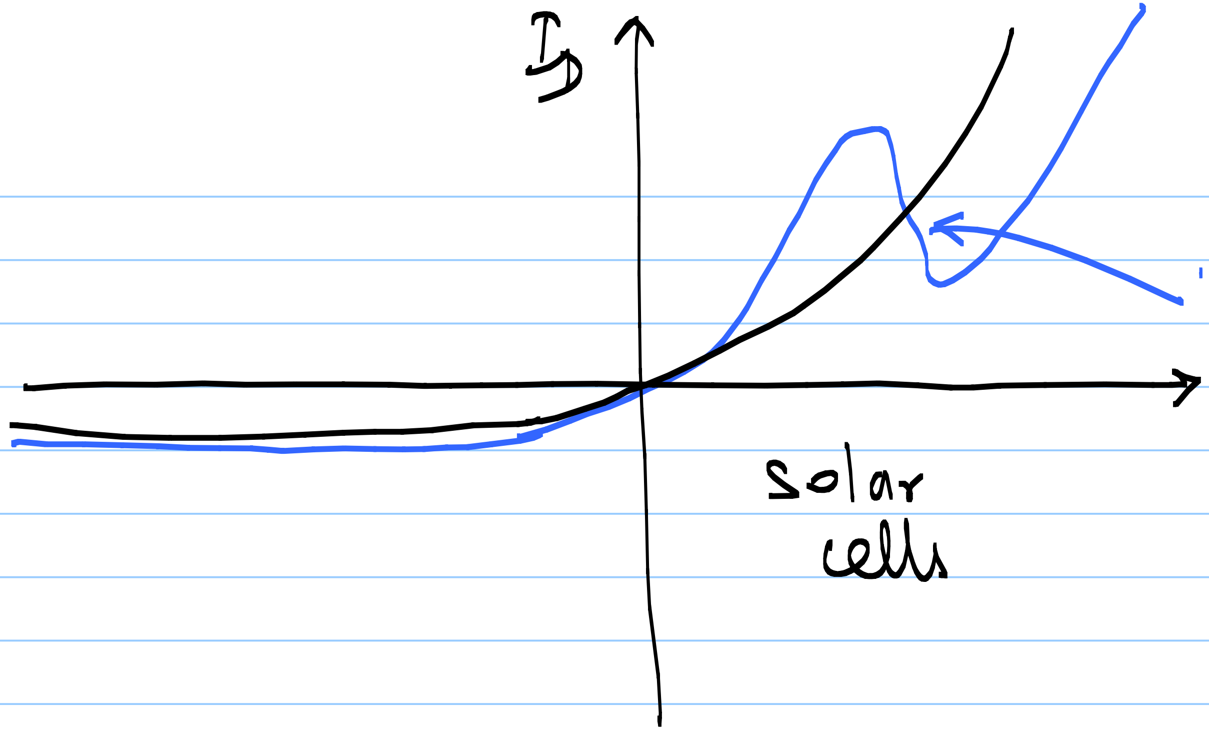


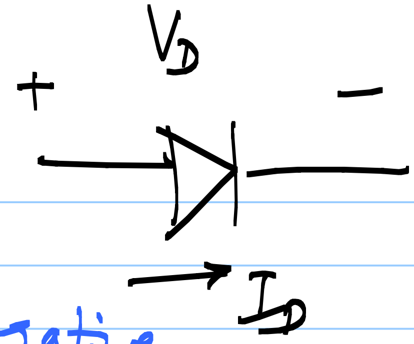
absorbs or dissipates electrical power = passive











"negative slope regim" bias the non-linear device in this regim to get "gain"

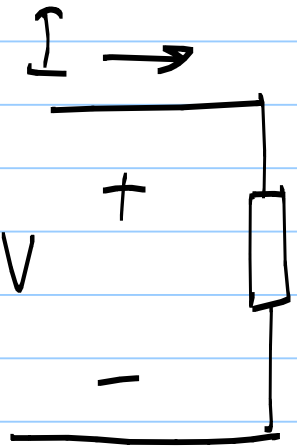
1) Is it possible to get "gain" from purely linear passive devices?

* No power gain possible
* v or i gain possible (transformers ...)

2) Non linear passive devices?

→ incremental / small-signal voltage and power gain possible

→ Battery power is used up (in overall power)



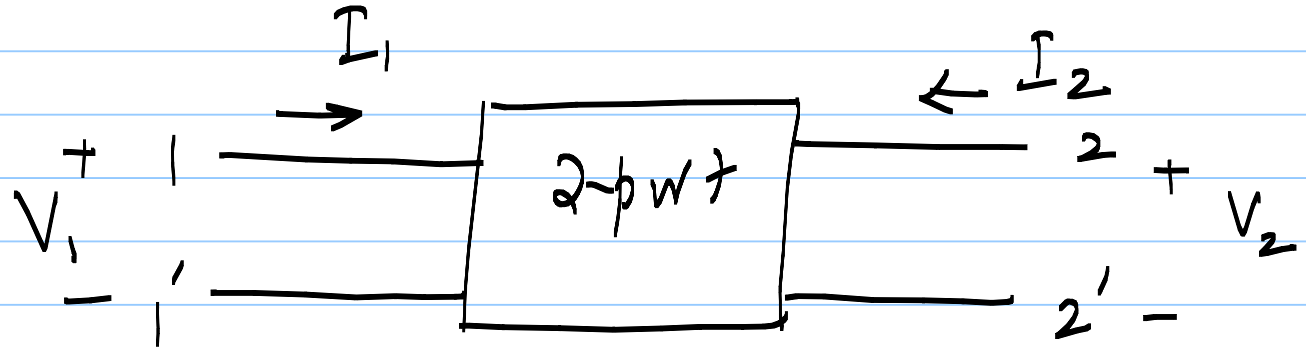
2-T

1-port

$$r = \frac{1}{f'(V_Q)}$$

1 parameter describes ss behaviour

2-port networks



4 parameters required to describe 2-port network

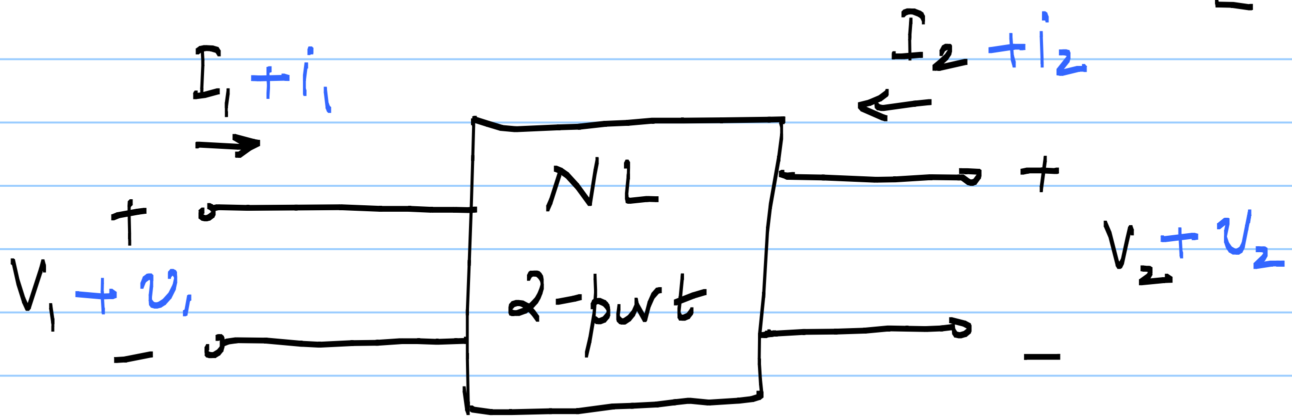
$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \underbrace{\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}}_{\text{(impedance) } Z\text{-parameters}} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \underbrace{\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

(Admittance) Y-parameters

13/8/20

Lecture 6



$$[I] = [Y][V]$$

$$I_1 = f(V_1, V_2)$$

$$I_2 = g(V_1, V_2)$$

$$I_1' = I_1 + i_1 = f(V_1 + v_1, V_2 + v_2)$$

$$I_1 = f(V_1, V_2)$$

* If v_1 & v_2 are small, $f()$ & $g()$ can be expanded in a 2-D Taylor Series around op. pt. {approx. 3D surface by a tangential plane @ op. pt.}

$$I_1' = I_1 + i_1 \approx I_1 + \frac{\partial f}{\partial V_1} \cdot v_1 + \frac{\partial f}{\partial V_2} \cdot v_2$$

$$\Rightarrow \left.\begin{aligned} i_1 &= \frac{\partial f}{\partial V_1} \cdot v_1 + \frac{\partial f}{\partial V_2} \cdot v_2 \\ i_2 &= \frac{\partial g}{\partial V_1} \cdot v_1 + \frac{\partial g}{\partial V_2} \cdot v_2 \end{aligned}\right\}$$

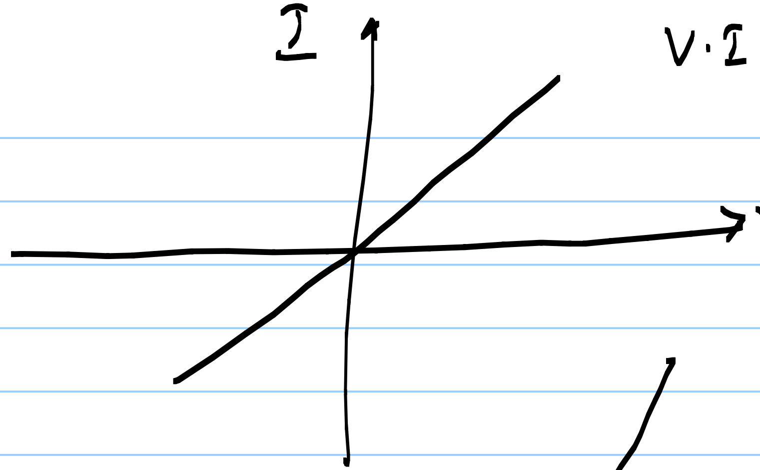
linear relationship between i_1, i_2, v_1, v_2

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \underbrace{\begin{bmatrix} \overset{y_{11}}{\partial f/\partial V_1} & \overset{y_{12}}{\partial f/\partial V_2} \\ \underset{y_{21}}{\partial g/\partial V_1} & \underset{y_{22}}{\partial g/\partial V_2} \end{bmatrix}}_{\text{incremental } y\text{-matrix}} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

Graphical Representation

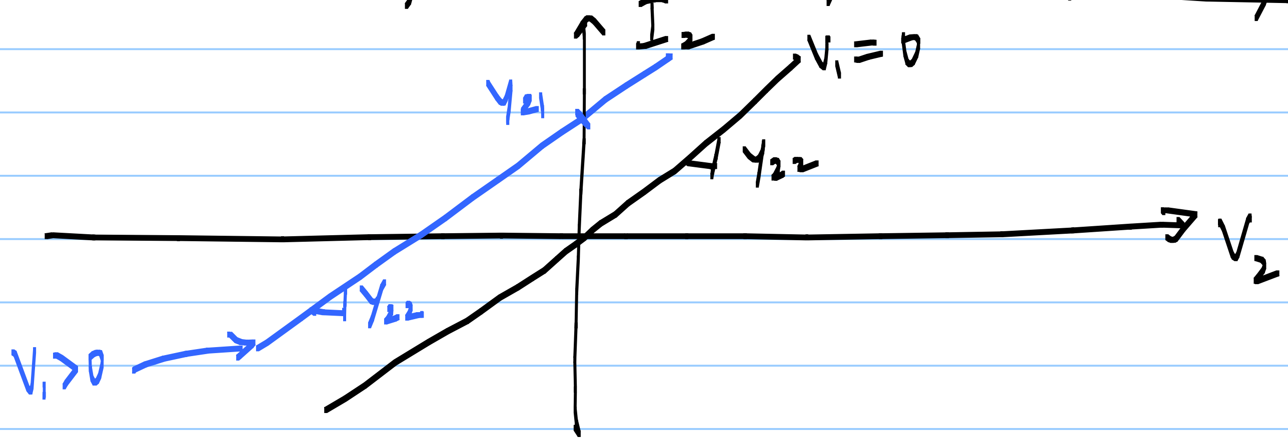
1) Linear passive 1-port

$V \cdot I \geq 0$



2) NL passive 1-port $V \cdot I \geq 0$

3) Linear 2-port : Input & output characteristics



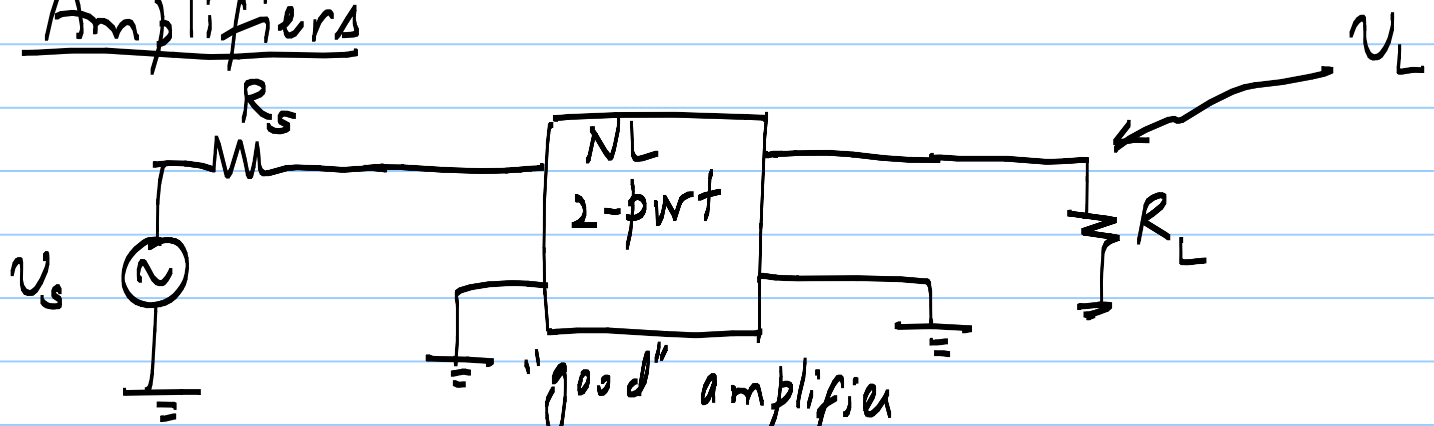
$$I_1 = Y_{11} V_1 + Y_{12} V_2$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2$$

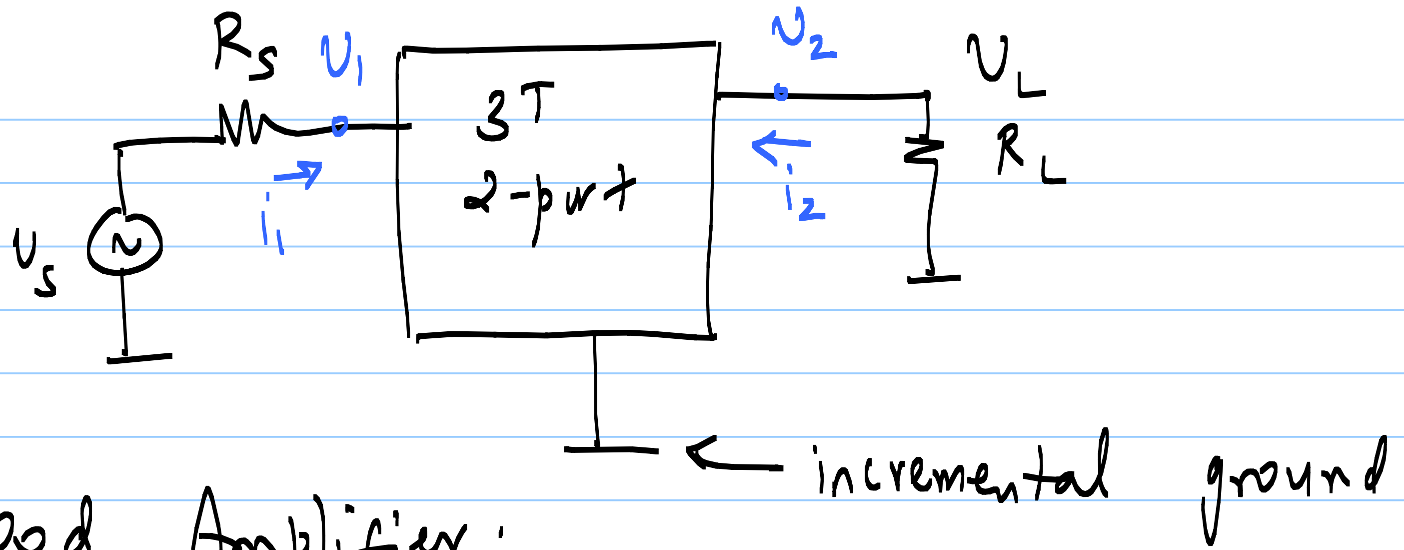
Input characteristics $\Rightarrow$ I_1 vs. V_1 for various V_2

passivity : $V_1 I_1 + V_2 I_2 \geq 0$
(Lin. or NonLin)

<u>Amplifiers</u>



"good" amplifier



Good Amplifier:

1) Large gain: $\frac{v_L}{v_s}$ as large as possible

2) Independent of source quality: v_L independent of R_s and gain independent of R_s

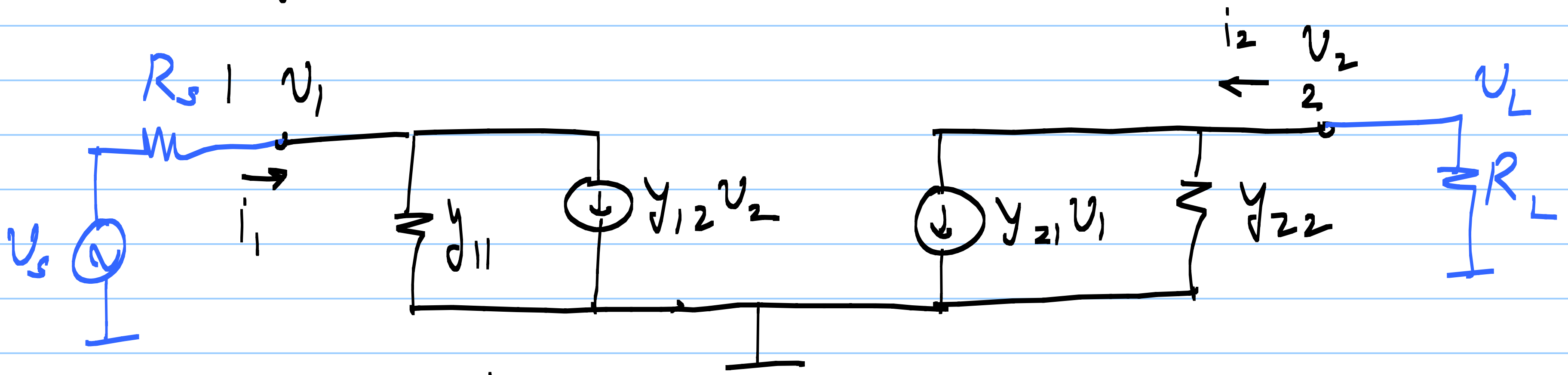
3) gain independent of R_L too.

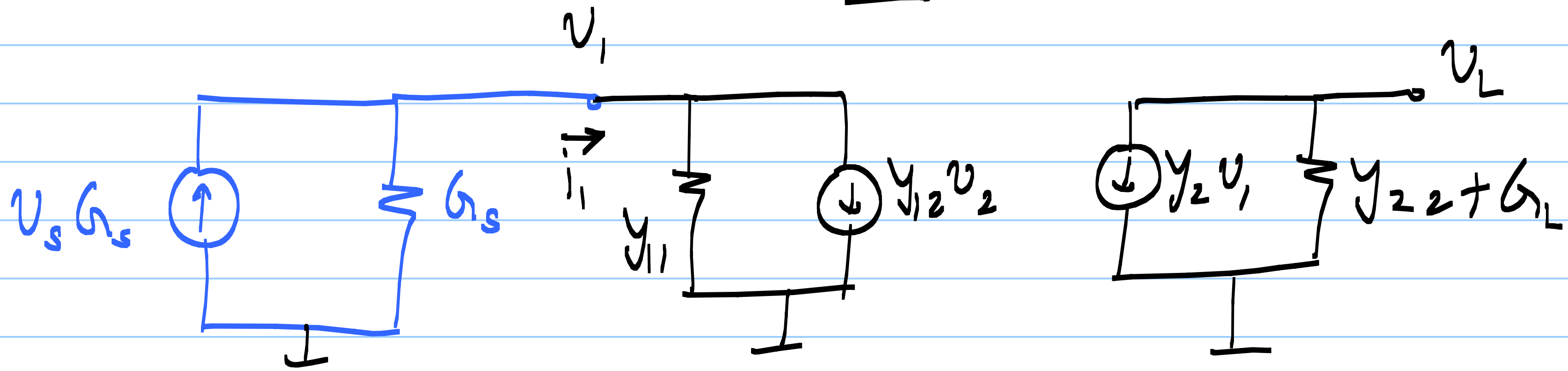
4) We want i_1 independent of v_2

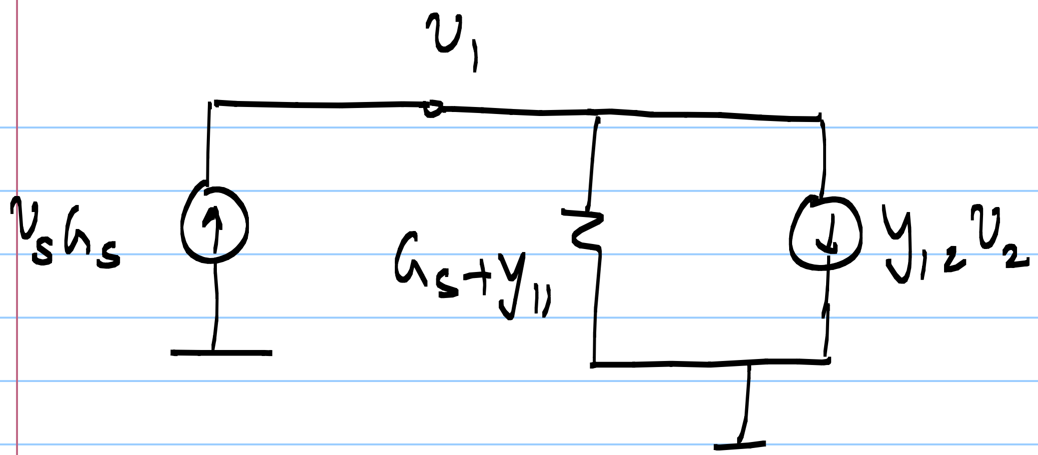
"unilateral" $y_{12} = 0$ is desired

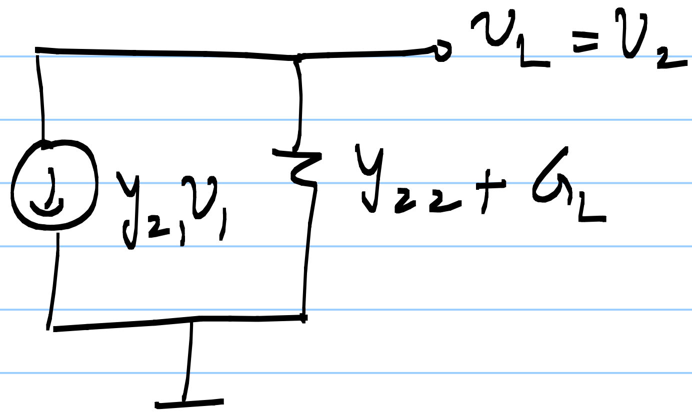
Derive constraints on [y] to achieve a "good" amp.

$$\left.\begin{aligned} i_1 &= y_{11}v_1 + y_{12}v_2 \\ i_2 &= y_{21}v_1 + y_{22}v_2 \end{aligned}\right\} \text{ replace by equivalent network}$$









KCL @ input & output.

@ input: $v_s G_s = v_1 (G_s + y_{11}) + y_{12} v_2$

$$\Rightarrow \quad v_1 = \frac{v_s G_s - y_{12} v_2}{G_s + y_{11}}$$

plug into

@ output: $y_{21} v_1 + v_2 (y_{22} + G_L) = 0$

$$y_{21} \left[\frac{v_s G_s - y_{12} v_2}{y_{11} + G_s} \right] + (y_{22} + G_L) \cdot v_2 = 0$$

$$v_s\left[\frac{y_{21}\cdot G_s}{y_{11}+G_s}\right] = v_2\left[\frac{y_{12}y_{21}}{y_{11}+G_s} - (y_{22}+G_L)\right]$$

$$= v_2\left[\frac{y_{12}y_{21} - (y_{22}+G_L)(y_{11}+G_s)}{y_{11}+G_s}\right]$$

$$\frac{v_2}{v_s} = \frac{v_L}{v_s} = \frac{y_{21}G_s}{y_{12}y_{21} - (y_{22}+G_L)(y_{11}+G_s)}$$

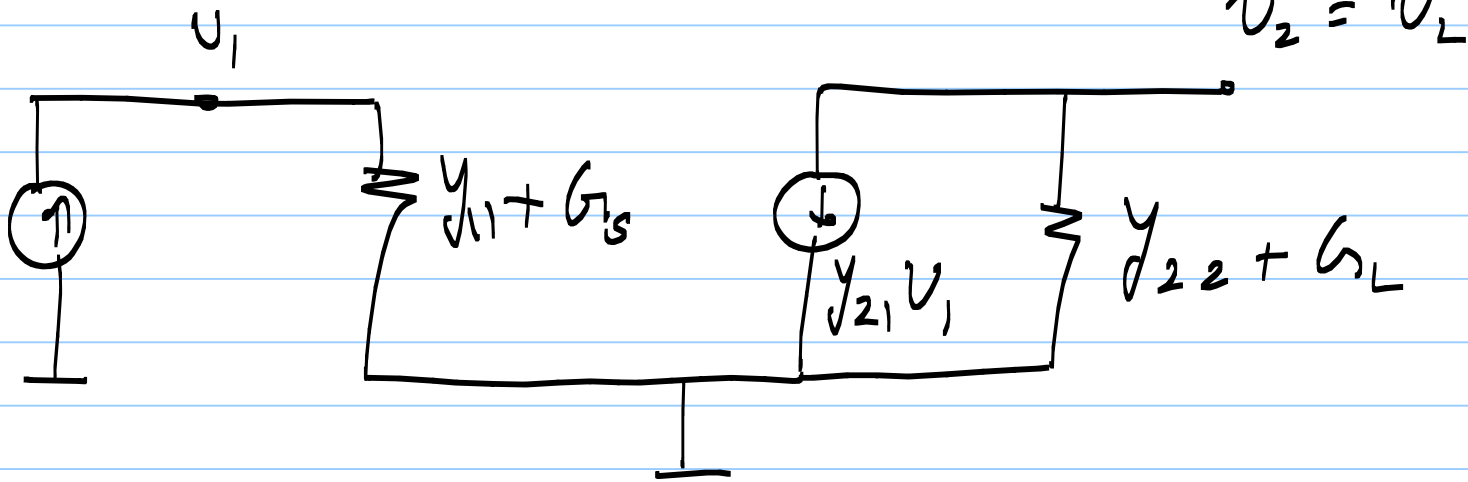
* If $y_{12}y_{21} = (y_{22}+G_L)(y_{11}+G_s)$, gain $=\infty$ undesired situation, gain needs to be a function of y_{11} etc.

* Make amplifier unilateral : $y_{12}=0$

$$\Rightarrow \quad \frac{v_L}{v_s} = \frac{-\,y_{21} G_s}{(y_{22} + G_L)(y_{11} + G_s)}$$

14/8/2020

Lecture 7



$$\text{Gain} = \frac{-y_{21}}{(y_{22}+G_L)} \cdot \left(\frac{G_s}{y_{11}+G_s} \right)$$

1) Gain independent of G_s : set $y_{11} = 0$

$\Rightarrow i_1 = 0$

$$\frac{v_L}{v_s} = \frac{-y_{21}}{y_{22}+G_L}$$

2) Gain as large as possible:

y_{21} as large as possible

and

$$\underbrace{y_{22} + G_L \text{ as small as possible}}$$

↳ set $y_{22} = 0$

$$\boxed{\frac{v_L}{v_s} = -\frac{y_{21}}{G_L}}$$

gain of amplifier still dep. on G_L

We need a 2-port network with inc $[y]$:

$$[y] = \begin{bmatrix} 0 & 0 \\ \text{as large as possible} & 0 \end{bmatrix} = \begin{bmatrix} \partial f/\partial v_1 & \partial f/\partial v_2 \\ \partial g/\partial v_1 & \partial g/\partial v_2 \end{bmatrix}$$

$I_1 = f(V_1, V_2) \quad ; \quad I_2 = g(V_1, V_2)$

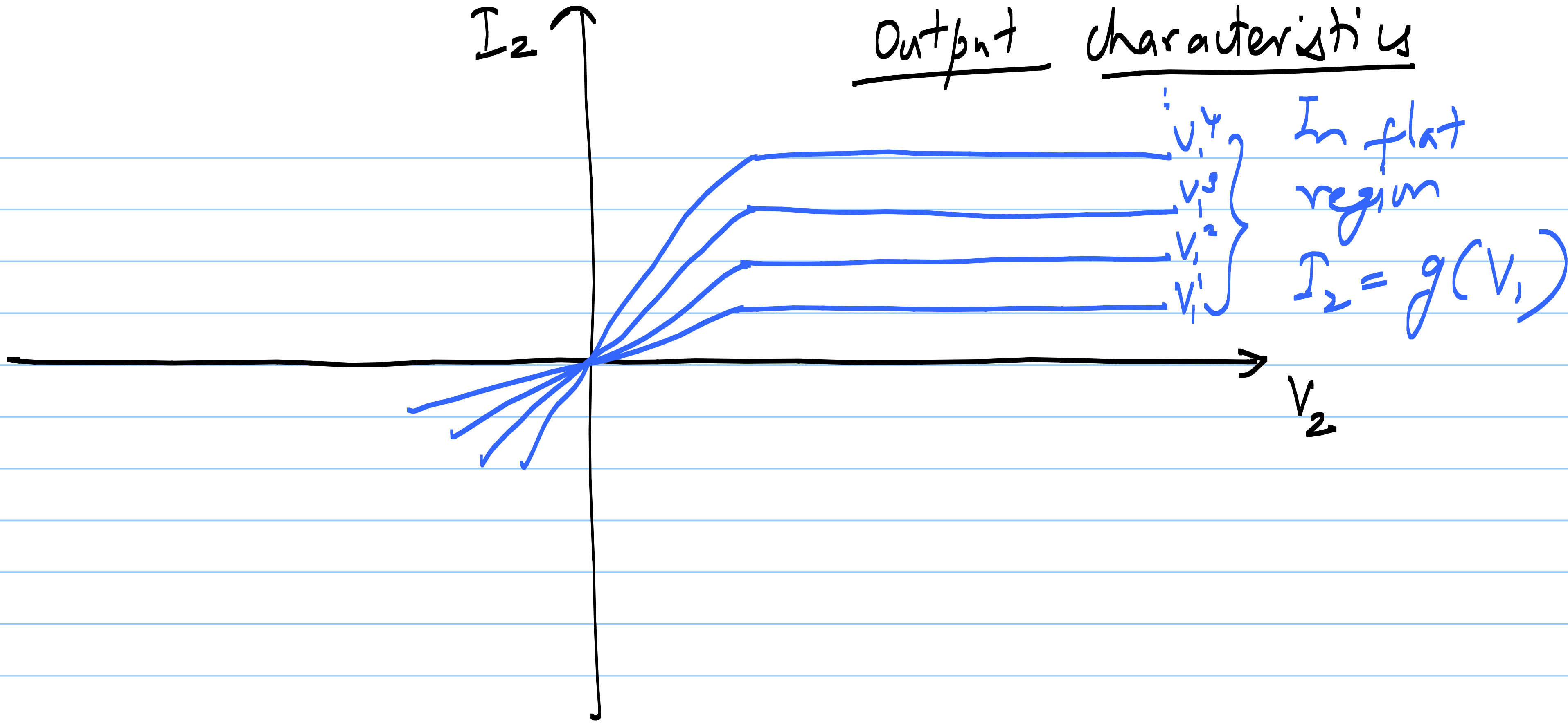
$y_{11}, y_{12} = 0 \Rightarrow \frac{\partial f}{\partial V_1}, \frac{\partial f}{\partial V_2} = 0 \Rightarrow I_1 = I_0$

constant current

$y_{22} = 0, \; y_{21} = \text{large} \Rightarrow I_2 = g(V_1)$ only

$I_1 = I_0$

$I_2 = g(V_1)$



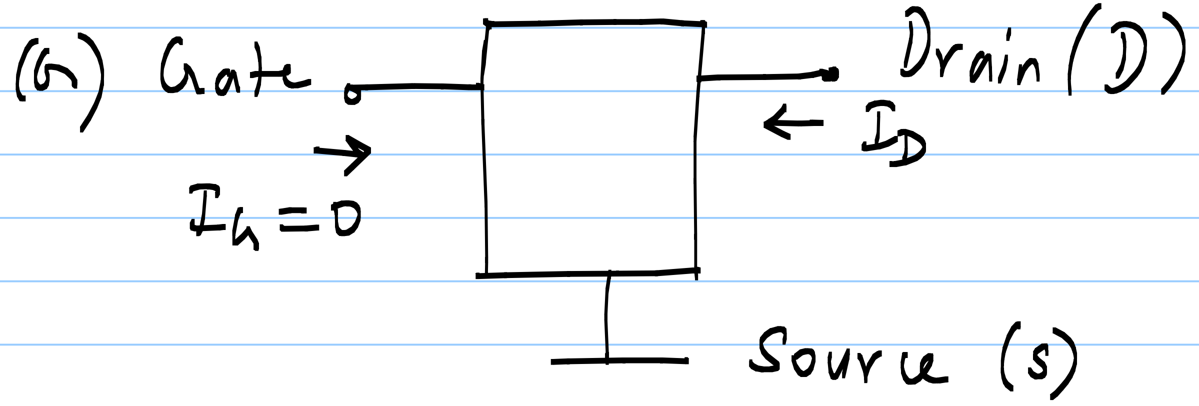
special case : $I_1 = 0 \Rightarrow$ passivity : $V_1 I_1 + V_2 I_2 \geq 0$

$\Rightarrow V_2 I_2 \geq 0$

All devices that exhibit "good" amplifier behaviour (high gain etc.) have such characteristics

MOSFET $\Rightarrow I_1 = 0$; $\left.\begin{array}{l}\text{BJT}\\ \text{JFET}\end{array}\right\} \Rightarrow I_1$ very small

MOSFET



NMOSFET

$$I_D = 0 \quad \text{if} \quad V_{GS} < V_T$$

$$I_D = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)(V_{GS}-V_T)^2 \quad \text{if} \quad V_{DS} \geq (V_{GS}-V_T)$$

"saturation"

$$I_D = \mu_n C_{ox}\left(\frac{W}{L}\right)\left[(V_{GS}-V_T)V_{DS} - \frac{V_{DS}^2}{2}\right] \quad \text{if } V_{DS} \leq (V_{GS}-V_T)$$

"triode", "linear"

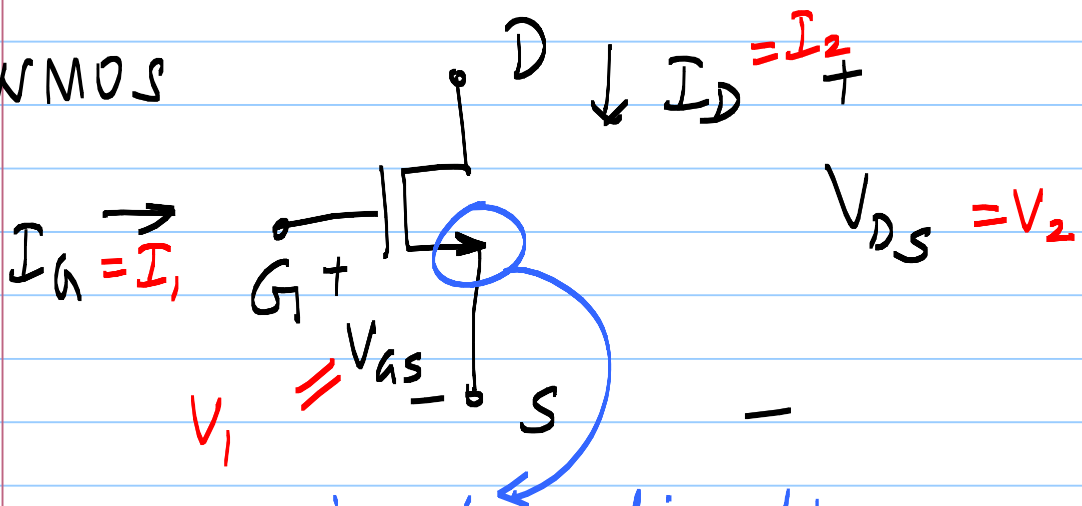
V_T = Threshold voltage

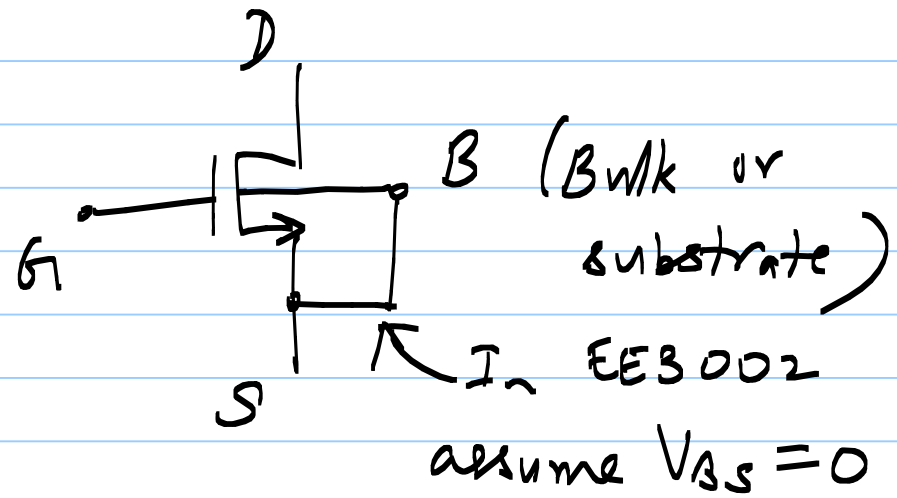
μ_n = mobility of electrons

C_{ox} = oxide cap. per unit area

W, L = geometric parameters of MOSFET

NMOS





current flow direction

$I_G = 0 \Rightarrow y_{11} = y_{12} = 0$

In sat., I_D is independent of V_{DS}

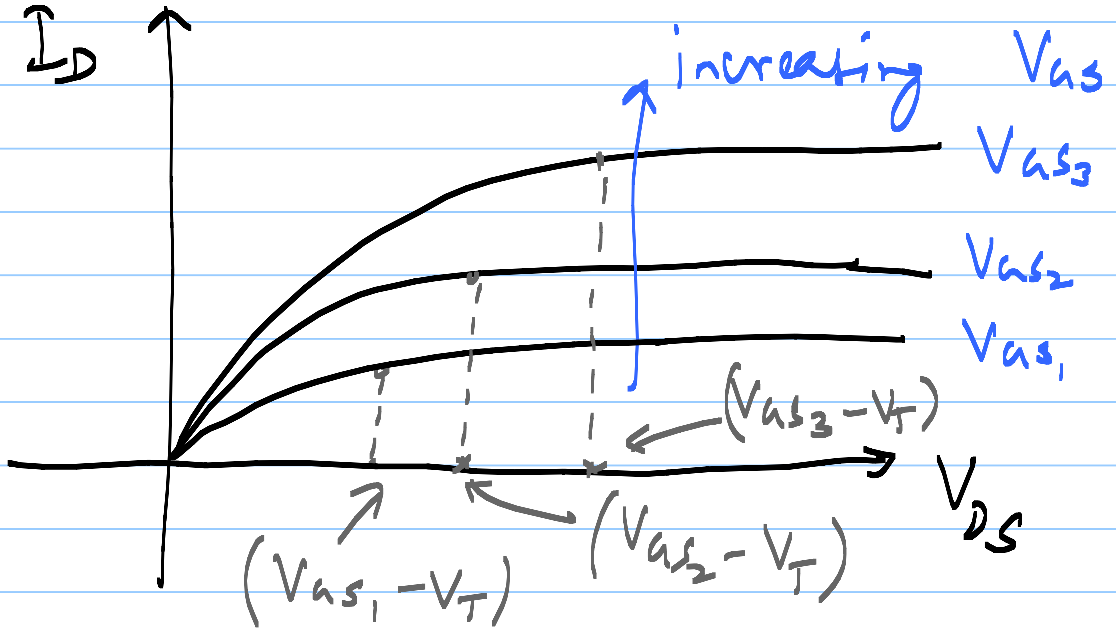
Operate MOSFET in sat. for building a good amplifier

I_2 is indep. of V_2 $\Rightarrow$ $y_{22} = 0$

$$y_{21} = \frac{\partial I_2}{\partial V_1} = \frac{\partial I_D}{\partial V_{GS}} = \mu_n C_{ox} \left(\frac{W}{L}\right)(V_{GS} - V_T)$$

for $V_{GS} > V_T$ and

$V_{DS} \geq (V_{GS} - V_T)$



⋮

$$I_{D_1} = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)(V_{GS_1} - V_T)^2$$

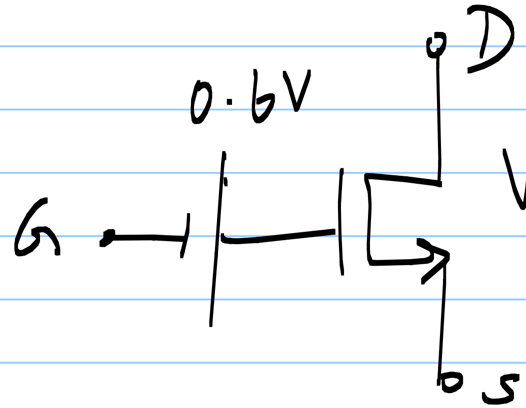
In this course, assume $V_T > 0$

1) "Enhancement mode" MOSFET (Normally OFF)

i.e. $V_T > 0$; @ $V_{GS} = 0$, $I_D = 0$

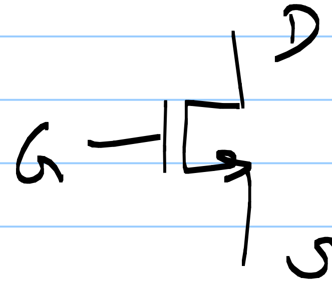
2) "Depletion mode" MOSFET (normally ON)

$V_T < 0$; @ $V_{GS} = 0$, $I_D > 0$



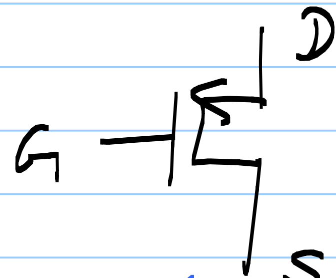
$V_T = 0.3V$ } Effective $V_T = -0.3V$

3)



NMOSFET

carriers $= e^-$



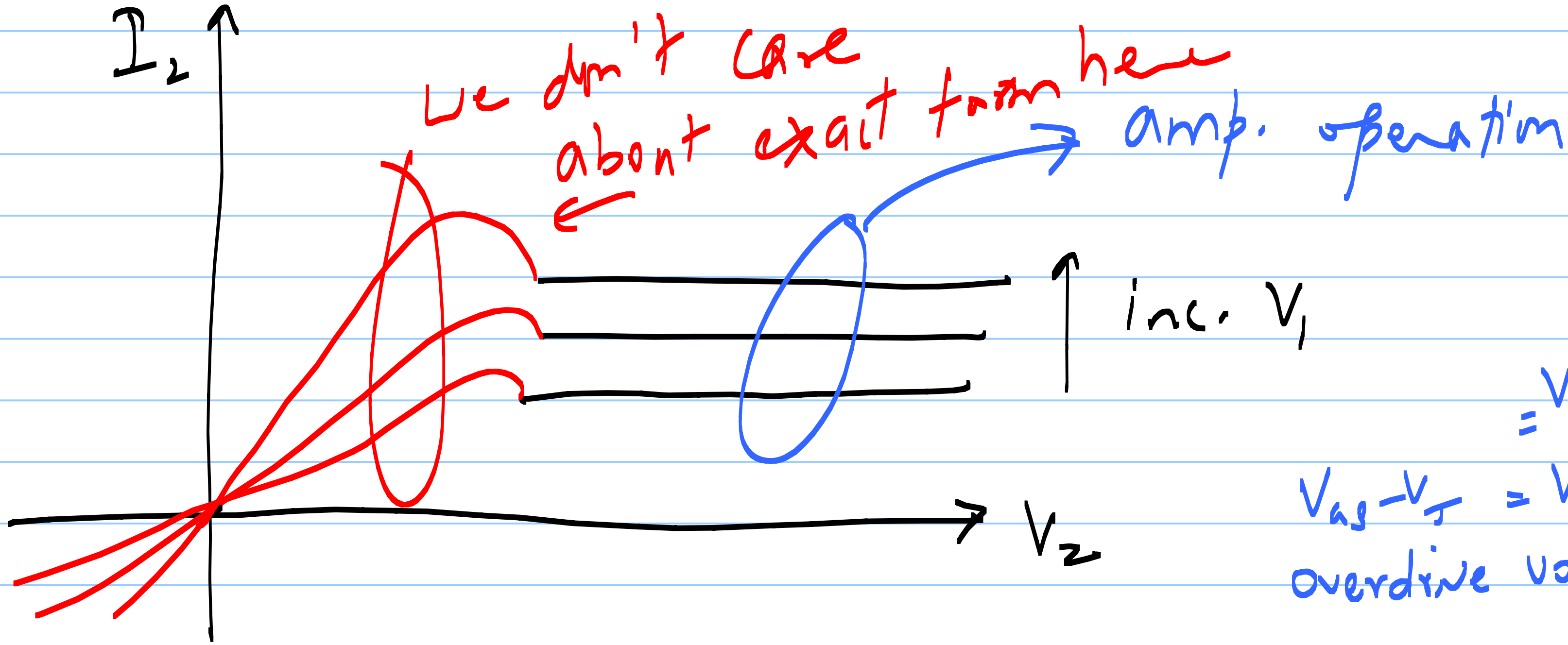
PMOSFET

carriers $= p^+$

later in the course

18|8|2020

Lecture 8



$= V_{Dsat}$

$V_{GS} - V_T = V_{ov.}$

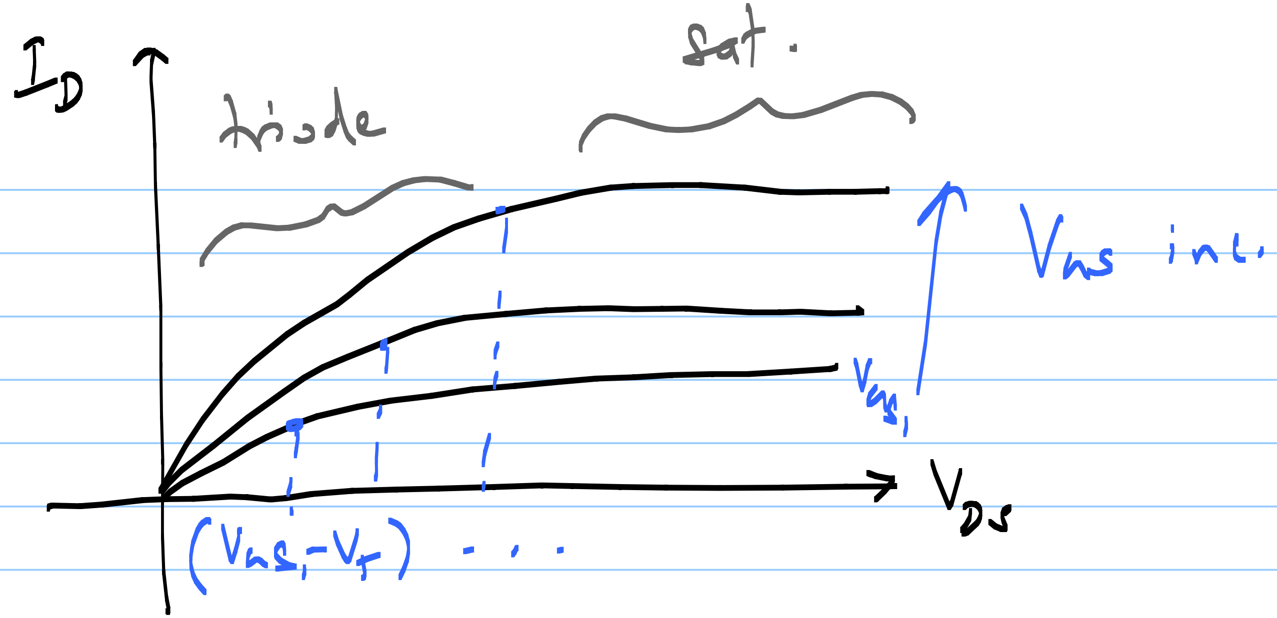
overdrive voltage

$I_G = 0$

$I_D = 0$ if $V_{GS} < V_T$ (off)

$I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_T)^2$ if $V_{GS} \geq V_T$ and $V_{DS} \geq V_{GS} - V_T$ (sat.)

$I_D = \mu_n C_{ox} \frac{W}{L} \left((V_{GS} - V_T) V_{DS} - \frac{V_{DS}^2}{2} \right)$ if $V_{GS} \geq V_T$ and $V_{DS} \leq V_{GS} - V_T$



V_T = threshold voltage

V_t = thermal voltage $= \frac{kT}{q}$

Amplifier: Use sat. region

$$I_G = 0 \; ; \quad I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS}-V_T)^2$$

use this to derive op. pt.

$$y_{11} = \frac{\partial I_G}{\partial V_{GS}} = 0 \; ; \quad y_{12} = \frac{\partial I_G}{\partial V_{DS}} = 0 \; ;$$

$$y_{22} = \frac{\partial I_D}{\partial V_{DS}} = 0 \; ; \quad y_{21} = \frac{\partial I_D}{\partial V_{GS}}$$

$$y_{21} = \frac{\partial I_D}{\partial V_{GS}} = \mu_n C_{ox} \left(\frac{W}{L}\right)(V_{GS} - V_T) = g_m$$

transconductance of MOSFET

1) $$g_m = \mu_n C_{ox} \left(\frac{W}{L}\right)(V_{GS} - V_T)$$

2) $$g_m = 2 \times \frac{1}{2} \times \mu_n C_{ox} \left(\frac{W}{L}\right)(V_{GS} - V_T)^2 \times \frac{1}{(V_{GS}\, V_T)}$$

$$= \frac{2 I_D}{V_{GS} - V_T}$$

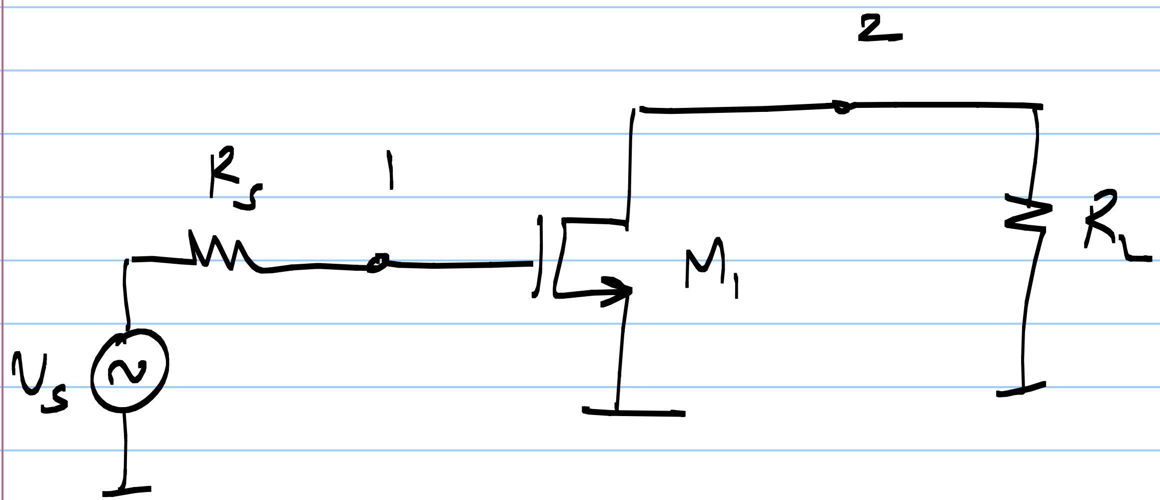
3) $$g_m^2 = 2 \times \frac{1}{2} \mu_n^2 C_{ox}^2 \left(\frac{W}{L}\right)^2 (V_{GS} - V_T)^2$$

$$= 2 \mu_n C_{ox} \left(\frac{W}{L}\right) I_D$$

$$g_m = \sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right) I_D}$$

$$[y] = \begin{bmatrix} 0 & 0 \\ g_m & 0 \end{bmatrix}$$

MOSFET Amplifier



Small-signal picture

(op. pt. should be such that M_1 is biased in sat.)

In triode region :

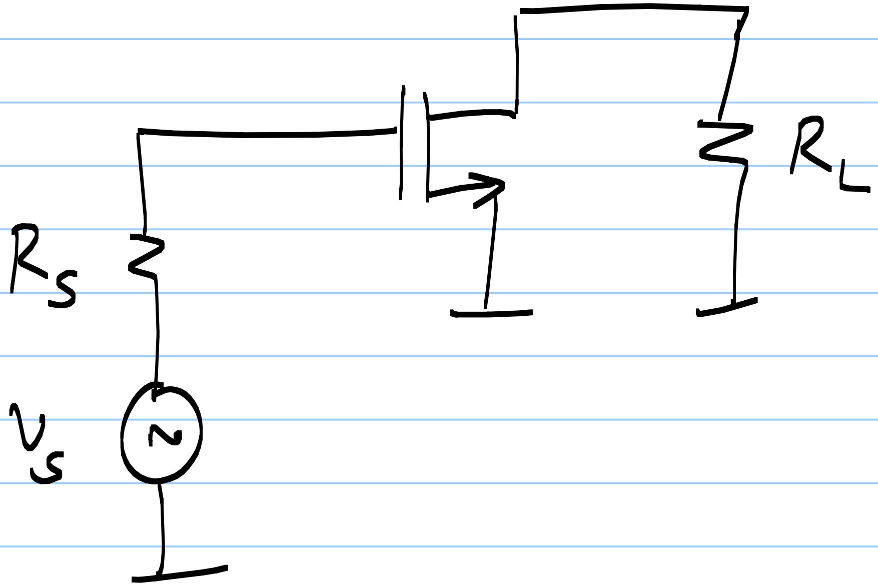
$$y_{11} = y_{12} = 0$$

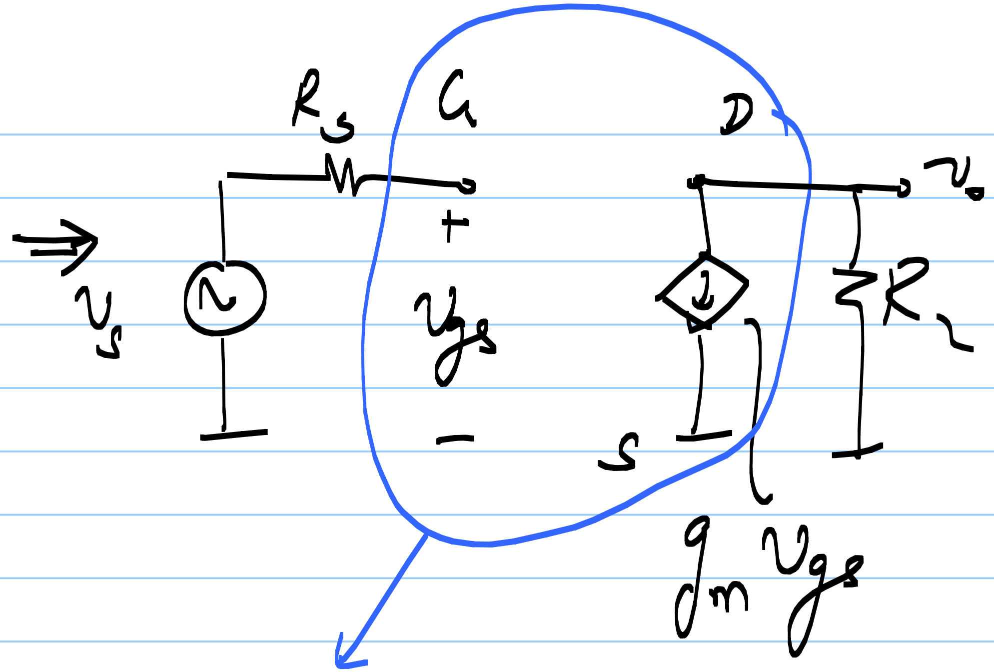
$$y_{22} = \frac{\partial I_D}{\partial V_{DS}} = \frac{\partial}{\partial V_{DS}} \left[\mu_n C_{ox} \left(\frac{W}{L}\right) \left\{ (V_{GS} - V_T) V_{DS} - \frac{V_{DS}^2}{2} \right\} \right]$$

$$= \mu_n C_{ox} \left(\frac{W}{L}\right) \left(V_{GS} - V_T - V_{DS} \right) \neq 0$$

$$y_{21} = \partial I_D / \partial V_{GS}$$

$$= \mu_n C_{ox} \left(\frac{W}{L}\right) \cdot V_{DS} \qquad \left\{ V_{DS} < (V_{GS} - V_T) \right\}$$

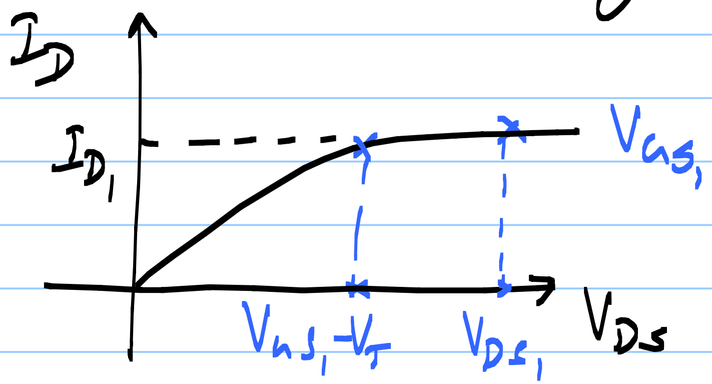


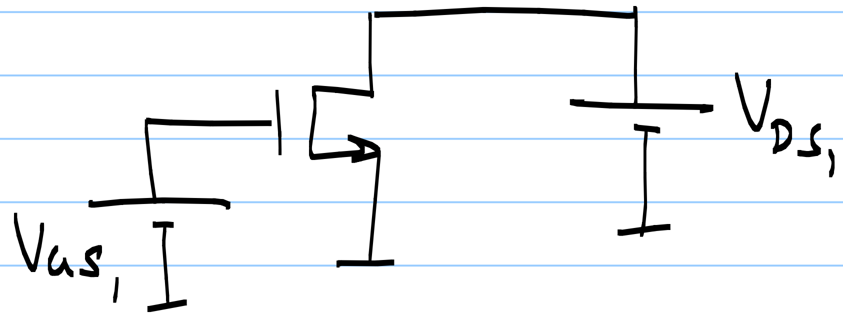


SS model of MOSFET

$$\text{gain} = \frac{v_o}{v_s} = -g_m R_L$$

MOSFET Biasing:

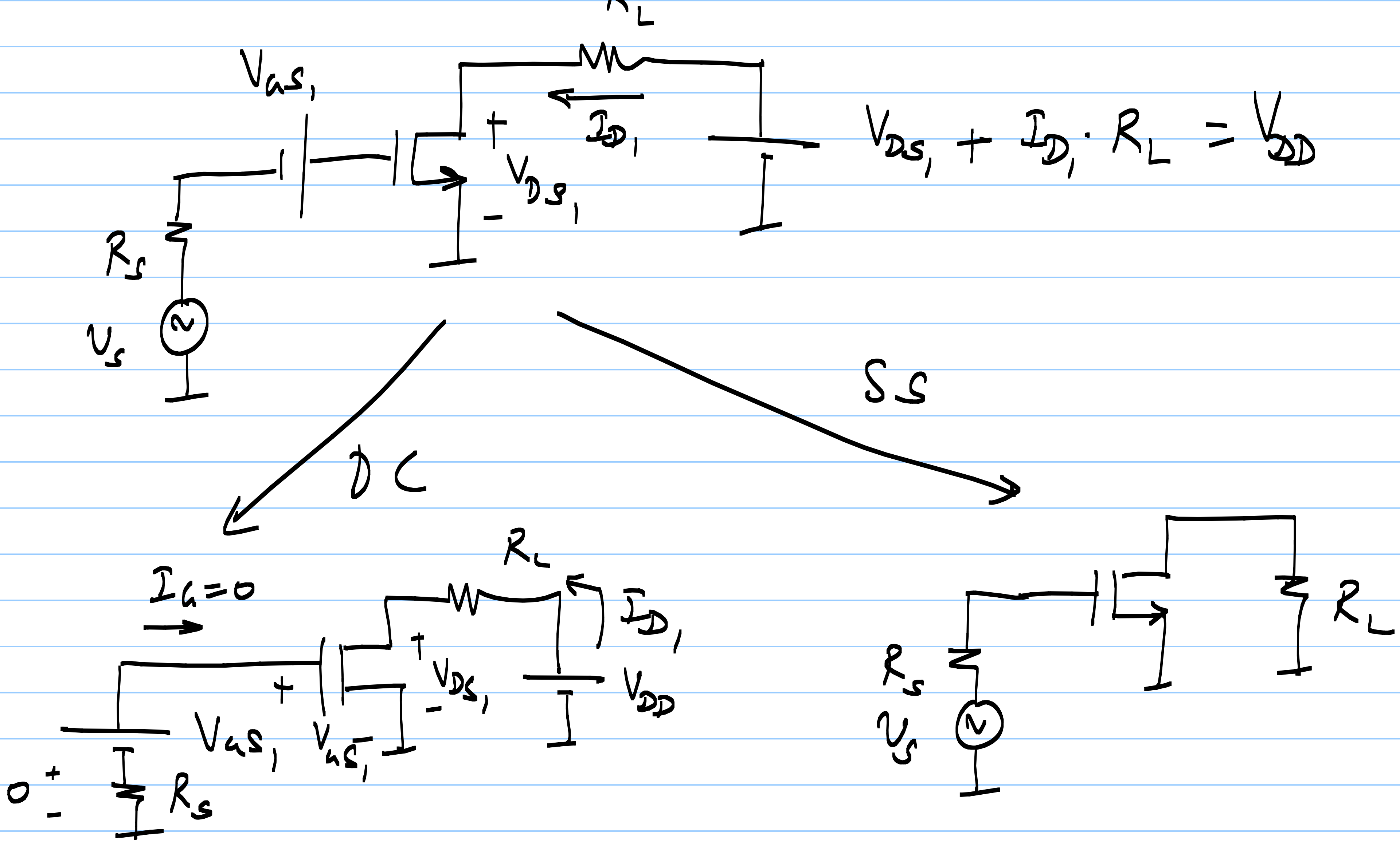




Add signal source & load to DC biased MOSFET:

$V_{R_L} = I_{D_1} \cdot R_L$

$V_{DS_1} + I_{D_1} \cdot R_L = V_{DD}$

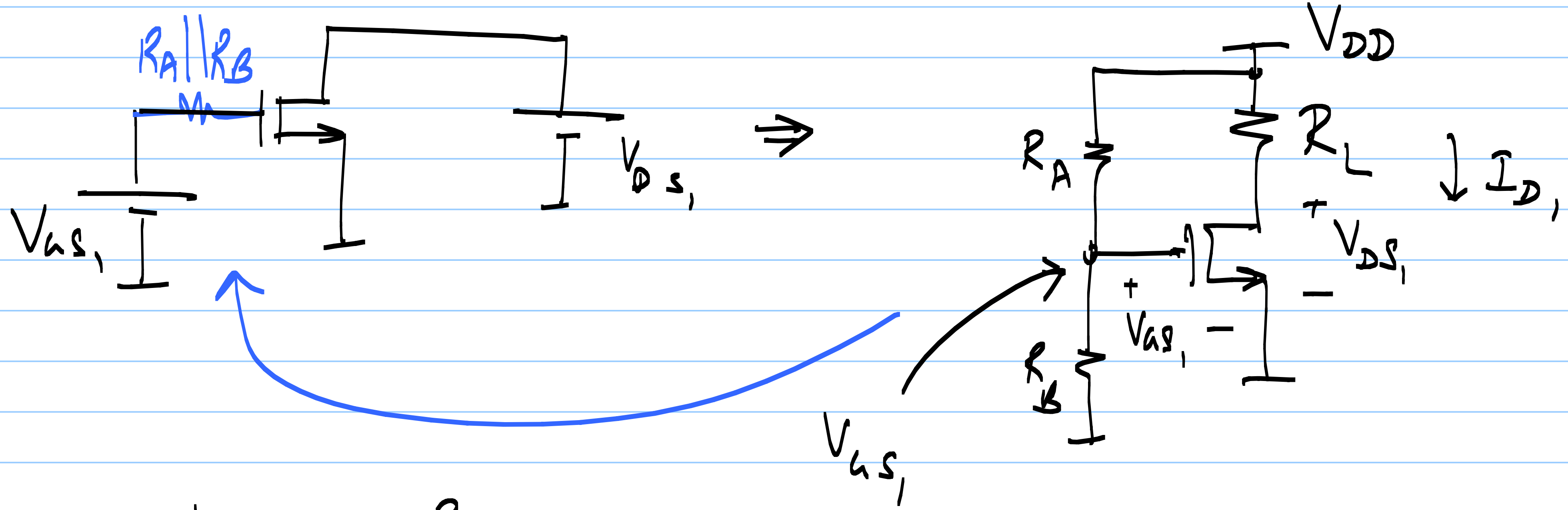


* Avoid use of 2 batteries :

⇒ generate V_{GS_1} from V_{DD} → largest voltage in the circuit

($V_{DD} > V_{GS_1}$)

DC :

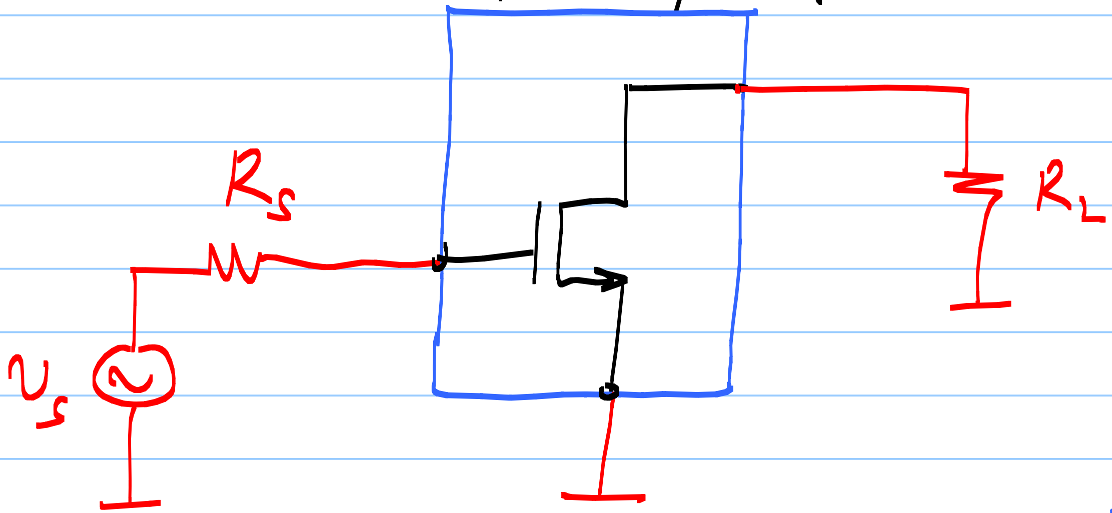


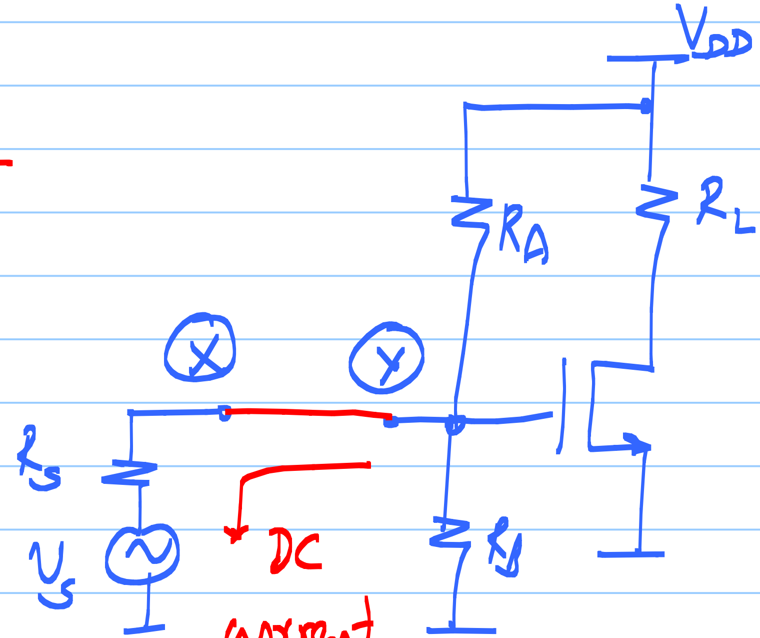
$$V_{GS_1} = \frac{R_B}{R_A + R_B} \cdot V_{DD}$$

19/8/2020

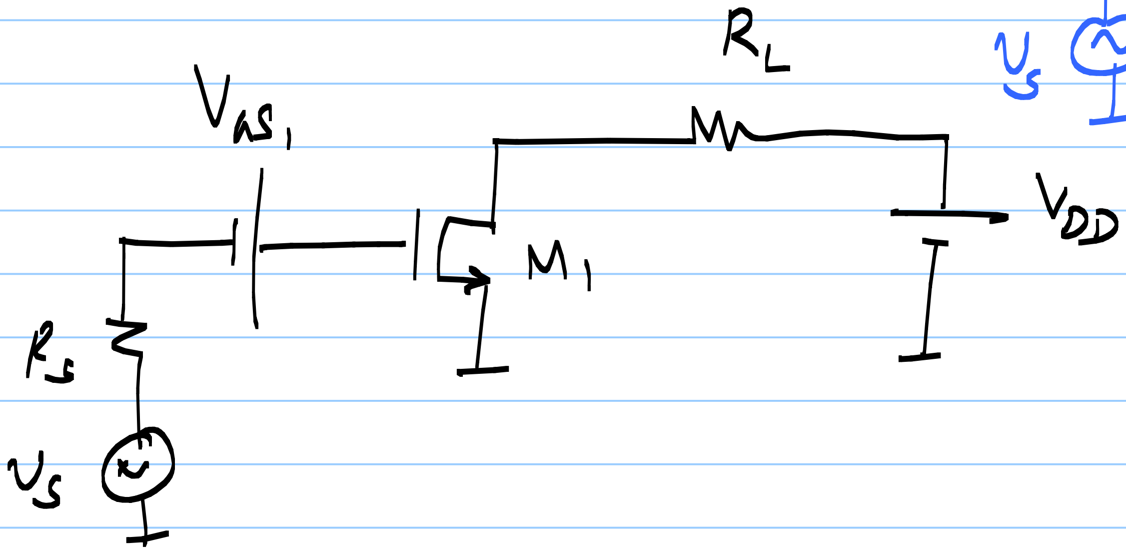
Lecture 9

NL 2-port (incremental view





DC current flow → * Might disturb op pt. of preceding circuit



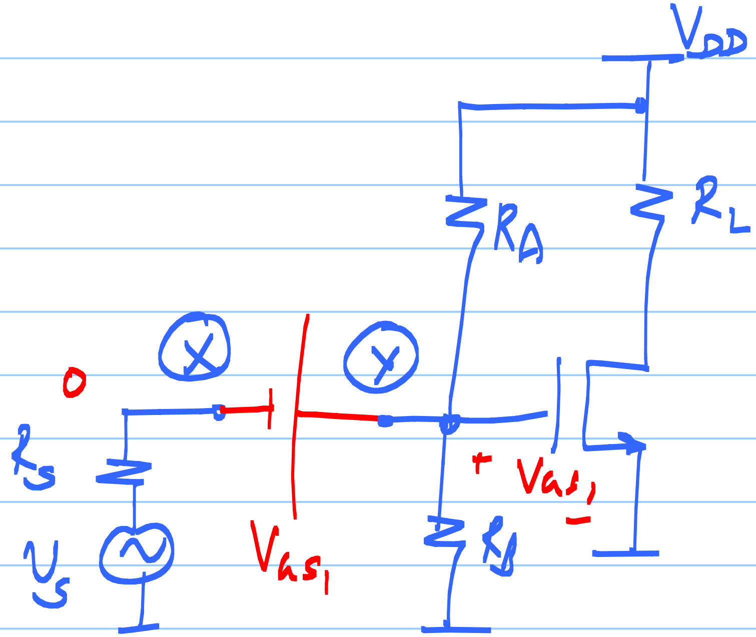
* V_{GS_1} itself changes

$$V'_{GS_1} = \frac{R_B \| R_s}{R_A + R_B \| R_s} \cdot V_{DD}$$

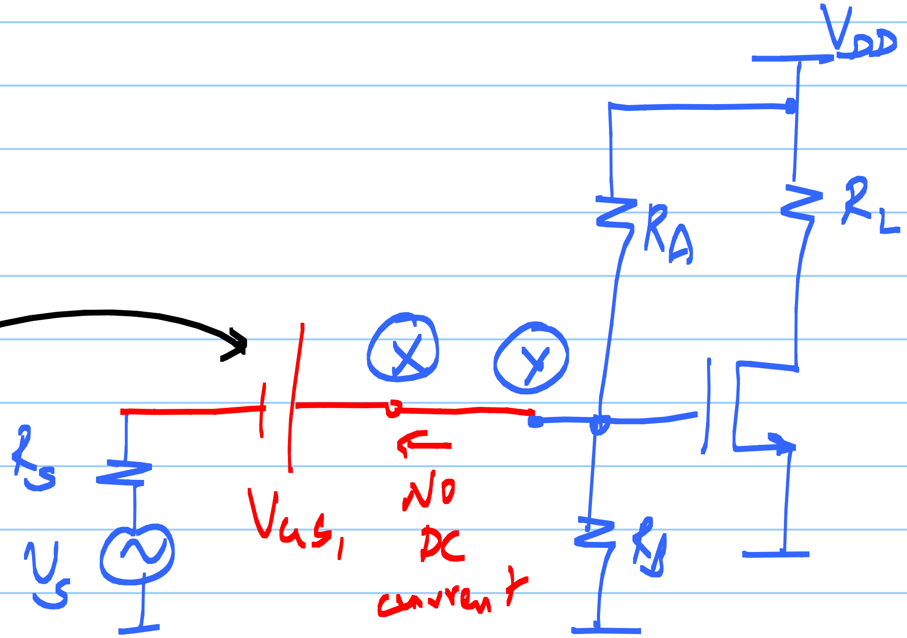
* We want no DC current flow through R_s

Ensure $V_x = V_y$

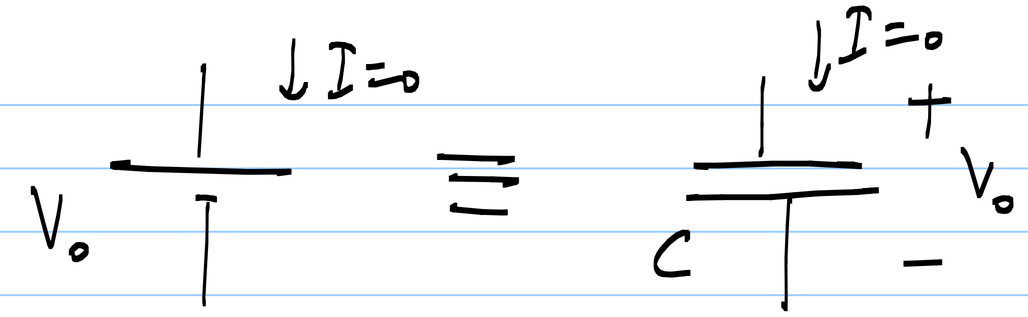
(N)



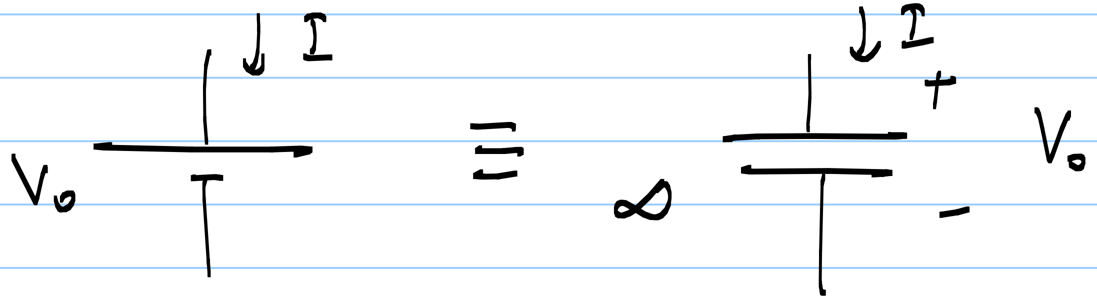
$V_x = V_y$



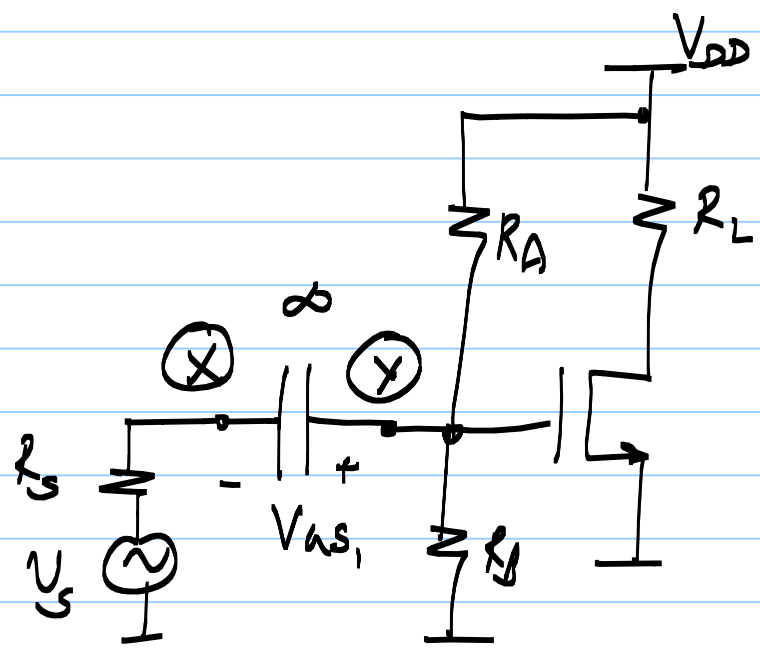
Don't add additional battery



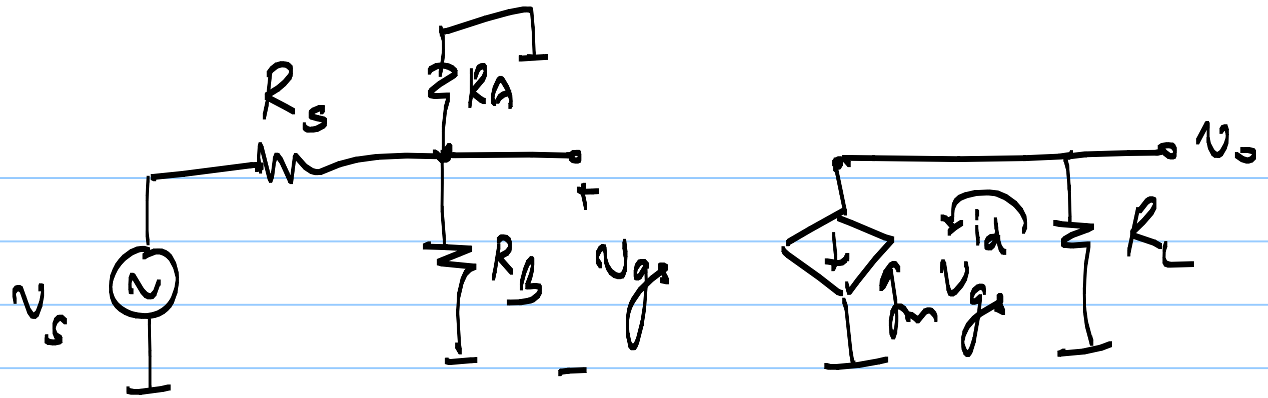
iff no current is drawn



even if you draw current



MOSFET Common-Source Amplifier



$$v_{gs} = \frac{R_A \| R_B}{R_s + R_A \| R_B} \cdot v_s$$

$$v_o = -g_m R_L \cdot v_{gs} = -g_m R_L \cdot \overbrace{\frac{R_A \| R_B}{R_s + R_A \| R_B}}^{y_{11} \neq 0} \cdot v_s$$

1) Choose $R_A \| R_B \gg R_s$

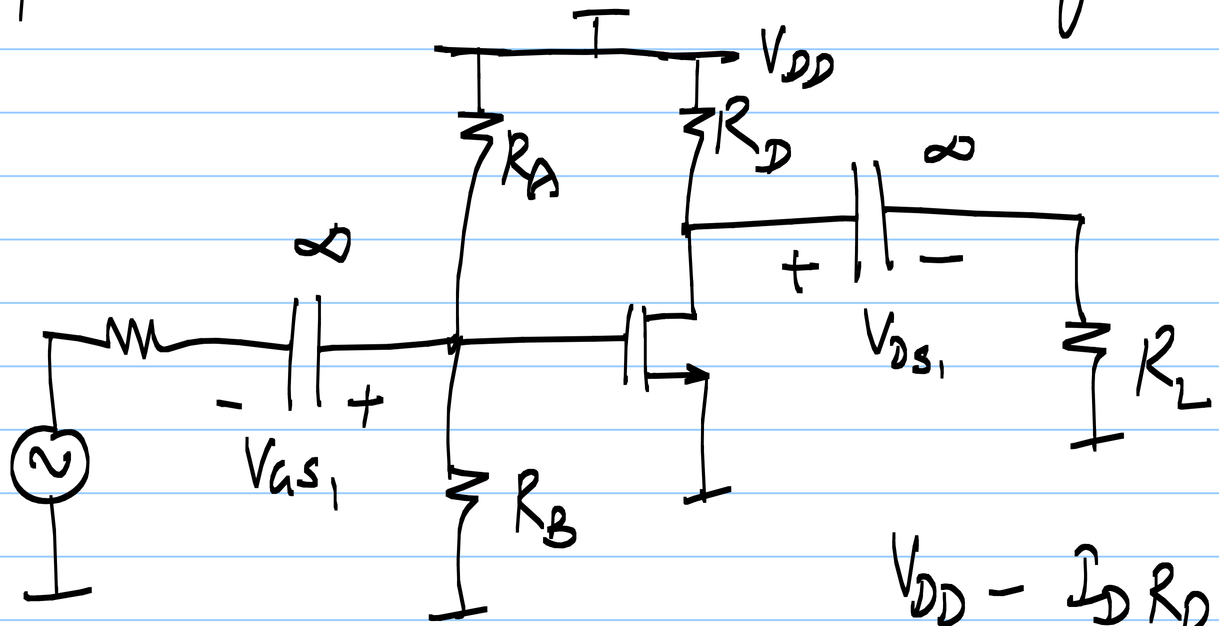
$$\frac{R_B}{R_A + R_B} \cdot V_{DD} = V_{GS},$$

2) then $\frac{v_o}{v_s} \approx -g_m R_L$

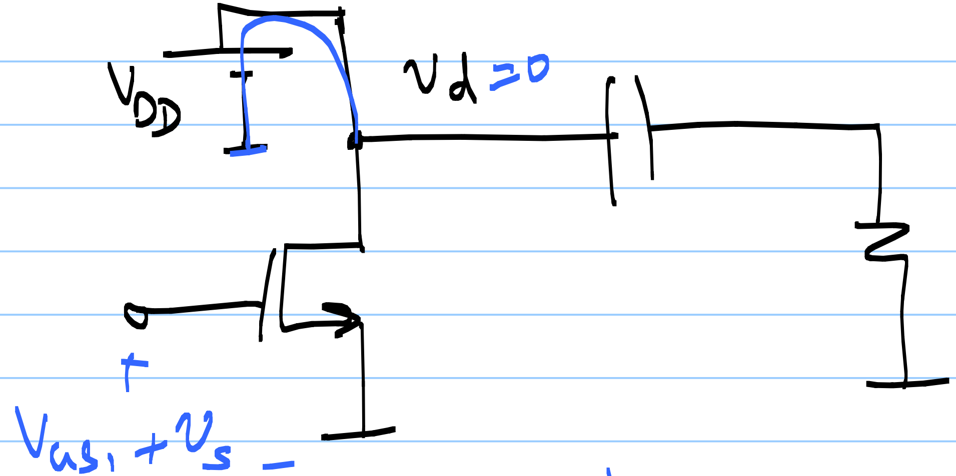
$$i_d = g_m v_{gs} = g_m \cdot \left(\frac{R_A \| R_B}{R_s + R_A \| R_B} \right) \cdot v_s$$

$$v_d = v_o = -i_d R_L$$

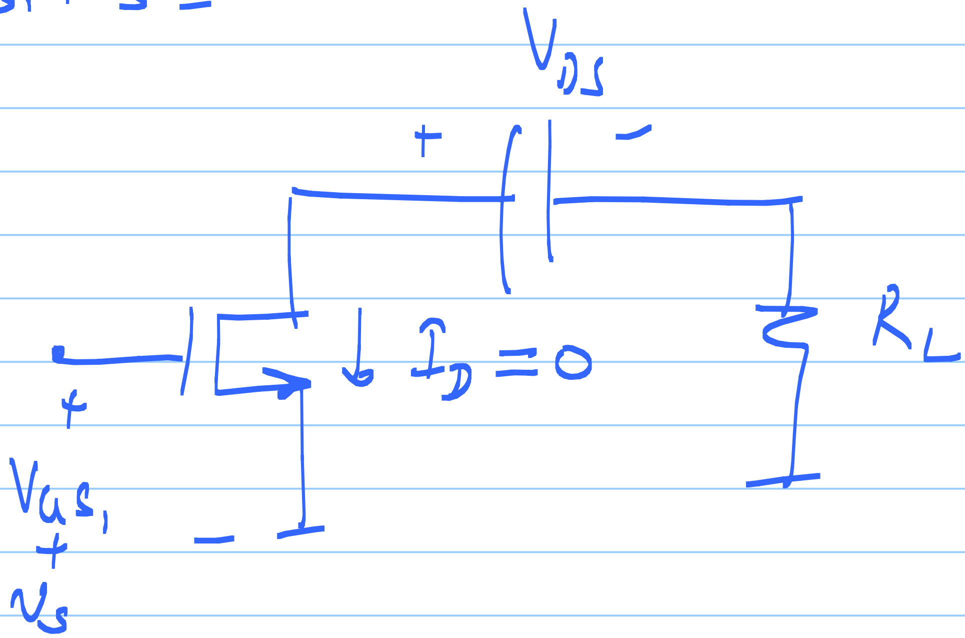
3) Suppose: No DC current through R_L:



$$V_{DD} - I_D R_D = V_{DS_1}$$



VDD
vd = 0
+
VGS1 + vs
−

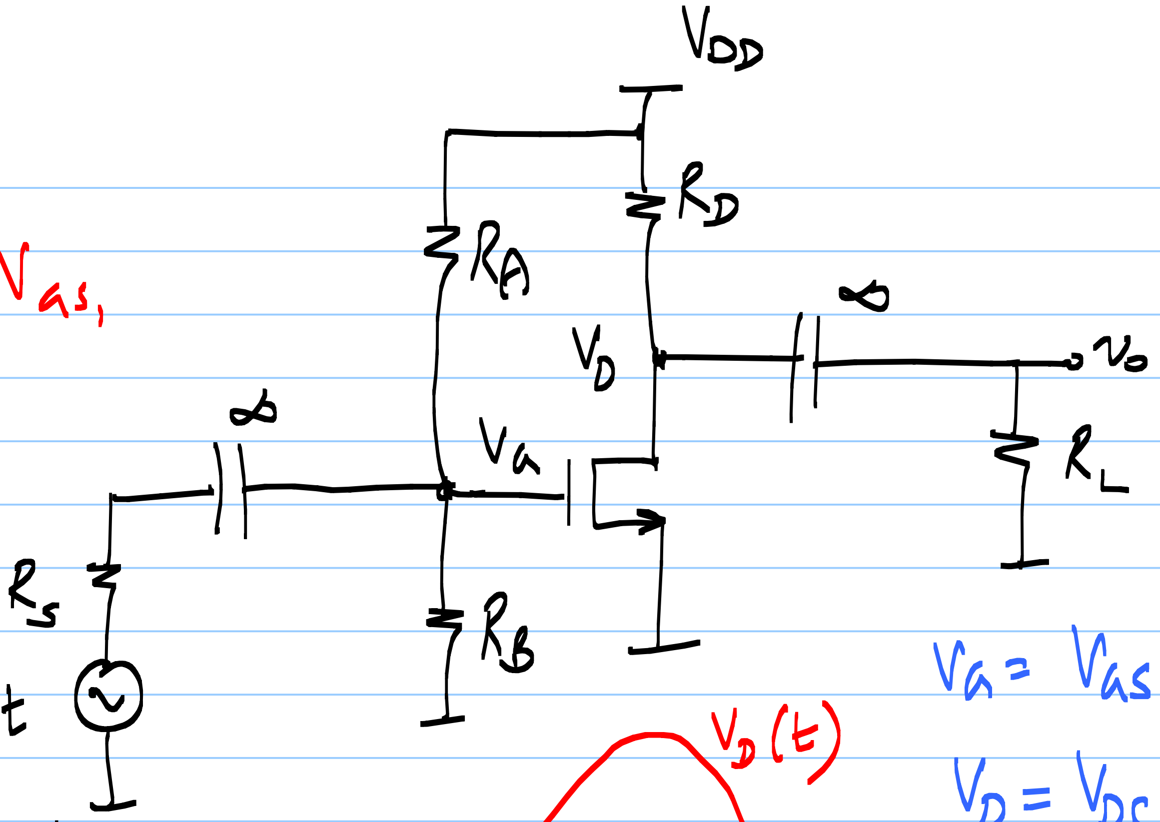


VDS
+
−
ID = 0
RL
+
VGS1
+
vs
−

Assume $V_{DS_1} > V_{GS_1}$

CSA

$$\frac{v_o}{v_s} = -g_m (R_D \| R_L)$$

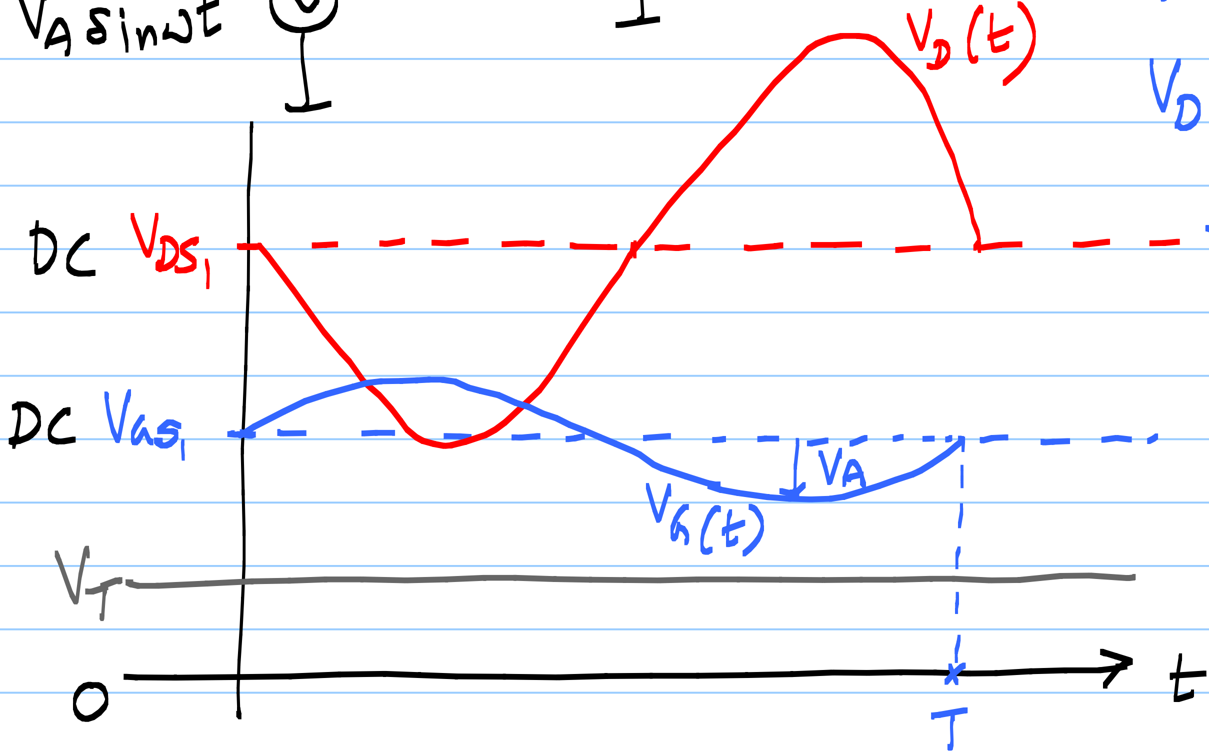


$v_s = V_A \sin \omega t$

$V_G = V_{GS_1} + V_A \sin \omega t$

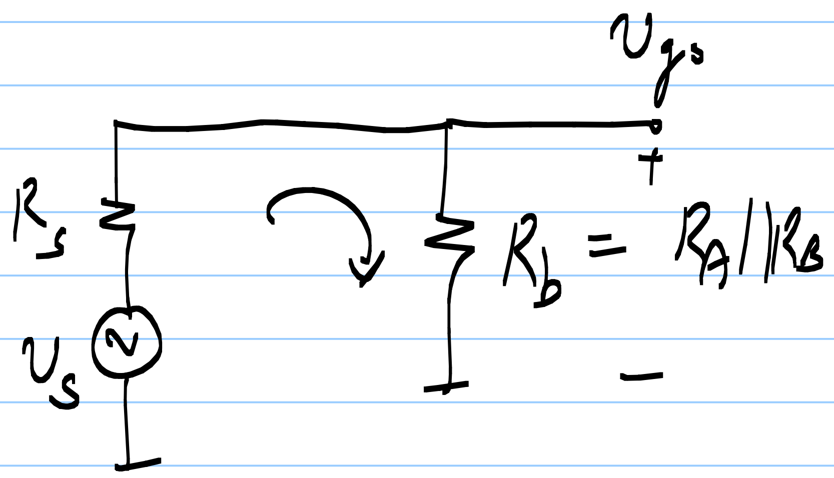
$V_D = V_{DS_1} - g_m (R_D \| R_L) V_A \sin \omega t$

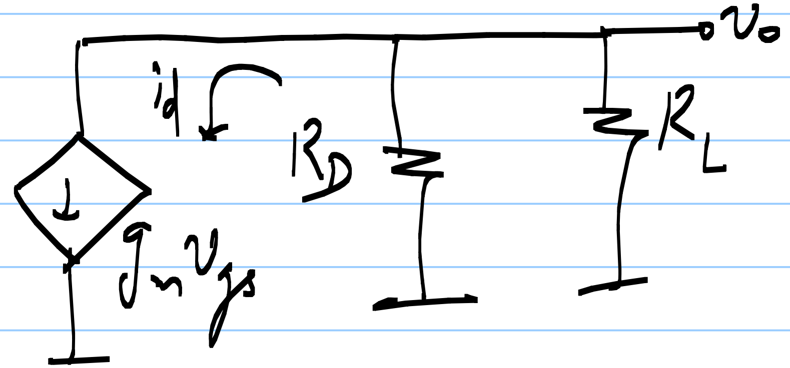
$= V_{DS_1} - G\, V_A \sin \omega t$



Triode boundary

$V_{DS} > V_{GS} - V_T$ at all time instants.





$$v_{gs} = \frac{R_b}{R_s + R_b} \cdot v_s \approx v_s \quad \text{if} \quad R_s << R_b$$

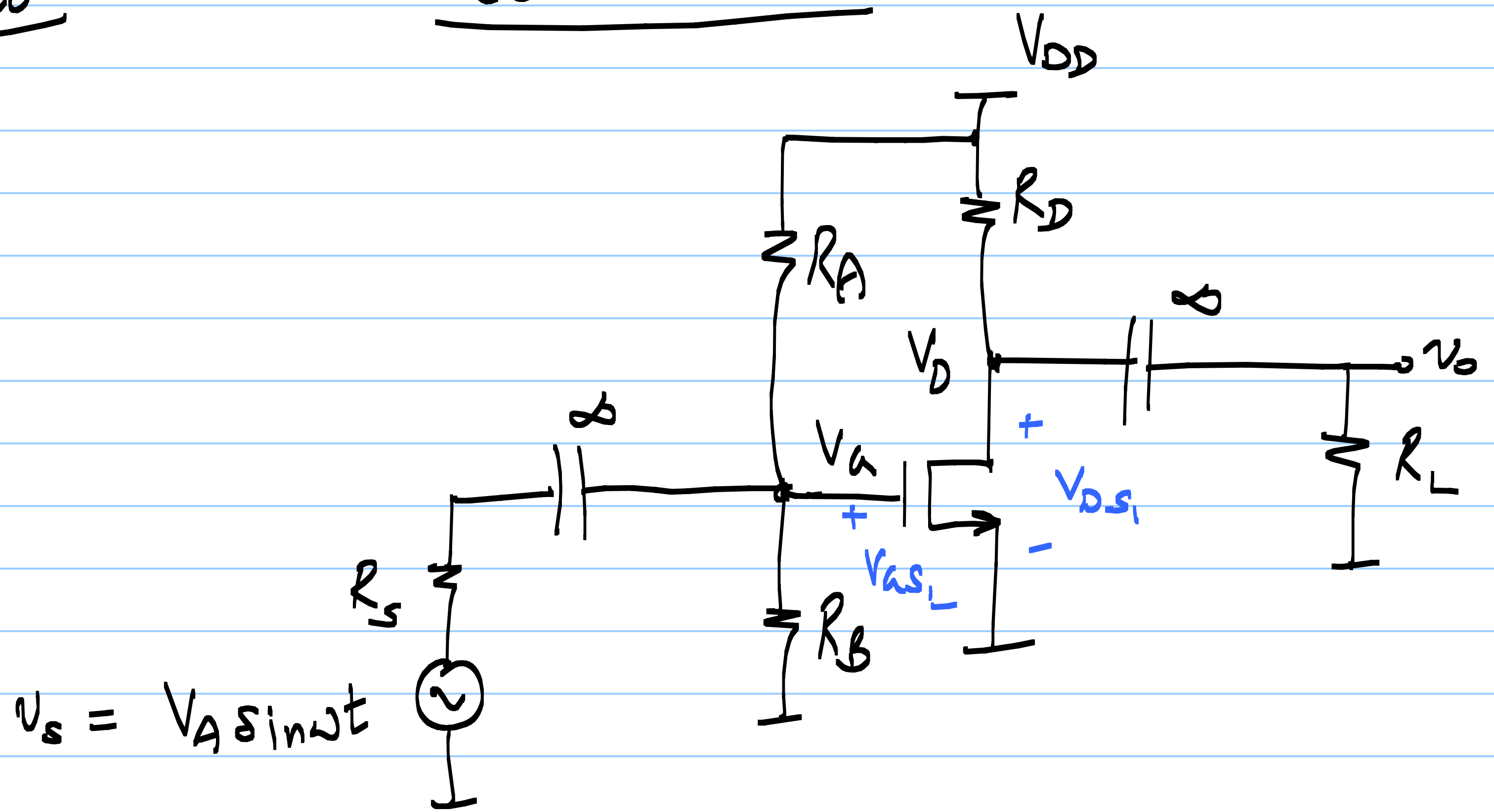
$$i_d = g_m v_{gs} \approx g_m v_s$$

$$v_o = -i_d \cdot (R_D \| R_L) = -g_m v_s (R_D \| R_L)$$

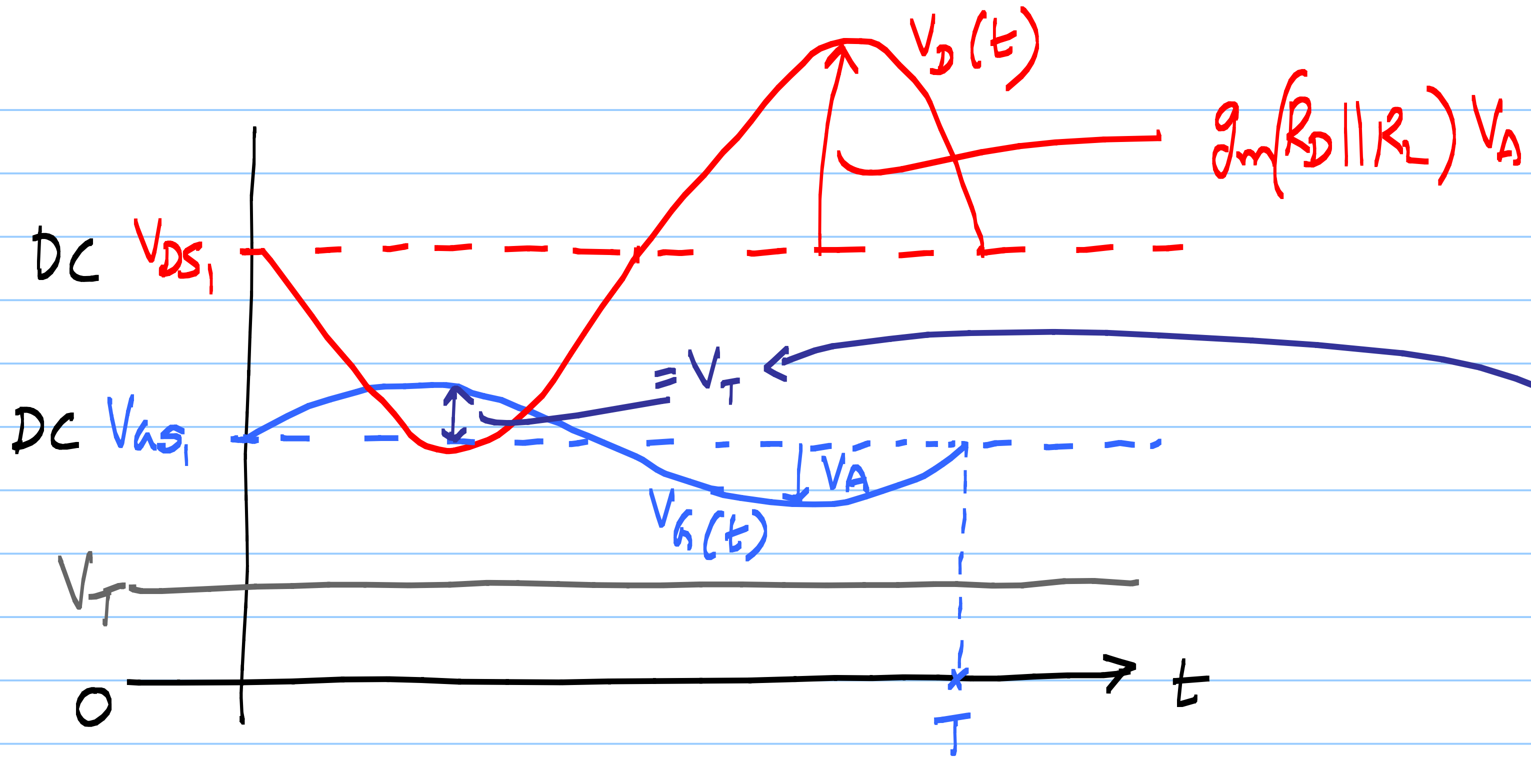
$$\frac{v_o}{v_s} = -g_m (R_D \| R_L) = -G$$

20/8/2020

Lecture 10



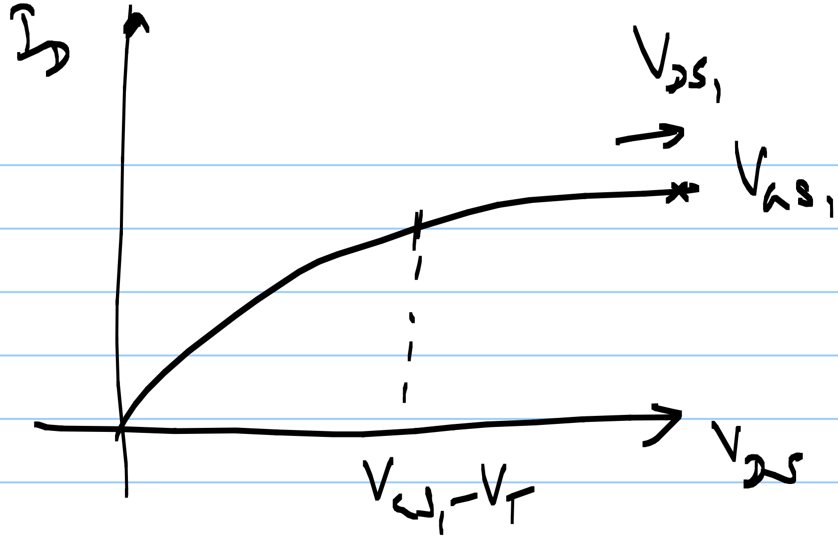
* $V_{DS}(t) > V_{GS}(t) - V_T$ at all times in the period so that device is in saturation.



1) Limit of V_A : instantaneous $V_{DS}(t) = V_{GS}(t) - V_T$

(or) $V_D(t) = V_G(t) - V_T$

$V_{A\,max_1}$ = maximum value of V_A that keeps M_1 from going into triode (+ve Half cycle of input sinusoid)

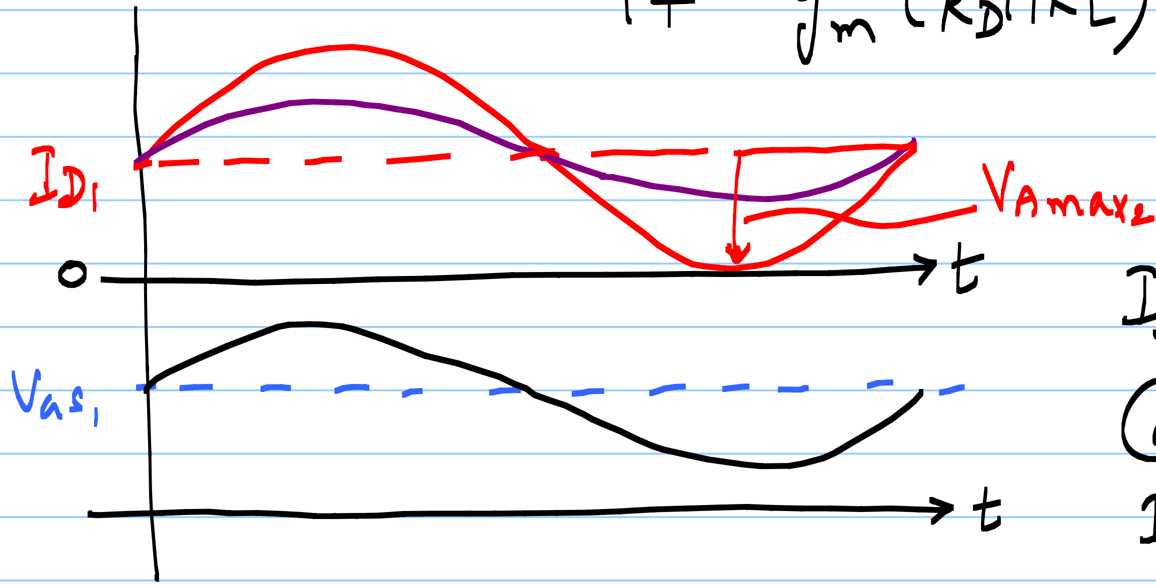


larger V_{DS_1}

$=$ larger VA_{max_1}

$$V_{DS_1} - g_m(R_D||R_L)\cdot VA_{max_1} = V_{GS_1} + VA_{max_1} - V_T$$

$$VA_{max_1} = \frac{V_{DS_1} - V_{GS_1} + V_T}{1 + g_m(R_D||R_L)}$$



$I_D = I_{D_1} + i_d$

$I_D(t) = I_{D_1} + g_m V_A \sin\omega t$

@ $V_A = VA_{max_2}$

$I_D(t) = 0$ @ neg. peak.

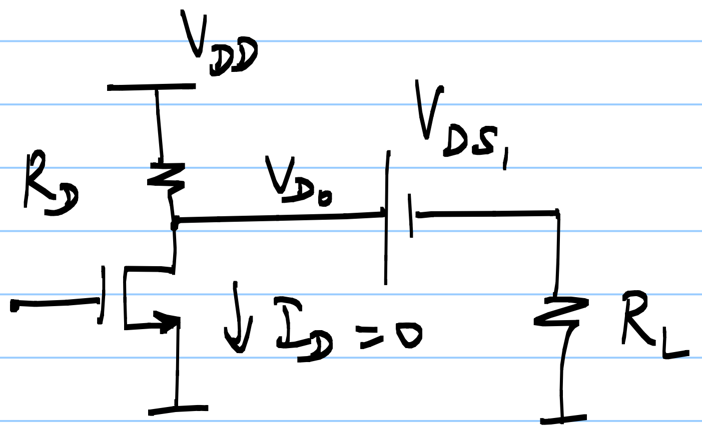
Device M_1 just cuts off. @ $V_{A_{max_2}}$

Any further increase in $V_A \rightarrow$ clipped sinusoid (current)

Set $I_D(t) = 0$ @ $-$ve peak

$$I_{D_1} - g_m V_{A_{max_2}} = 0 \Rightarrow V_{A_{max_2}} = \frac{I_{D_1}}{g_m}$$

What is $V_D(t)$ when $I_D(t) = 0$?



* $I_D = 0$
* KCL @ drain

$$\frac{V_{DD} - V_{D_0}}{R_D} = \frac{V_{D_0} - V_{DS_1}}{R_L}$$

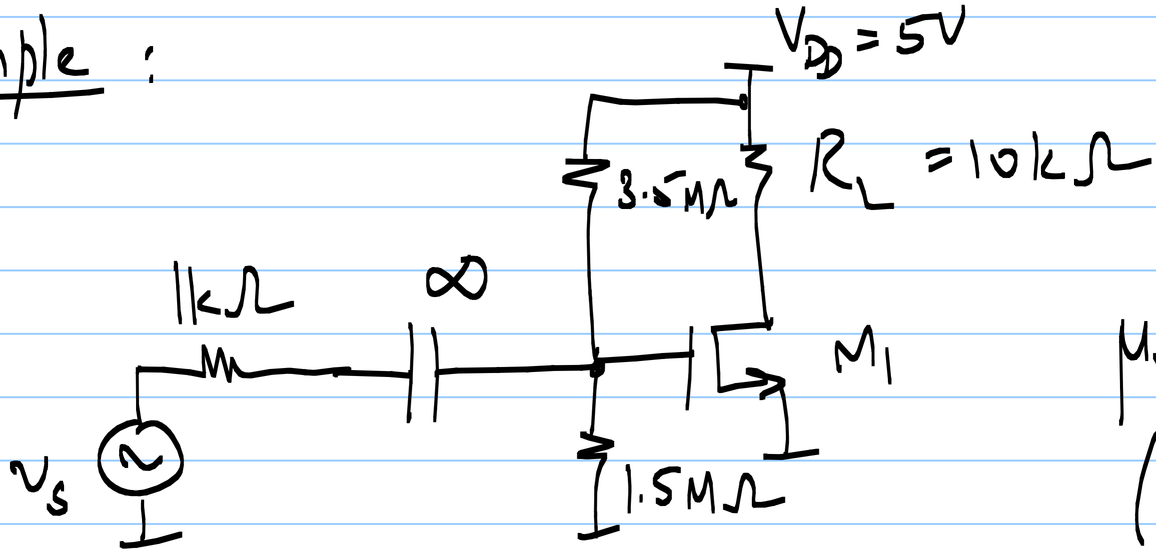
$$V_{D_0} = \frac{R_L V_{DD} + R_D V_{DS_1}}{R_L + R_D}$$

Swing limits of CSA

* In general, $V_{A_{max_1}} \neq V_{A_{max_2}}$
* swing limit $= \min\{V_{A_{max_1}}, V_{A_{max_2}}\}$
* Good design : choose $V_{A_{max_1}} = V_{A_{max_2}}$

choose (I_{D_1}, V_{DS_1}) {
@ +ve H.C. peak, M_1 just enters triode
@ −ve H.C. peak, M_1 just cuts off

Example :



$V_T = 1V$

$\mu_n C_{ox} = 100\mu A/V^2$

$\left(\frac{W}{L}\right) = 10$

$V_{GS_1} = 1.5V$

$I_{D_1} = \frac{1}{2} 100 \times 10^{-6} \times 10 \times (0.5)^2 = 125 \mu A$

$V_{DS_1} = V_{DD} - I_{D_1} \cdot R_L = 5 - (125 \times 10^{-6})(10 \times 10^3)$

$= 3.75V$

$g_{m_1} = \frac{2 I_{D_1}}{V_{GS_1} - V_T} = \frac{0.25 mA}{0.5} = 0.5\ mS$

inc. gain $\quad G = -g_{m_1} R_L = -5$

<u>Triode limit</u> (+ve HC)

$$\left.\begin{aligned} V_G &= 1.5V + V_A \sin \omega t \\ V_D &= 3.75V - 5V_A \sin \omega t \end{aligned}\right) \quad V_D = V_G - V_T$$

$$3.75 - 5V_{A_1} = 1.5 + V_{A_1} - 1$$

$$V_{A_1} = \frac{3.25}{6} = 541.67\ mV = V_{A_{max_1}}$$

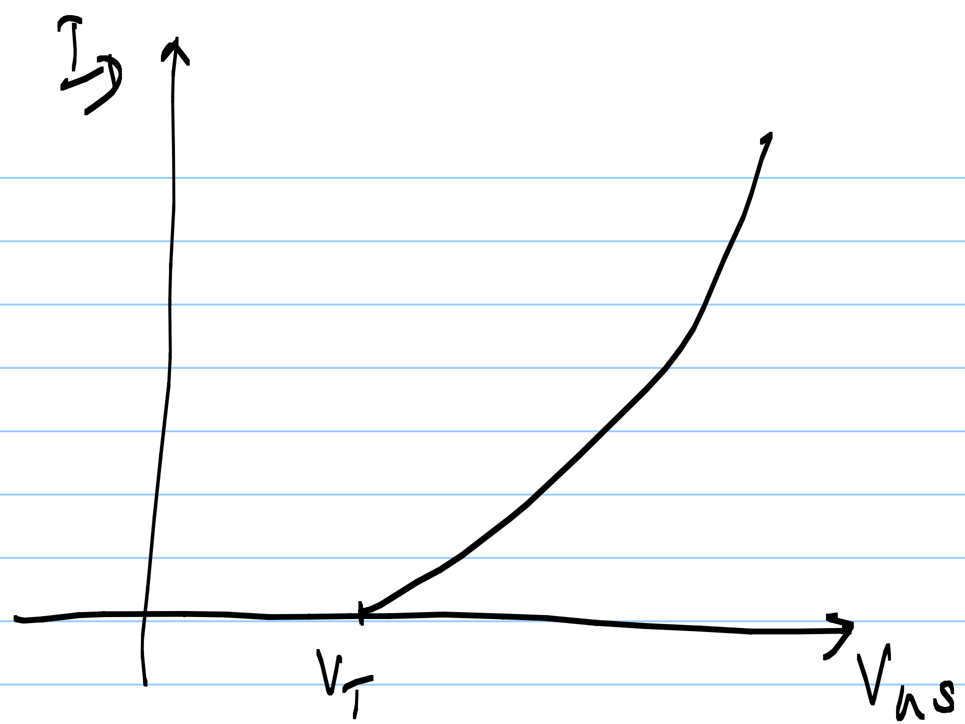
<u>Cut off limit</u> (−ve H.C.)

$$I_D = I_{D_1} + g_m V_A \sin\omega t$$

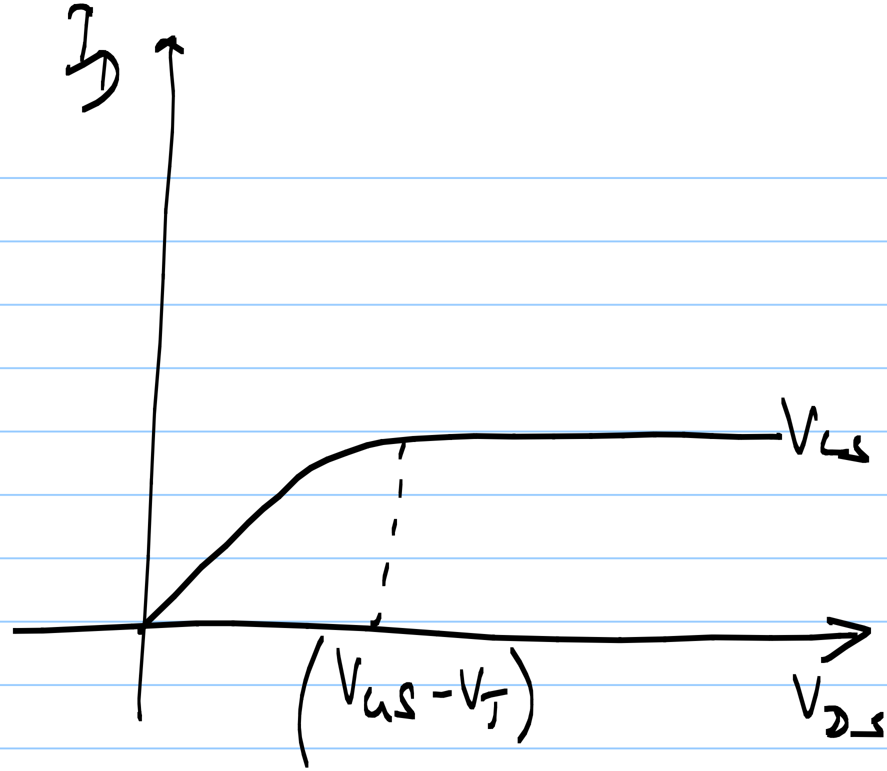
$$= 125\,\mu A + (0.5\,mS)\, V_A \sin\omega t$$

$$V_{A_2} = \frac{I_{D_1}}{g_m} = \frac{125\,\mu A}{0.5\,mS} = 250\,mV = V_{A_{max_2}}$$

$$V_{A_{max}} = \text{min. } f\left(V_{A_1}, V_{A_2}\right) = 250\,mV$$



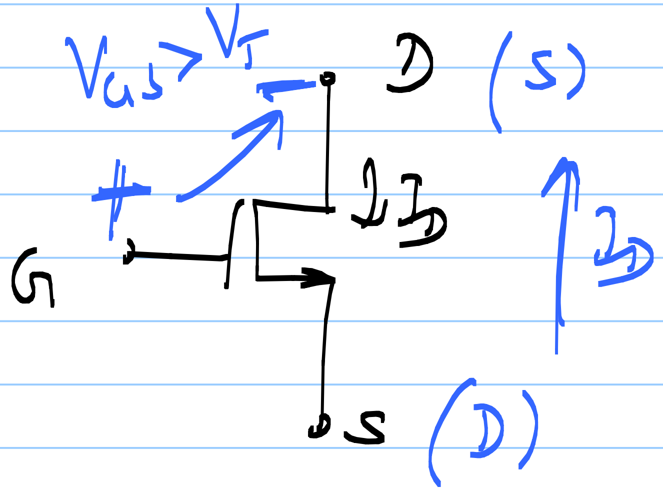
I_D
V_T
V_{GS}

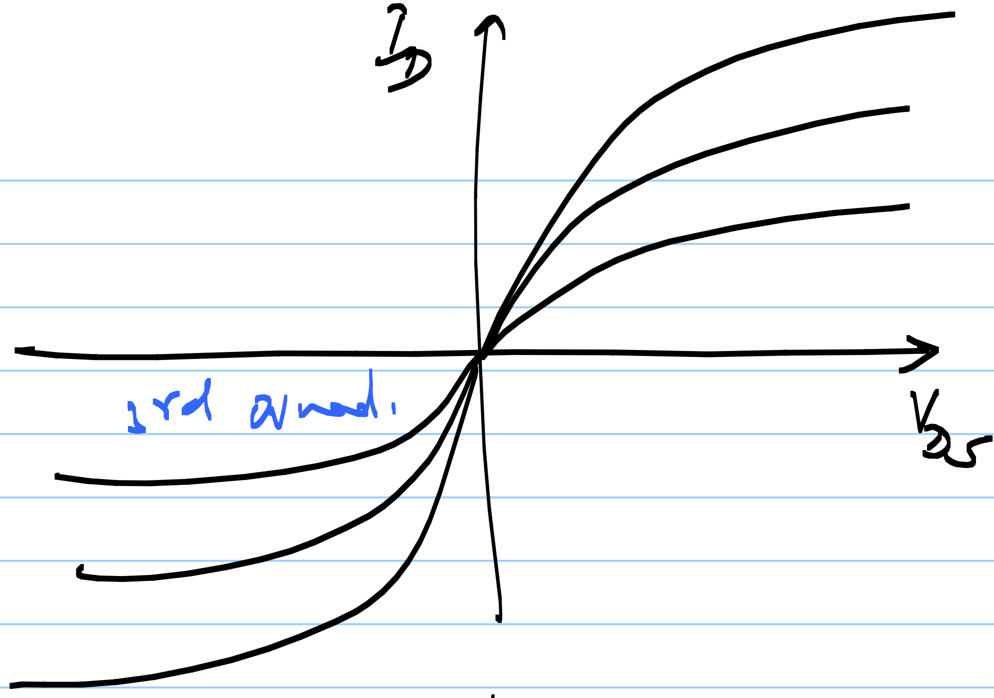


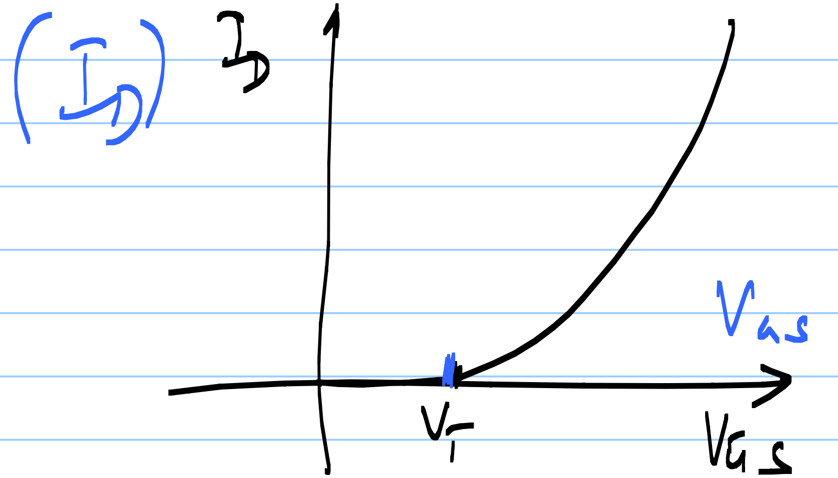
I_D
V_{GS}
$(V_{GS} - V_T)$
V_{DS}

21/8/2020

Lecture 11







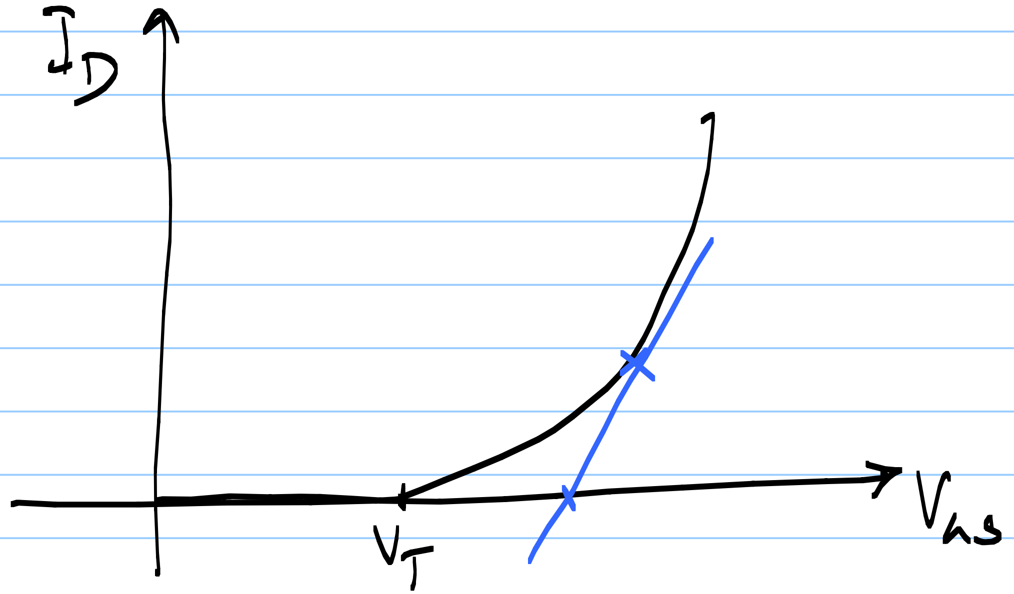
Cutoff condition @ V_{GS}

$I_D = 0 \Rightarrow V_{GS} \le V_T$

$V_{GS} = V_T$ limit

$$V_{GS_1} + V_A \underbrace{\sin \omega t}_{-1} = V_T$$

$V_{A_3} = V_{GS_1} - V_T = 1.5V - 1V = 500mV$



In sat. region

$$I_D = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)(V_{GS}-V_T)^2$$ No dep. on V_{DS}

$$I_D = f(V_{GS}, V_{DS})$$

$$I_D + i_d = f(V_{GS}+v_{gs}, V_{DS}+v_{ds})$$

$$= f(V_{GS}, V_{DS}) + \underbrace{\frac{\partial I_D}{\partial V_{GS}}}_{g_m} \cdot v_{gs} + \cancel{\frac{\partial I_D}{\partial V_{DS}}}^{0} \cdot v_{ds}$$

$$+ \frac{1}{2}\frac{\partial^2 I_D}{\partial V_{GS}^2}\cdot v_{gs}^2 + \frac{1}{2}\cancel{\frac{\partial^2 I_D}{\partial V_{DS}^2}}^{0}\cdot v_{ds}^2$$

$$+ \cancel{\frac{\partial^2 I_D}{\partial V_{GS}\,\partial V_{DS}}}^{0}\cdot v_{gs}\cdot v_{ds} + \cdots$$

$$\cancel{I_D} + i_d = \cancel{I_D} + \underbrace{g_m v_{gs}}_{\text{desired term}} + \underbrace{\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)\cdot v_{gs}^2}_{\text{undesired term (NL)}}$$

A

B

B << A

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)\cdot v_{gs}^2 << g_m v_{gs}$$

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)\cdot v_{gs}^2 << \mu_n C_{ox}\left(\frac{W}{L}\right)(V_{GS}-V_T)\cdot v_{gs}$$

$$\underline{\underline{v_{gs} << 2\,(V_{GS}-V_T)}}$$

In our example : $v_{gs} = \min\{V_{A_1}, V_{A_2}\} = 0.25V$

$$2(V_{GS_1} - V_T) = 1V$$

$\textcircled{B} \approx 25\%$ of $\textcircled{A}$ $\quad$ {Not small enough}

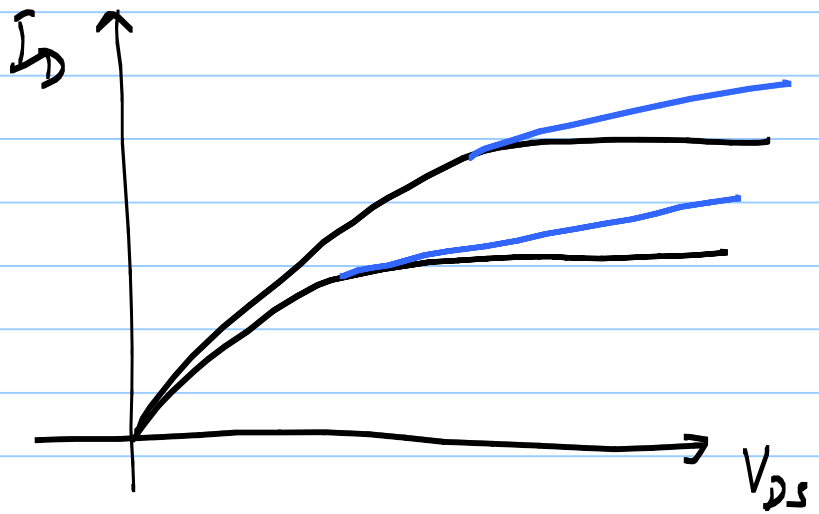
THD : "Total Harmonic Distortion"

%age.

25/8/20

Lecture 12

Real MOSFET Characteristics



We assumed that in sat.,

$I_D = f(V_{GS})$ only

In reality,

$I_D = f(V_{GS}, V_{DS})$

$$I_D = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)(V_{GS}-V_T)^2(1+\lambda V_{DS})$$

for a good MOSFET, $\lambda << 1$

"Channel Length Modulation"

for op. pt. calculations, use: $I_D = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)(V_{GS}-V_T)^2$

2-port parameters:

$$y_{11} = y_{12} = 0$$

$$y_{21} = g_m = \frac{\partial I_D}{\partial V_{GS}} = \mu_n C_{ox}\left(\frac{W}{L}\right)(V_{GS} - V_T)(1 + \lambda V_{DS})$$

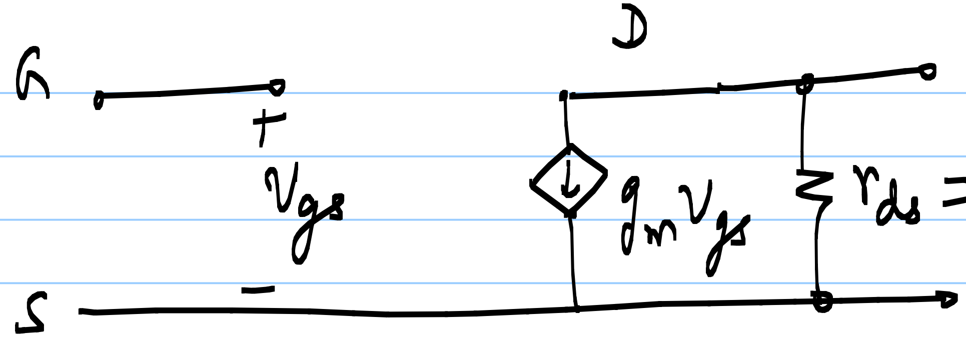
$$\approx \mu_n C_{ox}\left(\frac{W}{L}\right)(V_{GS} - V_T)$$ {assume same as before}

$$g_{ds} = y_{22} = \frac{\partial I_D}{\partial V_{DS}} = \underbrace{\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)(V_{GS} - V_T)^2}_{\approx I_D} \cdot \lambda$$

output conductance of MOSFET

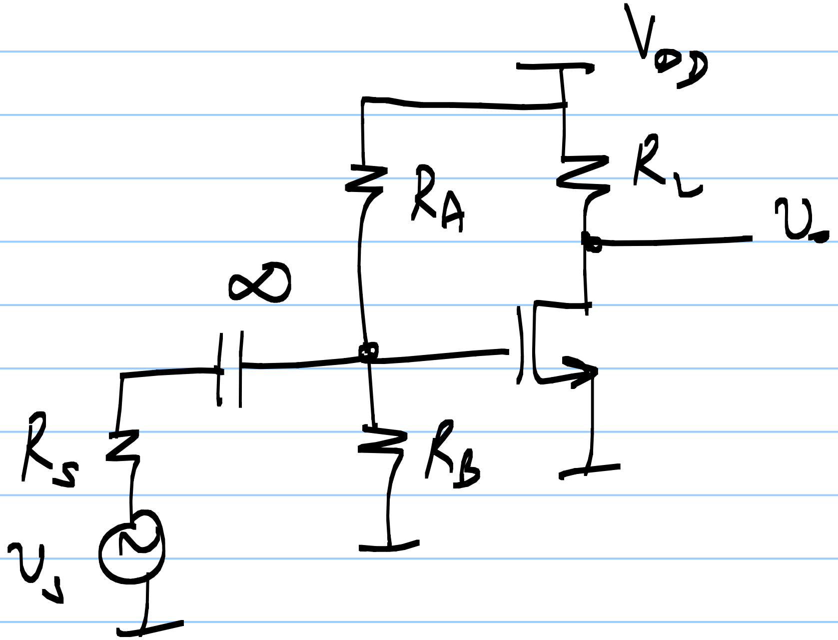
$$g_{ds} \approx \lambda \cdot I_D$$

$$r_{ds} = \frac{1}{g_{ds}} = \frac{1}{\lambda I_D}$$



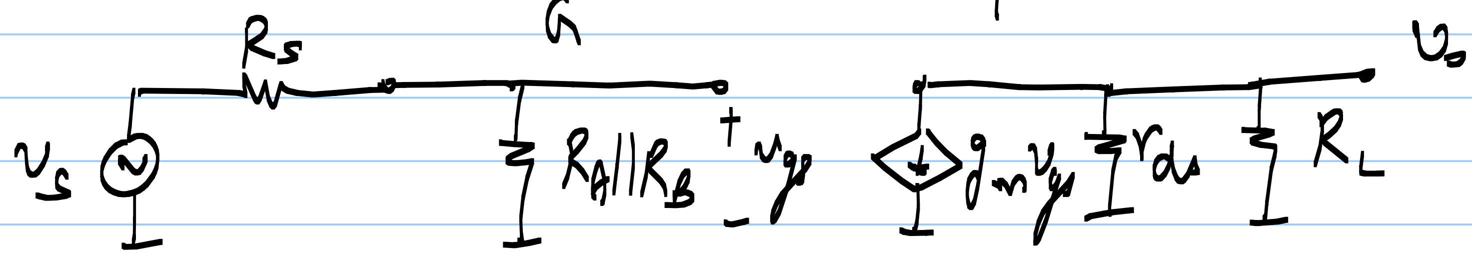
$$r_{ds} = \frac{1}{\lambda I_D}$$

Common-Source amp.



$$\frac{v_o}{v_s} = \frac{v_o}{v_{gs}} \cdot \frac{v_{gs}}{v_s}$$

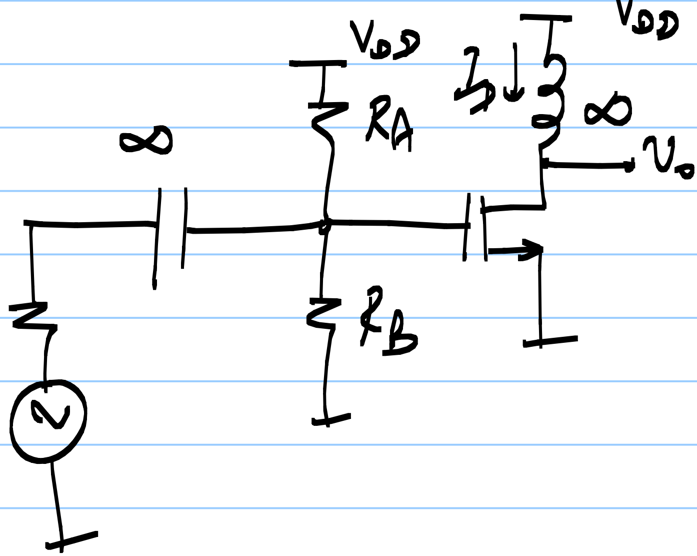
$$= -g_m (r_{ds} \| R_L) \cdot \underbrace{\frac{R_A \| R_B}{R_s + R_A \| R_B}}_{\approx 1}$$

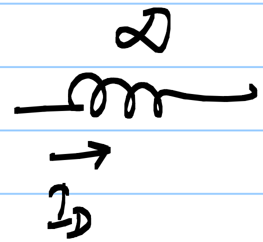


$$\frac{v_o}{v_s} \approx -g_m (r_{ds} \| R_L)$$ gain is smaller

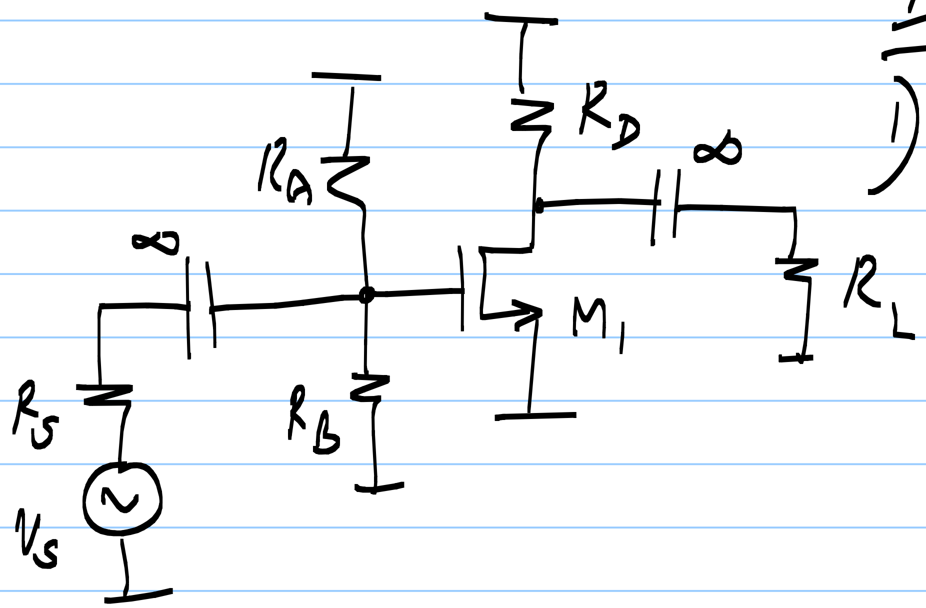
Max possible gain : $R_L = \infty$

$|\text{max. gain}| = g_m r_{ds} \equiv$ "intrinsic gain" of MOSFET





: DC short
AC open



<u>Issues:</u>

1) M_1 parameters vary with
   a) ambient temp.
   b) time
   c) device to device

i.e. random variations in device properties

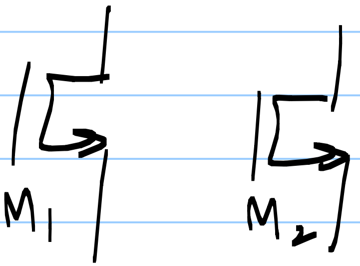
e.g. if V_T changes, $\longrightarrow$ I_D changes $\rightarrow$ changes in g_m, r_{ds}, V_{DS}, V_{ov} etc.

$\Rightarrow$ Undesirable

2) Tolerance in R_A, R_B etc

On an IC:

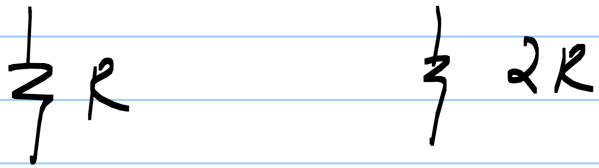
1) Multiple copies of the same device (R, C, MOSFET etc.) nominally have same properties, (On the same IC)



$$\left.\begin{matrix} V_{T1}, \; V_{T2} \\ \mu_{n1}, \; \mu_{n2} \\ \text{etc.} \end{matrix}\right\} \text{vary together}$$

2) Properties may vary across multiple ICs

3) Ratios of like components vary similarly



$$\frac{R_B}{R_A + R_B}$$ — used to generate V_{GS} from V_{DD}

Issues still exist:

1) V_{GS} is constant ($\propto V_{DD}$), but V_T etc. can vary $\Rightarrow$ I_D varies $\Rightarrow$ gain, swing limits vary.

$\Rightarrow$ Make V_{GS} vary with V_T to get desired I_D

Use "negative feedback" to generate V_{GS}

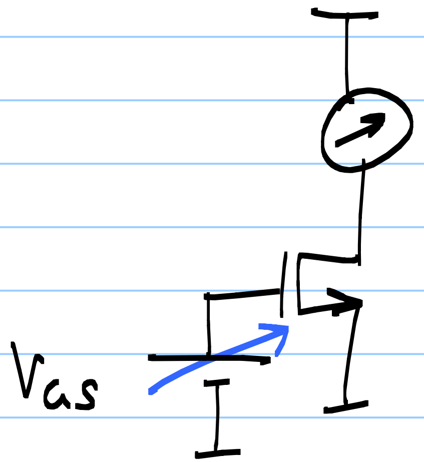
* Desired Value (desired bias Drain current $= I_0$)
* Measure actual value (I_D)
* Compare actual with desired ($I_D \leftrightarrow I_0$)

* Use error to move actual value towards desired value (make $I_D \to I_o$, using V_{GS})

26/8/2020

Lecture 13

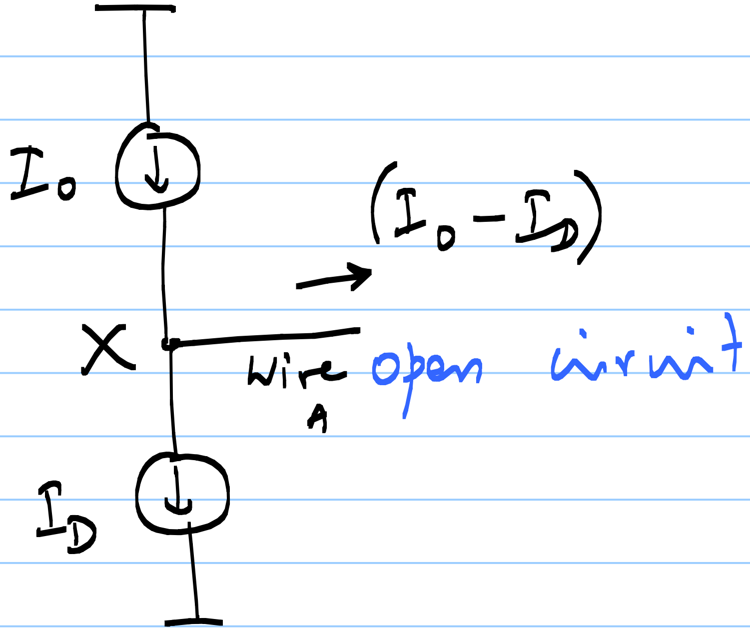
Negative feedback biasing



Ammeter = I_D

1) Apply V_{GS}
2) Measure I_D
3) If $I_D > I_0$, $\downarrow V_{GS}$
4) If $I_D < I_0$, $\uparrow V_{GS}$
5) If $I_D = I_0$, V_{GS} = same as before

$(I_0 - I_D) \equiv$ comparing I_0 with I_D



In a MOSFET:

V_{GS} is cause

I_D is effect

If wire A is open circuited:

1) If $I_0 > I_D$: $V_X \uparrow$

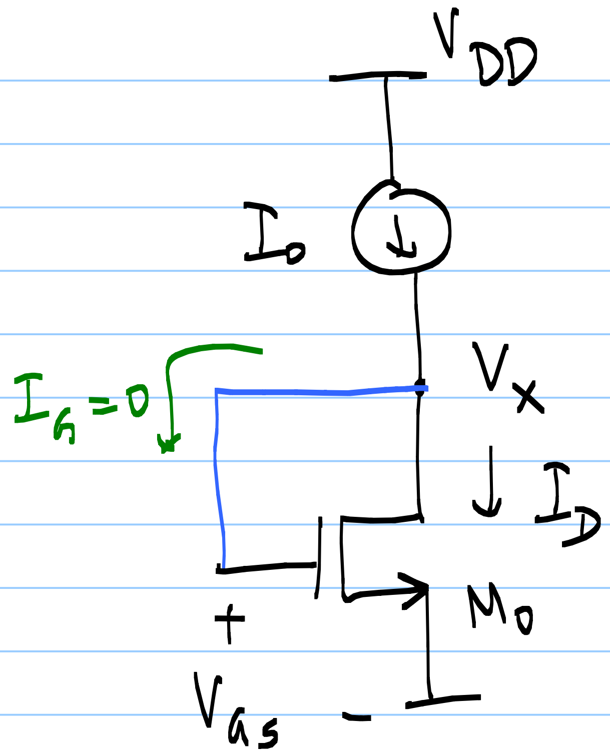
(need to $\uparrow V_{GS}$)

2) If $I_0 < I_D$: $V_X \downarrow$

(need to $\downarrow V_{GS}$)

3) If $I_0 = I_D$: V_X stays constant

(need to keep V_{GS} at same value)



* Connect X to G : Valid

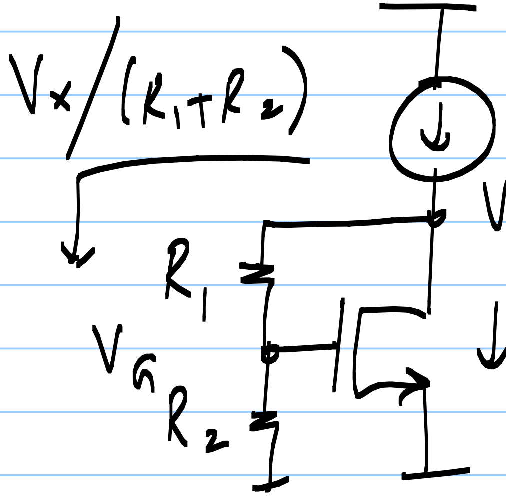
* $I_G = 0 \Rightarrow I_D = I_0$

$$V_X = V_{GS}\Big|_{I_D = I_0}$$

* Verify that M_0 is in saturation:

$$V_D = V_G \Rightarrow V_D > V_G - V_T$$

M_0 is in sat.

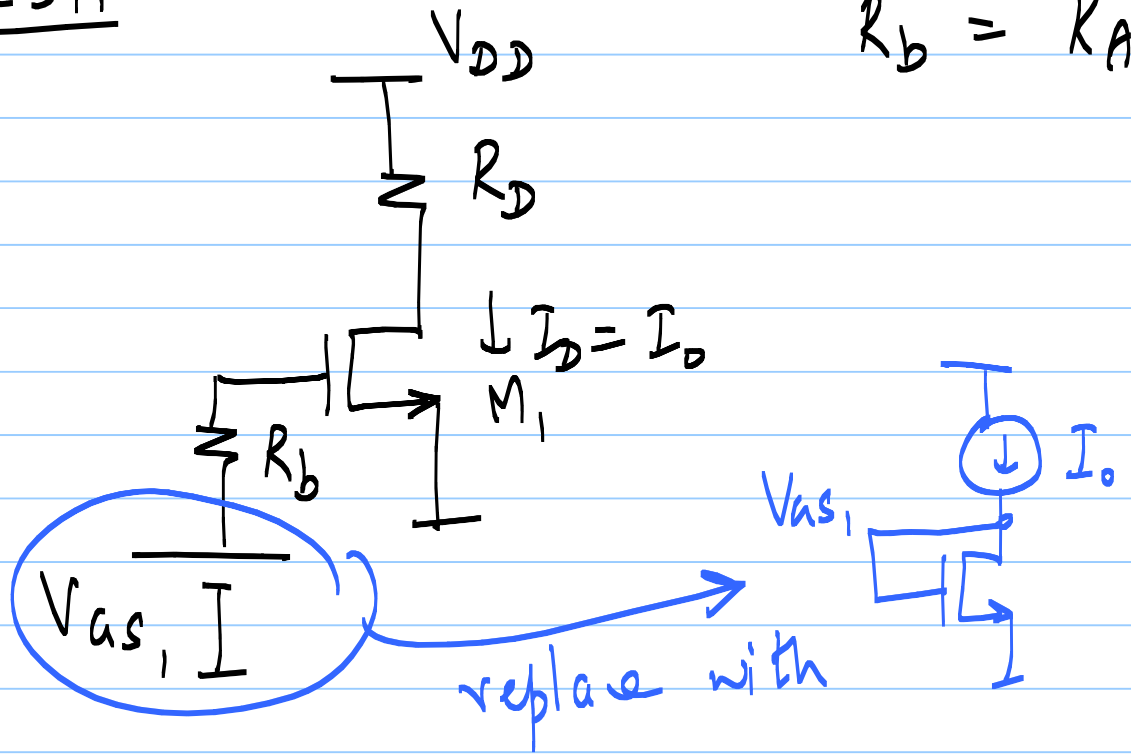


$$V_X = \left(\frac{R_1+R_2}{R_2}\right)\cdot V_{GS}$$

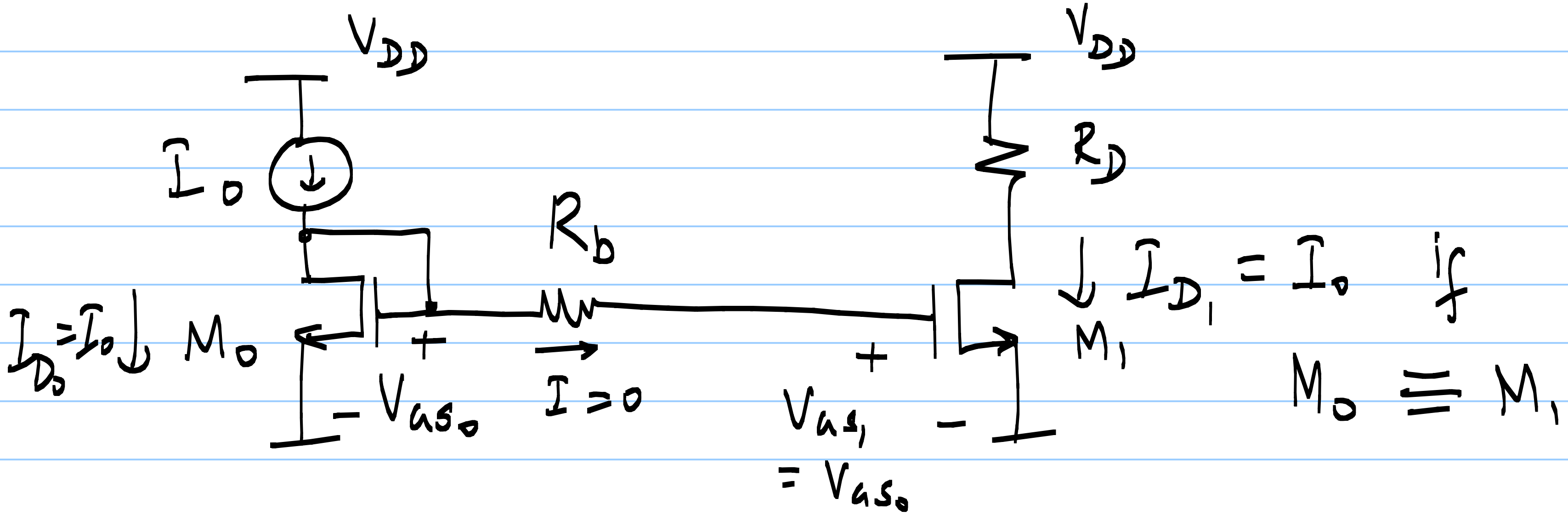
$$\downarrow I_D = I_0 - \left(\frac{V_X}{R_1+R_2}\right)$$

CSA

$R_b = R_A \| R_B$



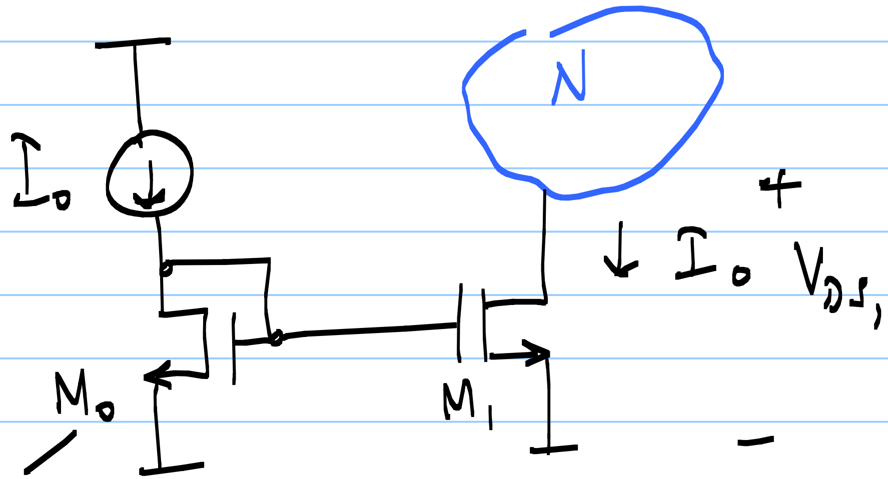
VDD
RD
ID = Io
M1
Rb
VGS1
replace with
VGS1
Io



VDD
Io
Rb
ID0 = Io
M0
+
− VGS0
I = 0
+
VGS1
= VGS0
−
VDD
RD
ID1 = Io if
M1
M0 ≡ M1

$M_0 \equiv M_1$ means same μ_n, C_{ox}, V_T, $\left(\frac{W}{L}\right)$

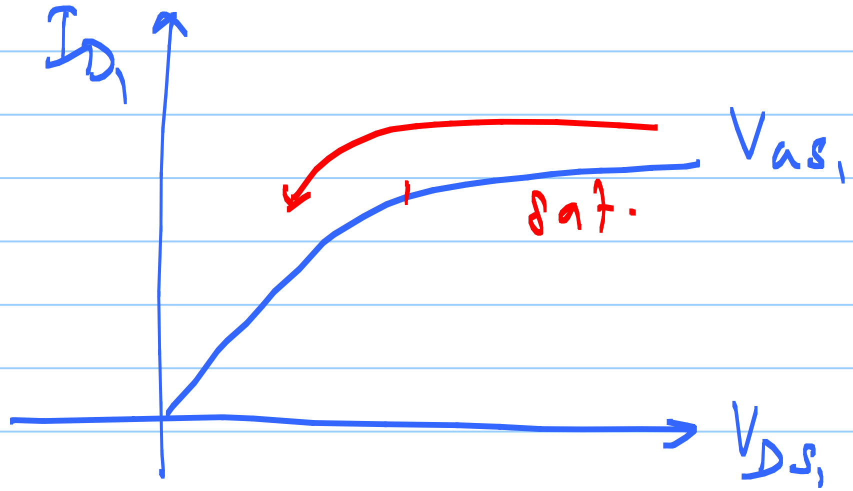
{same W and L}



always in sat.

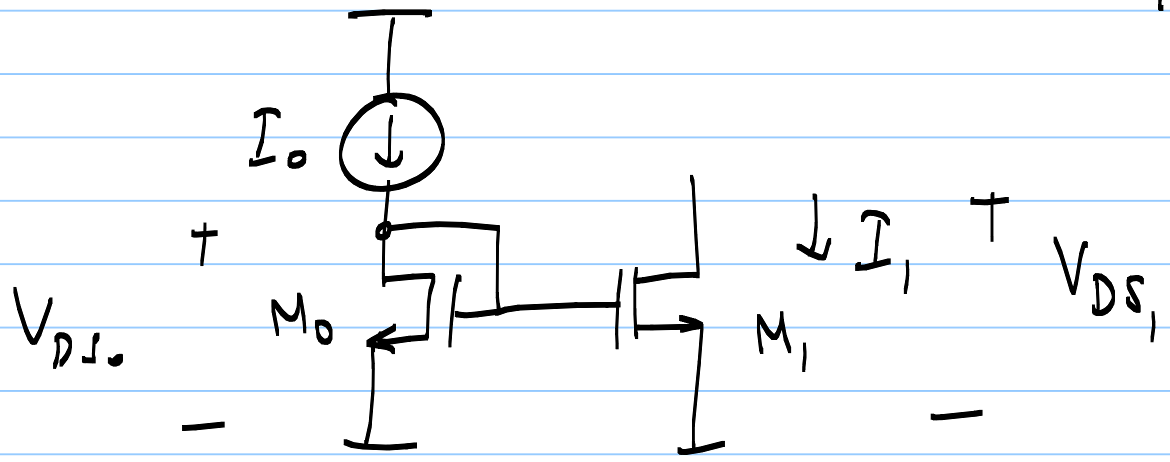
"Current Mirror" works well as long as $V_{DS_1} \geq V_{GS_1} - V_T$

(M_1 is in sat.)



27/8/2020

Lecture 14



$I_1 = I_0$ if M_1 is in sat.

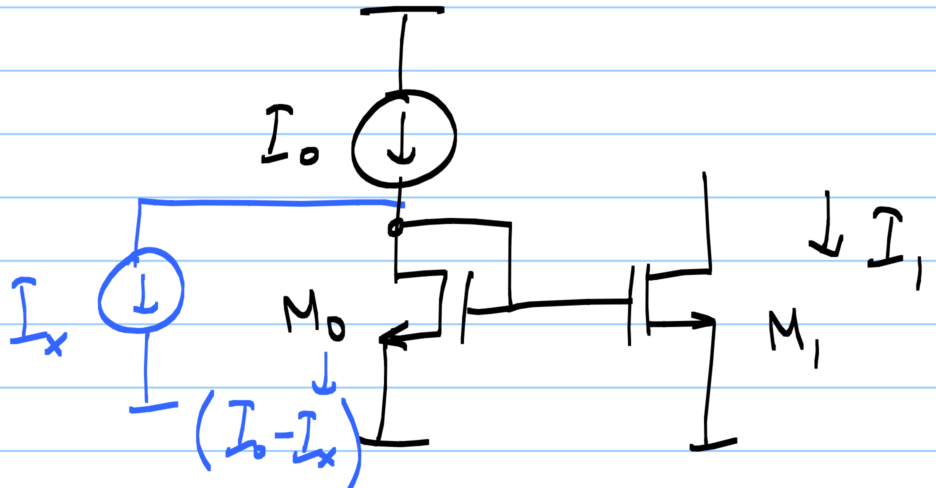
$V_{DS_1} \geq V_{GS_1} - V_T$

$V_{GS_0} = V_{GS_1} = V_{DS_0}$

* small deviations due to CLM (r_{ds})

$V_{DS_1} \neq V_{DS_0}$

$V_{DS_1} \neq V_{GS_1}$



1) I want $I_1 = k I_0$

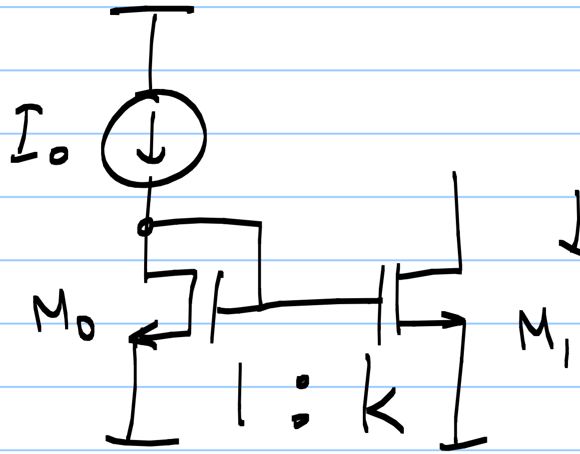
$$I_1 = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{GS_1} - V_T)^2$$

$$V_{GS_1} = V_{GS_0}$$

$$V_{GS_1} - V_T = V_{GS_0} - V_T$$

$$(V_{GS_1} - V_T)^2 = (V_{GS_0} - V_T)^2 = \frac{2 I_0}{\mu_n C_{ox} \left(\frac{W}{L}\right)_0}$$

$$\boxed{I_1 = I_0 \cdot \underbrace{\frac{(W/L)_1}{(W/L)_0}}_{k}}$$

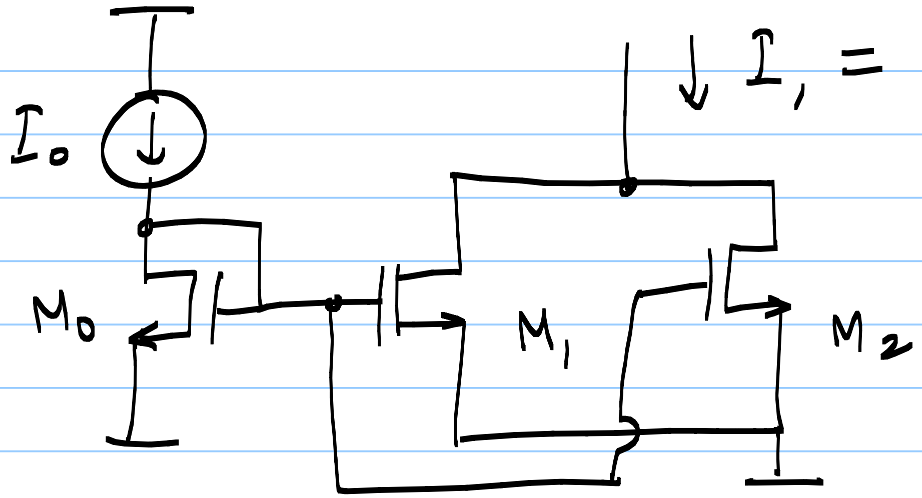


$$\Downarrow \; I_1 = k I_0$$

$$\boxed{\begin{array}{ll} \underline{\text{Ensure}}: & L_1 = L_0 \\ & W_1 = k W_0 \end{array}}$$

λ — models $\Delta L / L$ (CLM)

larger L $\rightarrow$ smaller $\Delta L/L$ $\rightarrow$ smaller λ $\rightarrow$ larger r_{ds}

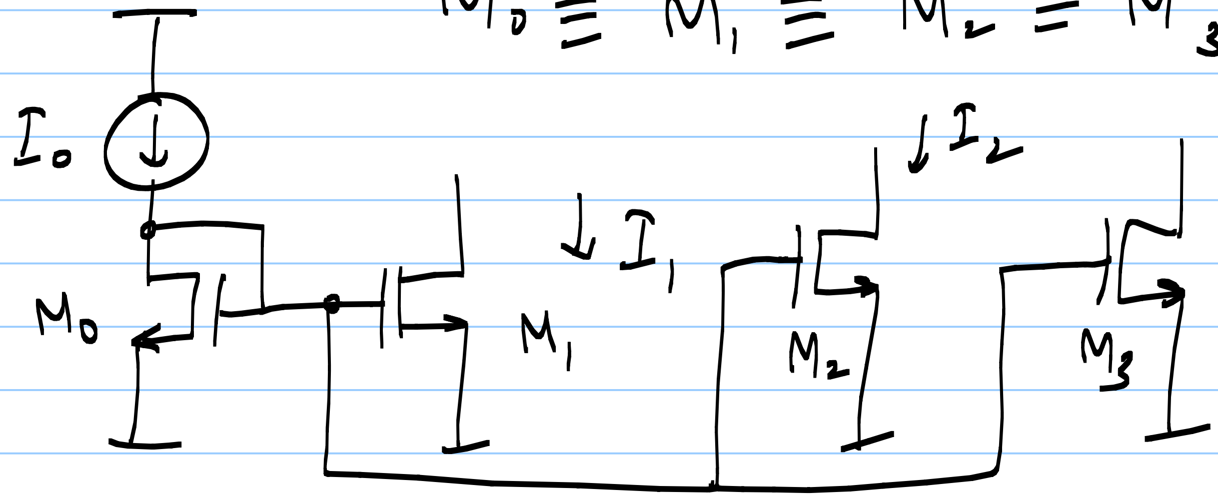


$\downarrow I_1 = I_{D_1} + I_{D_2} = 2I_0$

$M_0 \equiv M_1 \equiv M_2$

2)

$M_0 \equiv M_1 \equiv M_2 \equiv M_3 \ldots$

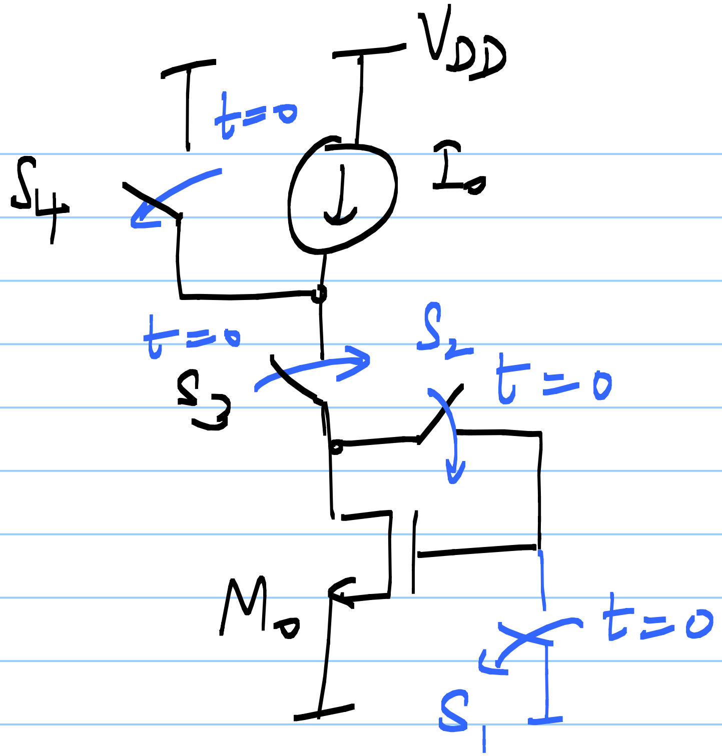


$\ldots$

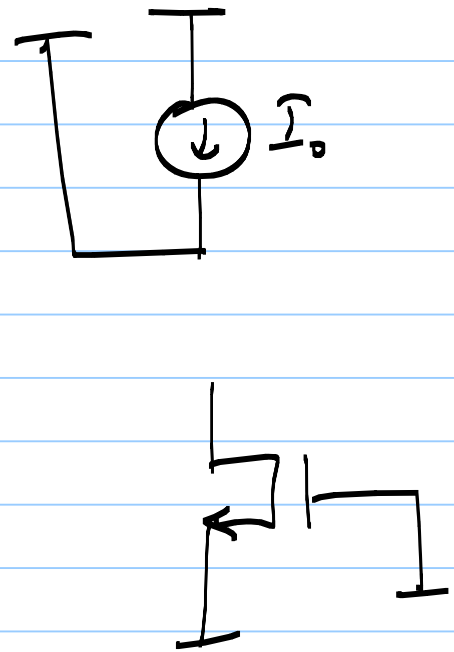
all V_{DS}'s $\geq V_{GS_1} - V_T$

[all devices in sat.]

Make as many copies as you want

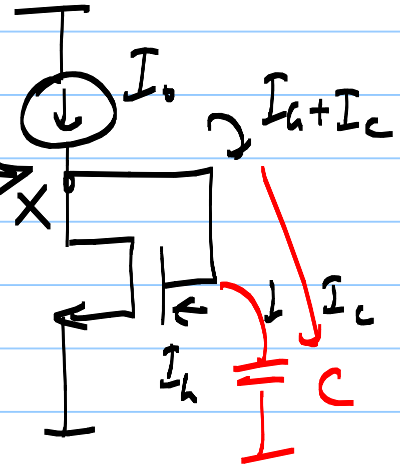


$t < 0$:



$t = 0^+$

KCL

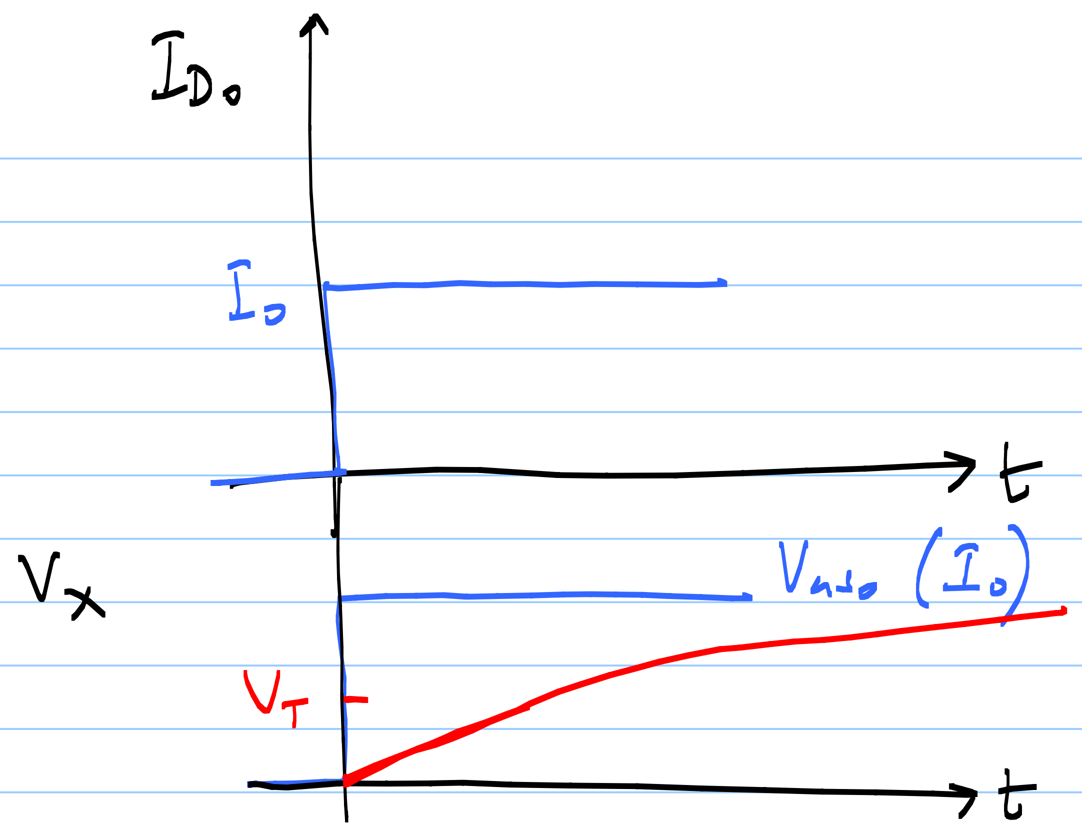


@ $t = 0^-$ $V_{GS} = 0$

@ $t = 0^+$: $I_G = 0$, $I_D = 0$

$\rightarrow V_X \uparrow \rightarrow V_{GS} \uparrow \rightarrow I_D \uparrow$

$\rightarrow I_D = I_0$

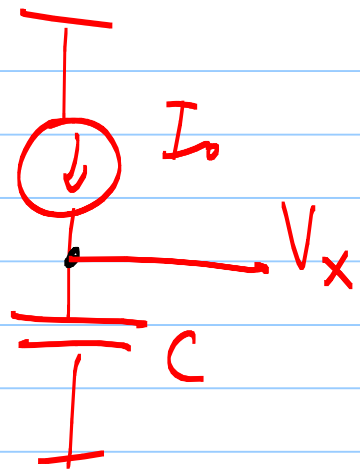


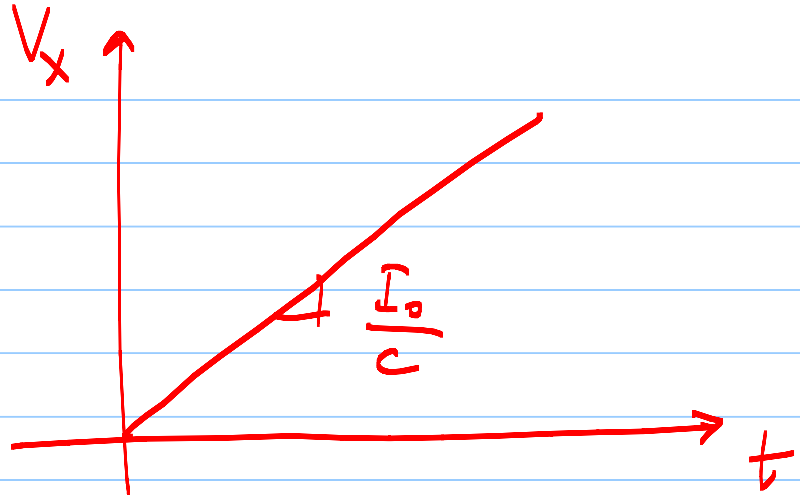
$$I_0 = I_C + I_{D0} + \cancel{I_G}^{\,0}$$

$= 0$ @ $t = 0^+$

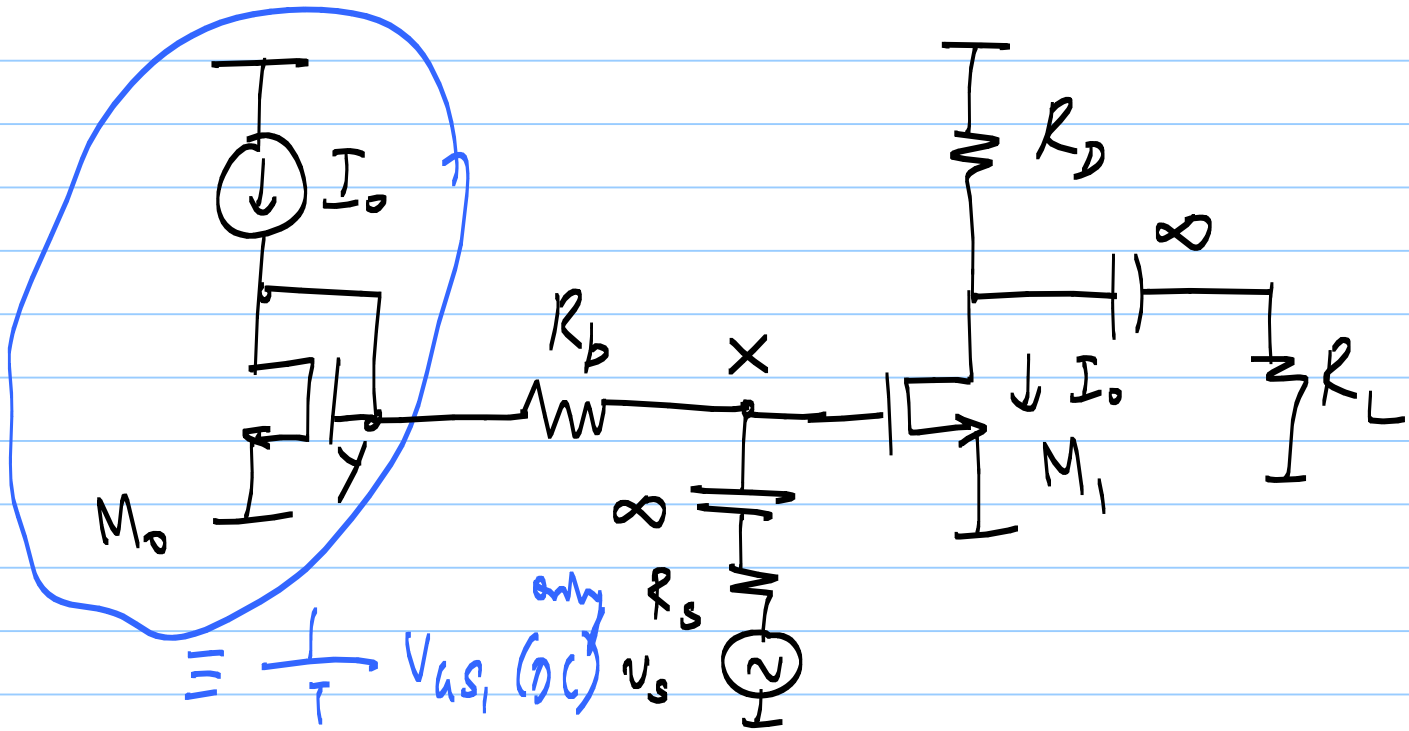
$I_D = 0$ till $V_{GS} = V_x = V_T \Rightarrow I_C = I_0$

$I = C\dfrac{dV}{dt}$

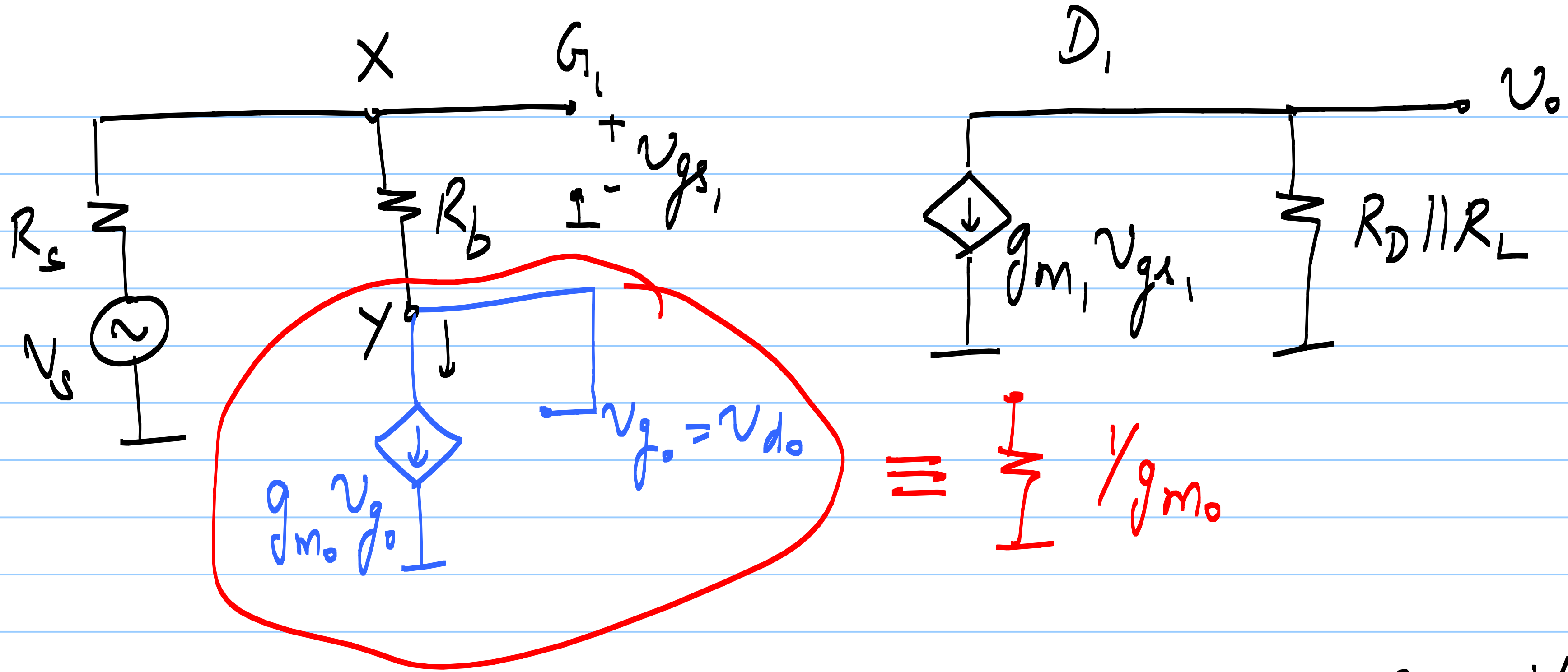


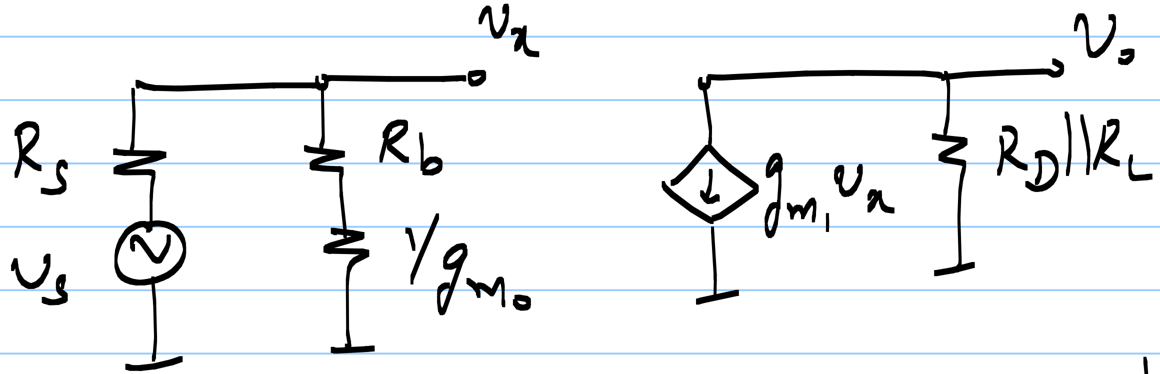


CSA



$M_o \equiv M_1$



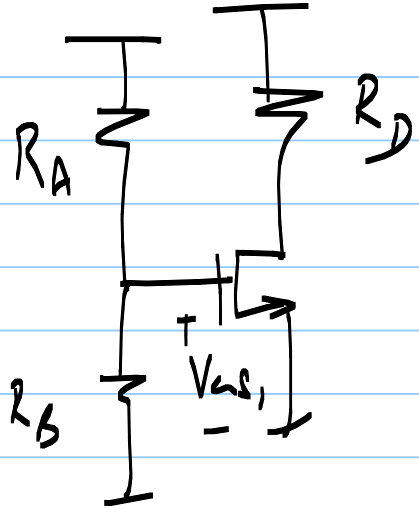


$$\frac{v_x}{v_s} = \frac{R_b + 1/g_{m_0}}{R_s + R_b + 1/g_{m_0}}$$

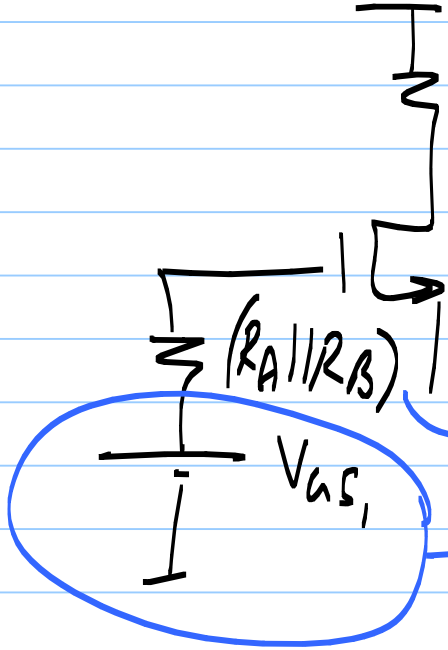
$g_{m_0} = g_{m_1} = \text{large}$

choose $R_b \gg 1/g_{m_0}, R_s$

$\Rightarrow v_x \approx v_s$



$\equiv$



replace with R_L

replace with I_o, M_o combo

$$v_o = -g_{m_1} v_x \left(R_D \| R_L\right)$$

$$\simeq -g_{m_1}\left(R_D \| R_L\right) \cdot v_s$$

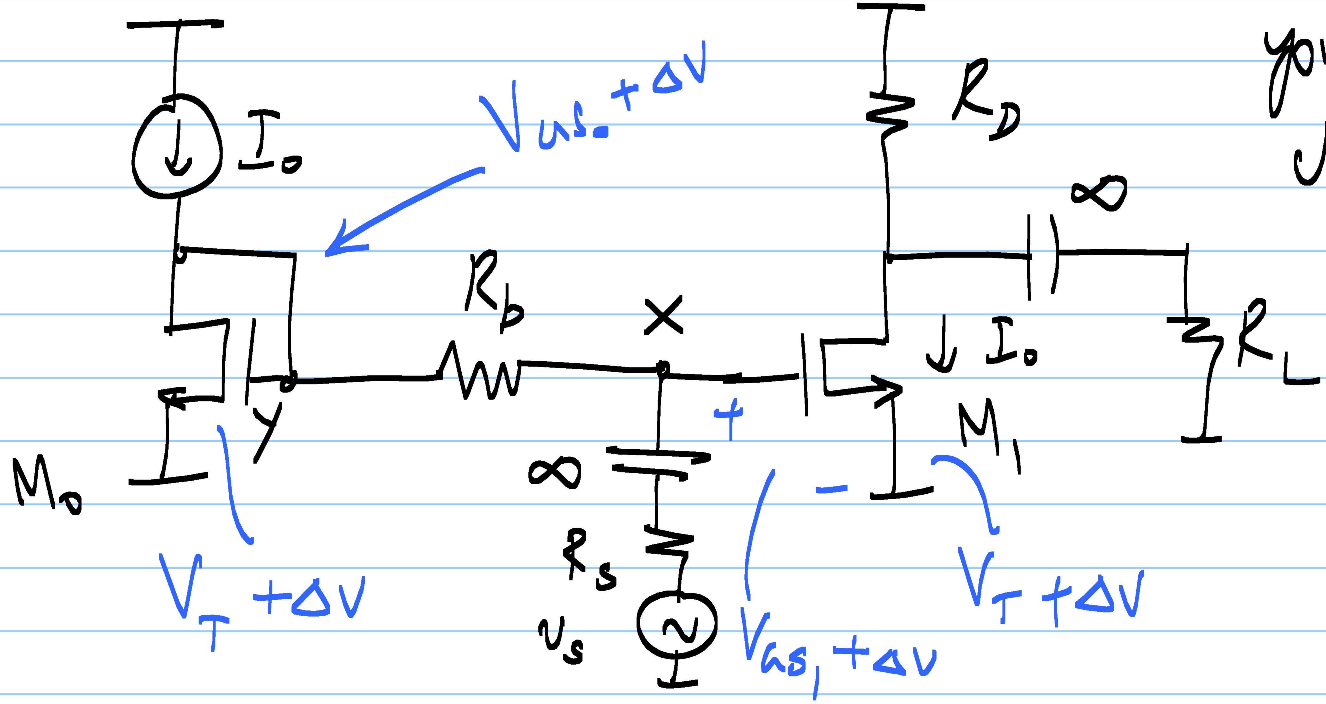
$$\frac{v_o}{v_s} \simeq -g_{m_1}\left(R_D \| R_L\right)$$ same as before

I_{D_1}, V_{GS_1}, V_{DS_1} – same as before $\Rightarrow$ Swing limits are the same too.

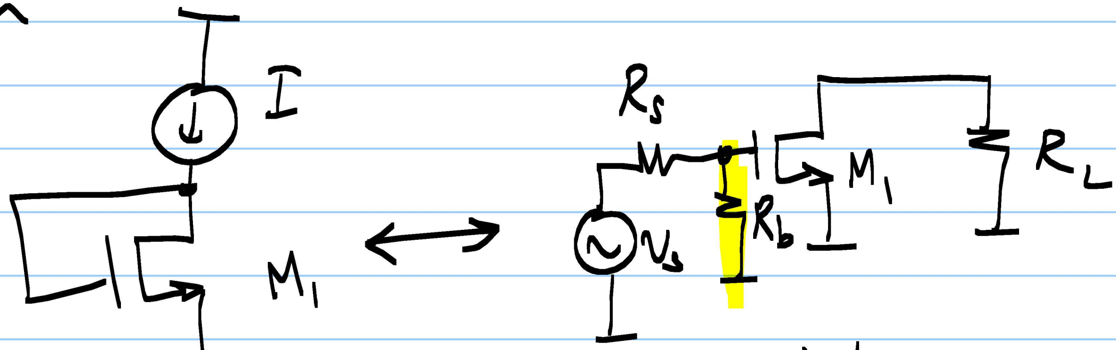
28/8/20

Lecture 15

* If V_{T_1} changes by ΔV, you expect V_{T_0} to also change by ΔV

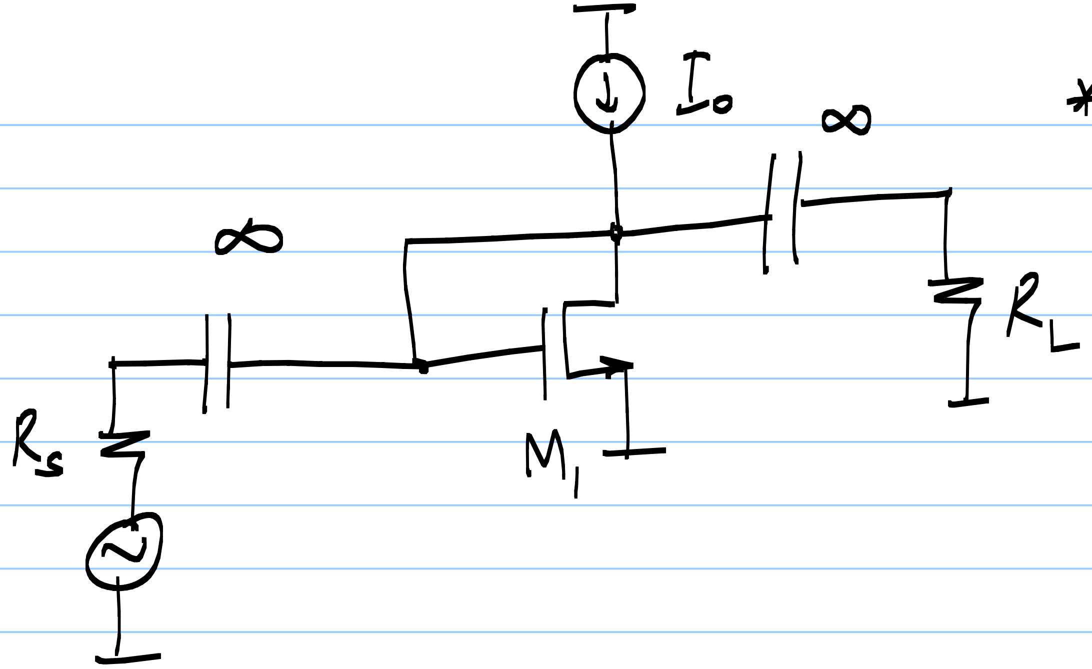


Can you do this with a single MOSFET?

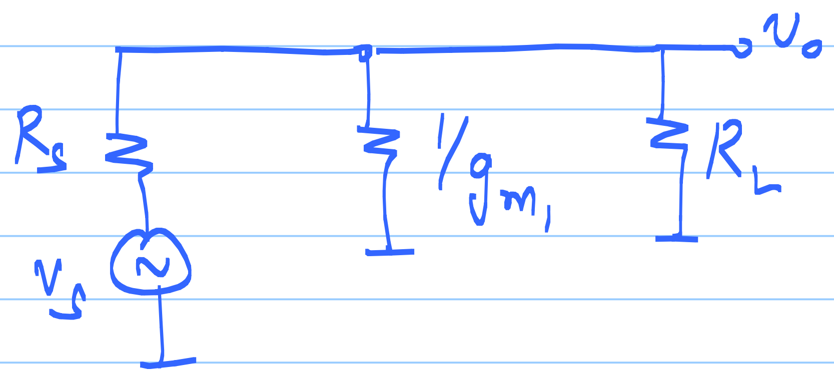


DC bias

Incremental



* Start with DC bias circuit
* Add source & load
* $V_{DS} = V_{GS}$
* M_1 is in sat.

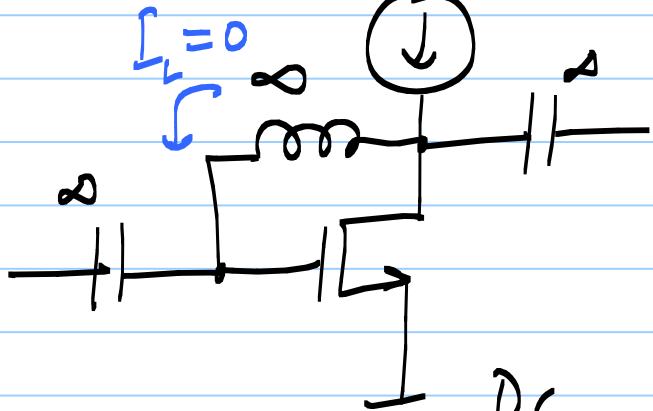


$$\frac{v_o}{v_s} = \frac{R_L \,||\, 1/g_{m_1}}{R_s + R_L \,||\, 1/g_{m_1}}$$

Not what we want!

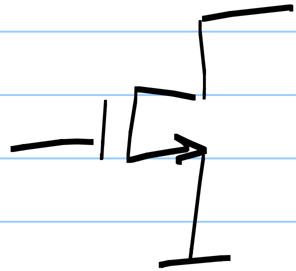
Options: 1) Add ∞ inductor between G & D

✓ AC

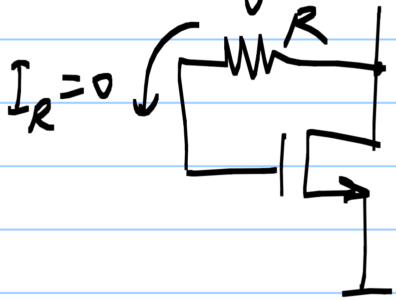


DC

works but impractical (L's are bulky)



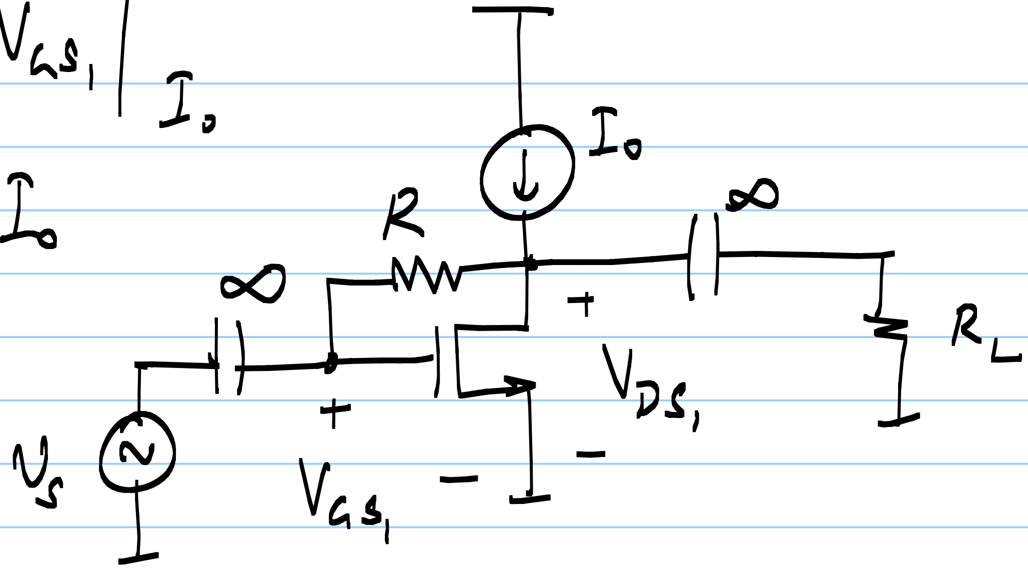
2) Use large resistor (because $I_L = 0$)



$V_{DS_1} = V_{GS_1}$, feedback maintained

$V_{DS_1} = V_{GS_1}\big|_{I_o}$

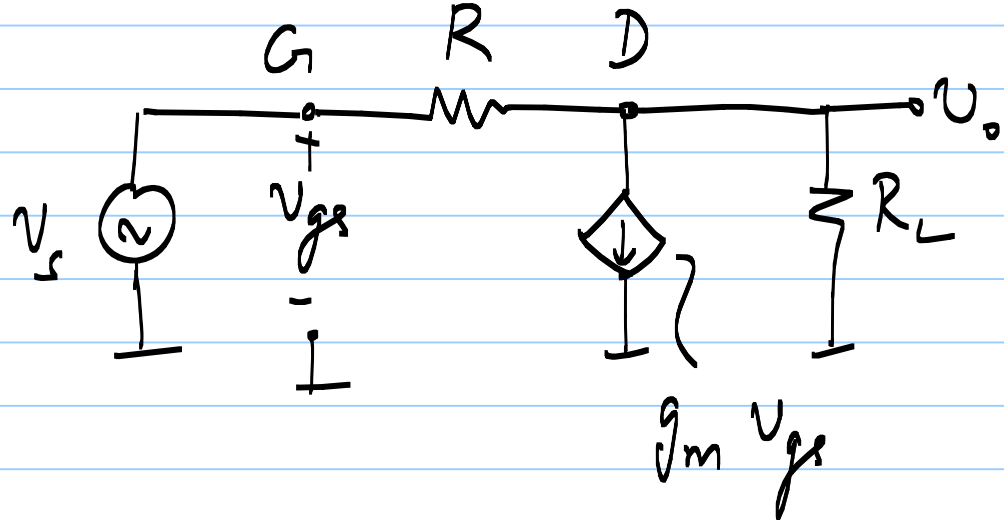
$I_{D_1} = I_o$



HW1 : Analyse with R_S

DC is okay

SS eq.:



$v_{gs} = v_s$

KCL @ drain node:

$$\frac{v_s - v_o}{R} = g_m v_s + \frac{v_o}{R_L}$$

$$\frac{1}{R} = G, \qquad \frac{1}{R_L} = G_L$$

$$v_s\left[G - g_m\right] = v_o\left[G_L + G\right]$$

$$\frac{v_o}{v_s} = \frac{G - g_m}{G + G_L} = \frac{-g_m}{G_L}\underbrace{\left[\frac{1 - G/g_m}{1 + G/G_L}\right]}_{\approx 1 \text{ (desired)}}$$

We want

* $g_m \gg G_L$ (large gain)

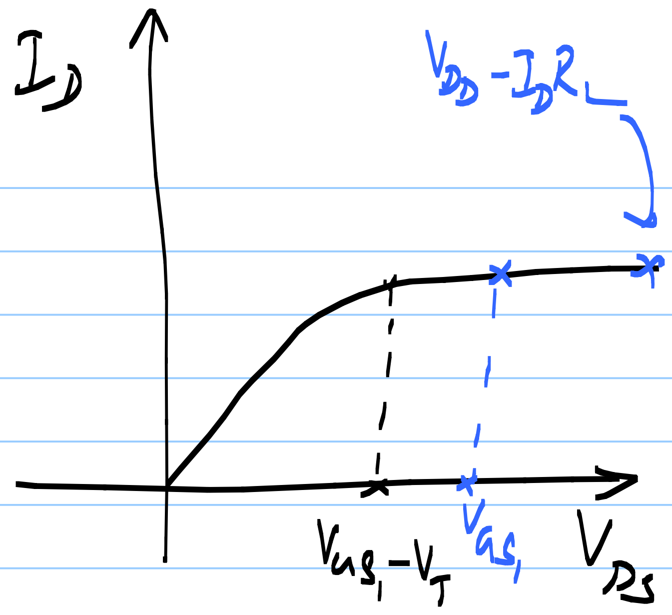
* $G \ll G_L \Rightarrow G \ll g_m$

By "large R", we mean $R \gg R_L$

$$\text{gain} \approx -g_m R_L$$

Swing limits :

* Triode limit is lower because $V_{DS} = V_{GS}$
* Cutoff limit is same as before ; I_D = same as before



1/9/20

Lecture 16

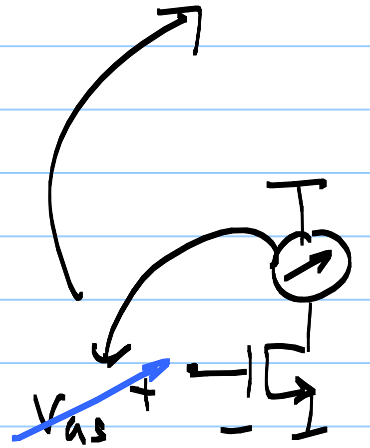
We want to bias the MOSFET with stable Queiscent current:

1) Measure I_D (I_S)

2) Compare I_D (I_S) with I_0

3) If $I_D > I_0 \Rightarrow$ reduce V_{GS} ($\downarrow V_G$ or $\uparrow V_S$)

If $I_D < I_0 \Rightarrow$ increase V_{GS} ($\uparrow V_G$ or $\downarrow V_S$)



Measure I_D

__For MOSFET__: $I_G = 0$

$\Rightarrow I_S = I_D$ always (1)

$\rightarrow$ Measure I_S or I_D

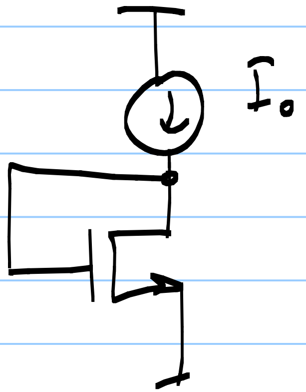
We control $V_{GS} = V_G - V_S$ (2)

keep S fixed, vary G

keep G fixed, vary S

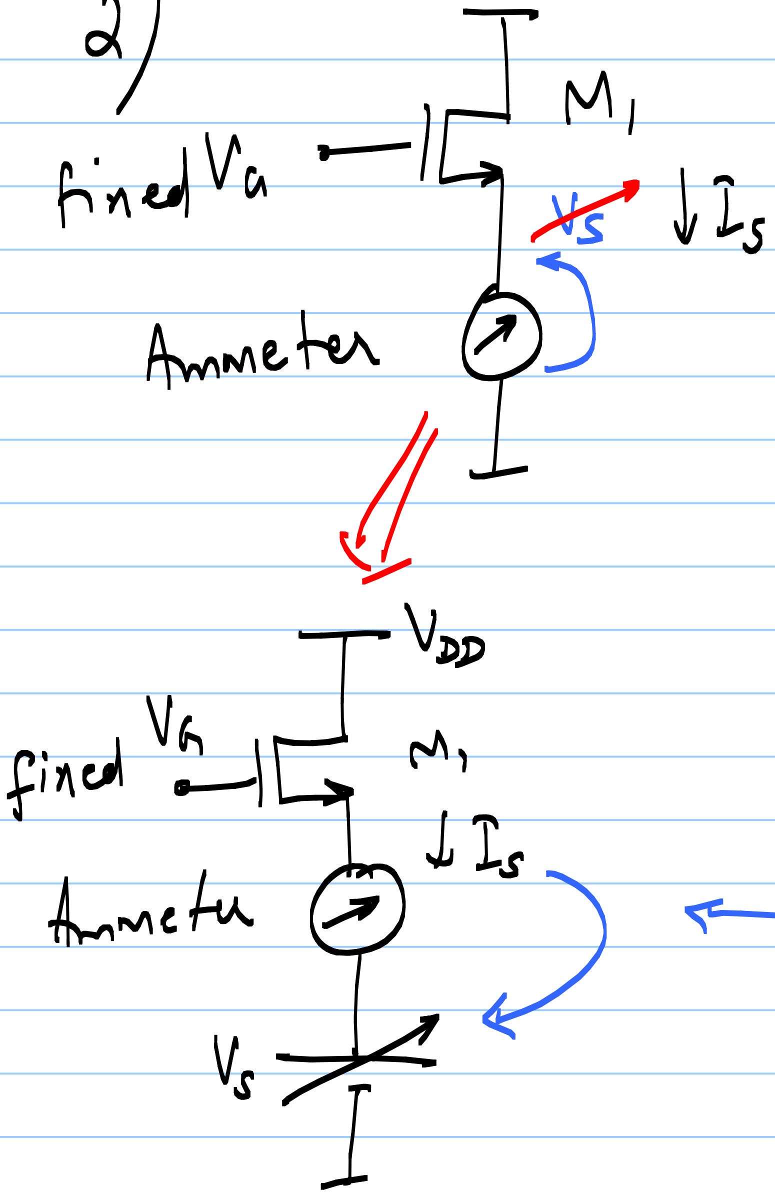
* 4 ways of negative feedback bias stabili-
-zation

1)



Measure I_D, f.b. to V_G,
keep V_S fixed

2)



Measure I_s, f.b to V_s,

keep V_G fixed

step 0: M_1 in sat.

$\Rightarrow V_{DD} > V_G - V_T$

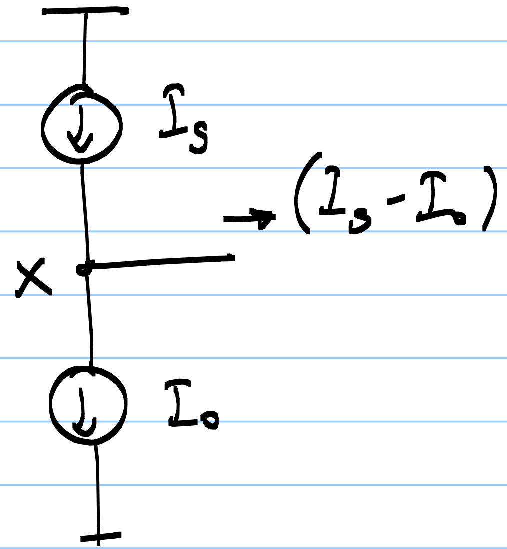
step 1: Measure I_s

step 2: If $I_s > I_0 \Rightarrow$ need to $\downarrow V_{GS}$

$\Rightarrow \uparrow V_S$

If $I_s < I_0 \Rightarrow$ need to $\uparrow V_{GS}$

$\Rightarrow \downarrow V_s$



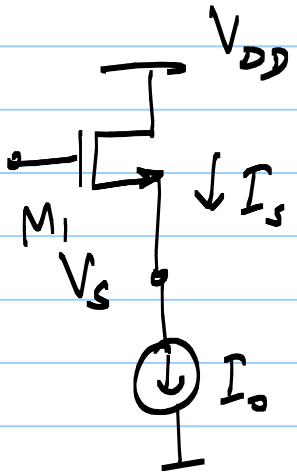
magnitude & sign of $(I_s - I_o)$ tells you what to do

If $I_s > I_o \Rightarrow V_x \uparrow$ [need to $\uparrow V_s$]

$I_s < I_o \Rightarrow V_x \downarrow$ [need to $\downarrow V_s$]

$I_s = I_o \Rightarrow V_x$ same [V_s same]

V_{G_0} = fixed V_G



Due to negative f.b. action:

$$I_s = I_o = I_D$$

$$V_s = ?$$

M_1 is in sat. $\Rightarrow$ $I_D = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)(V_{GS} - V_T)^2$

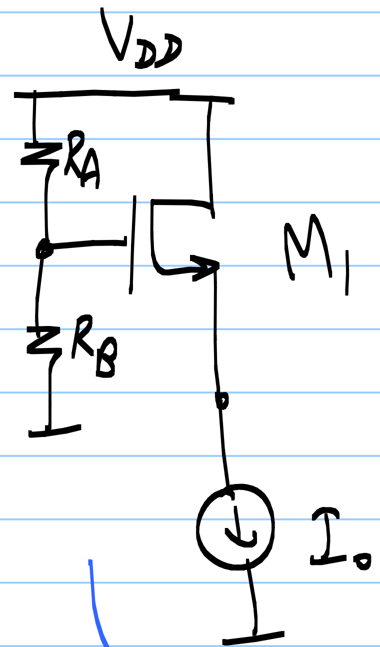
$$V_G - V_s = V_{GS} = V_T + \sqrt{\frac{2I_D}{\mu_n C_{ox}\left(\frac{W}{L}\right)}}$$

$= V_{OV}$ or V_{Dsat}.

$$V_S = V_G - V_{GS}$$

$$V_S = V_{G0} - \sqrt{\frac{2 I_0}{\mu_n C_{ox} \left(\frac{W}{L}\right)}} - V_T$$

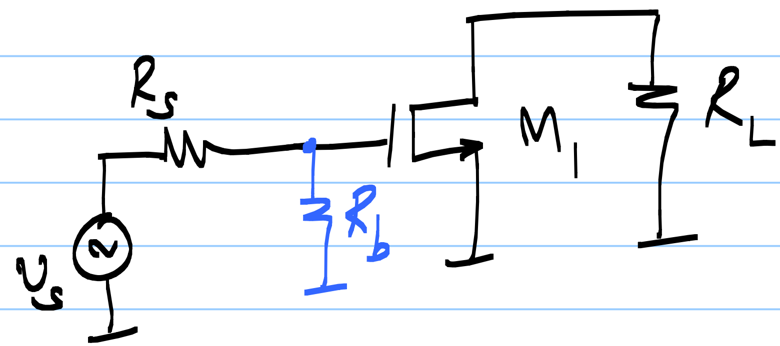
fixed V_{G0}

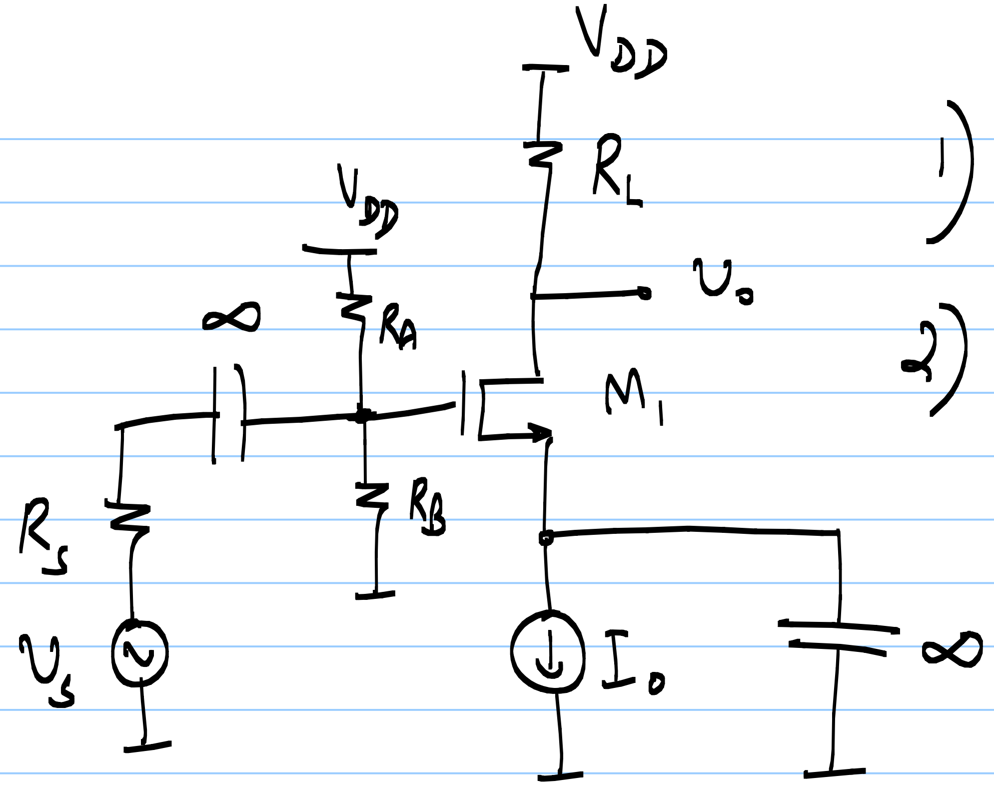


$$V_{G0} = \frac{R_B}{R_A + R_B} \cdot V_{DD}$$

<u>Small-signal:</u>

?





1) $R_A \| R_B \gg R_s$

2) M_1 in sat.:

$$V_D \geq V_G - V_T$$

$$V_{DD} - I_o R_L \geq \frac{R_B}{R_A + R_B} \cdot V_{DD} - V_T$$

Swing limits

1) Cut off limit: $I_o + g_m V_A \sin\omega t = 0$

$$V_{A,1} = \frac{I_o}{g_m}$$

2) Triode limit $\quad V_D(t) = V_G(t) - V_T$

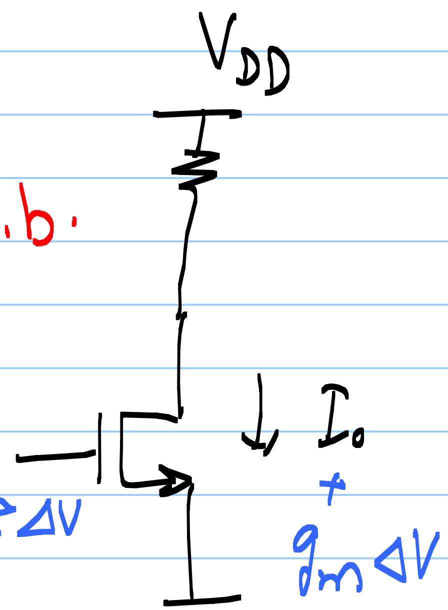
$$V_{DD} - I_o R_L - g_m R_L V_A \sin\omega t = \frac{V_{DD} R_B}{R_A + R_B} + V_A \sin\omega t - V_T$$

$$V_{A_2} = \left[V_T + \frac{V_{DD} \cdot R_A}{R_A + R_B} - I_o R_L \right] \frac{1}{(1 + g_m R_L)}$$

2/9/20

Lecture 17

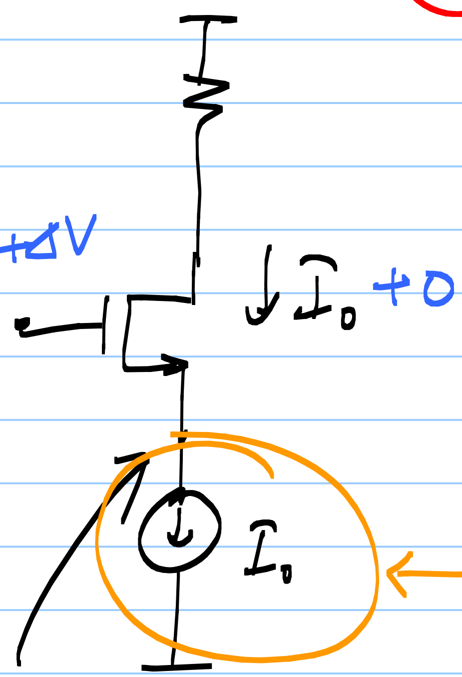
1 No f.b.

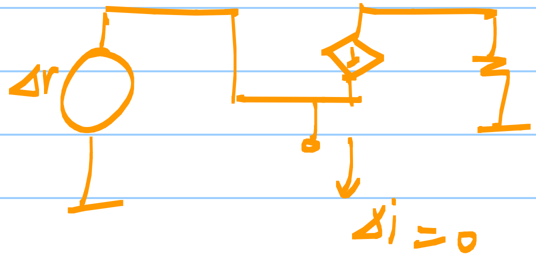


$V_{G_1} + \Delta V$

$$= V_T + \sqrt{\frac{2I_o}{\mu_n C_{ox}\left(\frac{W}{L}\right)}}$$

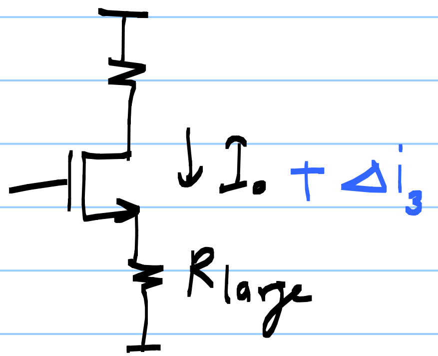
2 Very Strong f.b.





open circuit for incremental signals

$$V_{S_2} = V_{G_2} - V_{GS_2}\Big|_{I_o} + \Delta V$$

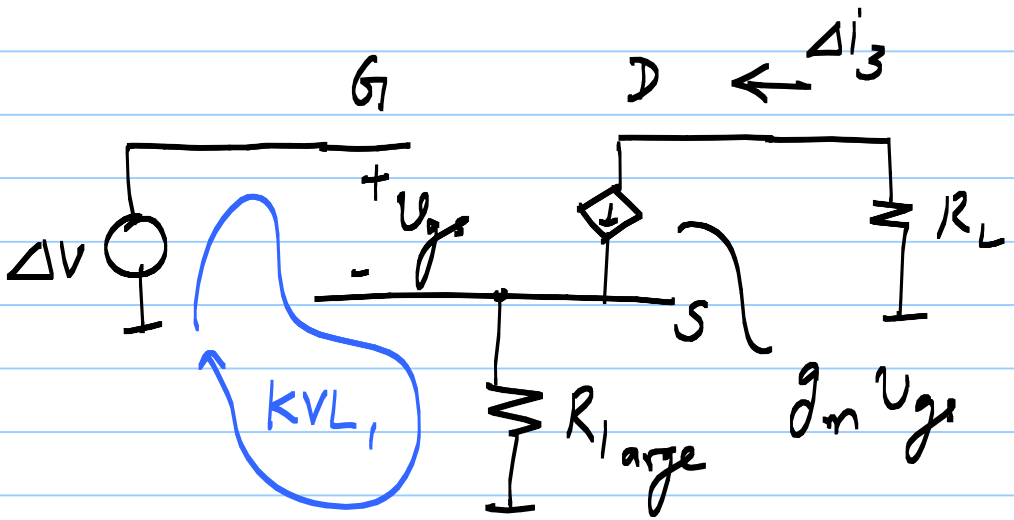


DC point: $V_{S_3} = I_o \cdot R_{large}$

$V_{G_3} = V_{S_3} + V_{GS}\Big|_{I_o}$

3 Intermediate f.b.

$\Delta i_3 = ?$



KVL$_1$

$$\Delta V = v_{gs} + \Delta i_3 \cdot R_{large}$$

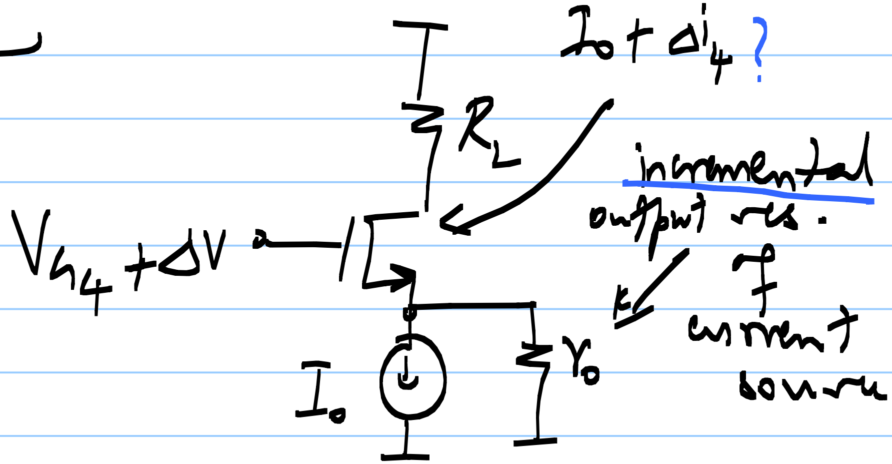
$$\Delta i_3 = g_m v_{gs}$$

$$\Delta V = \frac{\Delta i_3}{g_m} + \Delta i_3 R_{large}$$

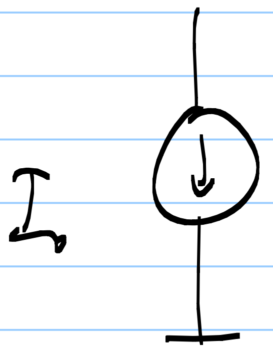
$$\boxed{\Delta i_3 = \frac{g_m}{1+g_m R_{large}} \cdot \Delta V}$$

f.b. strength is a function of R_{large}

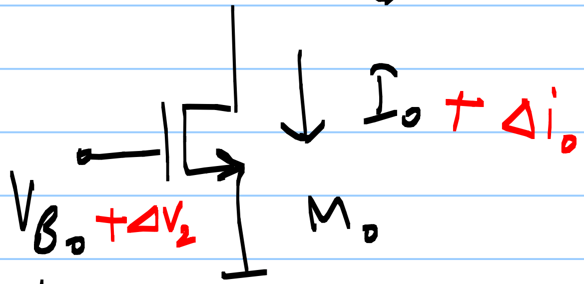
$$\Delta i_4 = \frac{g_m}{1+g_m r_o} \cdot \Delta V$$



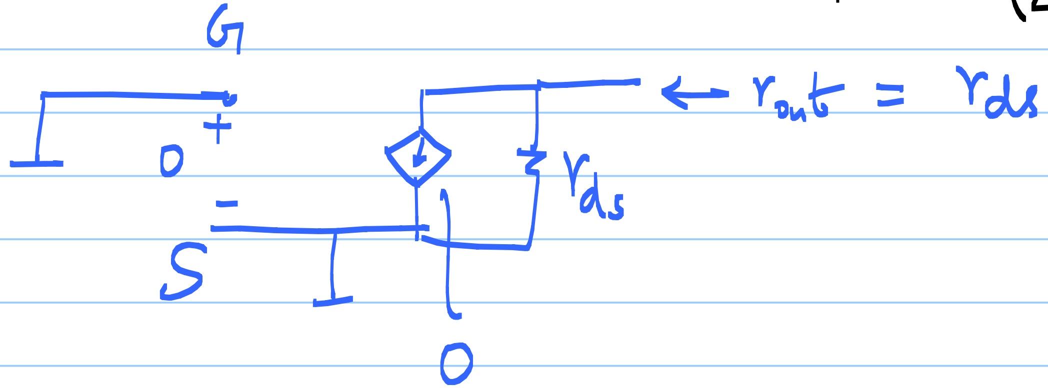
$\downarrow r_{out} = r_{ds}$

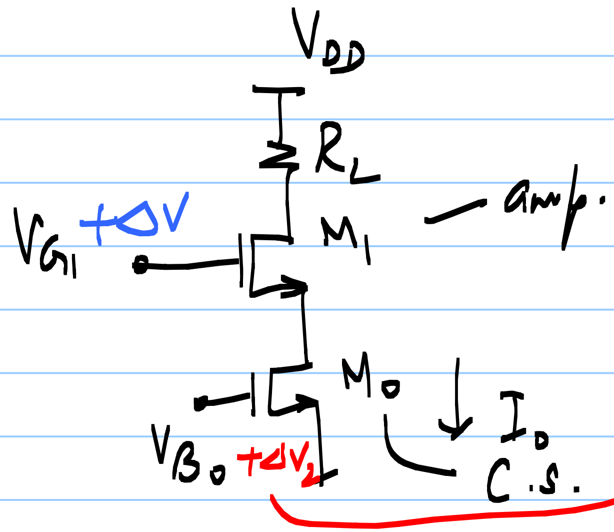


$\equiv$

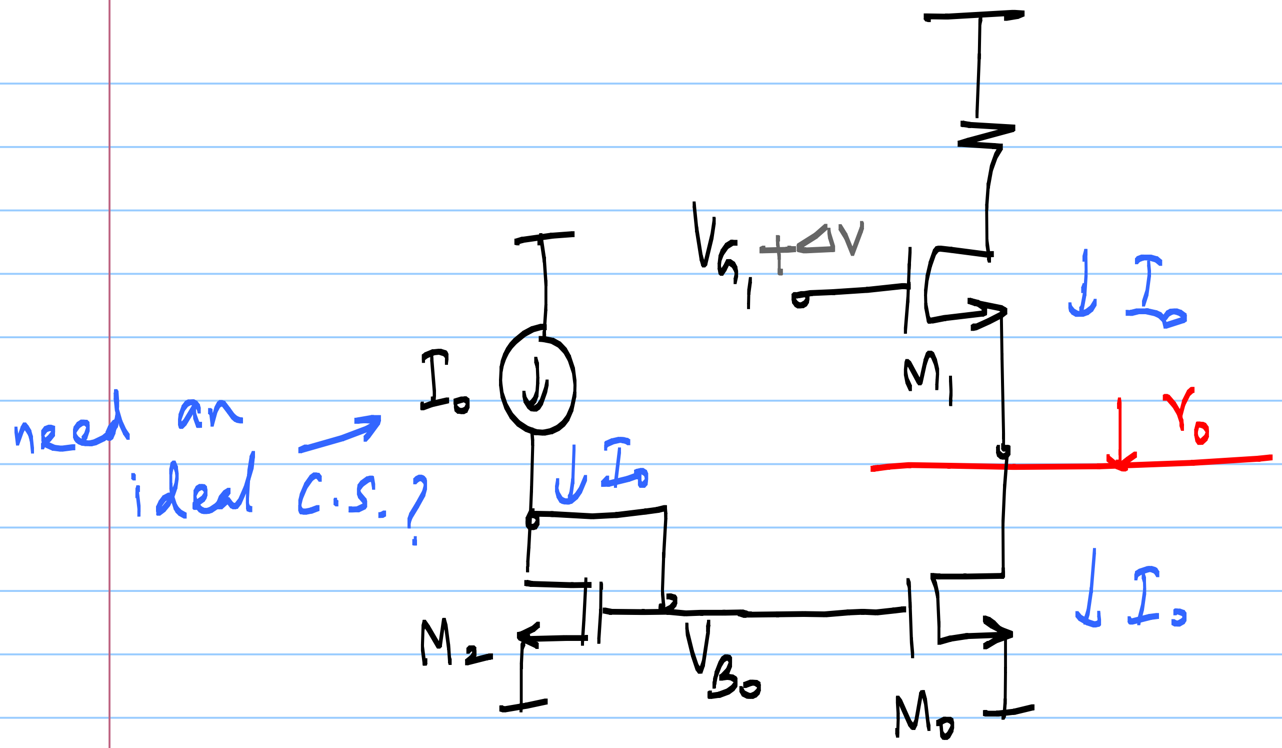


$$V_{B_o} + \Delta V_2 \equiv V_T + \sqrt{\frac{2 I_o}{\mu_n C_{ox} \left(\frac{W}{L}\right)}}$$





need f.b. to generate V_{B_o}



HW2

draw incremental eq. circuit and verify $r_o = r_{ds_o}$

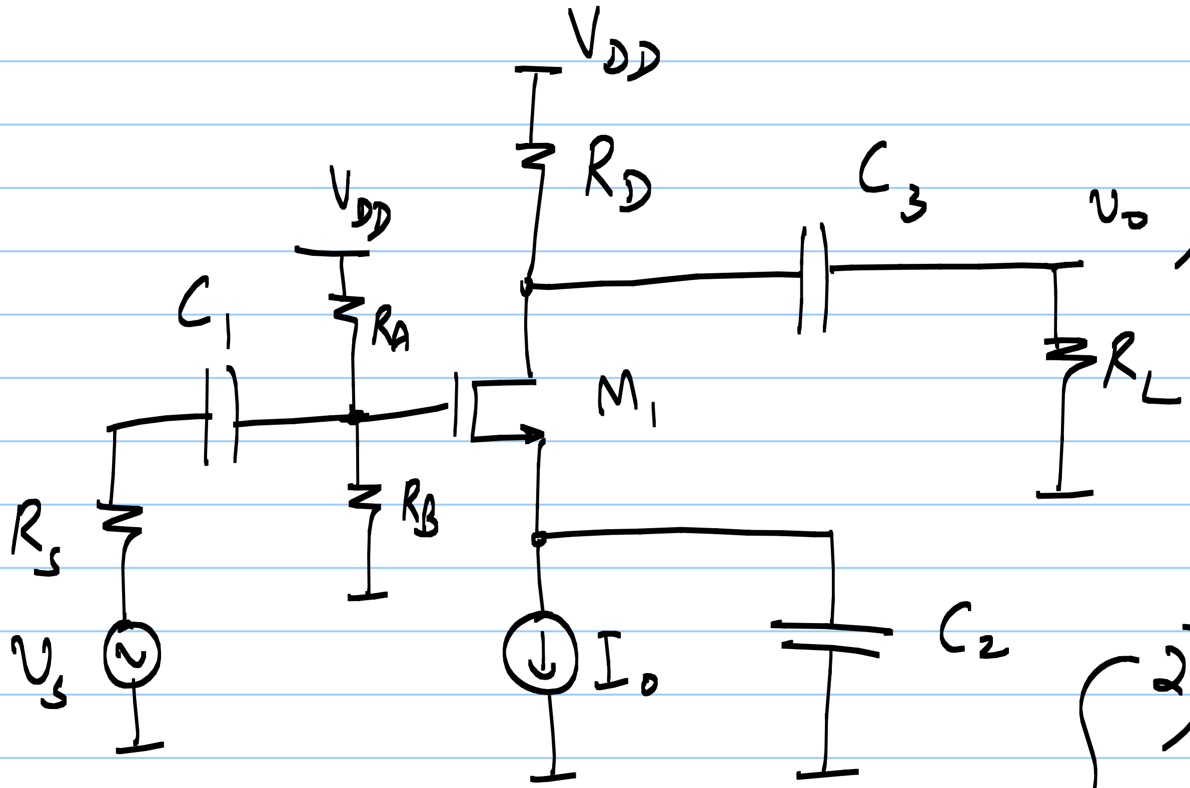
* On any IC: one reference V_{ref} & I_{ref} generated using a "Bandgap" reference circuit

e.g. $V_{ref} = f$ (Si bandgap voltage)

* Often need 1 single off-chip Resistor → low tolerance, low tempco

3/9/20

Lecture 18

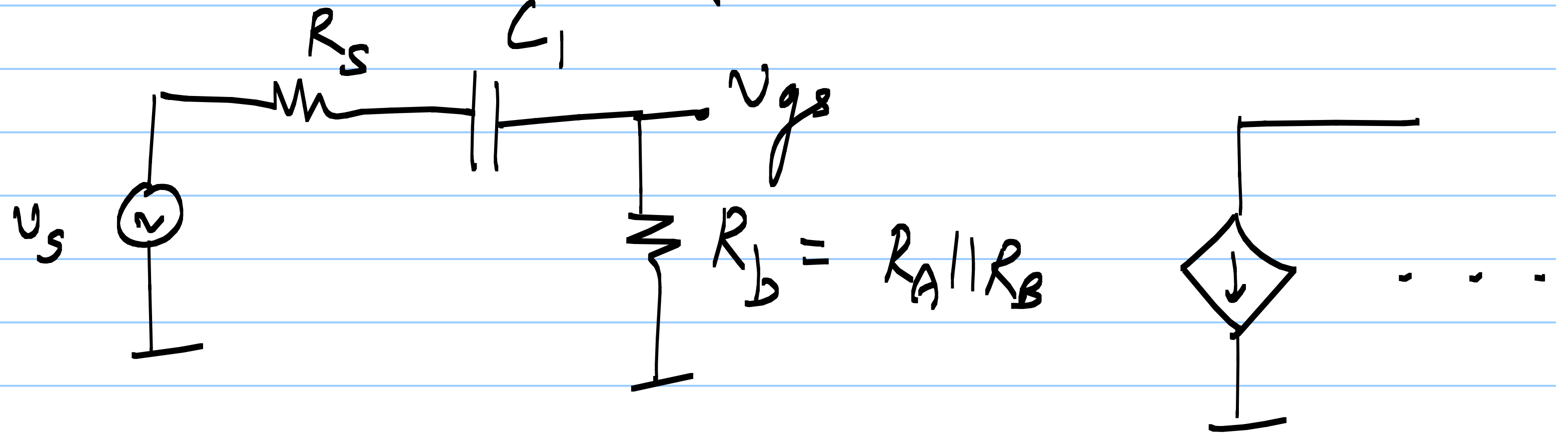


So far: $C_1, C_2, C_3 = \infty$

1) Choose C_1 based on ac resistance @ freq. of operation $\Rightarrow$ small ac resistance

2) Eg. @ input, you have HPF $\rightarrow$ plot Bode curves & choose based on freq. response

SS eq. ckt. @ input: (assume $C_2 = \infty$, $C_3 = \infty$)



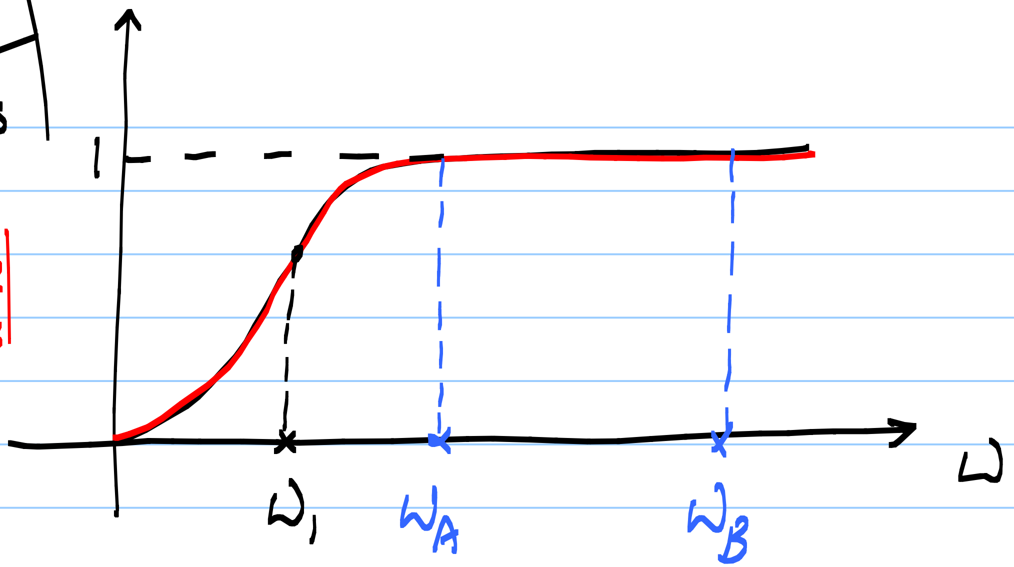
Want → $|v_{gs}| = |v_s|$ $\quad (R_b \gg R_s)$

→ $|v_o| = g_m(R_D || R_L) \cdot |v_s|$

$$v_{gs} = \frac{R_b \cdot v_s}{R_s + R_b + \frac{1}{j\omega C_1}} = \frac{j\omega C_1 R_b}{1 + j\omega C_1 (R_s + R_b)} \cdot v_s$$

Plot $\left| \frac{v_{gs}}{v_s} \right|$

for C_1: $\left|\dfrac{v_{gs}}{v_s}\right|$



$\omega_1 = -3\text{ dB}$ freq of HPF $\left\{ C_1 \ \& \ R_S + R_b \right\}$

$\dfrac{1}{g_m(R_D \| R_L \| r_{ds})} \cdot \left|\dfrac{v_o}{v_s}\right|$

for C_3:

for C_2: HW3

* choose $\omega_1 \ll \omega_A$ e.g. $\omega_1 = \frac{1}{10}\omega_A$ or $\frac{1}{20}\omega_A$ etc.

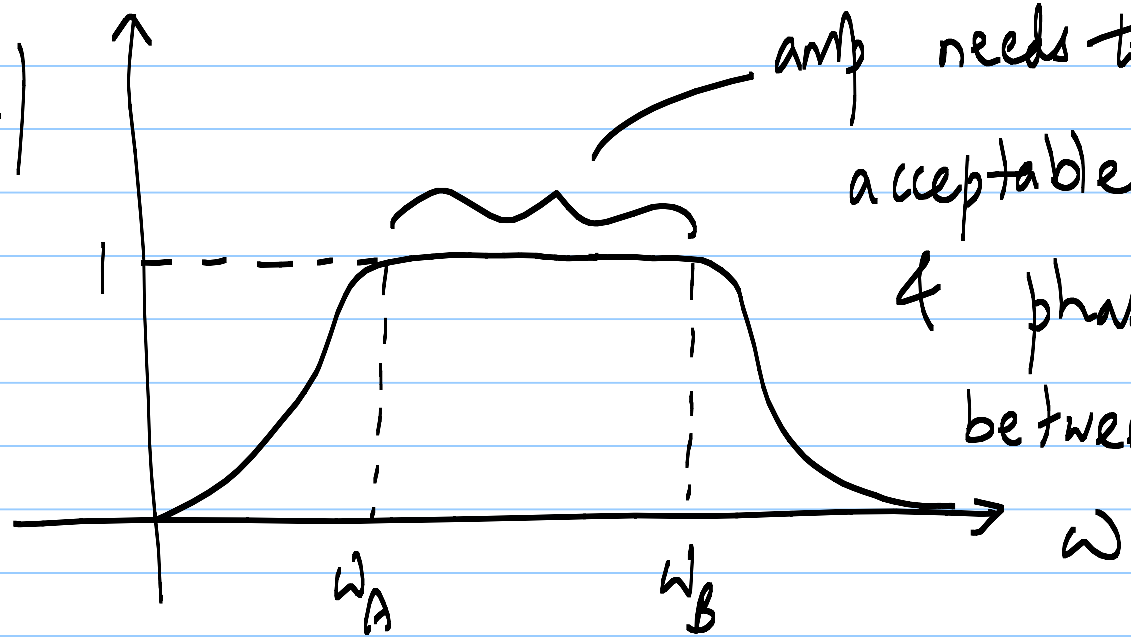
* pole for C_3 depends on R_D, R_L & r_{ds}

$r_{ds} - f(\lambda, I_D)$

$\lambda - f(L)$

* pole for C_2 depends on ? HW3

$$\frac{1}{g_m(R_D \| R_L \| r_{ds})} \cdot \left|\frac{v_o}{v_s}\right|$$



amp needs to have acceptable magnitude & phase response between ω_A & ω_B

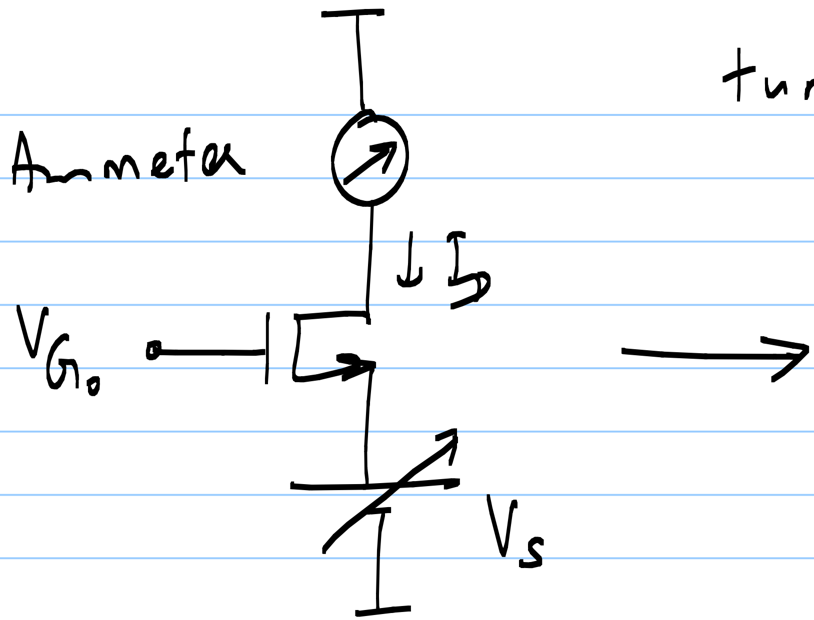
Bias stab.

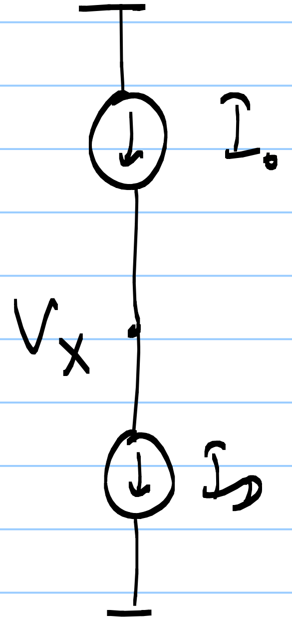
case 1 : Measure I_D, f.b. to V_G

case 2 : Measure I_S, f.b. to V_S

case 3 : Measure I_D, f.b. to V_S

tune V_s so that $I_D = I_0$





$I_0 < I_D$: $V_x \downarrow$

$I_0 > I_D$: $V_x \uparrow$

$I_0 = I_D$: $V_x \leftrightarrow$

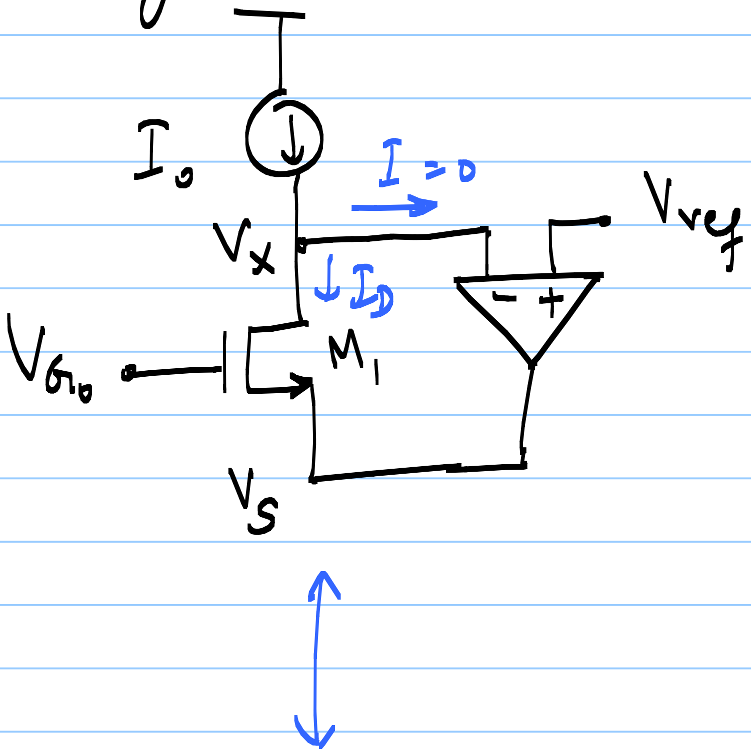
e.g. $I_0 < I_D$: $V_x \downarrow$

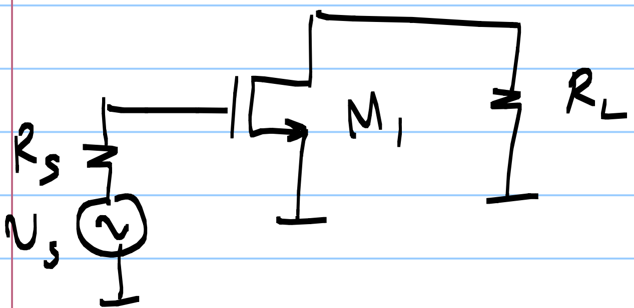
We want to $\downarrow V_{GS} \Rightarrow \uparrow V_s$ } change in direction of control

||

inversion in polarity of f.b.

DC biasing for case 3:



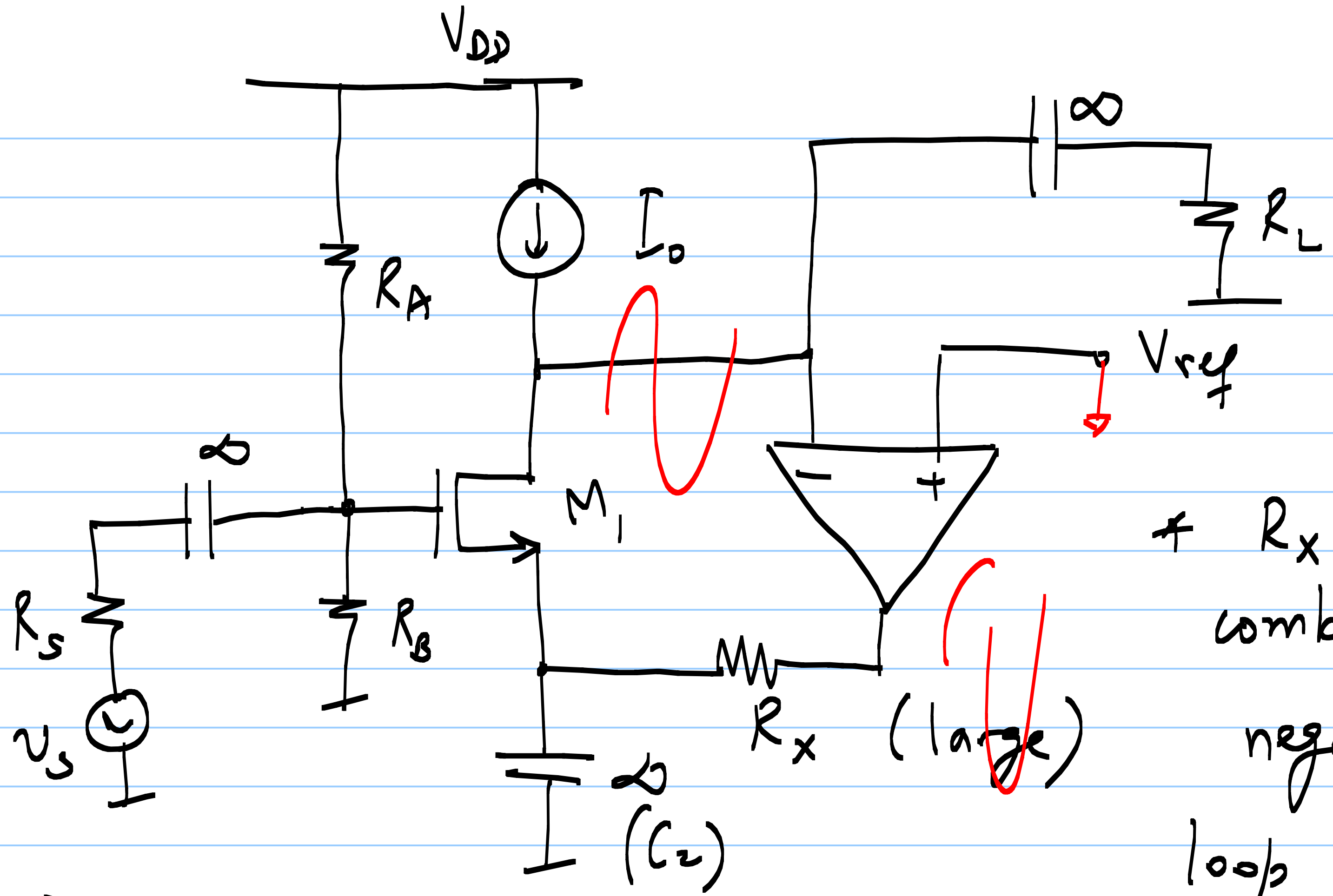


no opamp

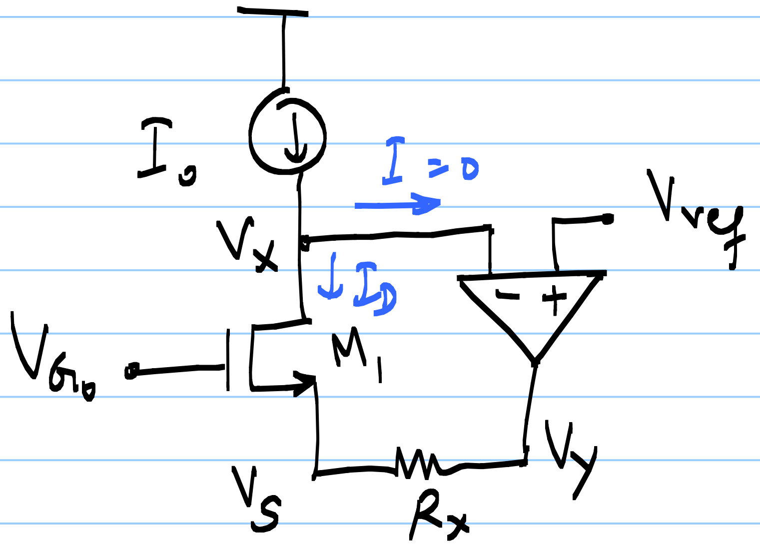
Ideal

* opamp changes V_S to a value so that $V_X = V_{ref}$ in steady state
* $V_S = V_{G_0} - V_{GS}\big|_{I_0}$
* opamp only for DC stab.
* V_{ref} chosen so that M_1 is biased in sat.

$V_{ref} \geq V_{G_0} - V_T$

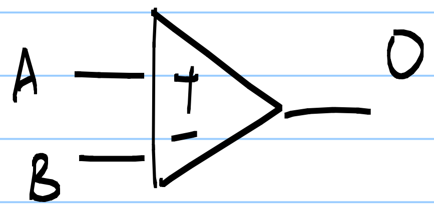


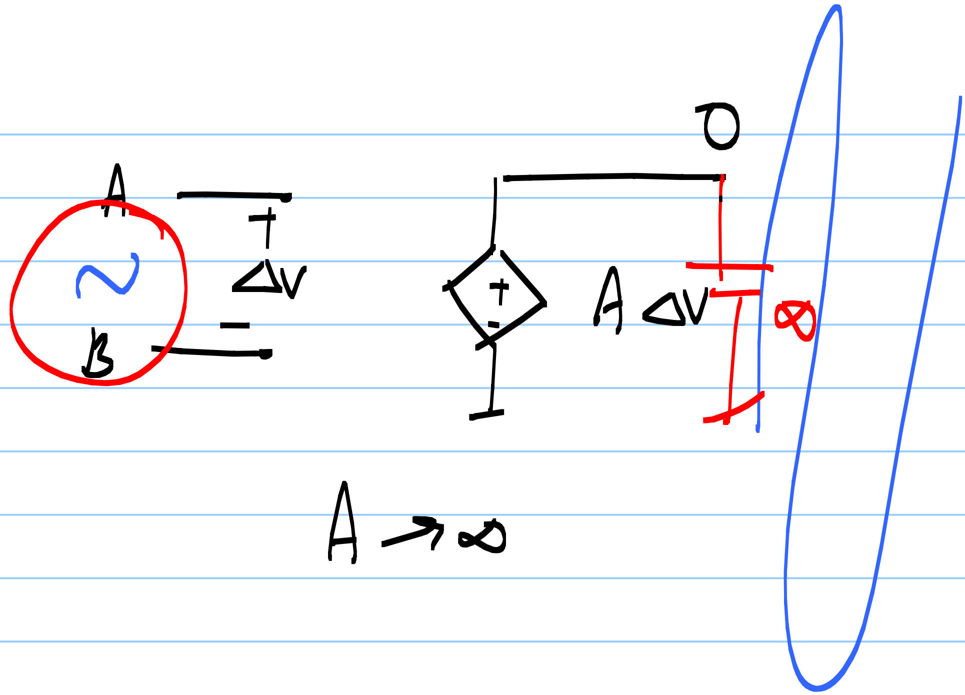
+ R_x + C_2 combo break negative f.b. loop for small signal operation



$$V_y = V_S - I_o R_x$$

as $R_x \uparrow$, $V_y \downarrow$





$A \to \infty$

4/9/20

Lecture 19

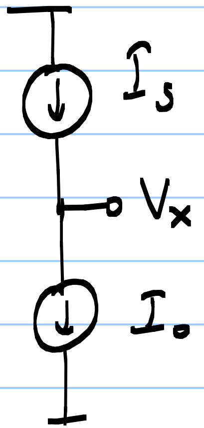
Bias Stab.

Sensed	Controlled
I_D	V_G
I_s	V_s
I_D	V_s
I_s	V_G

no opamps required in general

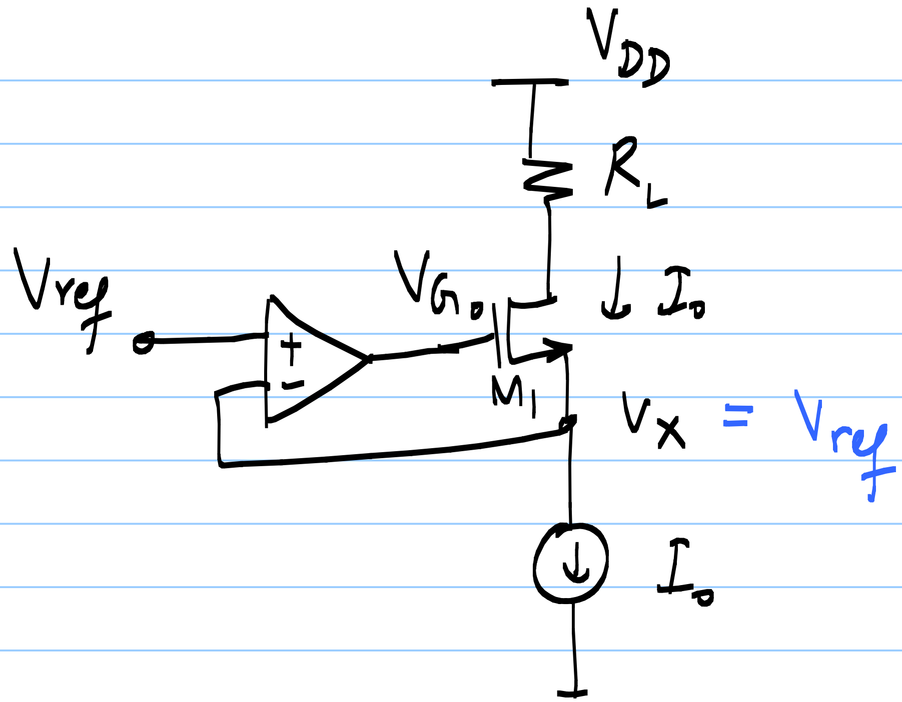
Need opamp for cases 3 & 4 because of polarity inversion in f.b.

Case 4



$I_s > I_o \Rightarrow V_x \uparrow$ (we need to $\downarrow V_G$)

$I_s < I_o \Rightarrow V_x \downarrow$ (we need to $\uparrow V_G$)



$V_{G_0} = V_{ref} + V_{GS}\big|_{I_0}$

$V_{D_0} = V_{DD} - I_0 R_L$

$V_{S_0} = V_{ref}$

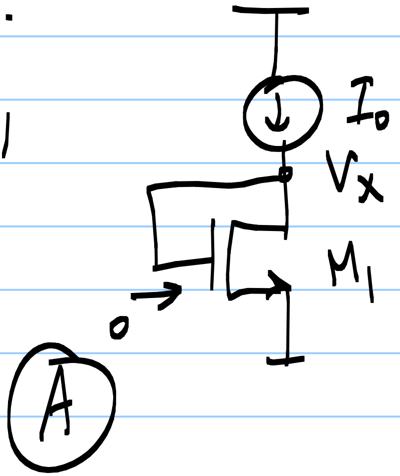
Choose V_{ref} so that M_1 in sat.

$V_{D_0} \geq V_{G_0} - V_T$

* You can use opamps in cases 1 & 2 also

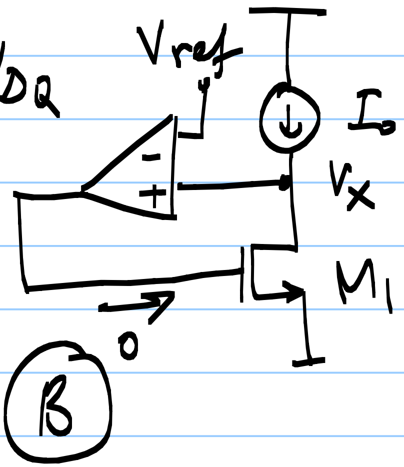
e.g.

case 1



$V_x = V_{G_Q} = V_{DQ}$

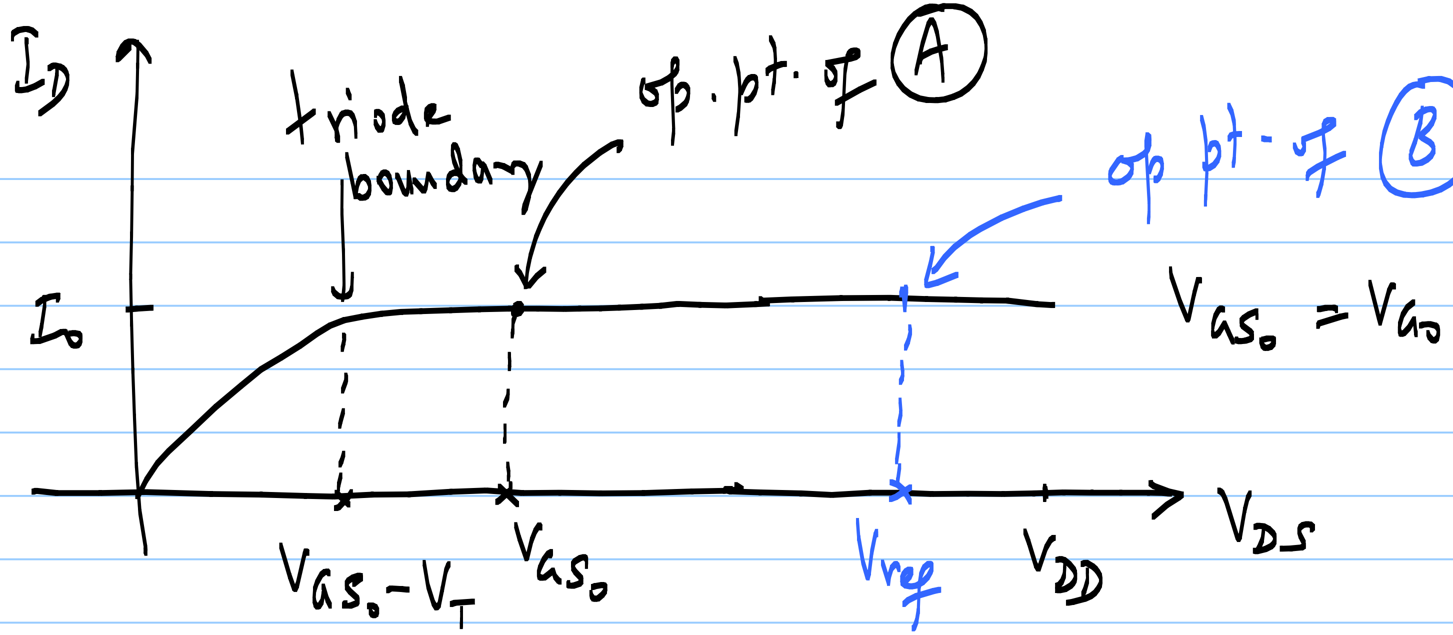
$V_{G_Q} = V_{GS}$

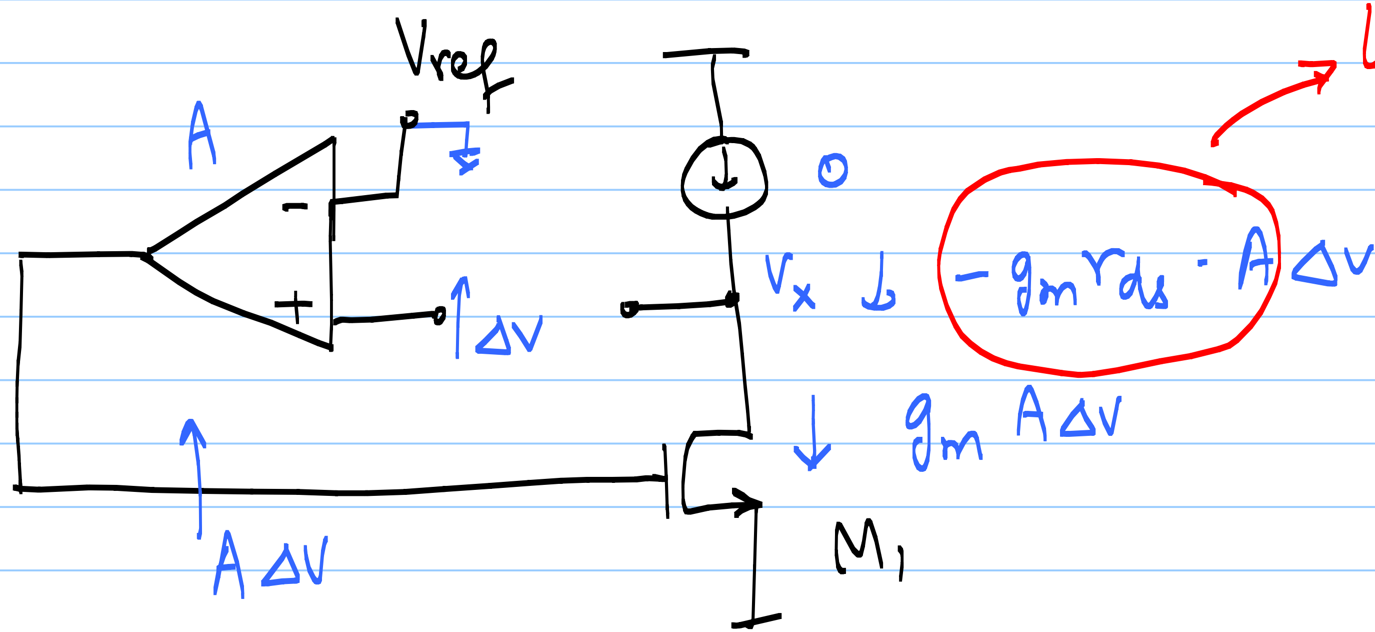


$V_x = V_{ref} = V_{DQ}$

$V_{G_Q} = V_{GS}$

can be biased further from triode boundary

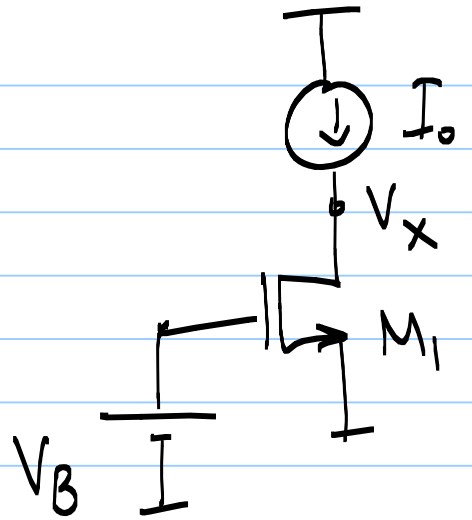




Loop gain of B

$\boxed{\text{−ve f.b.}}$

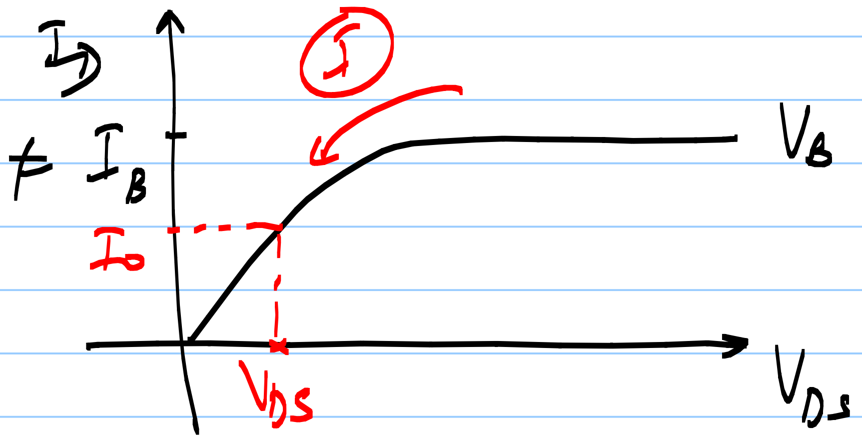
Loop gain of A $= -g_m r_{ds}$



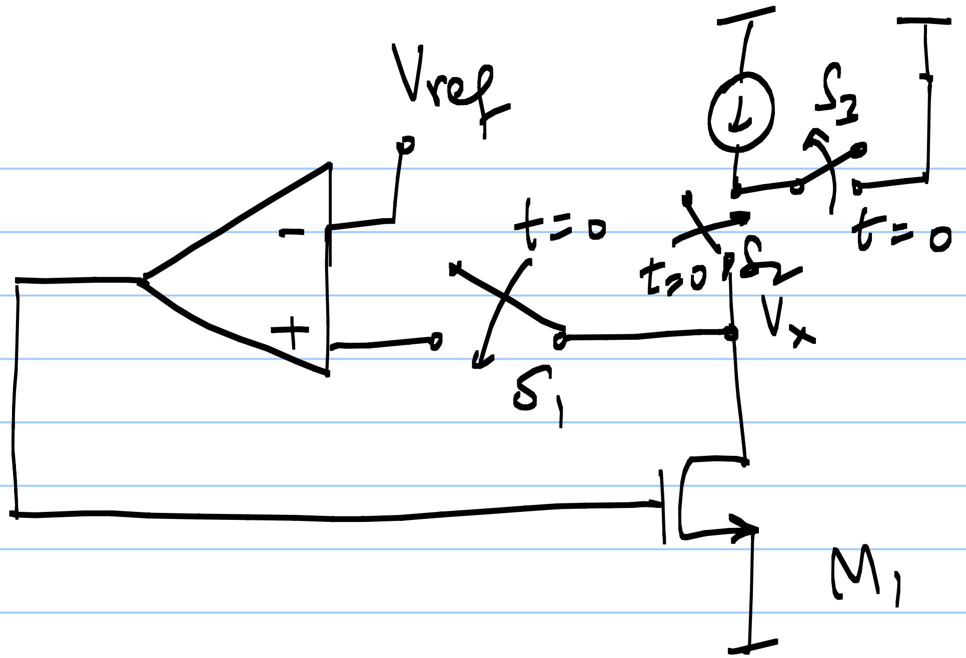
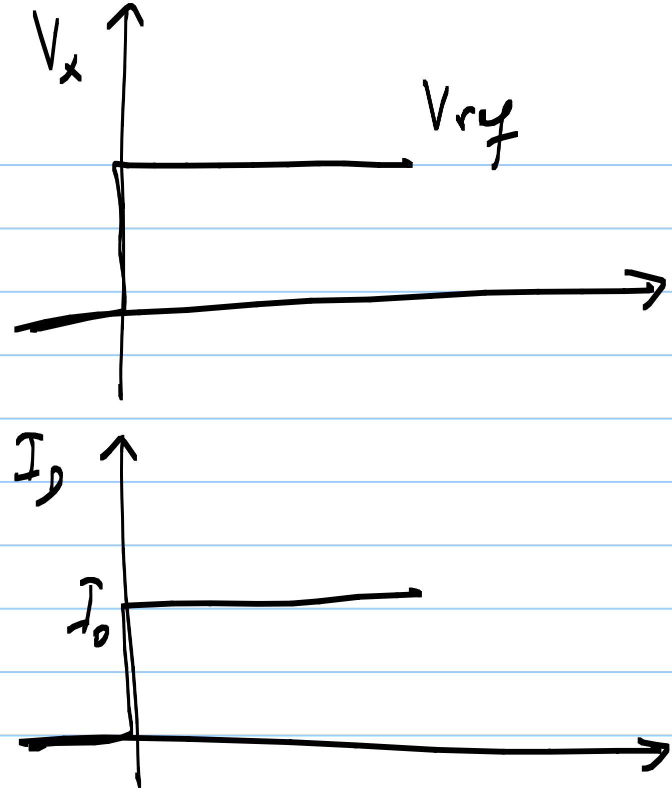
$$V_B \neq V_{GS}\big|_{I_o}$$

I $\quad V_B < V_{GS}\big|_{I_o} \Rightarrow V_x \uparrow$ to ∞

II $\quad V_B > V_{GS}\big|_{I_o} \Rightarrow V_x \downarrow$



$V_B \longrightarrow I_B$ II

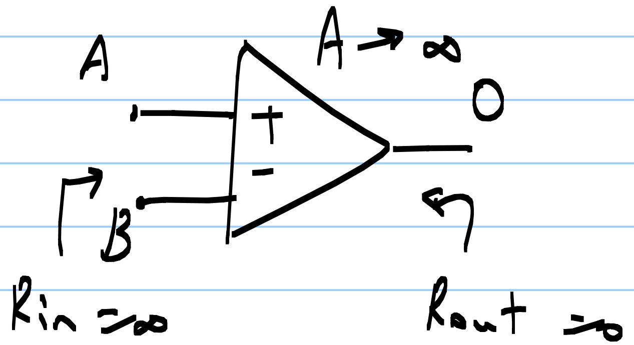

Vref
-
+
t=0
S1
t=0
S2
S3
t=0
Vx
M1

Vx
Vref
ID
I0

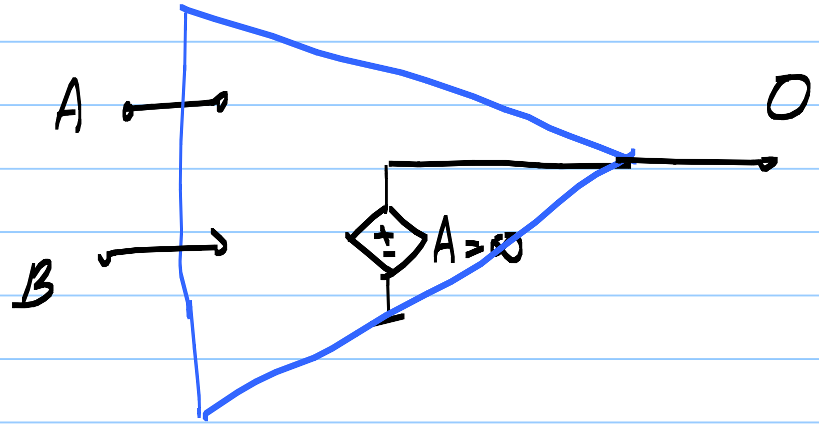
8/9/20

Lecture 20

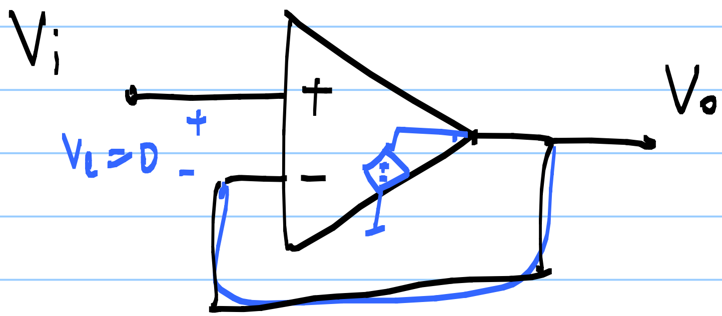
Negative f.b. to create small signal controlled sources

VCVS using opamp



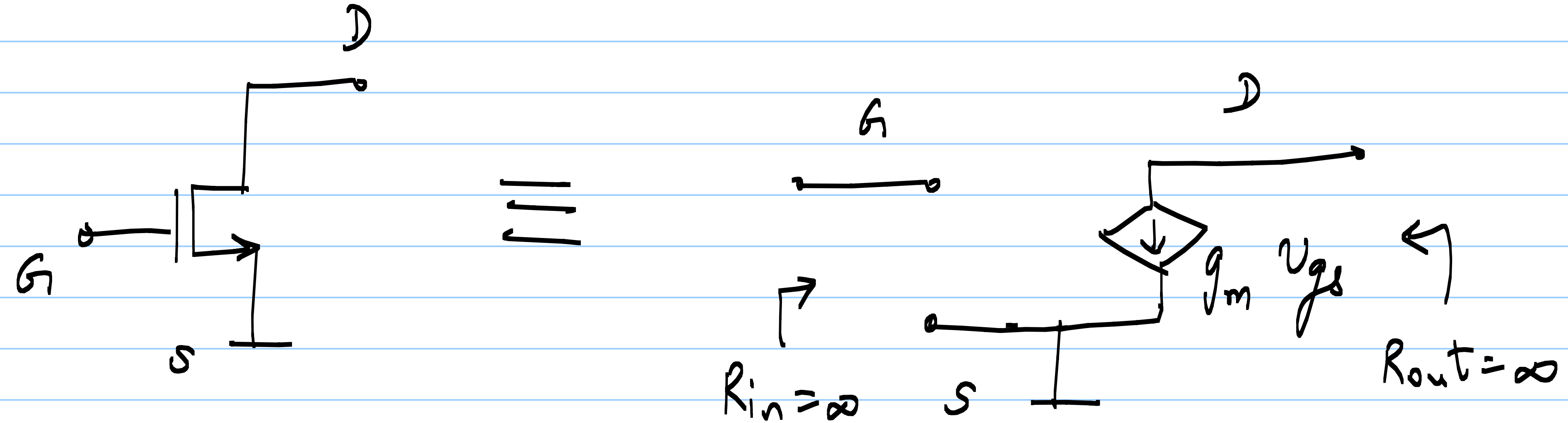


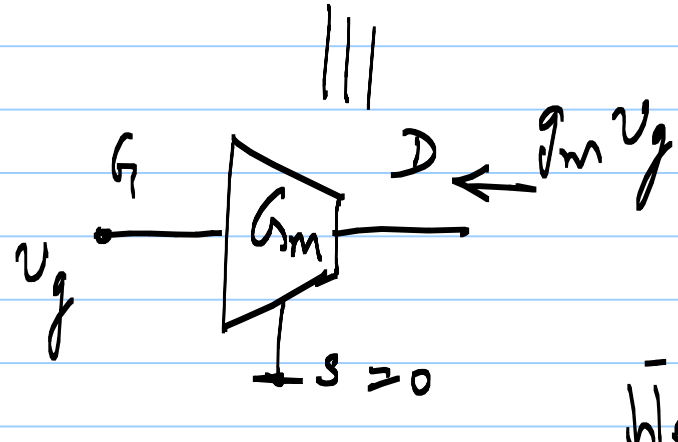
opamp
VCVS = 1



VCVS using MOSFET (any controlled source)

1) Works only for small-signal
2) Idealized view for MOSFET:





Ideal transcond-
-uctor
block

$$\boxed{G_m = g_m}$$

allow $g_m \to \infty$

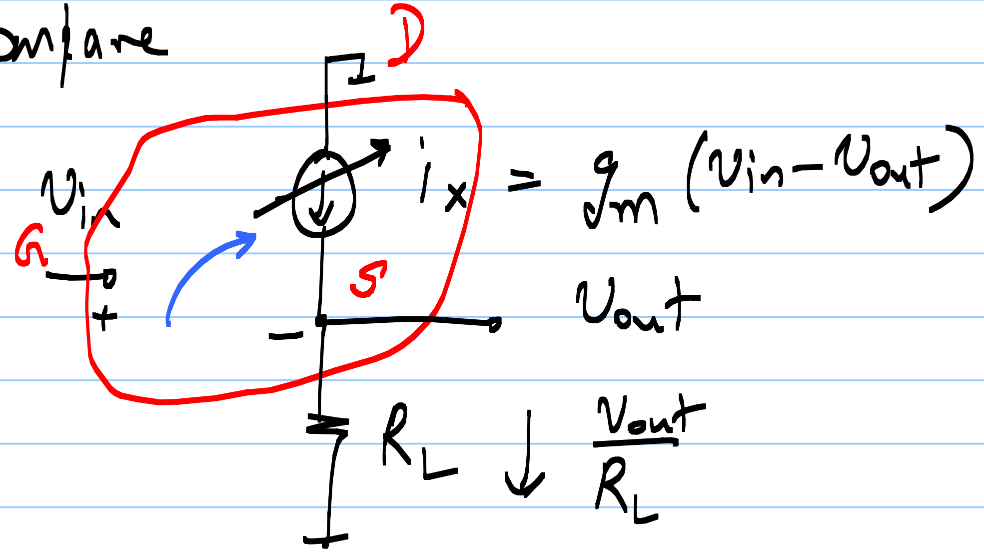
i.e. $\left(\frac{W}{L}\right)$ & I_D can be as large as required

to set g_m = as large as required.

<u>Ideal VCVS =1</u> : $v_{out} = v_{in}$ $(g_m \to \infty)$

* measure v_{in}, v_{out}; compare
* change v_{out}
* If $v_{out} > v_{in}$, reduce i_x

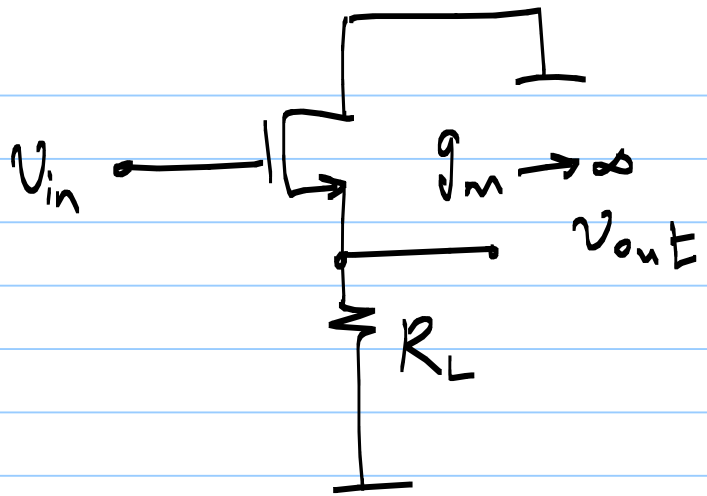
If $v_{out} < v_{in}$, increase i_x

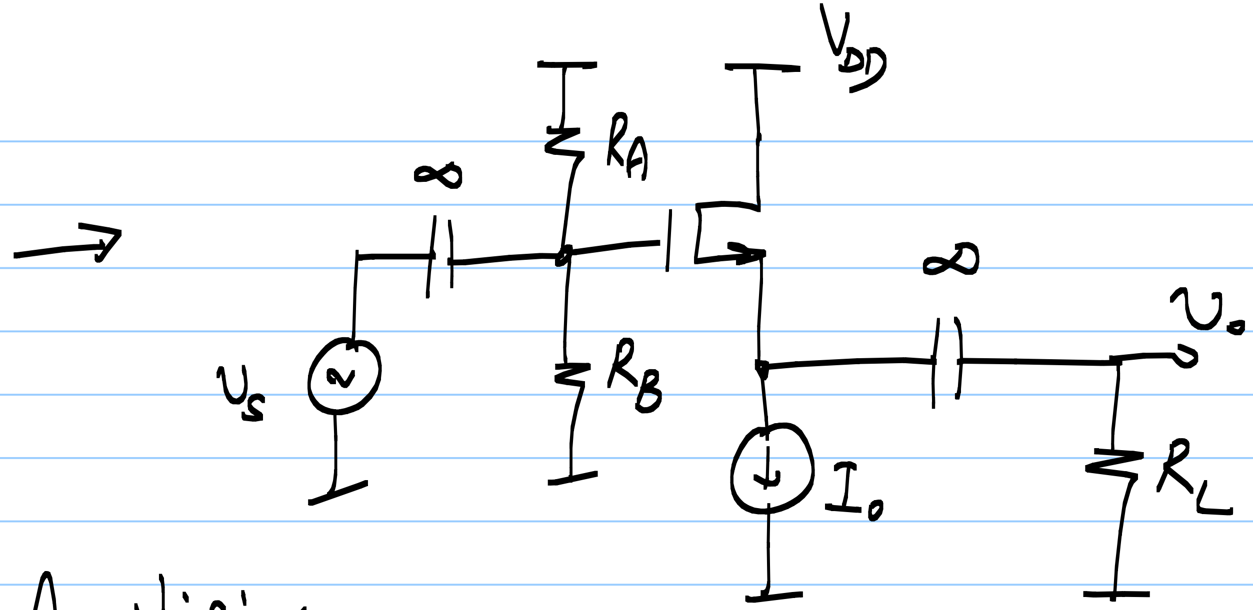


output current $\frac{v_{out}}{R_L} = g_m(v_{in} - v_{out})$ if in neg. f.b.

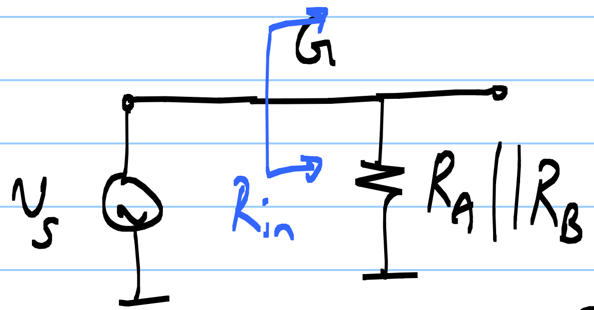
we want $v_{out} = v_{in}$ $\Rightarrow$ $g_m \to \infty$ so that

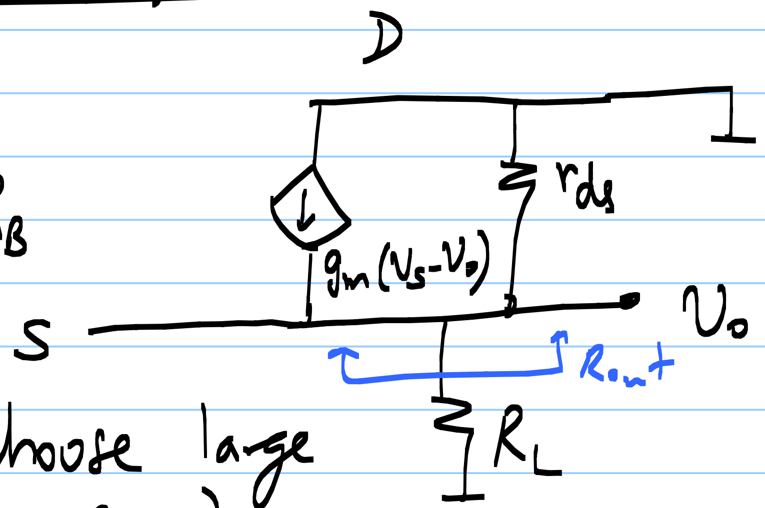
$v_{in} - v_{out} \to 0$





Common Drain Amplifier





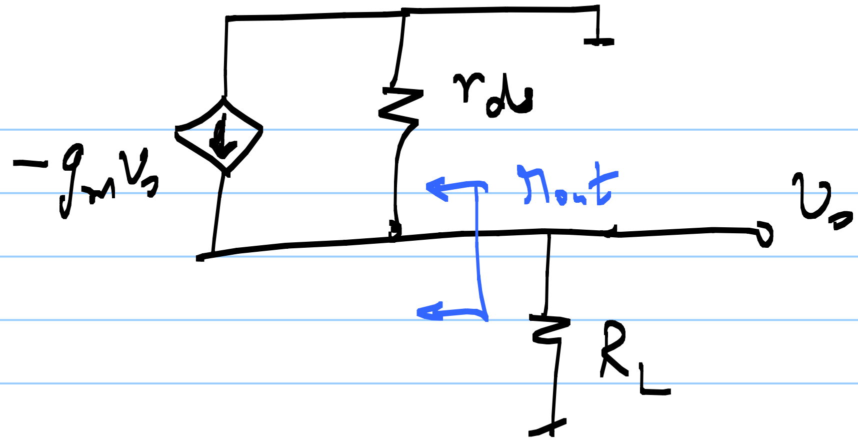
$v_{gs} = v_s - v_o$

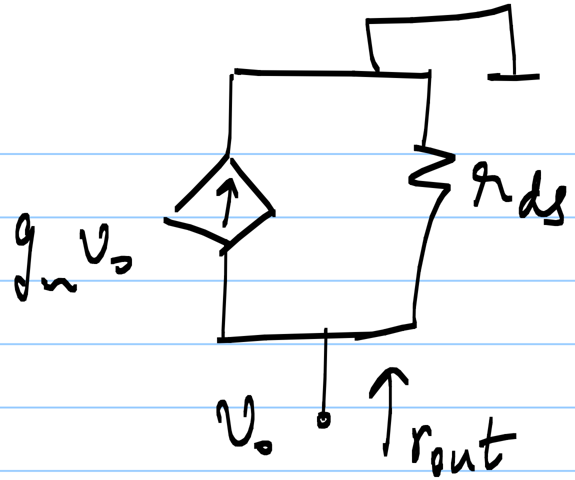
VCVS

$Z_{in} = R_A || R_B$ (choose large R_A, R_B)

$Z_{out} = ?$

set $v_s = 0$





$$r_{out} = \frac{1}{g_m} \parallel r_{ds}$$

$$Z_{out} = \frac{r_{ds}}{1 + g_m r_{ds}} = \frac{1}{g_m} \cdot \frac{g_m r_{ds}}{1 + g_m r_{ds}}$$

If $g_m r_{ds} \gg 1$, $Z_{out} = 1/g_m$

KCL @ S node:

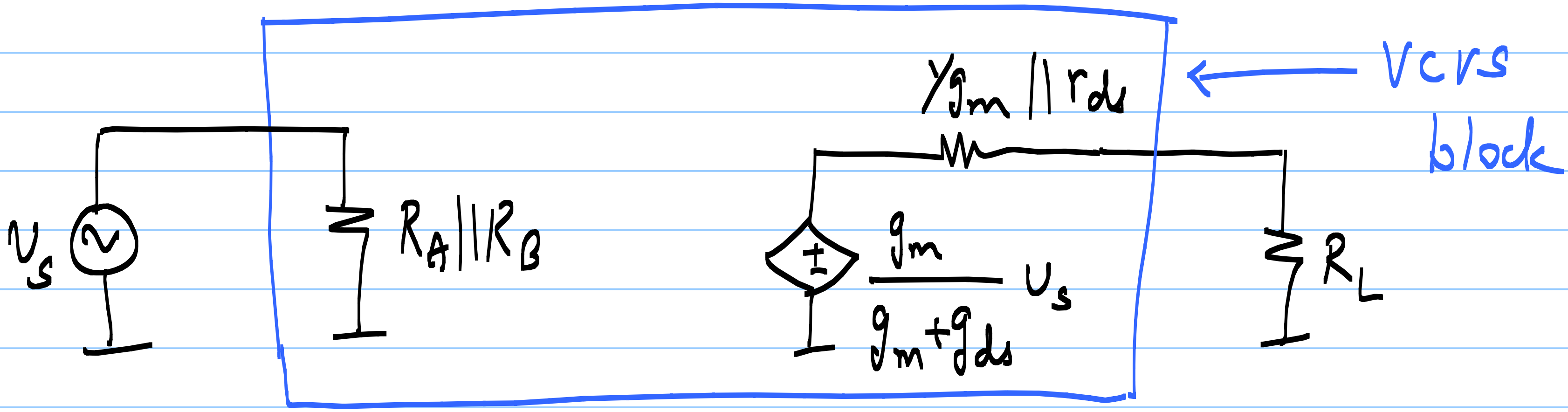
$$g_m (v_s - v_o) = v_o (G_L + g_{ds})$$

$$v_o(g_m + g_{ds} + G_L) = g_m v_s$$

$$\frac{v_o}{v_s} = \frac{g_m}{g_m + g_{ds} + G_L} < 1$$

$\rightarrow 1$ if $g_m \rightarrow \infty$

$\hookrightarrow Z_{out} \rightarrow 0$

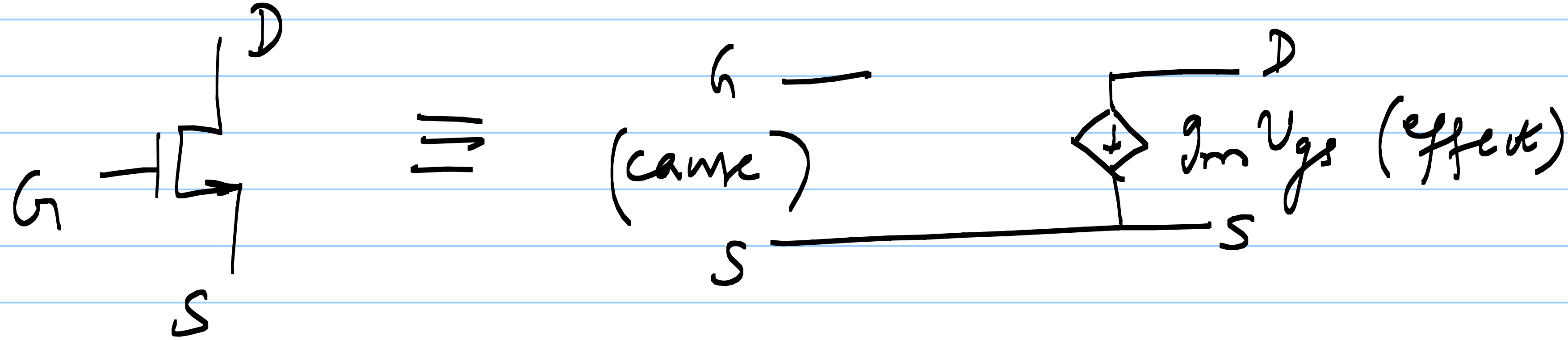


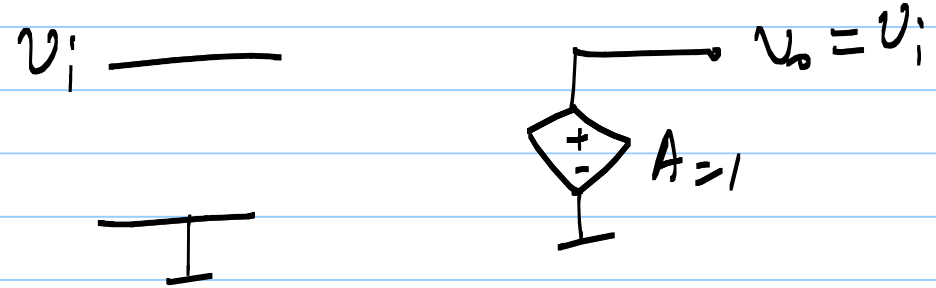
9/9/20

Lecture 21

<u>MOSFET VCVS</u>, gain $=1$, $v_o = v_i$

using negative f.b.





$v_o = v_i$

$\Rightarrow \underbrace{v_i - v_o}_{\text{comparison action}} = 0$

use this $(v_i - v_o)$ to change v_o

Sense i_d or i_s (relate to v_o)

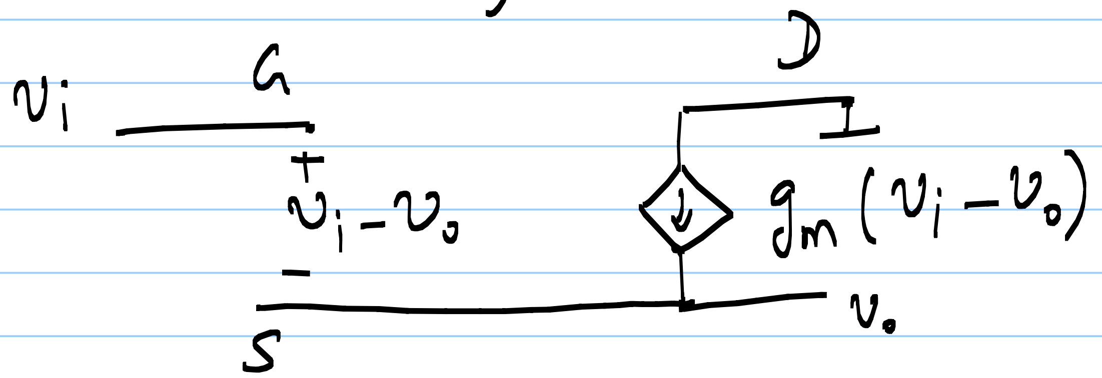
drive back to v_{gs} (relate to $v_i - v_o$)

i_d, i_s, v_g, v_s etc. are small signal quantities

$v_i, v_o \longleftrightarrow v_g, v_s$ (cause)

$v_o \longleftrightarrow v_d, v_s$ (effect i_d or i_s)

$$\left.\begin{array}{l} v_o \to v_s \\ v_i \to v_g \end{array}\right) \quad v_i - v_o = v_{gs}$$



$\downarrow g_m(v_i - v_o)$

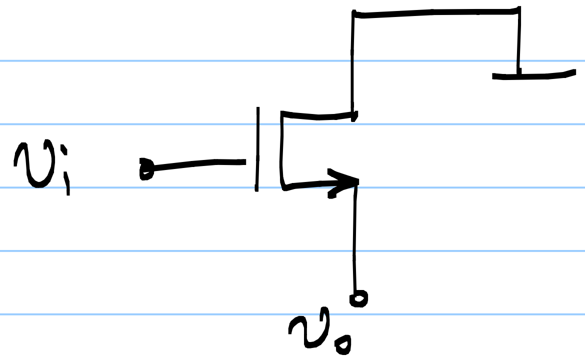
$\uparrow g_m(v_o - v_i)$

Check for -ve f.b.:

If $v_o > v_i \Rightarrow g_m(v_o - v_i)$ will flow out of S node $\Rightarrow v_o \downarrow$

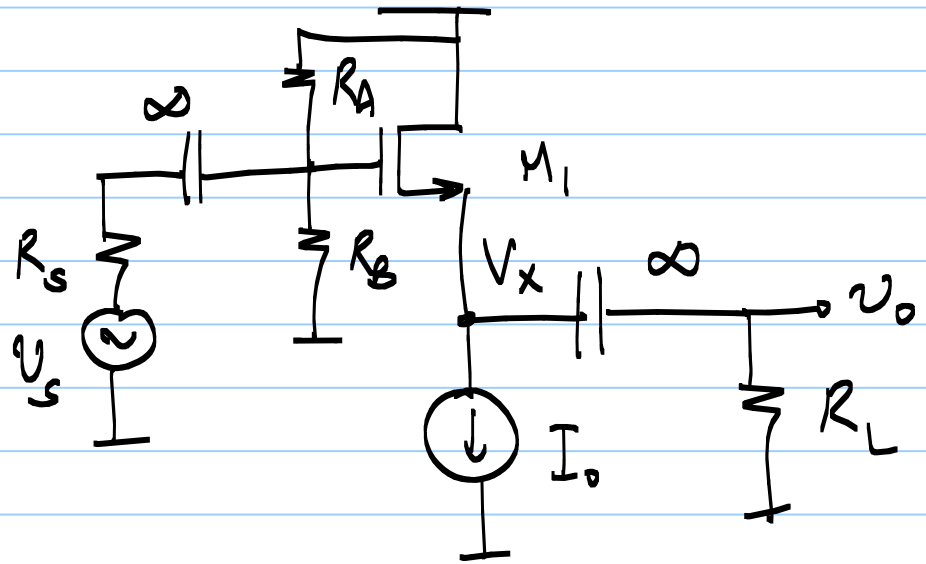
If $v_o < v_i \Rightarrow g_m(v_i - v_o)$ will flow into S node $\Rightarrow \uparrow v_o$

$v_o = v_i \Rightarrow i_d = 0$



Common Drain Amplifier

Common Drain Amplifier
(or)
Source follower



$$V_x = \frac{R_B}{R_A+R_B} V_{DD} - V_T - \sqrt{\frac{2I_o}{\mu_n C_{ox}\left(\frac{W}{L}\right)}}$$

1) choose large R_A, R_B (large Z_{in})

2) As $g_m \to \infty$, gain $\to 1$ (gain <1 if r_{ds} is significant)

3) As $g_m \to \infty$ $Z_{out} \to 0$

$$\frac{v_o}{v_s} = \frac{g_m}{g_m + g_{ds} + G_L}$$

$g_m \gg G_L$

$g_m \gg g_{ds}$

Swing limits

$$v_s = V_A \sin\omega t$$

1) Cutoff limit

$$I_D = I_Q + i_d$$

$$= I_0 + i_d$$

assume g_{ds} is very small

$$i_d = \frac{v_o}{R_L} = \frac{1}{R_L} \cdot \frac{g_m R_L}{1+g_m R_L} \cdot v_s$$

$$I_D = I_0 + \frac{g_m R_L}{1+g_m R_L} \cdot \frac{V_{A_1} \sin\omega t}{R_L}$$

Set $I_D = 0 \Rightarrow V_{A_1} = I_o \cdot R_L \left[1 + \frac{1}{g_m R_L} \right]$

2) Triode limit

$$V_D - \cancel{V_S} = V_G - \cancel{V_S} - V_T$$

$$V_D = V_G - V_T$$

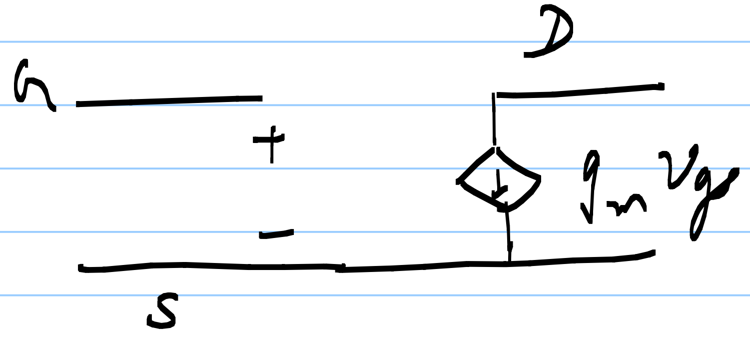
$$V_{DD} = \frac{V_{DD} \cdot R_B}{R_A + R_B} + V_{A_2} \sin \omega t - V_T$$

$$V_{A_2} = \frac{R_A}{R_A + R_B} \cdot V_{DD} + V_T$$

$$V_{A_{max}} = \min. \{ V_{A_1}, V_{A_2} \}$$

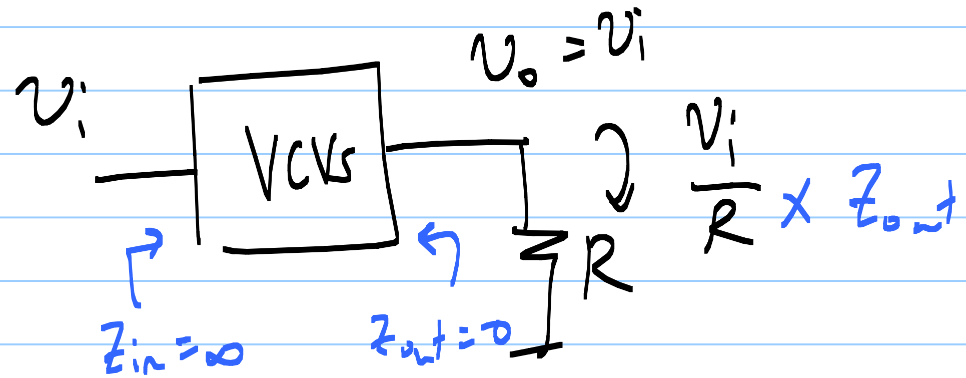
MOSFET VCCS

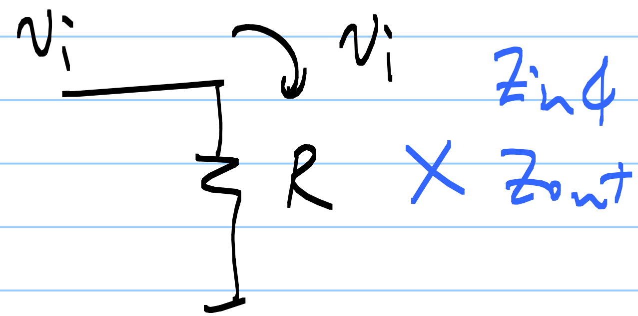
$i_o = G \cdot v_i = \frac{v_i}{R}$; $Z_{in} = \infty$, $Z_{out} = \infty$

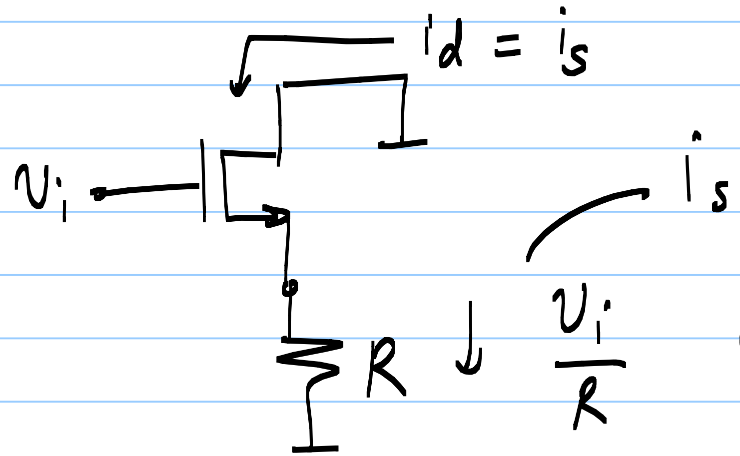


controlling – v_{gs}

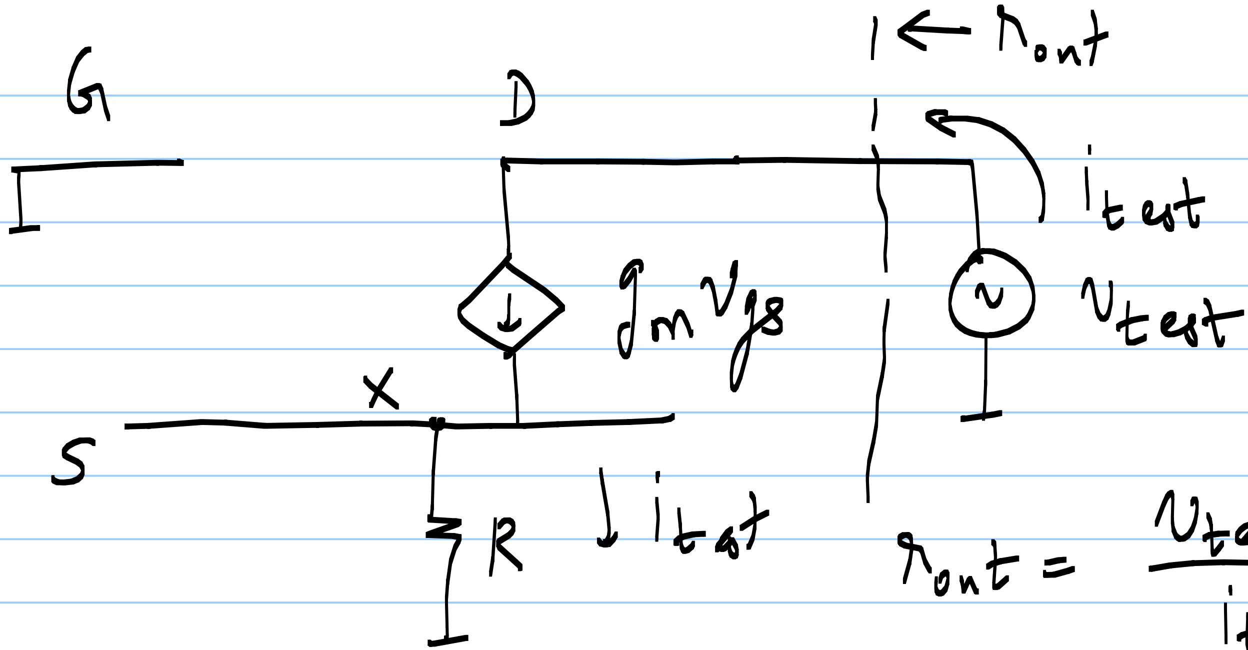
controlled – i_d







if $g_m R >> 1$



$$r_{out} = \frac{v_{test}}{i_{test}}$$

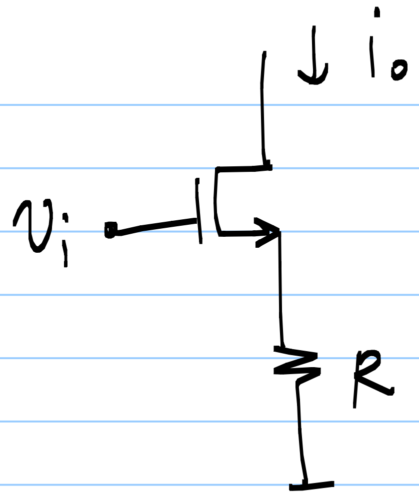
$$v_x = i_{test} \cdot R$$

$$g_m v_{gs} = -g_m v_x = -g_m R \cdot i_{test} = i_{test}$$

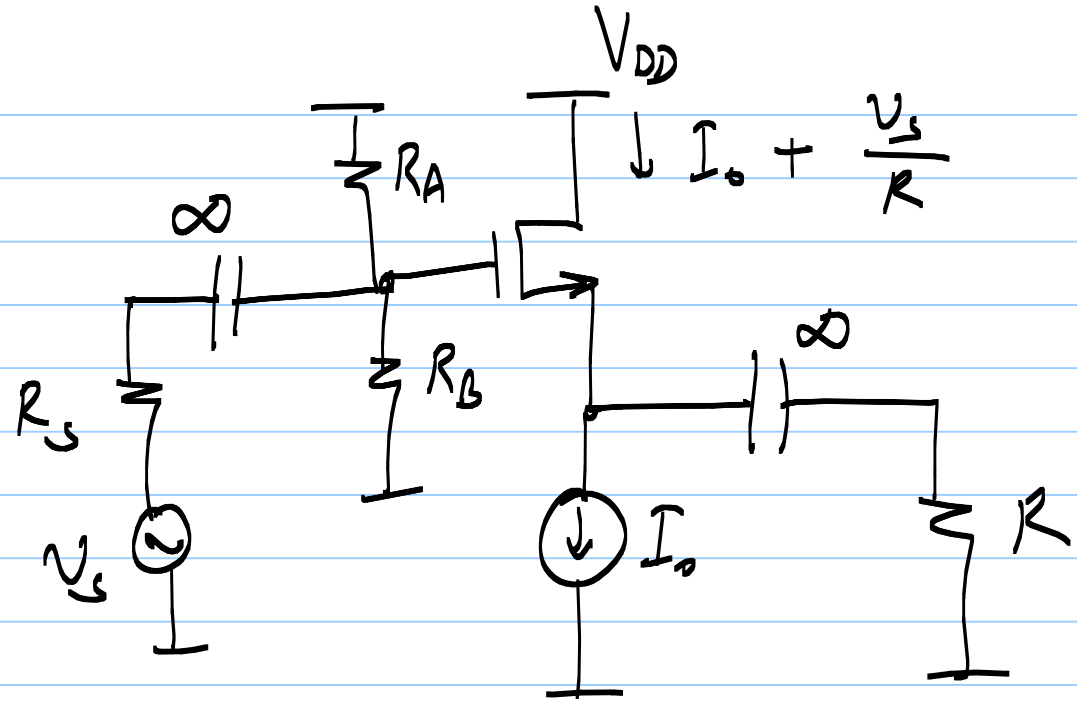
$$i_{test} = 0 \Rightarrow Z_{out} = \infty$$

HW #6 Calculate Z_{out} if r_{ds} is finite

$$Z_{out} = r_{ds} + g_m R r_{ds} + R$$



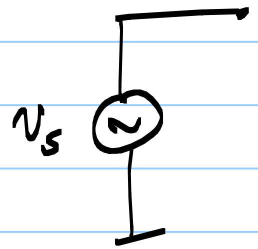
Trans admittance amplifier

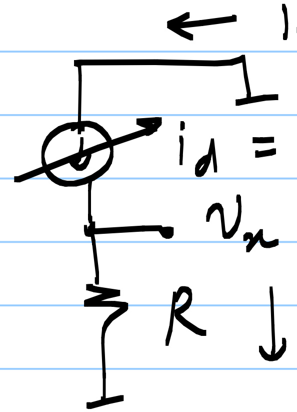


10/9/20

Lecture 21

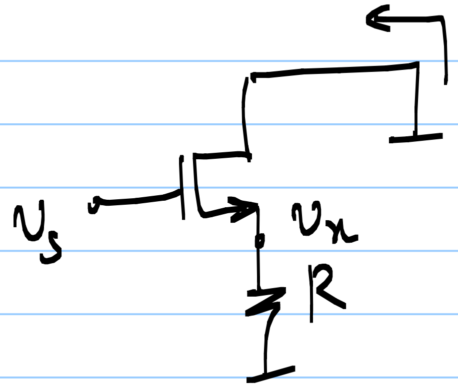
VCCS : $i_o = \dfrac{v_s}{R} = G \cdot v_s$





$i_d = \dfrac{v_s}{R}$ $\quad g_m$ large : $v_x = v_s$

$\dfrac{v_x}{R} = \dfrac{v_s}{R}$



$i_o = \dfrac{v_s}{R}$ if $g_m \to \infty$

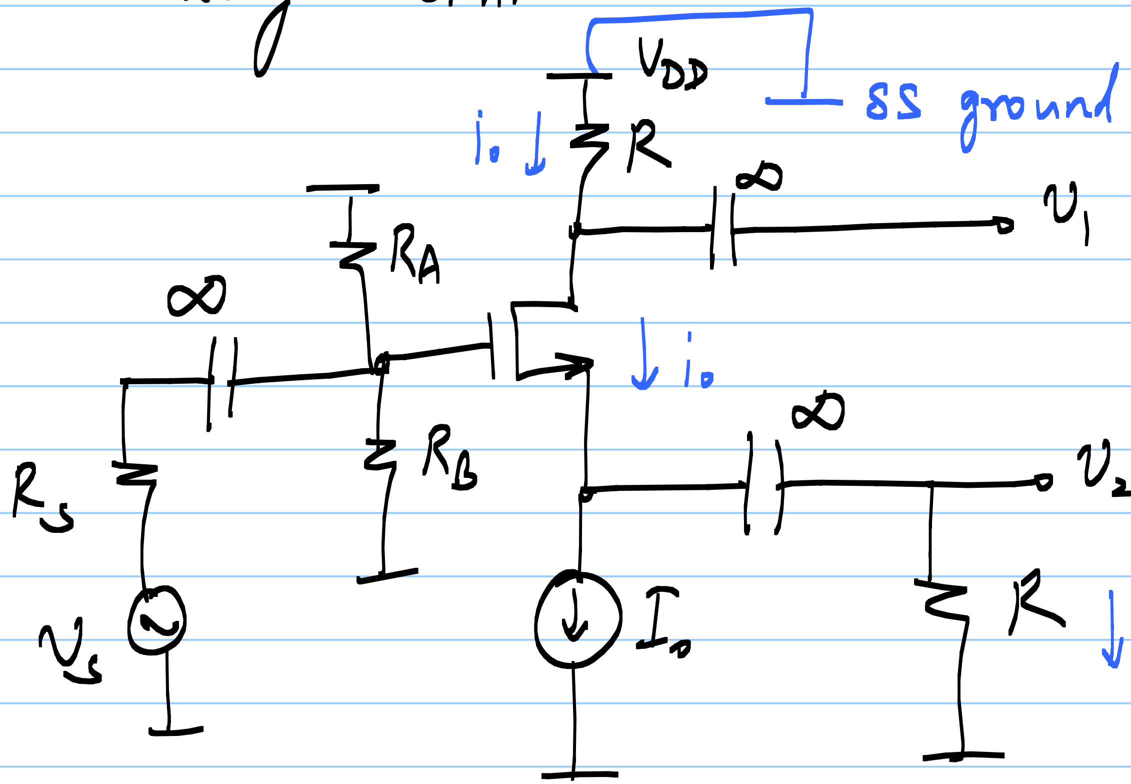
actual $i_o = \dfrac{v_x}{R} = \dfrac{v_s}{R} \cdot \left(\dfrac{g_m}{g_m + G} \right)$

$$\boxed{i_o = \frac{g_m}{1 + g_m R} \cdot v_s}$$

Large $g_m \Rightarrow g_m R \gg 1$ (or) $g_m \gg G$

HW 7 : Swing Limits

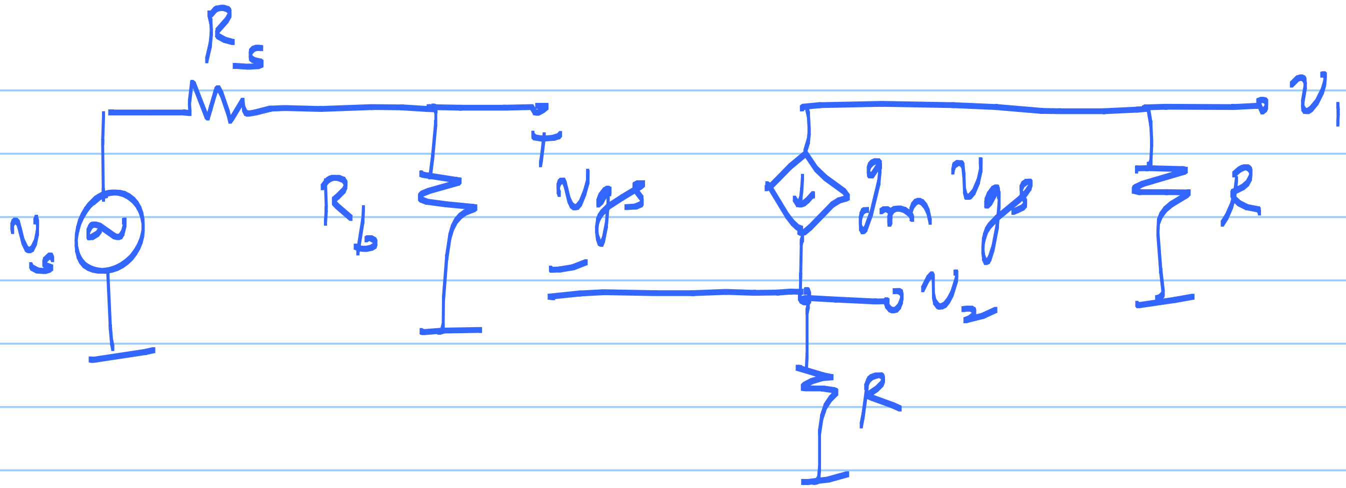
Phase Splitter

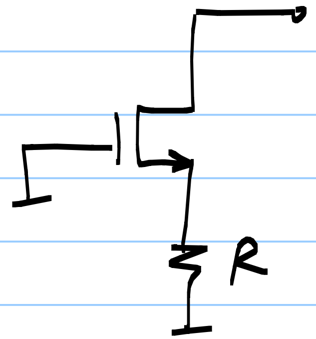


$$v_1 = -\frac{g_m R}{1+g_m R} \cdot v_s = -v_2$$

$$v_2 = \frac{g_m R}{1+g_m R} \cdot v_s$$

$$i_o = \frac{g_m}{1+g_m R} \cdot v_s$$



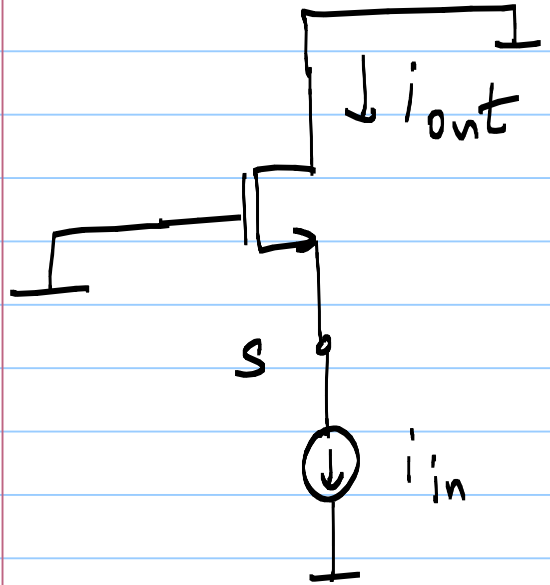


$\leftarrow r_{out} \approx (g_m r_{ds}) R$

MOSFET incremental CCCS

$i_o = i_{in}$

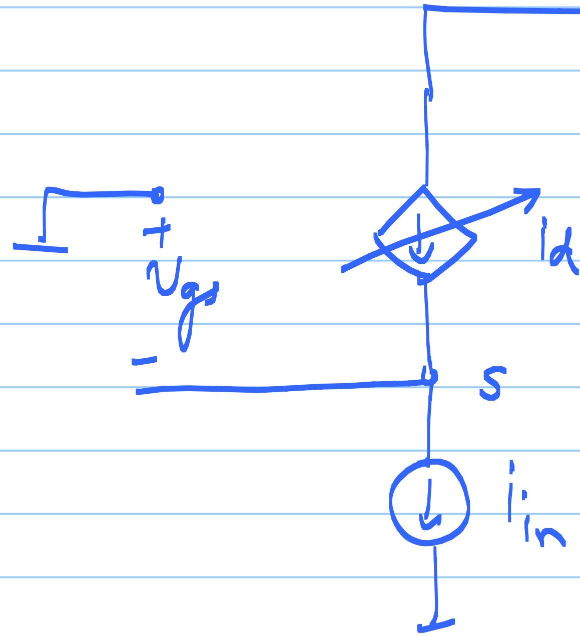
$Z_{in} = 0$; $Z_{out} = \infty$



$i_G = 0$

$i_d = i_s$

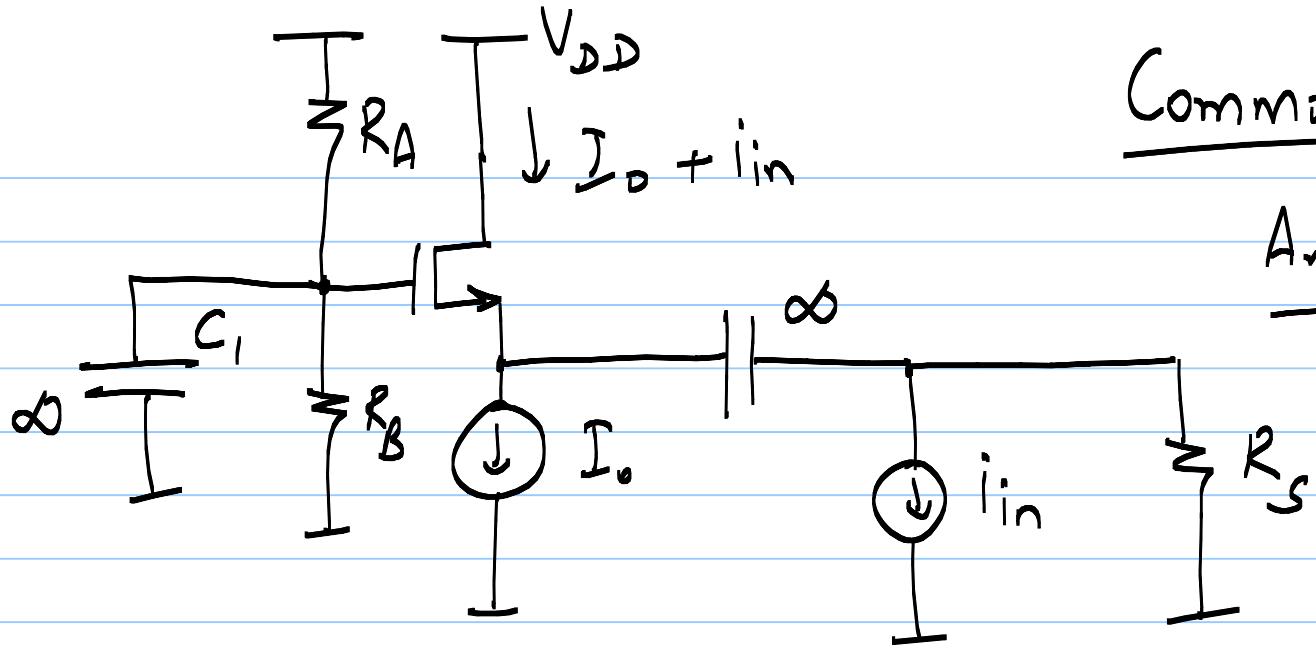
v_{gs} — const



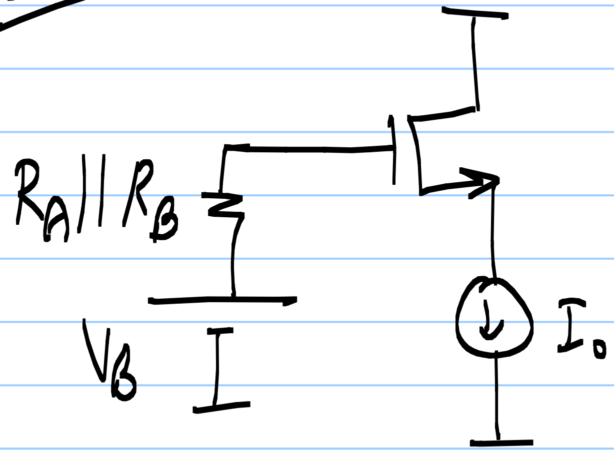
$i_d = g_m v_{gs} = i_{in}$

Neg. f.b. biasing : Source f.b.

Common Gate Amplifier

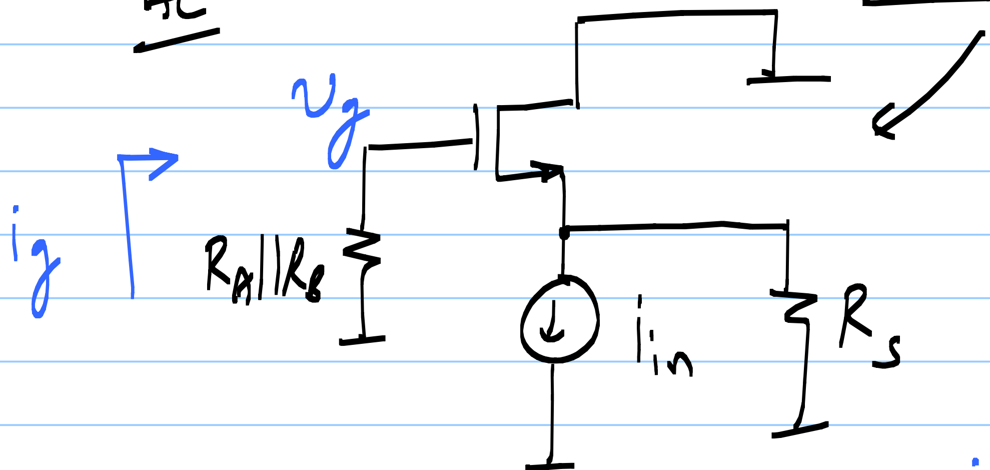


DC



AC

No C_1



$v_g = 0$ if $i_g = 0$

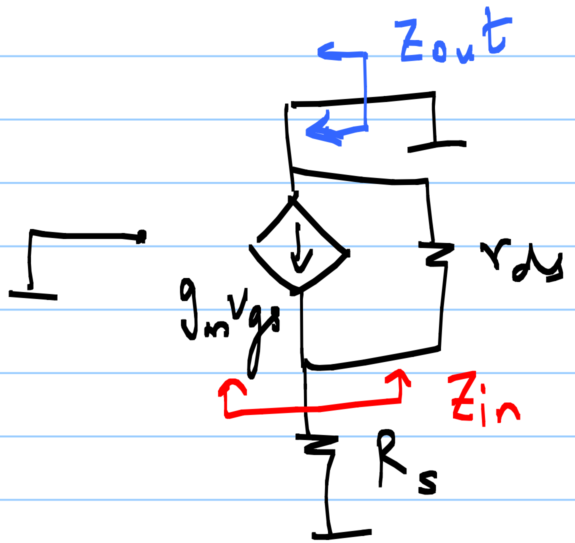
Unfortunately AC $i_g \neq 0$

C_1 is necessary

$$\frac{i_{out}}{i_{in}} = 1 \quad \text{independent of } g_m$$

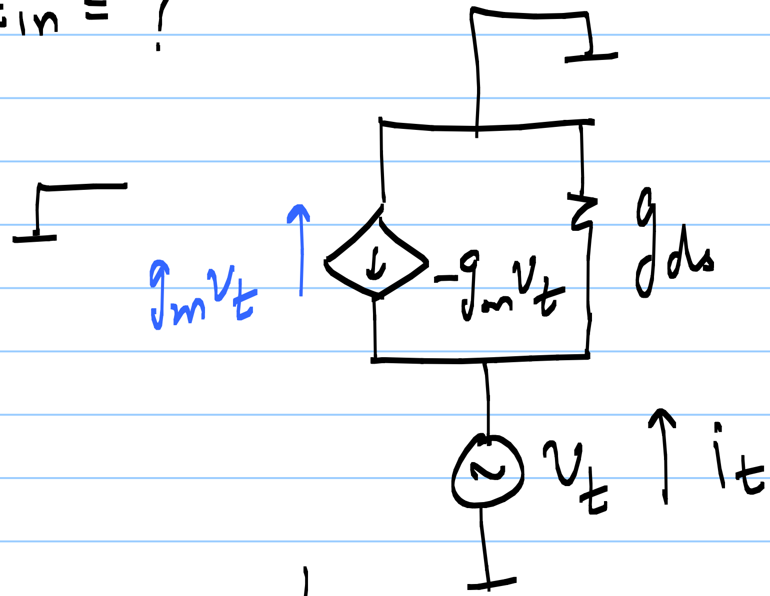
$$Z_{out} = R_s + r_{ds} + g_m R\, r_{ds} \approx g_m R_s\, r_{ds}$$

(very high)



$Z_{in} = ?$

$v_{gs} = -v_t$



$$Z_{in} = \frac{1}{g_m + g_{ds}}$$

large intrinsic gain : $g_m r_{ds} \gg$

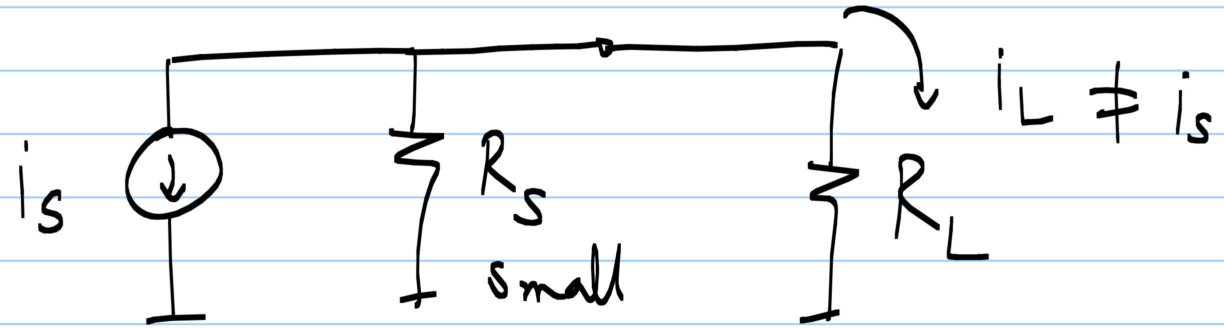
$g_m \gg g_{ds}$

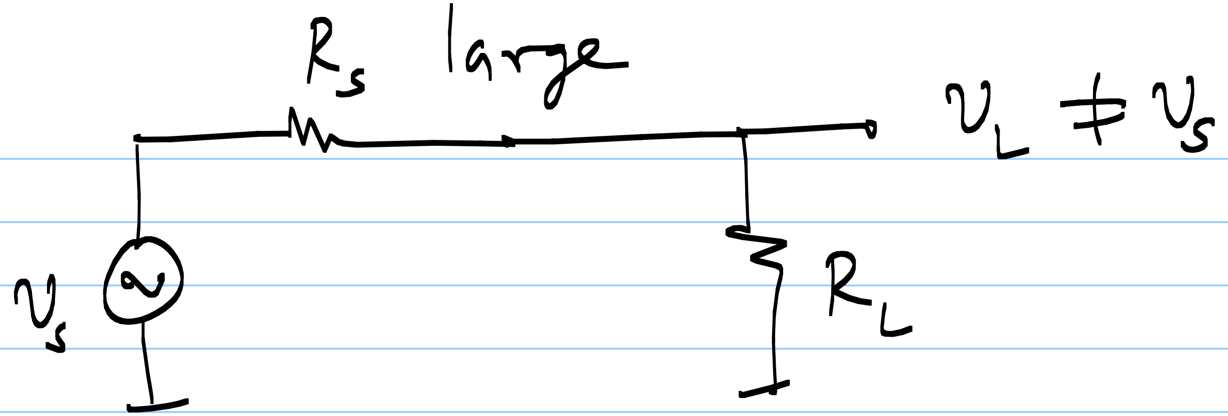
$$Z_{in} \approx \frac{1}{g_m}$$

large g_m : Z_{in} as small as required

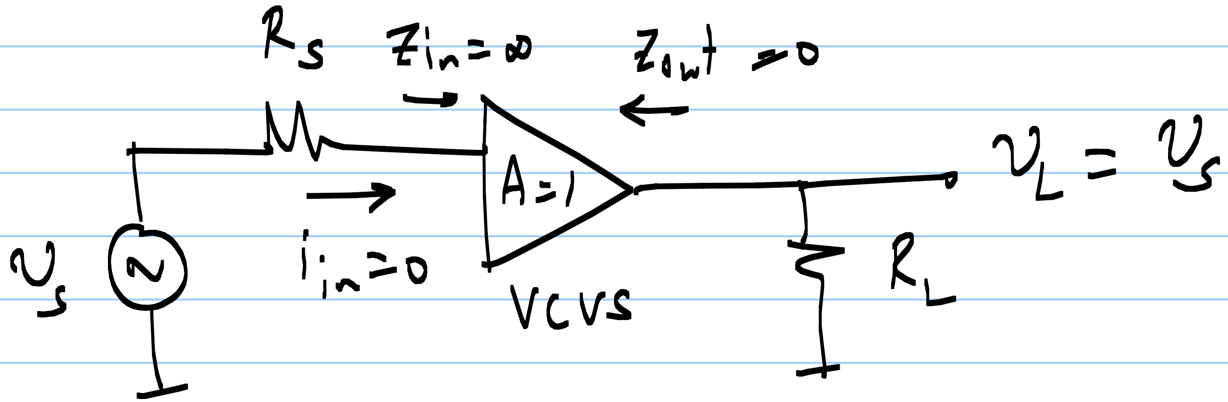
Z_{out} as large as required.

Why do you need a CCCS of gain 1?

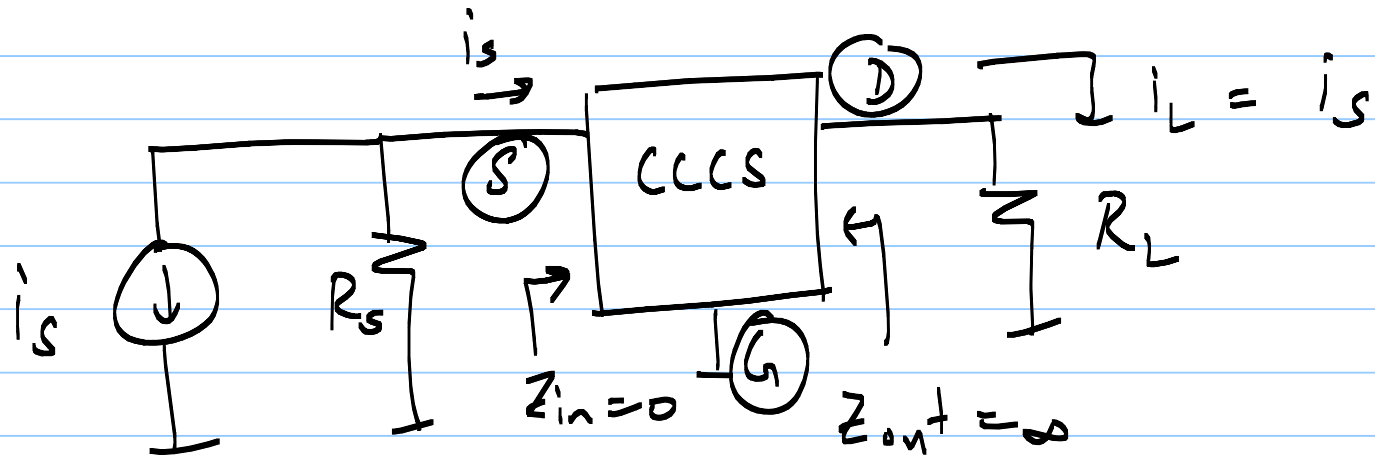




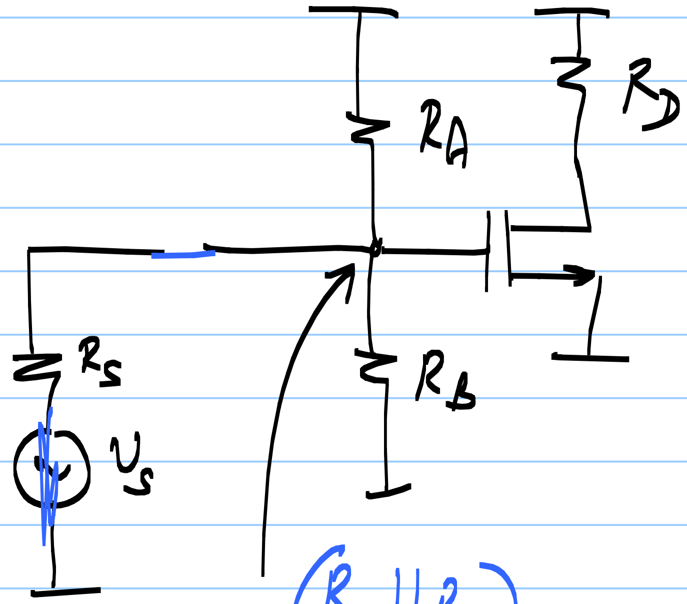
Rs large
vL ≠ vs
vs
RL



Rs
Zin = ∞
Zout = 0
A = 1
vL = vs
vs
iin = 0
VCVS
RL



is
D
iL = is
S
CCCS
RL
is
Rs
Zin = 0
G
Zout = ∞

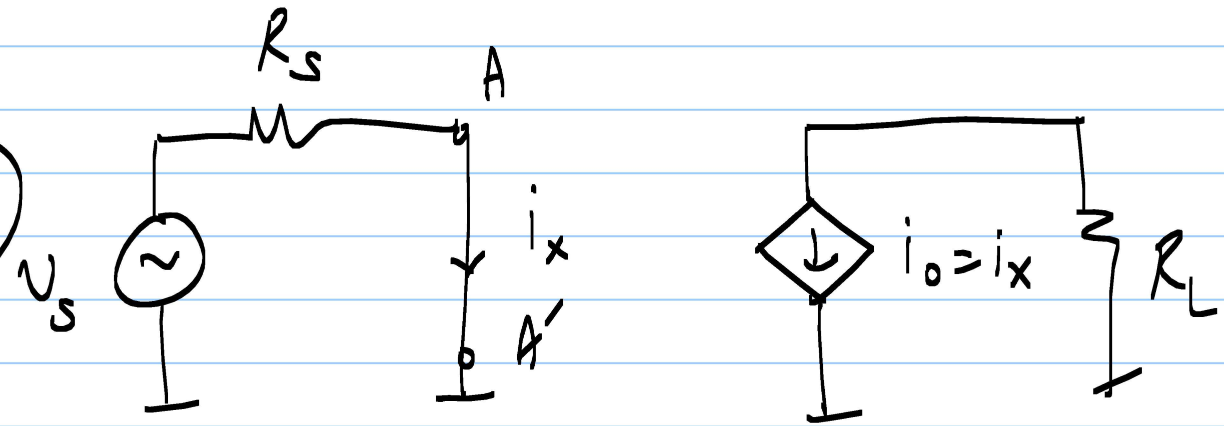


$$\frac{(R_B \| R_S)}{R_A + (R_B \| R_S)} \cdot V_{DD}$$

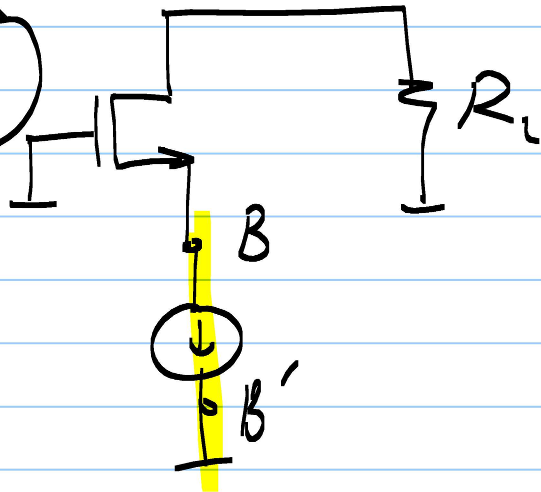
11|9|2020

Lecture 23

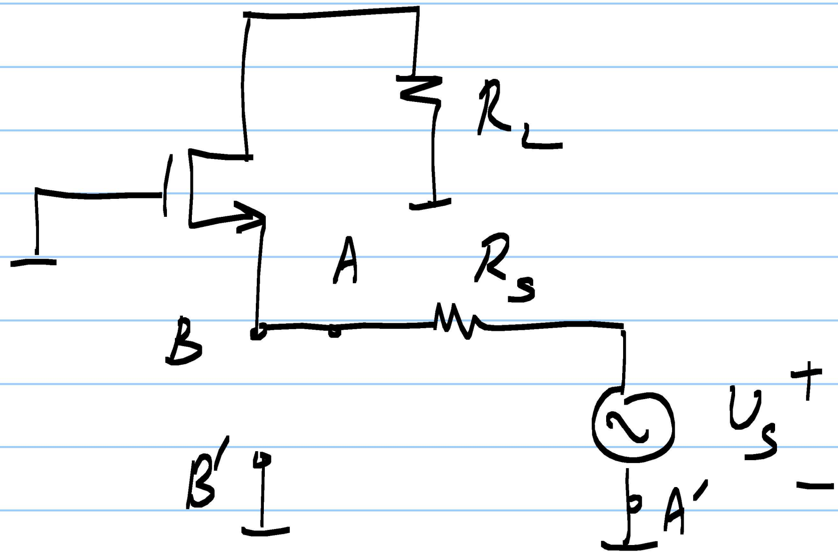
I



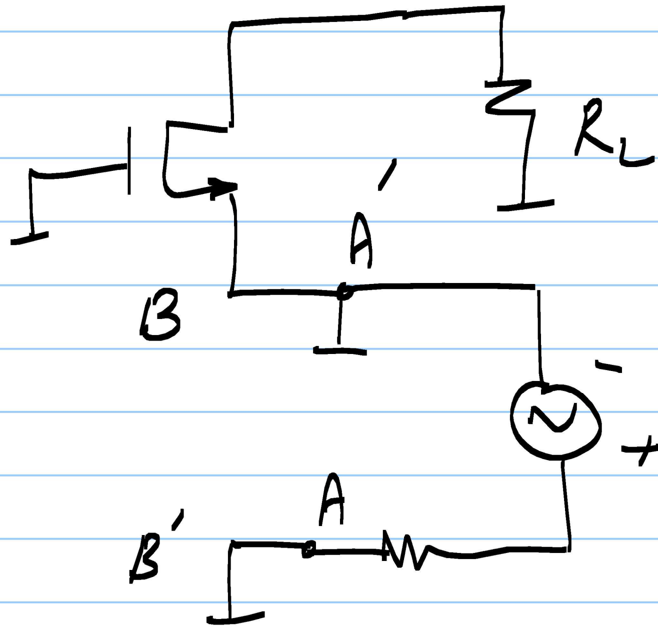
II

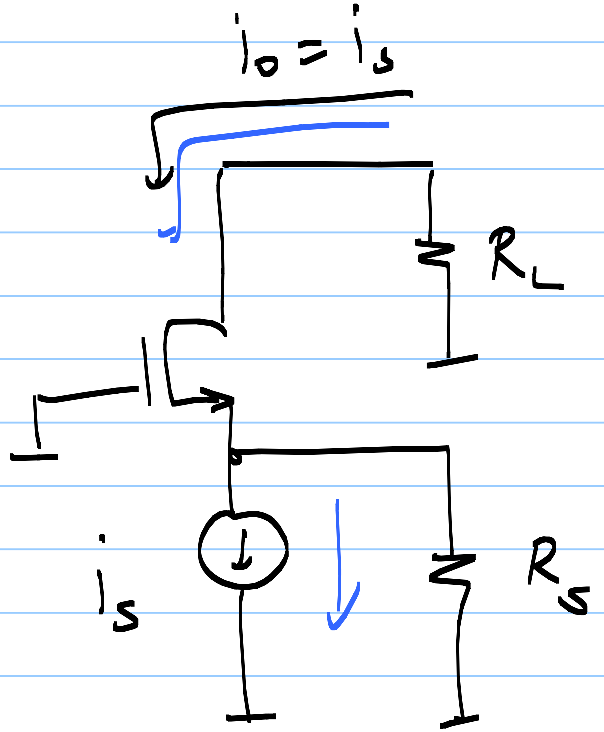


III

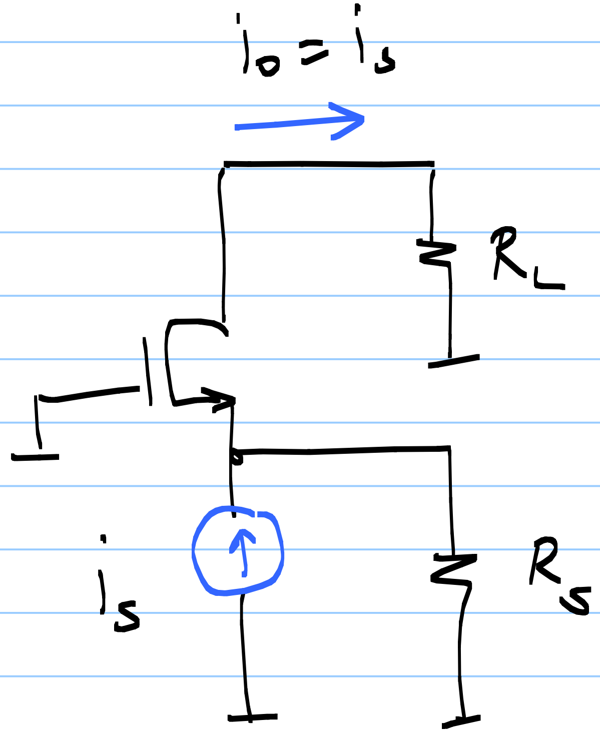


IV

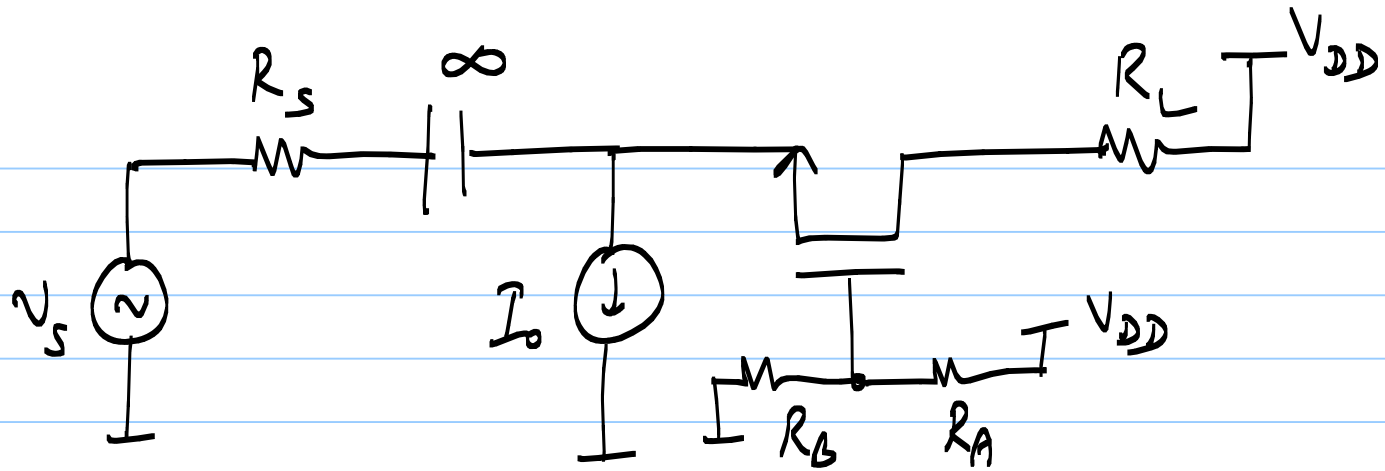


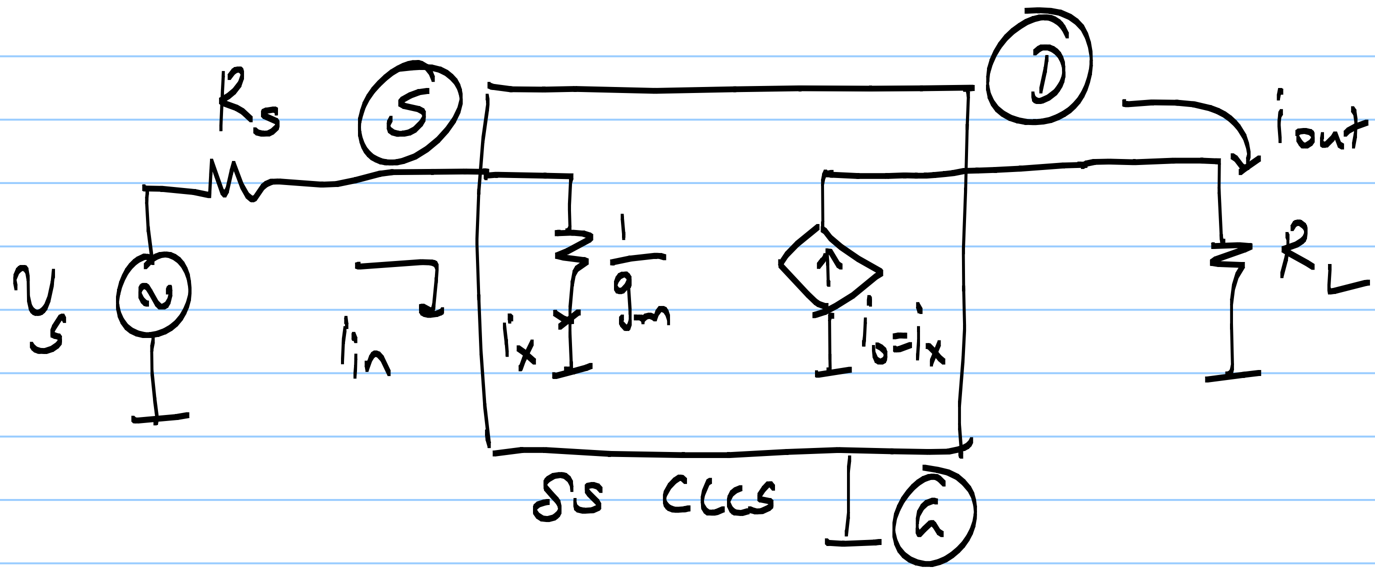


$i_o = i_s$
R_L
i_s
R_s



$i_o = i_s$
R_L
i_s
R_s

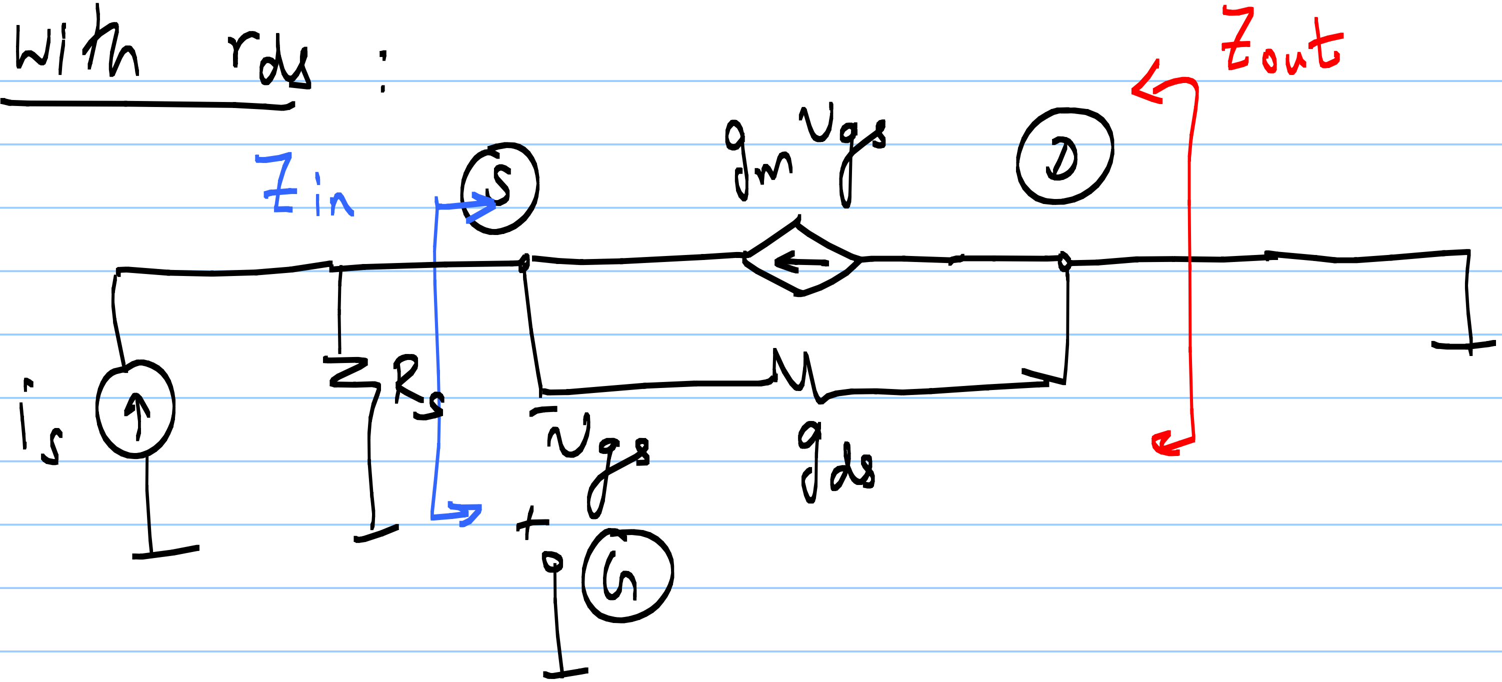




$$i_{in} \approx \frac{v_s}{R_s} \quad \text{if} \quad g_m \gg \frac{1}{R_s}$$

$$\frac{i_{out}}{v_s} = \frac{1}{R_s} \quad ; \quad \frac{v_{out}}{v_s} = \frac{R_L}{R_s}$$

with r_{ds} :



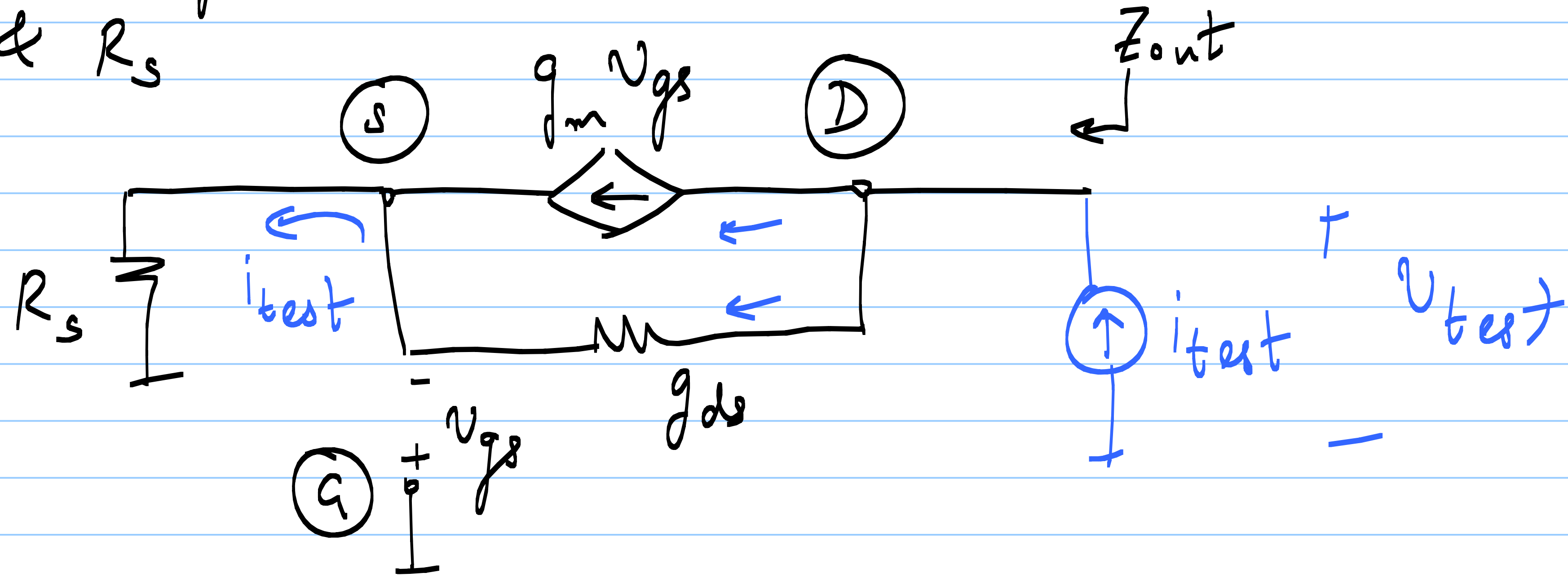
without g_{ds} : $Z_{in} = 1/g_m$

with g_{ds}, without R_L : $Z_{in} = \dfrac{1}{g_m + g_{ds}}$

with g_{ds}, R_L : HW 8 $Z_{in} = ?$

without g_{ds}: $Z_{out} = \infty$

with g_{ds}:
& R_s



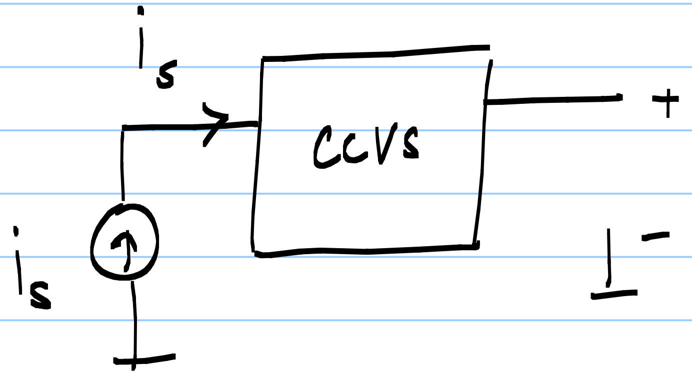
$$Z_{out} = R_s + r_{ds} + g_m r_{ds} R_s$$

$v_{S} = R_s \cdot i_{test} = -v_{gs}$ ...

HW9 : swing limits for CGA

MOSFET incremental CCVS

Trans-impedance amplifier

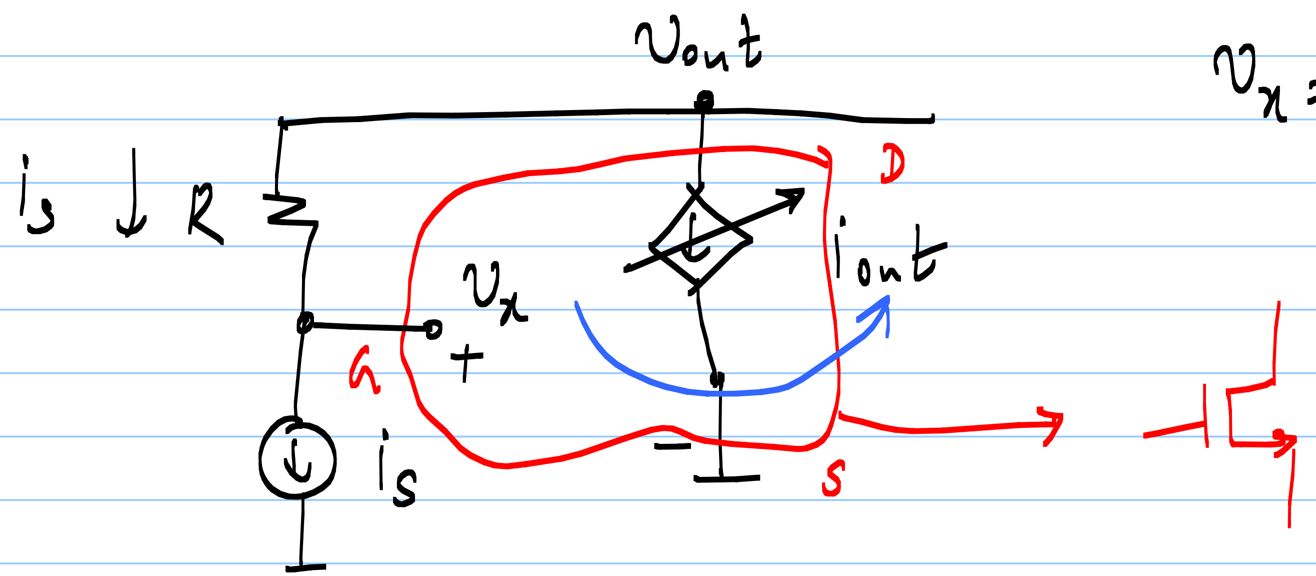


$v_{out} = R i_s$; $Z_{in} = 0$

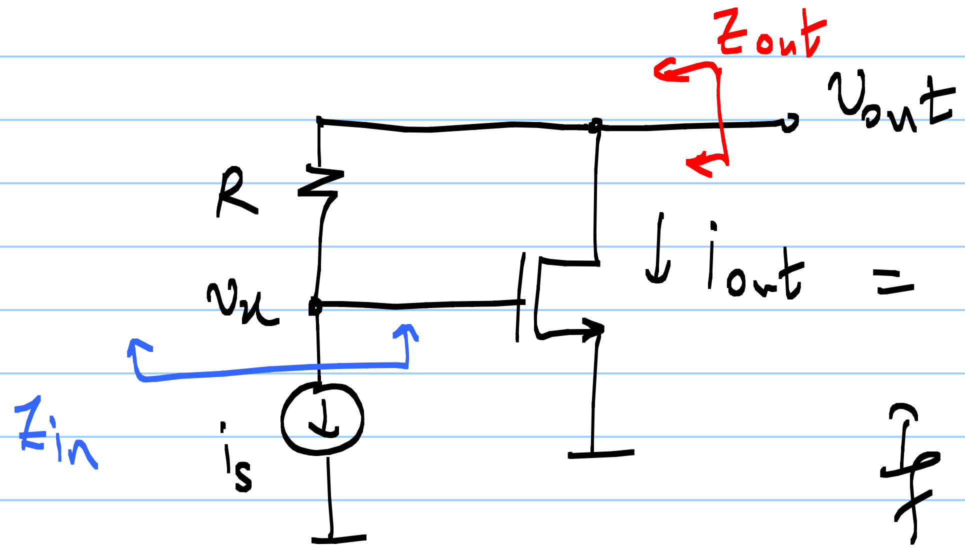
$Z_{out} = 0$

(if $g_m \to \infty$)

$v_{out} = R i_s \Rightarrow v_{out} - R i_s = 0$

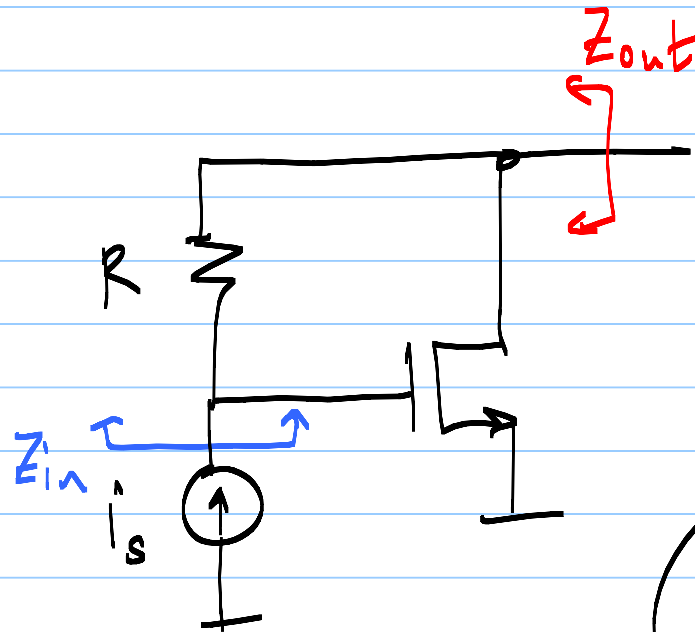


$v_x = v_{out} - R i_s$



$\downarrow i_{out} = g_m v_x$

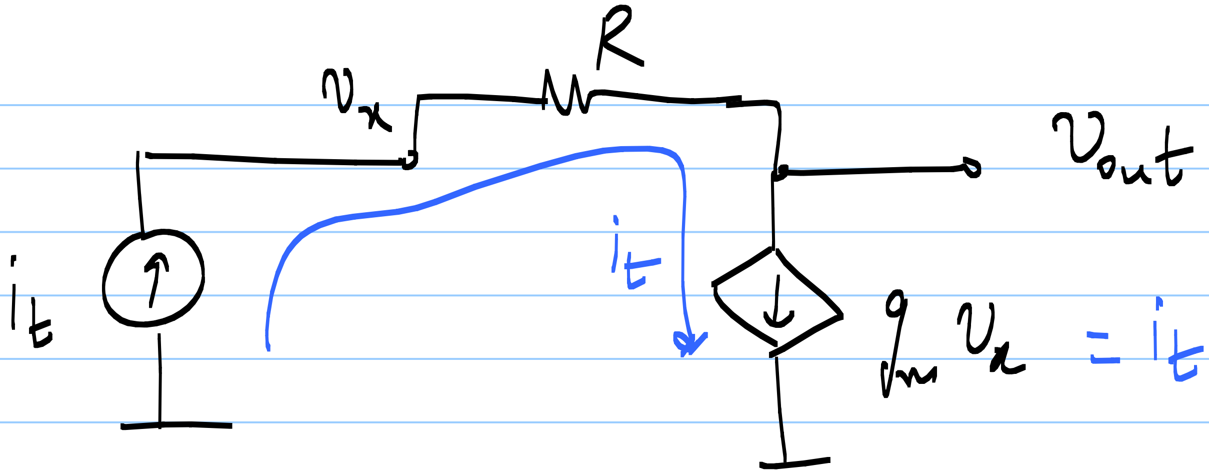
If $g_m \to \infty$, $v_x \to 0$



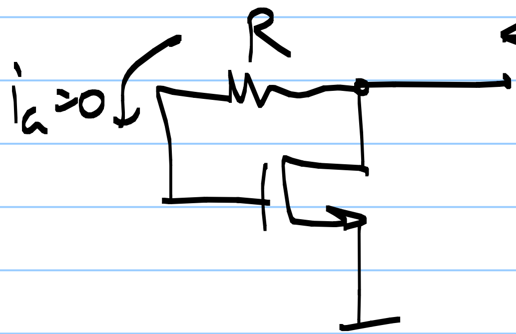
$v_{out} = -R i_s$

In class: Z_{in} & Z_{out}
without R_s & R_L

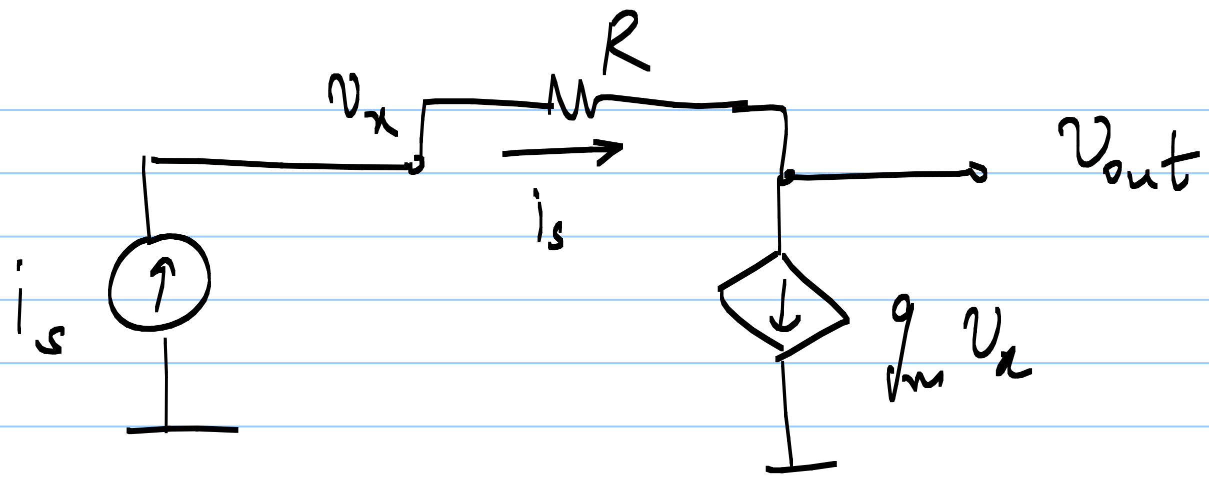
HW9: Z_{in} & Z_{out}
with R_s & R_L



$$i_t = g_m v_x \Rightarrow Z_{in} = \frac{v_x}{i_t} = \frac{1}{g_m}$$



$$\leftarrow Z_{out} = \frac{1}{g_m}$$



$$i_s = g_m v_x$$

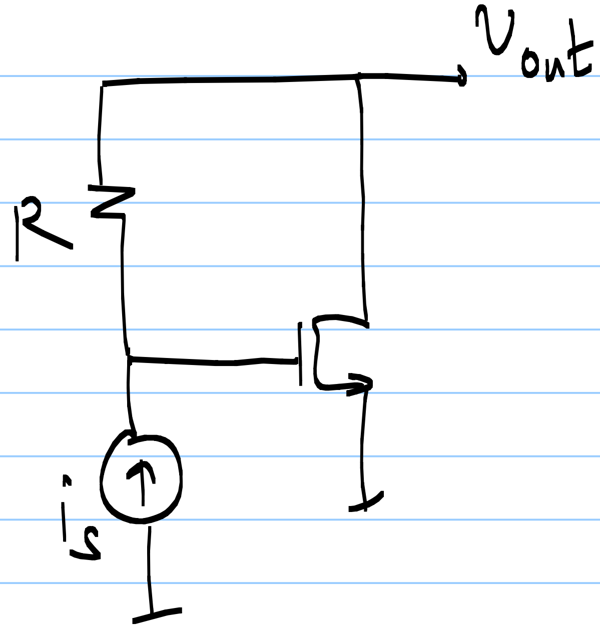
$$v_{out} = v_x - i_s \cdot R$$

$$v_{out} = i_s \left[\frac{1}{g_m} - R\right]$$

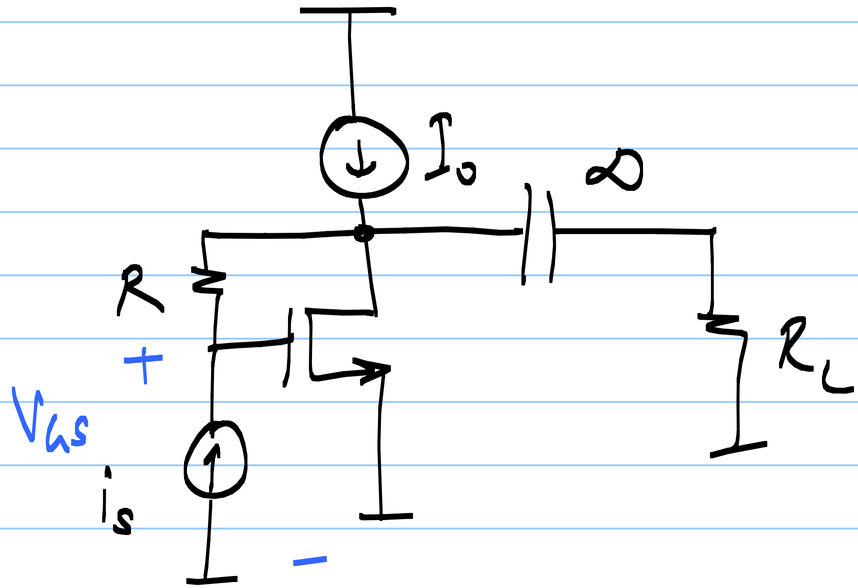
$$\frac{v_{out}}{i_s} = \left[\frac{1}{g_m} - R\right] = -R\left[1 - \frac{1}{g_m R}\right]$$

If $g_m R \to \infty$, $\frac{v_{out}}{i_s} \to -R$

large $g_m \Rightarrow g_m R \gg 1$ (or) $\boxed{g_m \gg \frac{1}{R}}$



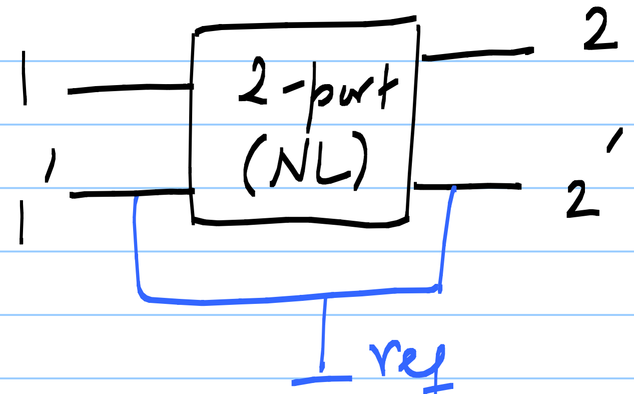
$\Longrightarrow$

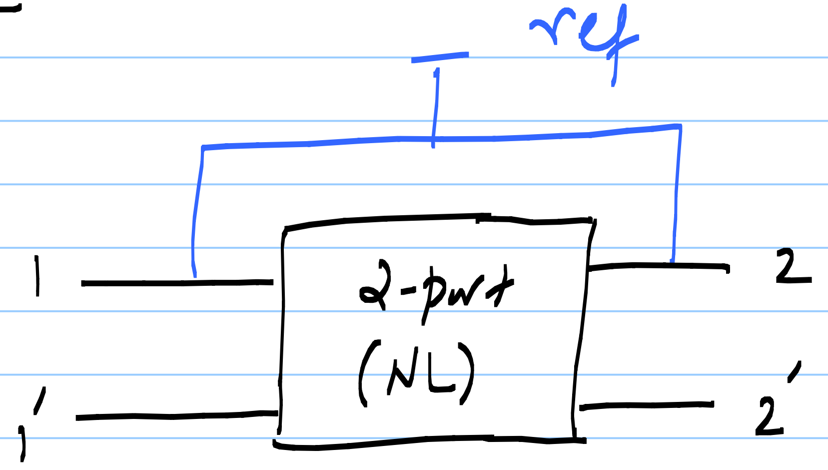


with biasing

15/9/20

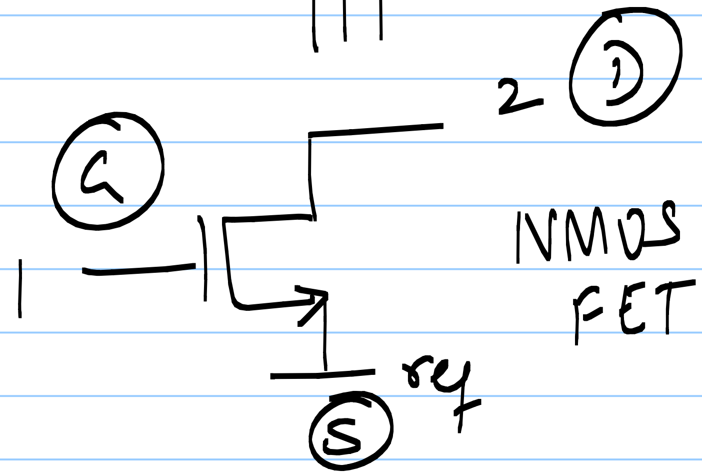
Lecture 24

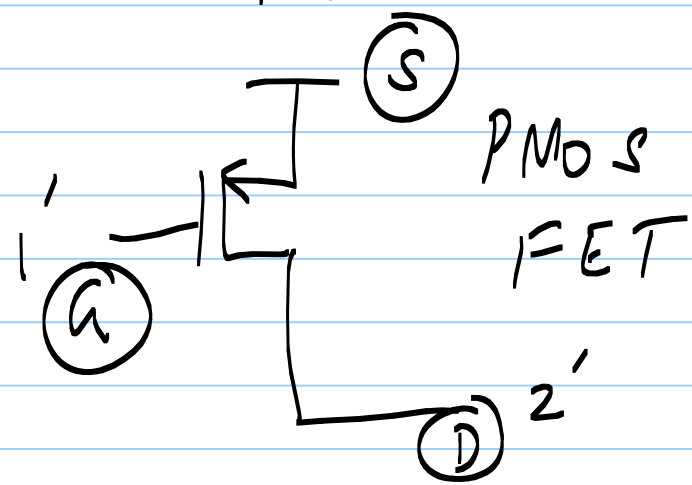


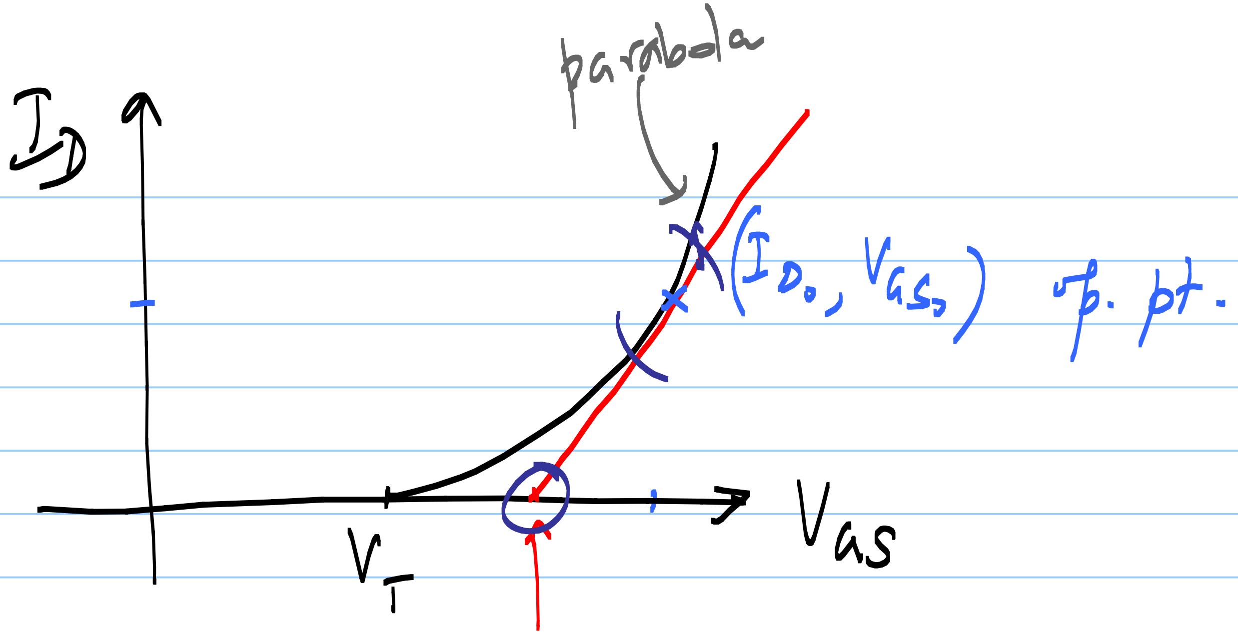


3T 2-port #1 ≡

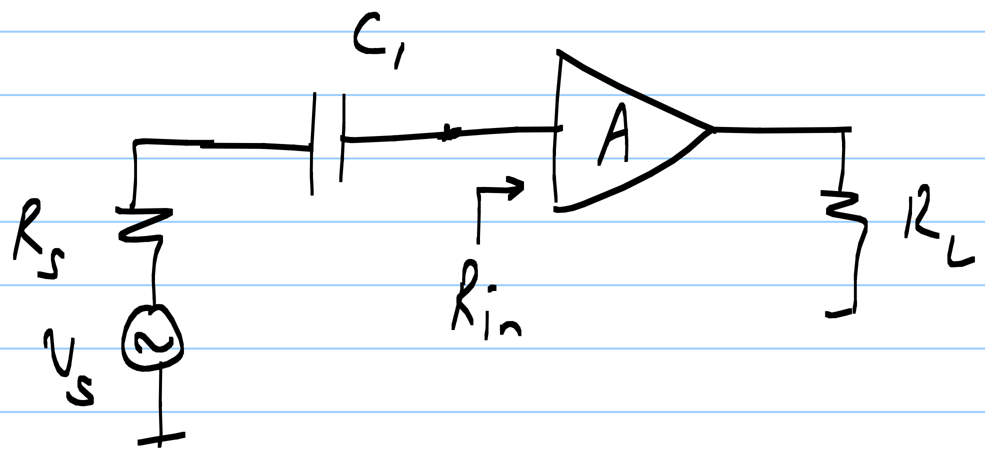
3T 2-port #2 ≡

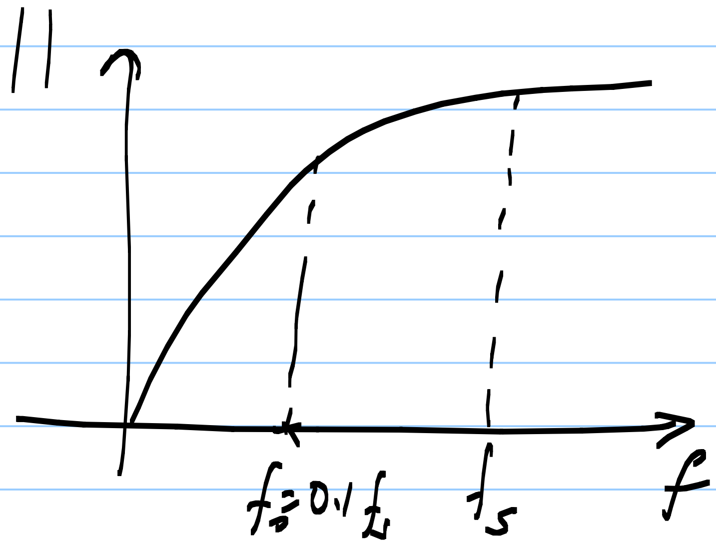


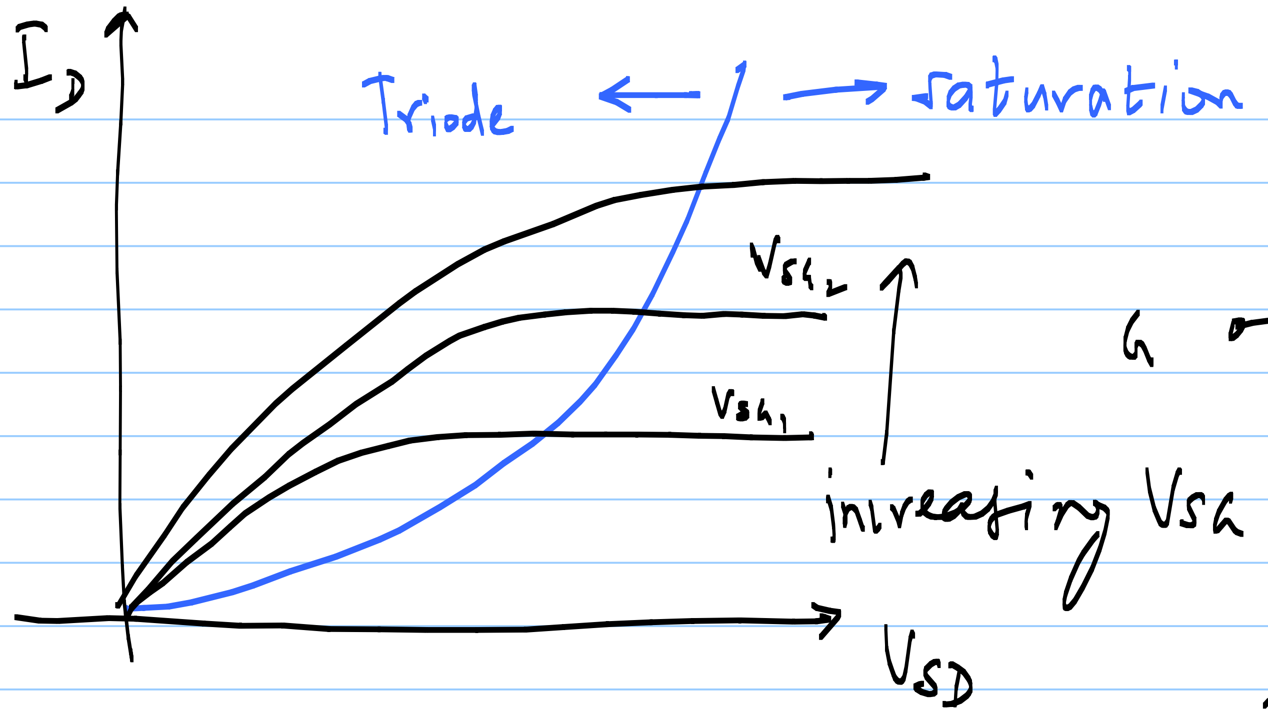


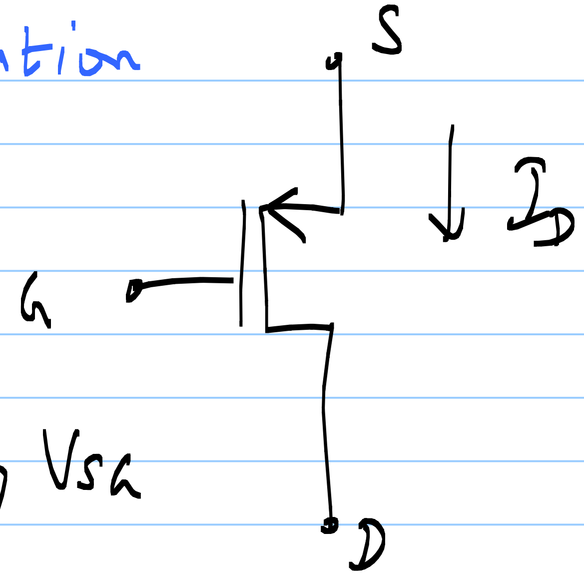


$I_D = 0$ point assuming linear approx.

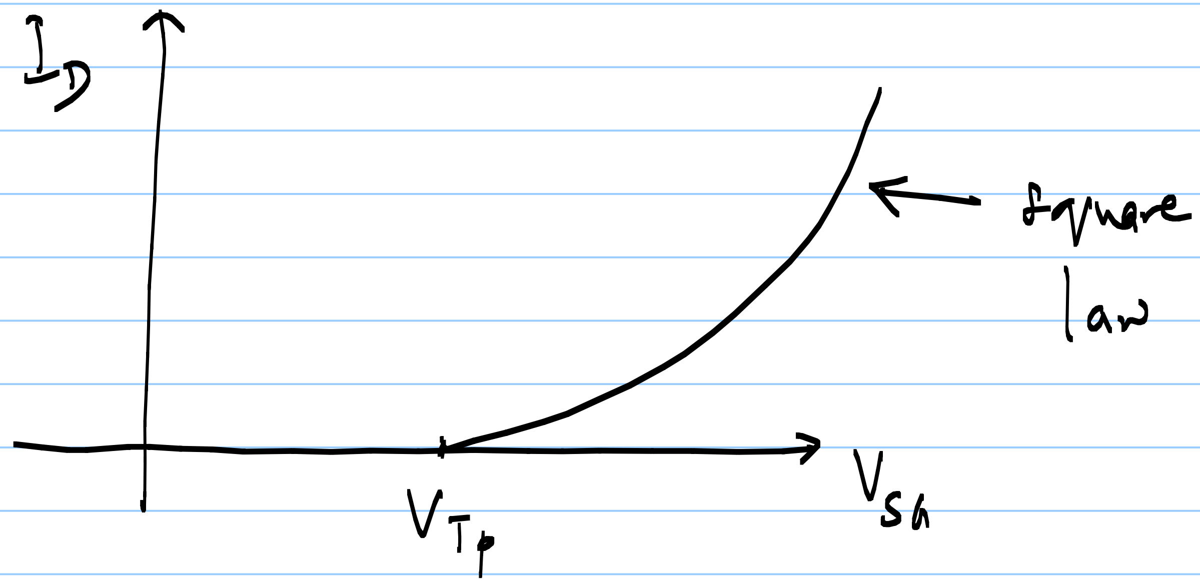





I_D
Triode
saturation
V_SG2
V_SG1
increasing V_SG
V_SD


S
G
D
I_D

$$I_G = 0$$


I_D
square law
V_Tp
V_SG

$$I_D = 0 \quad \text{if} \quad V_{SG} < V_{Tp} \qquad \text{(OFF)}$$

$$= \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)\left[V_{SG} - V_{Tp}\right]^2 \cdot (1+\lambda_p V_{SD}) \qquad \text{(SAT) if } V_{SG} > V_{Tp} \text{ and } V_{SD} \geq V_{SG} - V_{Tp}$$

i.e. $V_D \leq V_G + V_{Tp}$

$$= \mu_p C_{ox}\left(\frac{W}{L}\right)\left[(V_{SG}-V_{Tp})V_{SD} - \frac{V_{SD}^2}{2}\right] \qquad \text{(TRIODE)}$$

$$\text{if} \quad V_{SG} > V_{Tp} \quad \text{and} \quad V_D > V_G + V_{Tp}$$

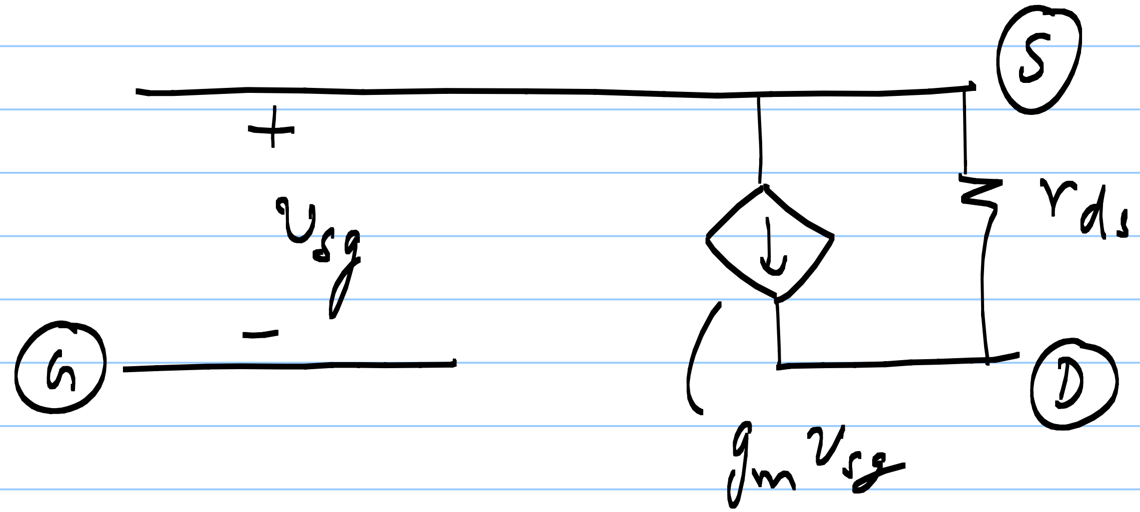
<u>Small-signal</u>

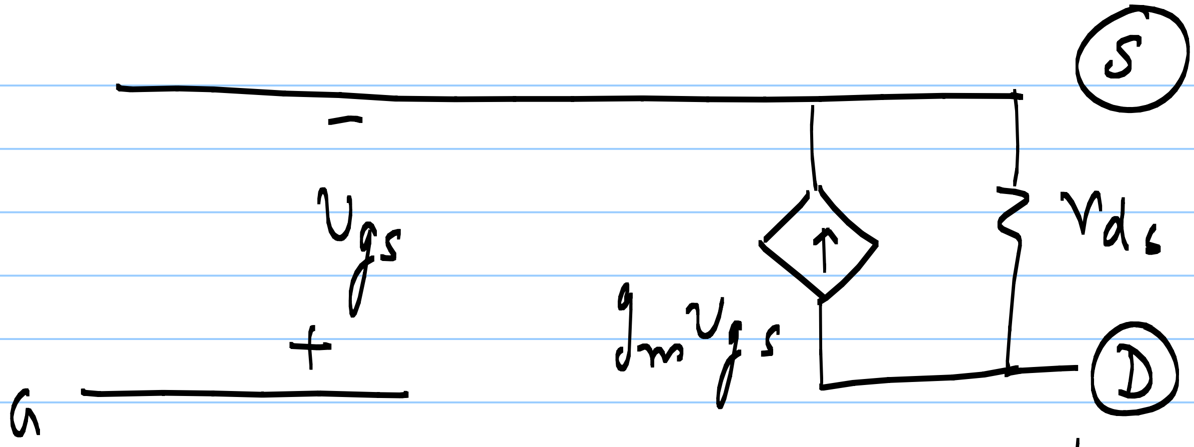
$y_{11} = 0 \; ; \; y_{12} = 0$

$$y_{21} = g_m = \mu_p C_{ox}\left(\frac{W}{L}\right)\left(V_{SG} - V_{Tp}\right)$$

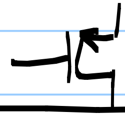
HW 10 2 other expressions for g_m

$$y_{22} = g_{ds} = \frac{1}{r_{ds}} = \lambda_p \cdot I_D$$



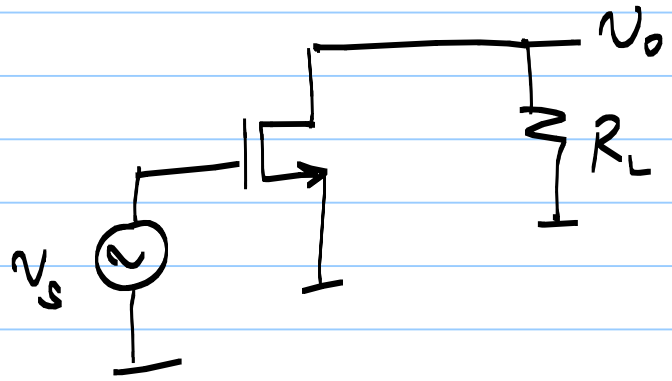


Exactly same ss model as for NMOS

 NMOS	 PMOS
* DC I_D flows into D	DC I_D flows out of D
* For sat., $V_D >> V_S$ $V_D \geq V_G - V_{Tn}$	For sat.: $V_D << V_S$ $V_D \leq V_G + V_{Tp}$
* $V_{GS} \geq V_{Tn}$ for $I_D > 0$	$V_{SG} \geq V_{Tp}$ for $I_D > 0$
* $V_{Tn} > 0$ for enhancement mode device	$V_{Tp} > 0$ for enhancement mode device

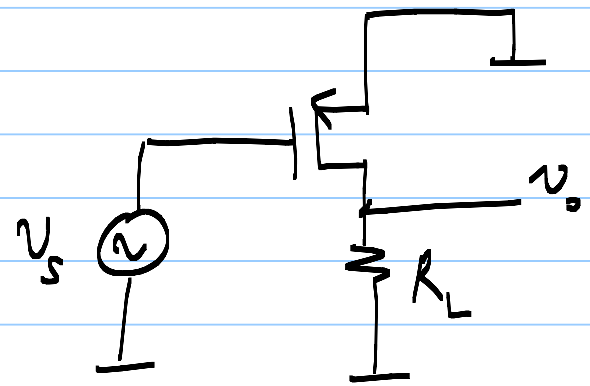
Common Source Amplifier

NMOS

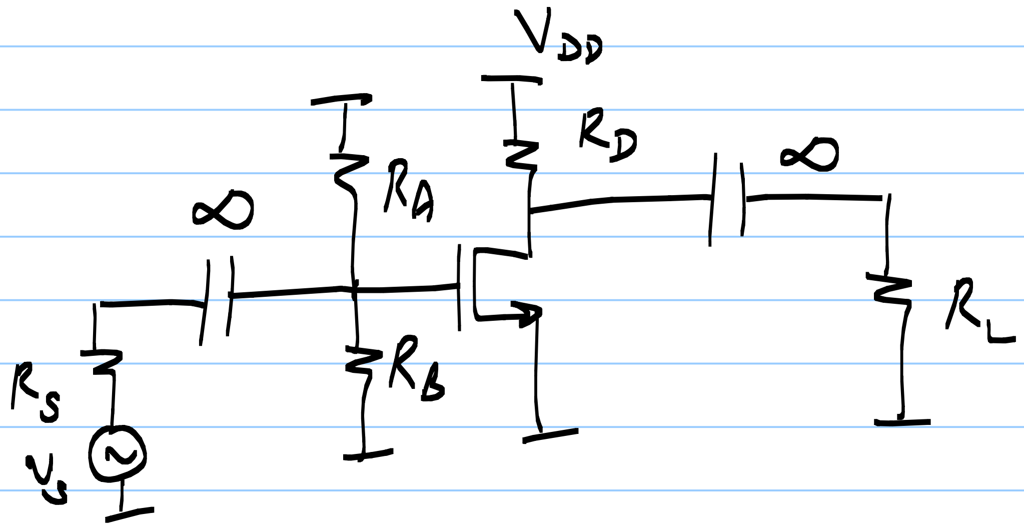


$$\frac{v_o}{v_s} = -g_{m_n} R_L$$

PMOS



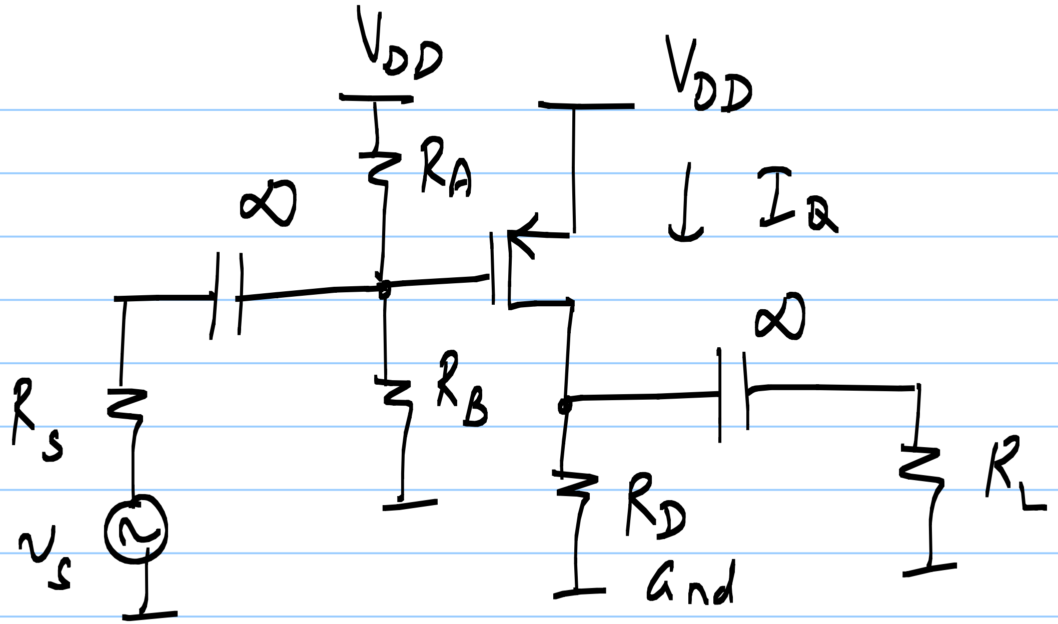
$$\frac{v_o}{v_s} = -g_{m_p} R_L$$



NMOS

CSA

$$V_{GS_n} = \frac{R_B}{R_A + R_B} V_{DD}$$



PMOS

Common-source

amplifier

$$I_Q = \frac{1}{2} \mu_p C_{ox} \left(\frac{W}{L}\right) \left(V_{SG_Q} - V_{T_p}\right)^2$$

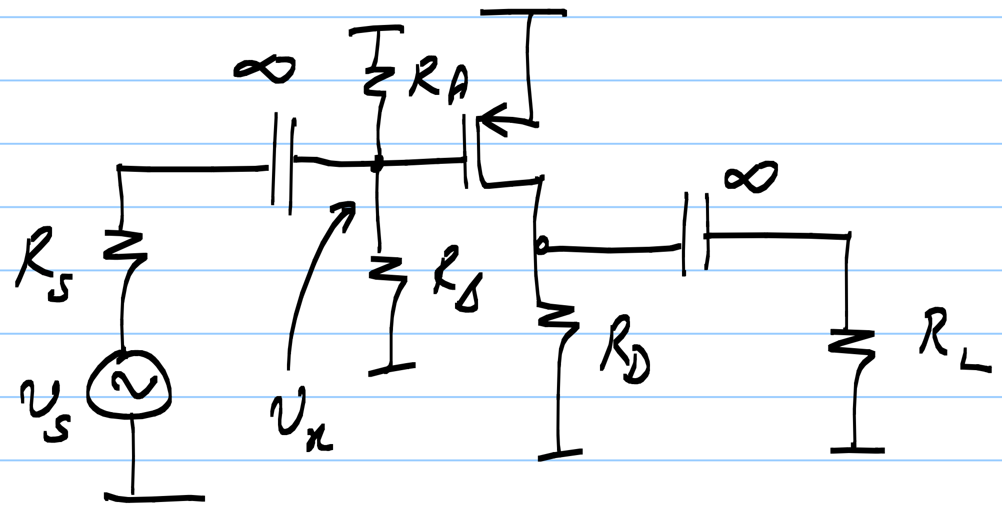
$$V_{SG_Q} = \frac{R_A}{R_A + R_B} \cdot V_{DD} \quad ; \quad V_{SD_Q} = V_{DD} - I_Q \cdot R_D$$

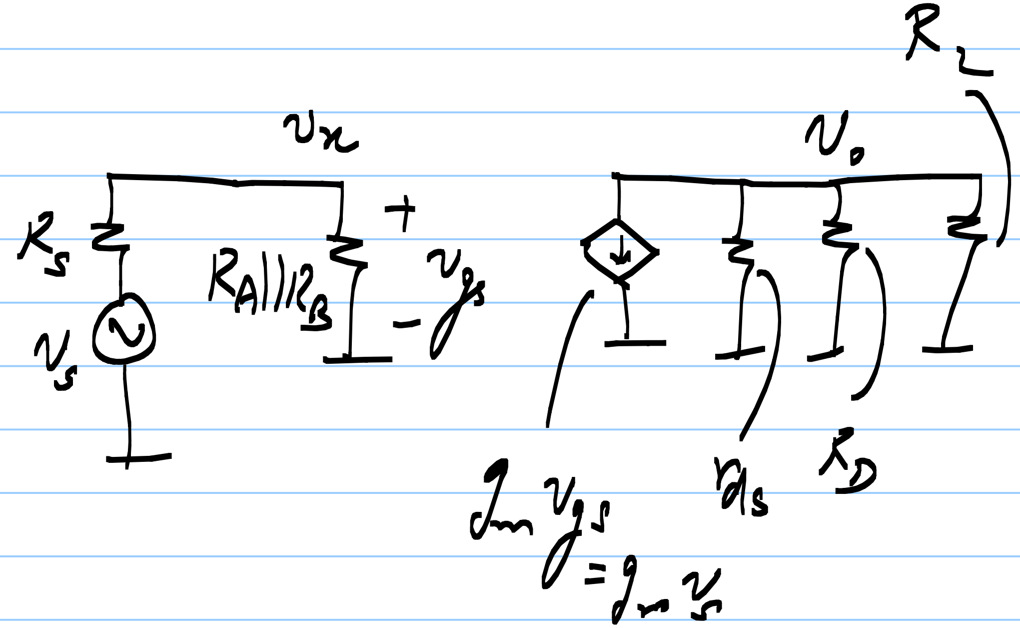
$$V_{S,Q} = V_{DD} \quad ; \quad V_{G,Q} = \frac{R_B}{R_A + R_B} \cdot V_{DD}$$

$$V_{D,Q} = I_Q \cdot R_D$$

16/9/20

Lecture 25





$$v_x = \frac{R_A||R_B}{R_s + R_A||R_B} \cdot v_s$$

choose $R_A||R_B >> R_s$

$\Rightarrow v_x \approx v_s$

$$v_o = -g_m (r_{ds}||R_D||R_L) \cdot v_s$$

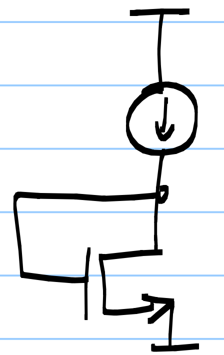
HW 11 : **Swing Limits** :

1) Triode Limit : $V_D > V_G + V_{Tp}$

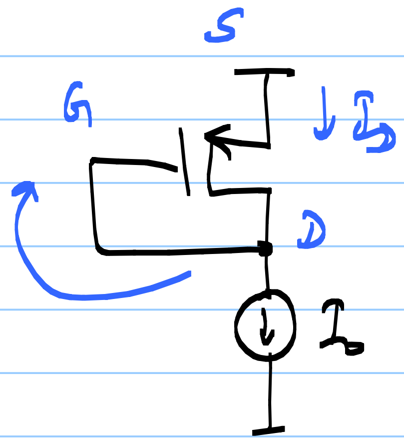
2) Cutoff limit : $I_D = 0$

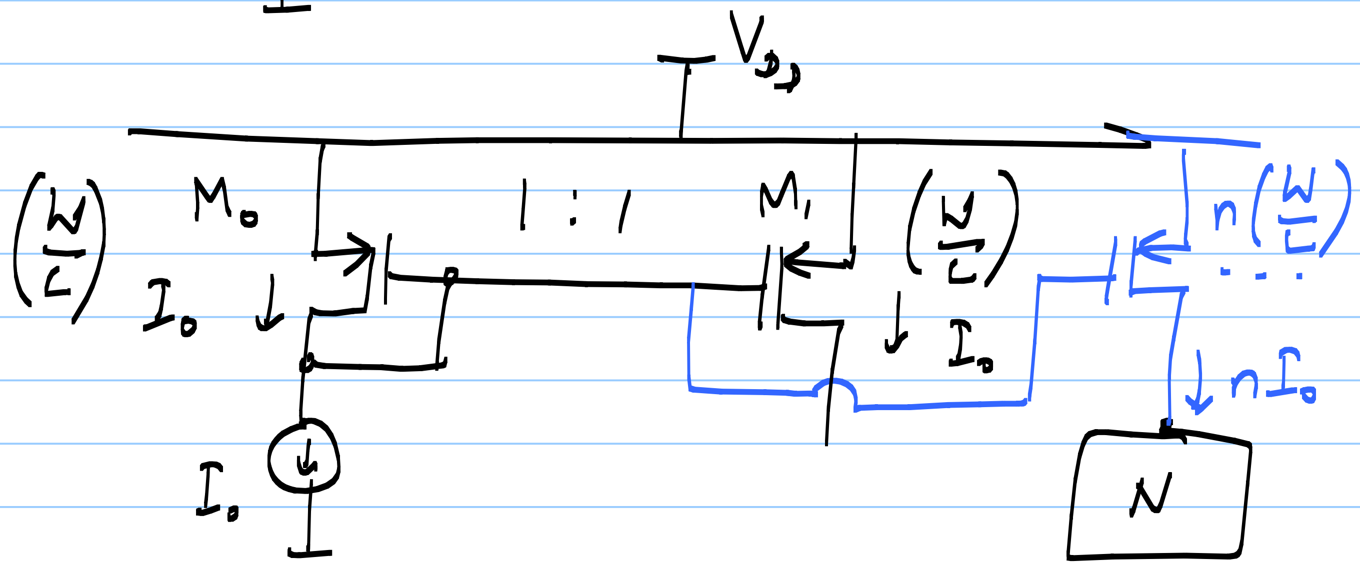
Bias Stabilization

(I) Drain to Gate f.b.



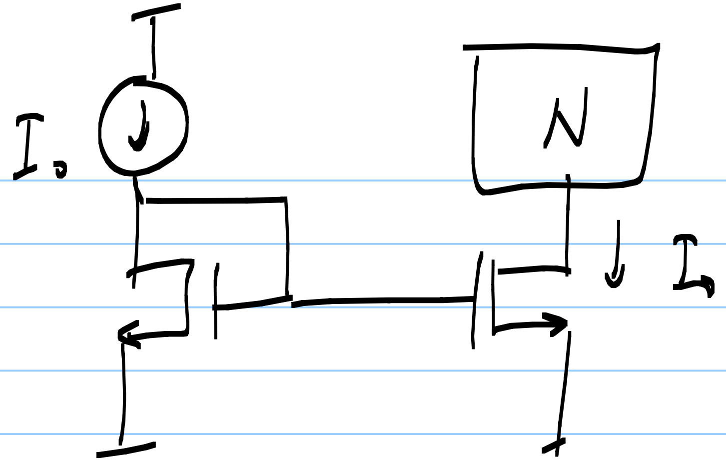
NMOS



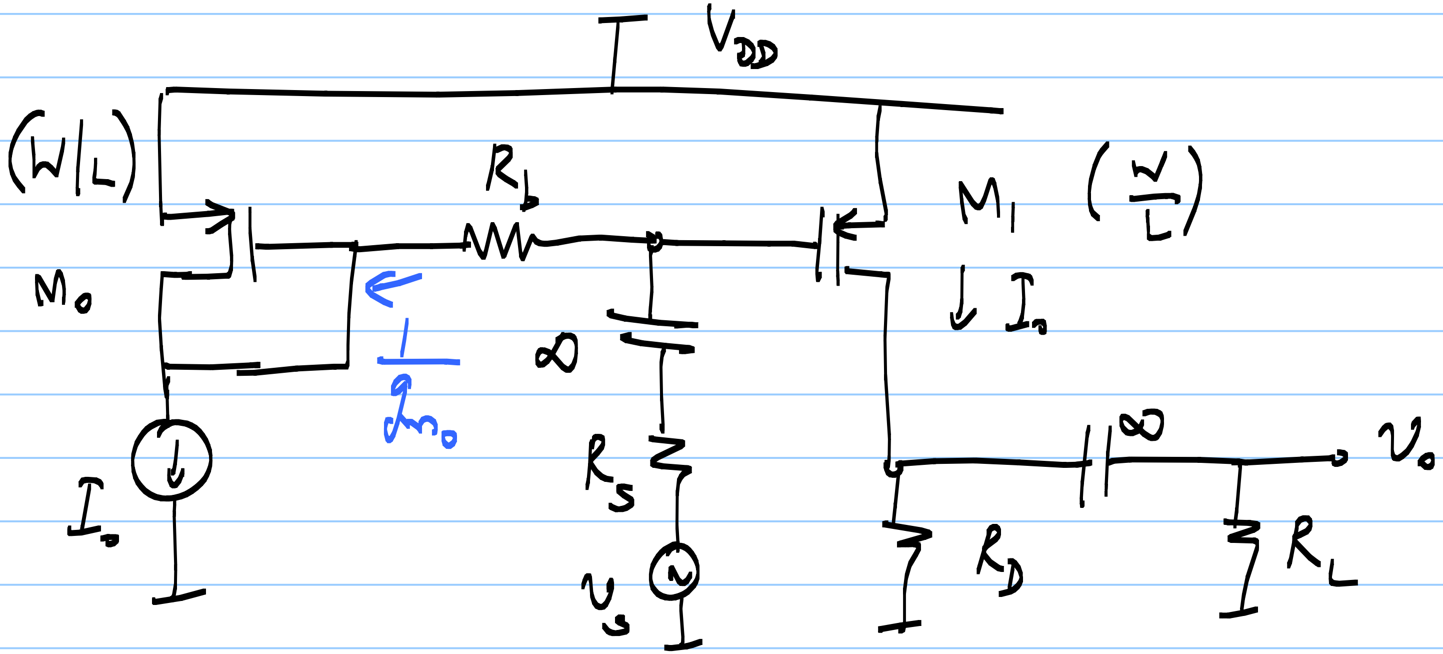


PMOS current mirror (pushes current into N)

NMOS CM → pulls current from N

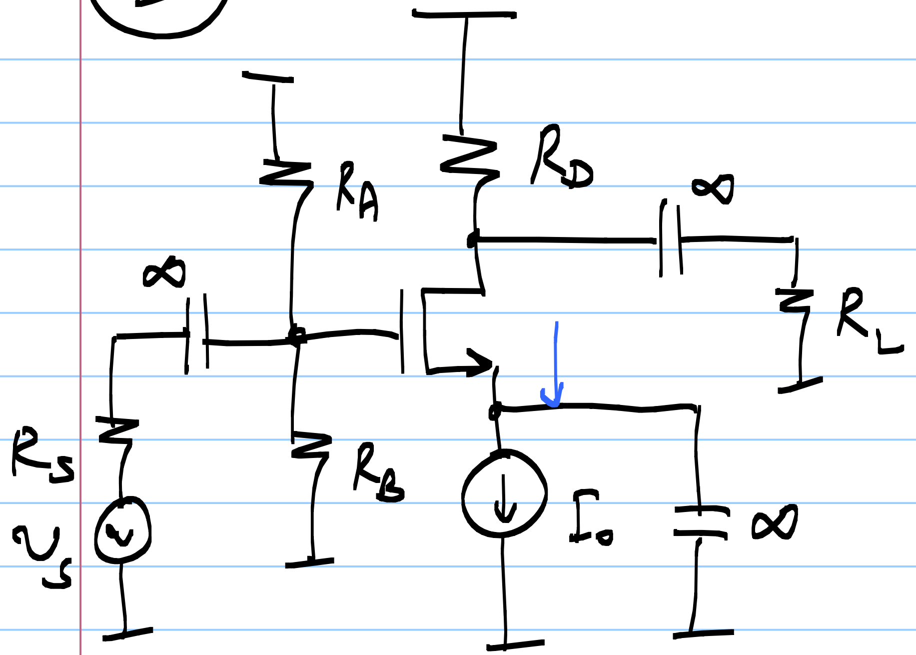


pulls current I_0 from N.

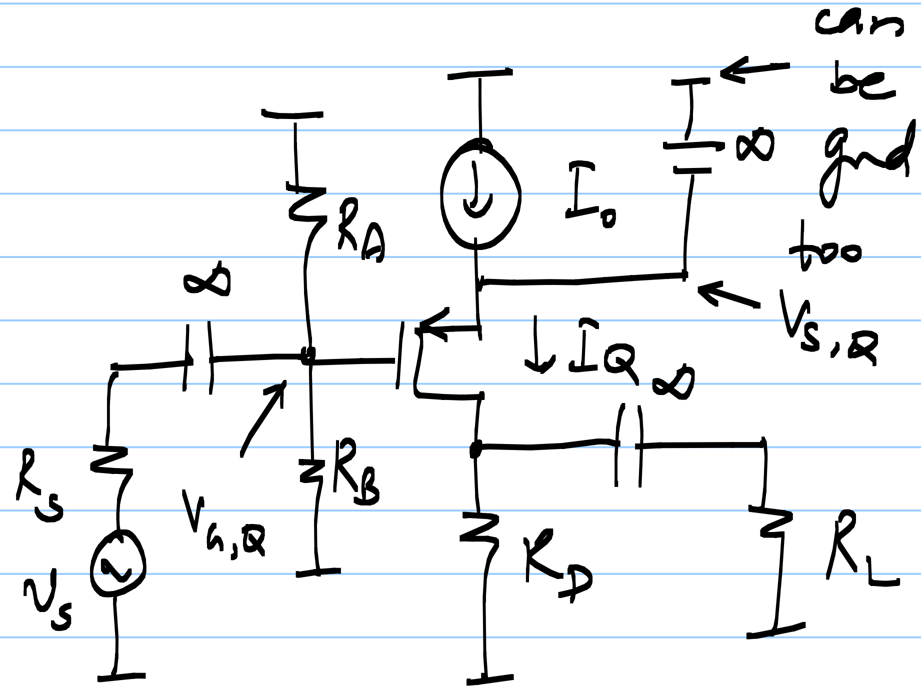


II

NMOS



PMOS



$$I_Q = I_o$$

$$V_{G,Q} = \frac{R_B}{R_A + R_B} \cdot V_{DD}$$

$$V_{D,Q} = I_o \cdot R_D$$

$$V_{S,Q} = V_{G,Q} + V_{SG,Q}$$

$$V_{SG,Q} = V_{T_p} + \sqrt{\frac{2 I_o}{\mu_p C_{ox} \left(\frac{W}{L}\right)}}$$

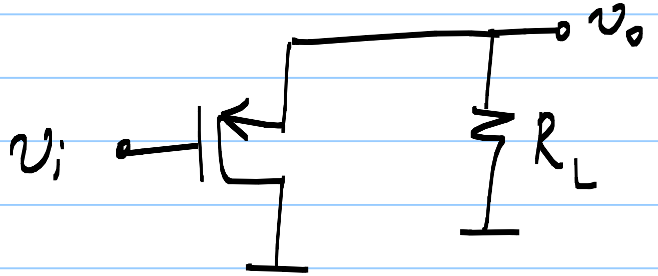
HW12 : SS analysis, swing limits

Case III & IV → HW13

SS Controlled Sources (SS eq. circuits are identical)

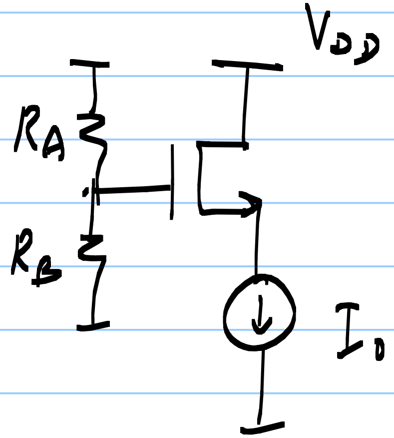
1) VCVS gain = 1 i.e. $v_o = v_i$

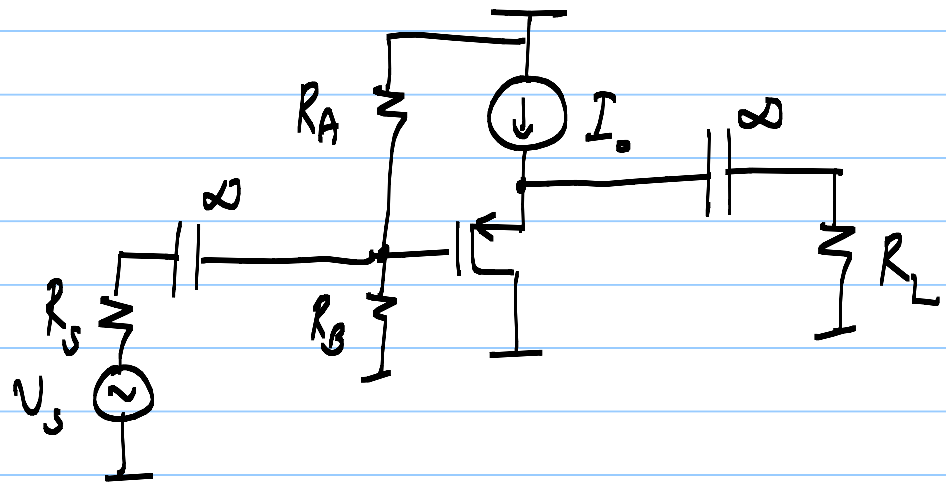
PMOS Common Drain Amplifier or PMOS source follower



$v_o = v_i$ as $g_m \to \infty$

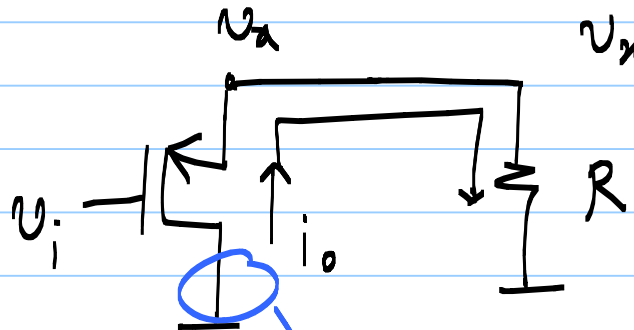
$$\frac{v_o}{v_i} = \frac{g_m}{g_m + g_{ds} + G_L}$$





PMOS CDA

2) <u>VCCS</u> – TCA (PMOS) $\quad i_o = \dfrac{v_i}{R}$



$v_x \approx v_i$

as $g_m \to \infty$, $v_x \approx v_i$

$$i_o = \frac{v_i}{R}$$

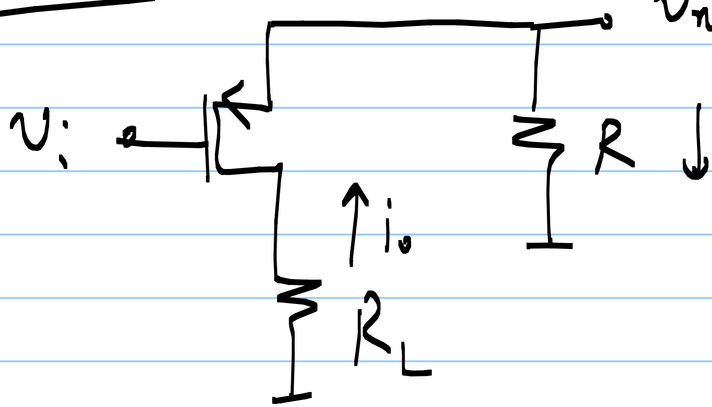
R_L can be placed here

17/9/2020

Lecture 26

PMOS based controlled sources

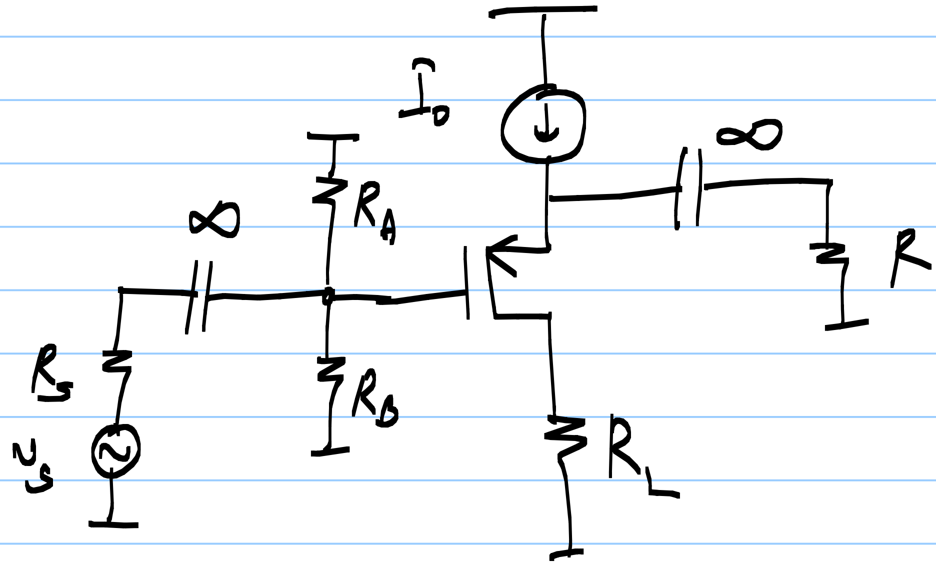
2) VCCS



$v_n \approx v_i$ if $g_m \to \infty$

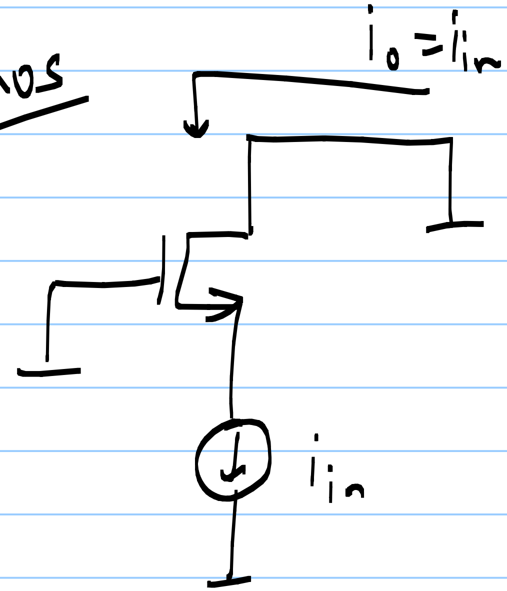
$i_o = \dfrac{v_i}{R}$

PMOS Transadmittance amplifier

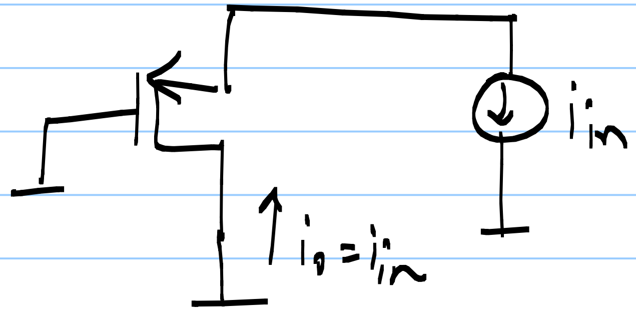


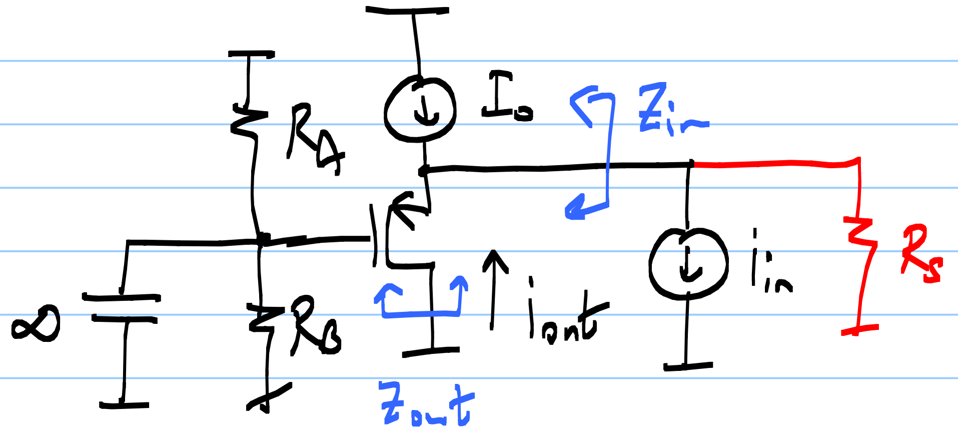
3) CCCS – Common Gate Amplifier

NMOS



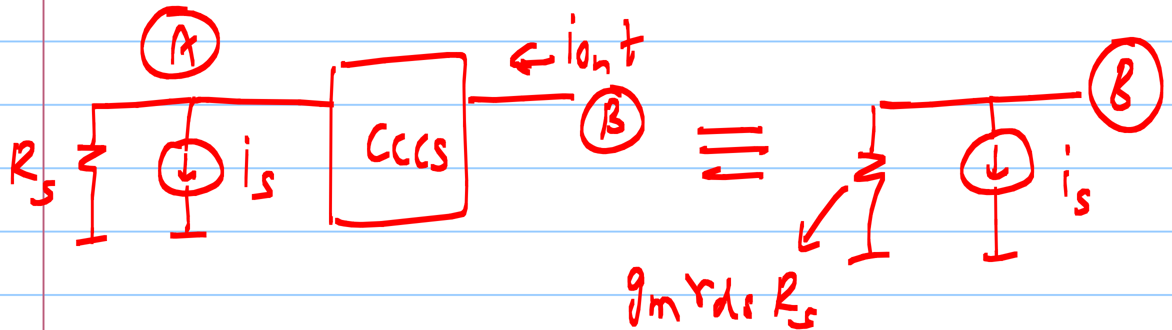
PMOS





$$Z_{in} = \frac{1}{g_m + g_{ds}} \approx \frac{1}{g_m}$$

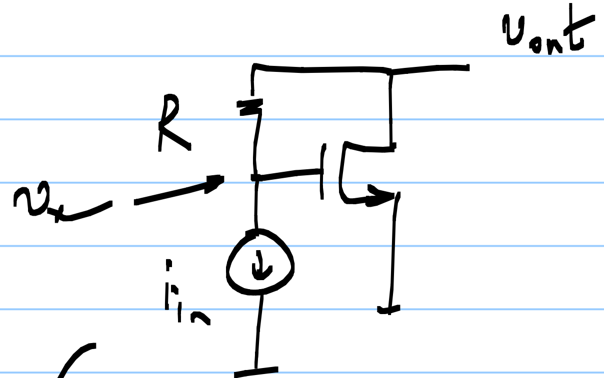
$$Z_{out} = \infty$$



$$Z_{out} = R_s + r_{ds} + g_m r_{ds} R_s$$

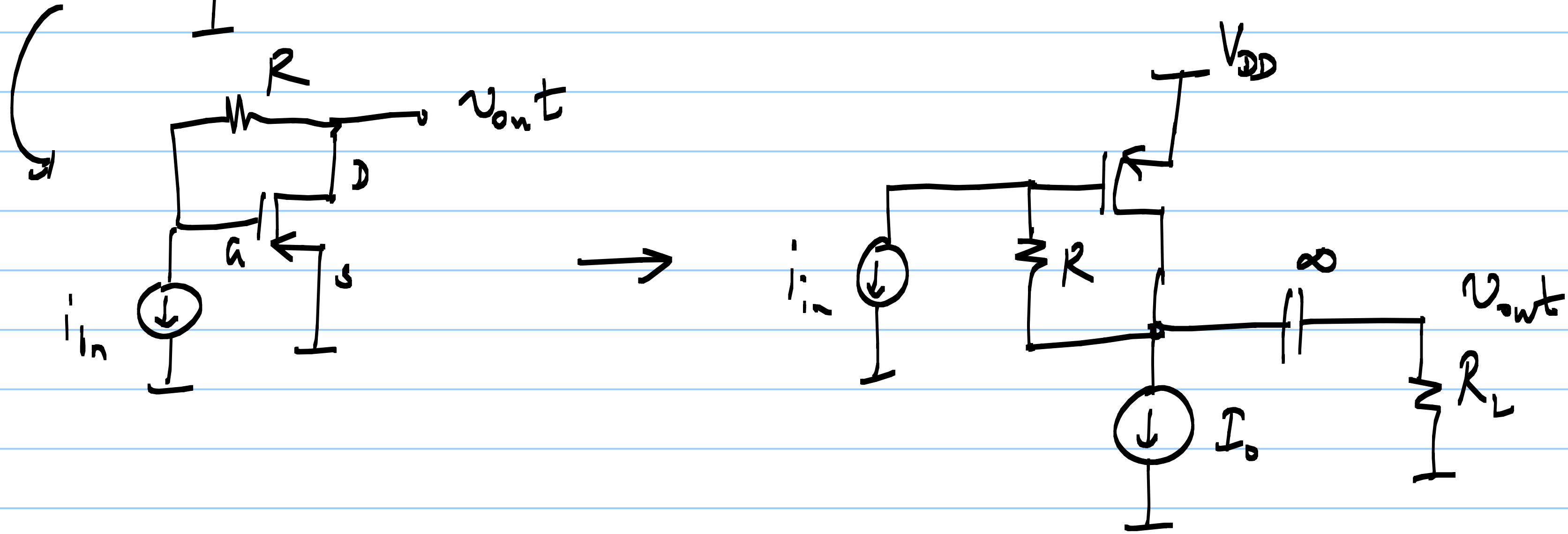
$$\approx g_m r_{ds} R_s$$

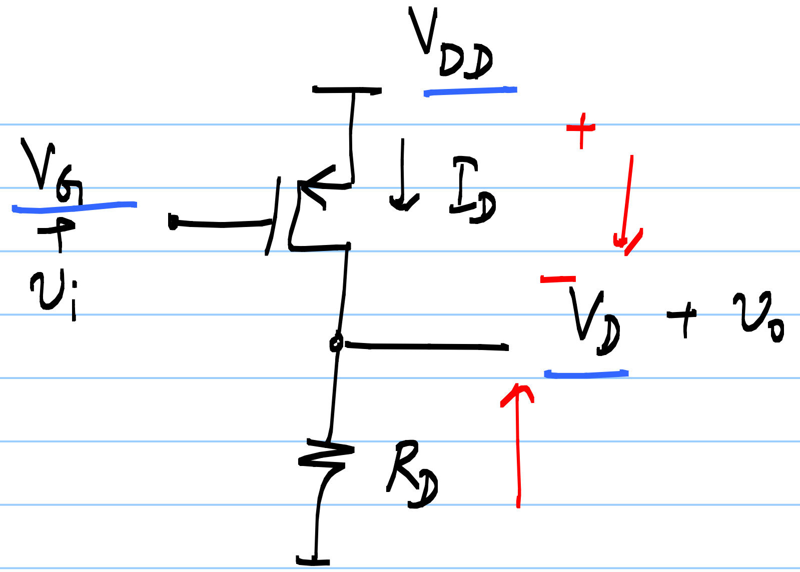
4) <u>CCVS</u> (Trans impedance amplifier) $v_{out} = R \cdot i_{in}$



$v_x = v_{out} - i_{in} \cdot R$

as $g_m \to \infty$, $v_x \to 0$

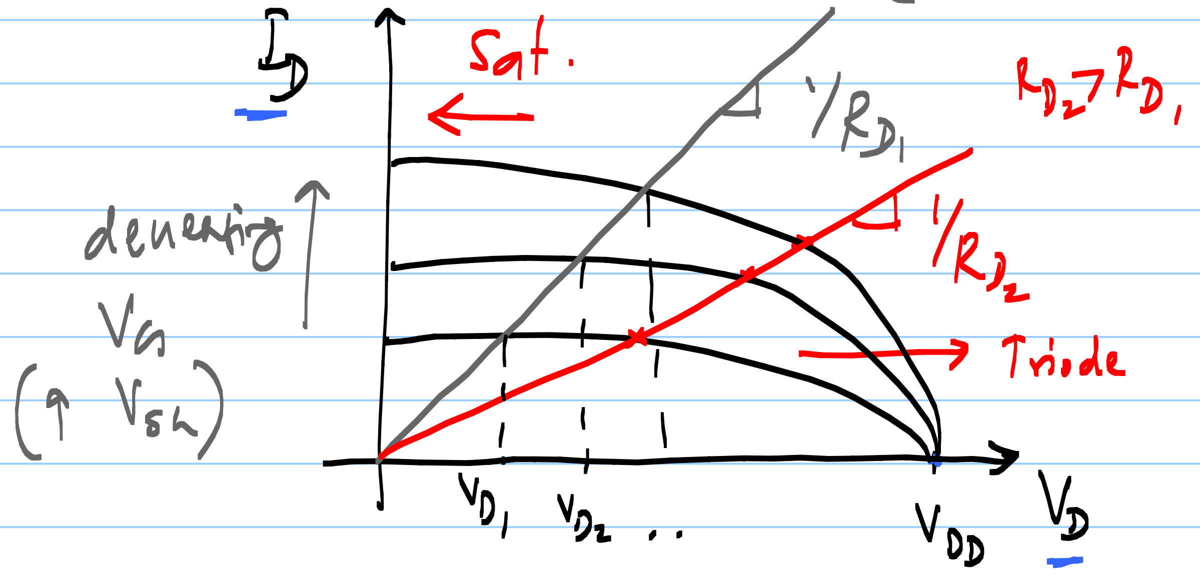


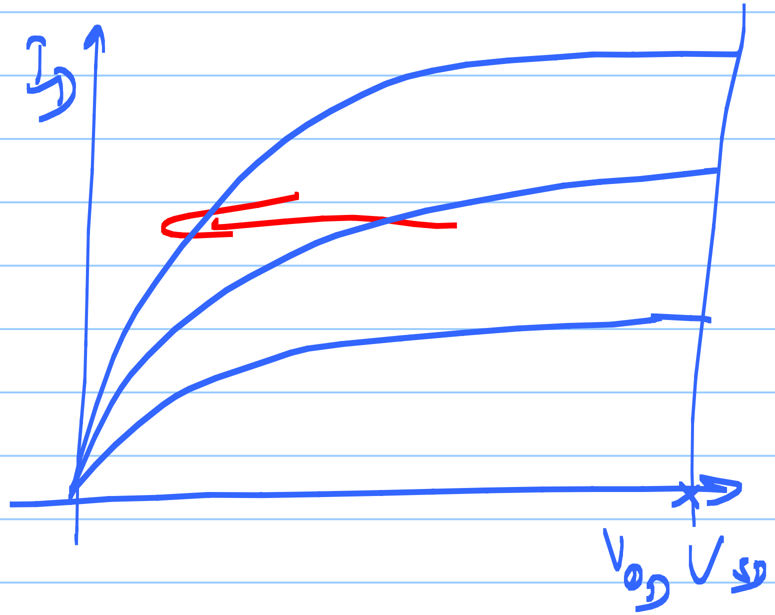


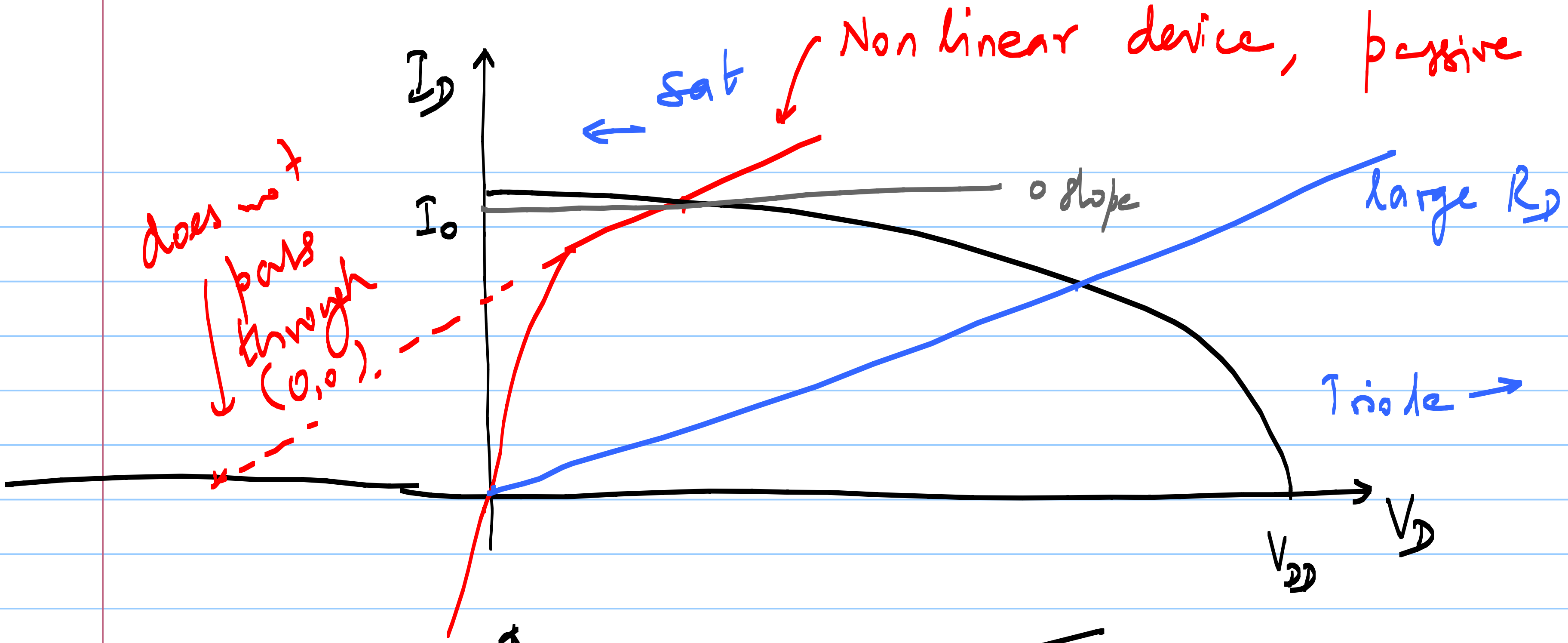
$$\frac{v_o}{v_i} = -g_m \cancel{R_D}$$

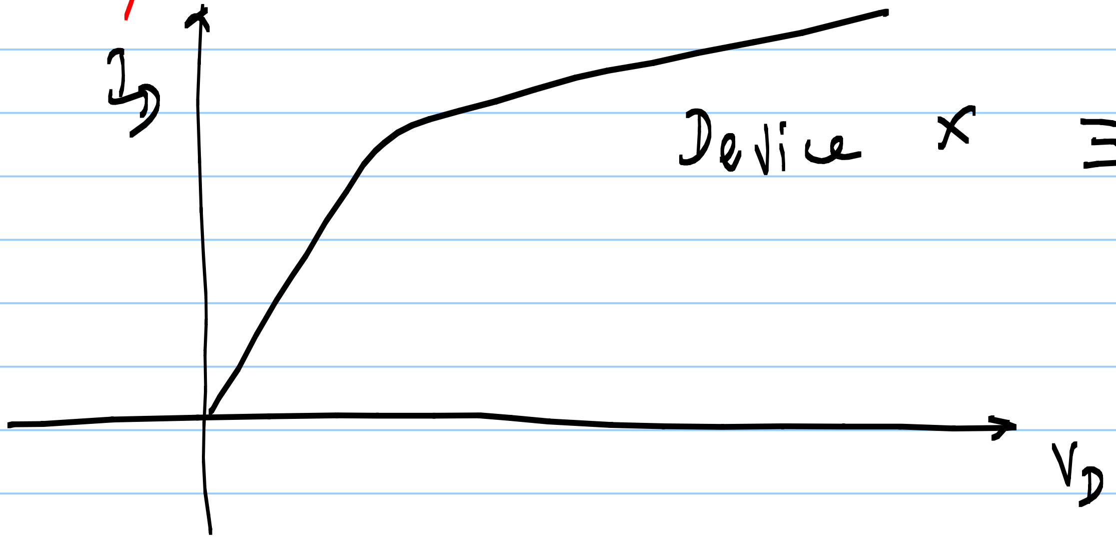
$$V_D = I_D R_D$$

$$V_{SD} = V_{DD} - V_D$$

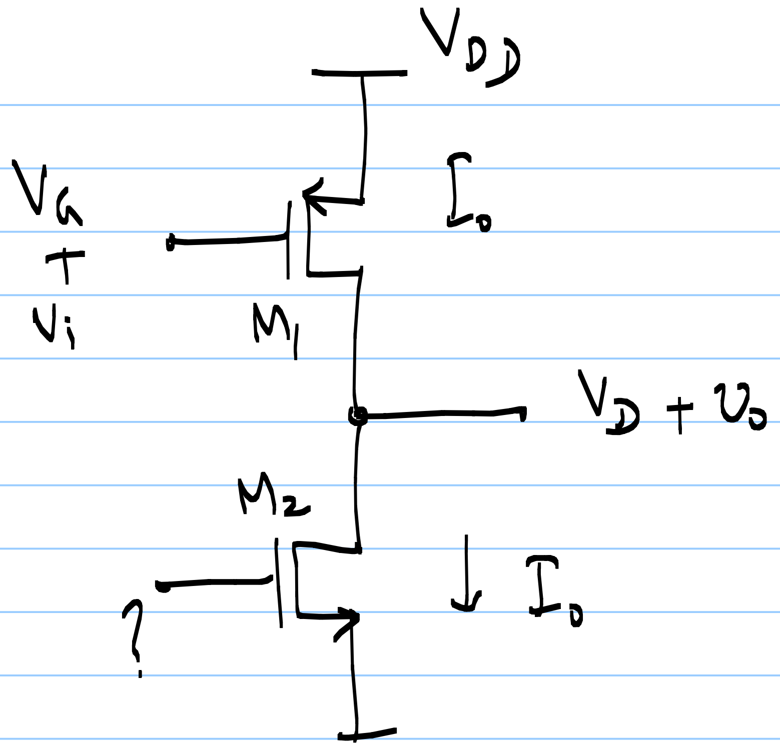




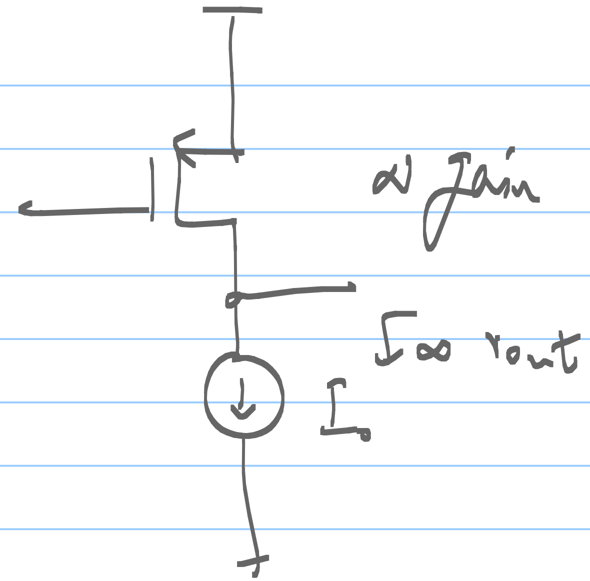




Device $x \equiv$ NMOS

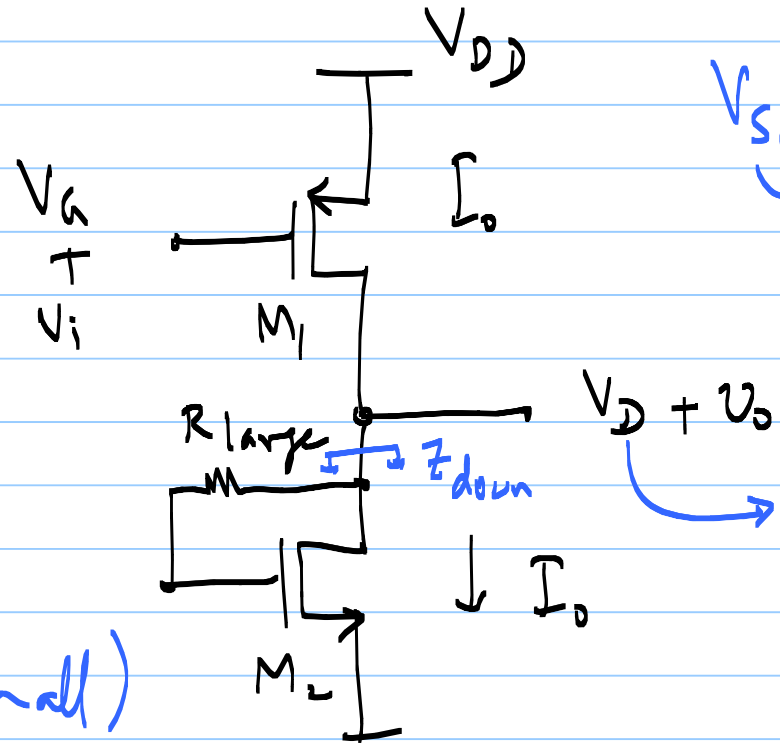


≡



$$\underbrace{V_{SG_2} = V_{DD} - V_{G_2}}_{\hookrightarrow \text{ decides } I_o}$$

I



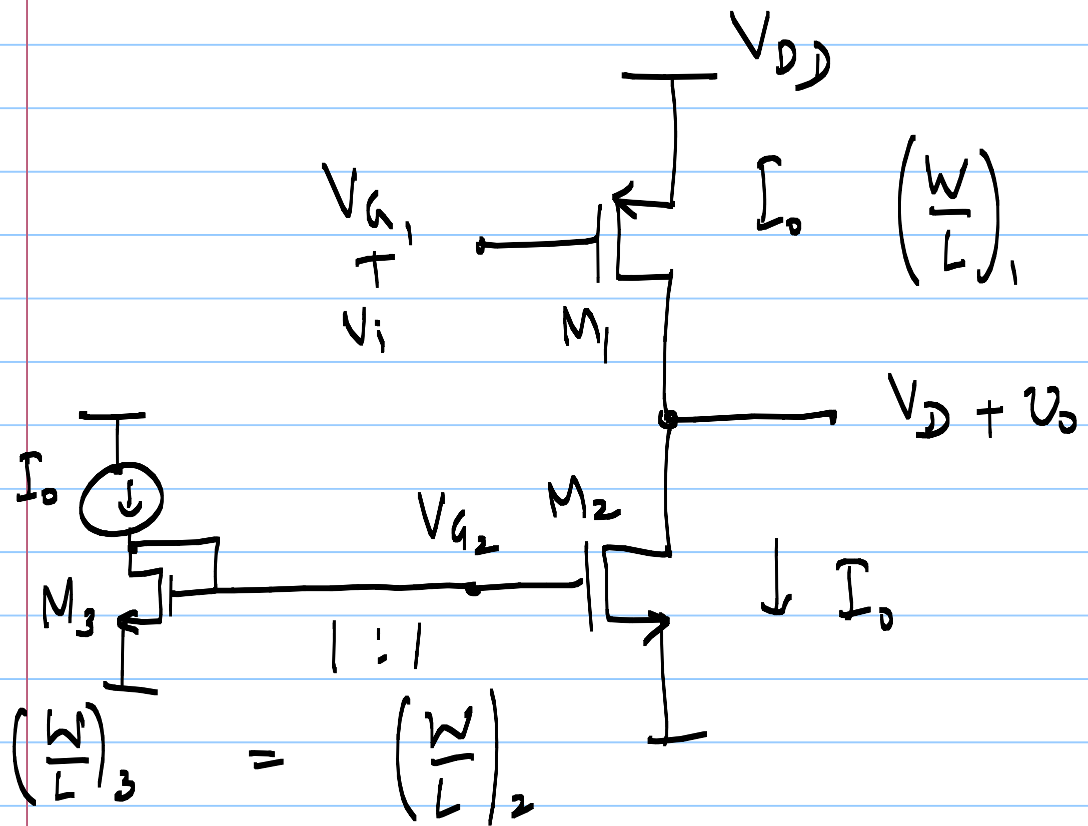
$$V_D = V_{G_2}\Big|_{I_o}$$

$$Z_{down} = \frac{1}{g_{m_2}}$$

$$\text{gain} = -\frac{g_{m_1}}{g_{m_2}} \quad (\text{small})$$

18|9|2020

Lecture 27

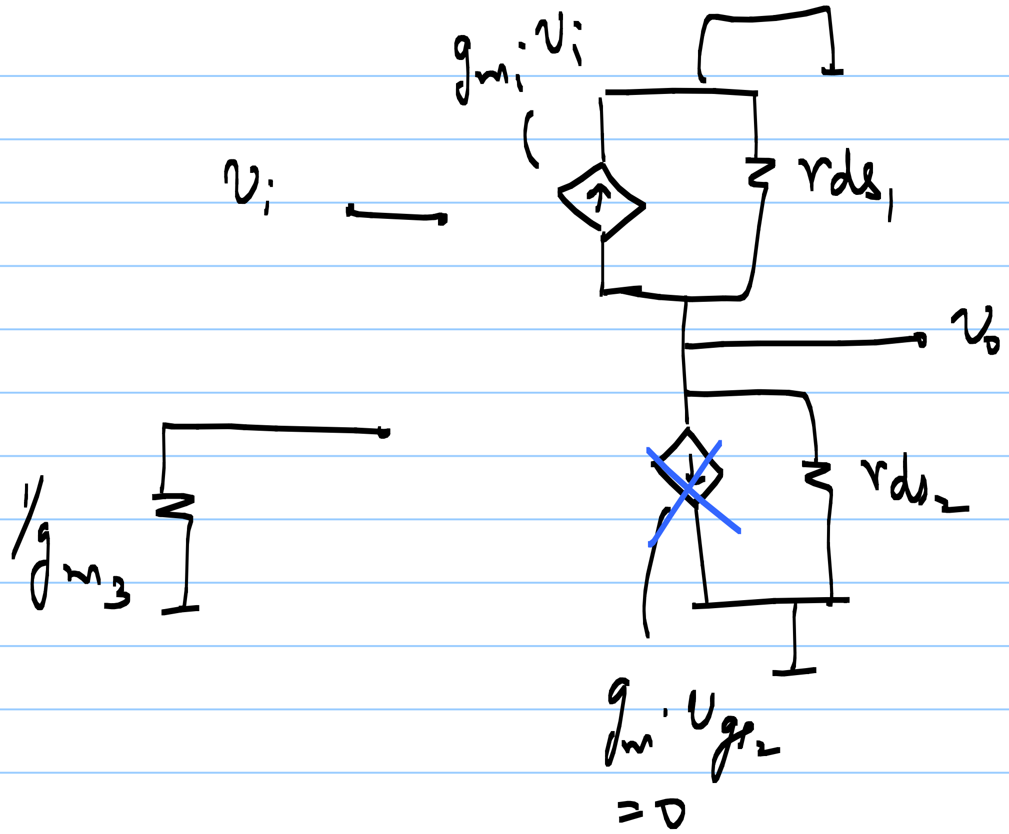


$I_0 \quad \left(\frac{W}{L}\right)_1$ – decided by gain, swing limit etc.

ideal M_1, M_2 & M_3 :

* all $I_D = I_0$
* ideal gain $= \infty$
* practical gain

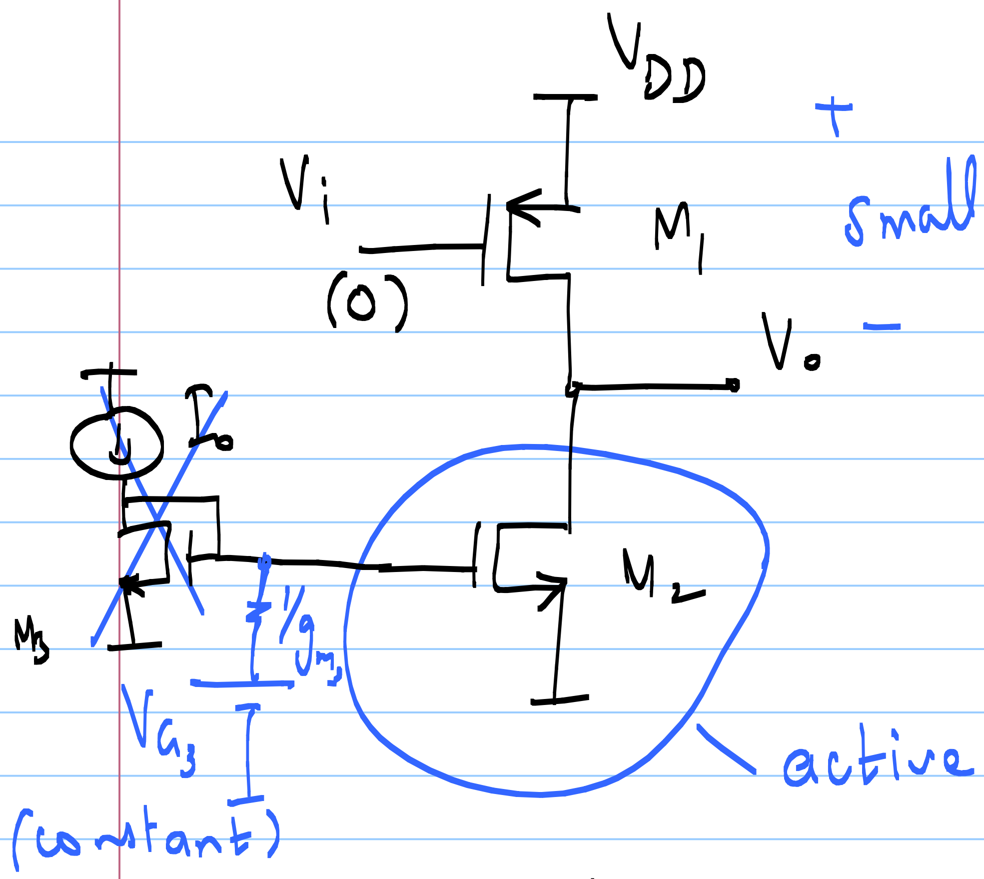
$$= -g_{m_1}\left(r_{ds_1} \| r_{ds_2}\right)$$



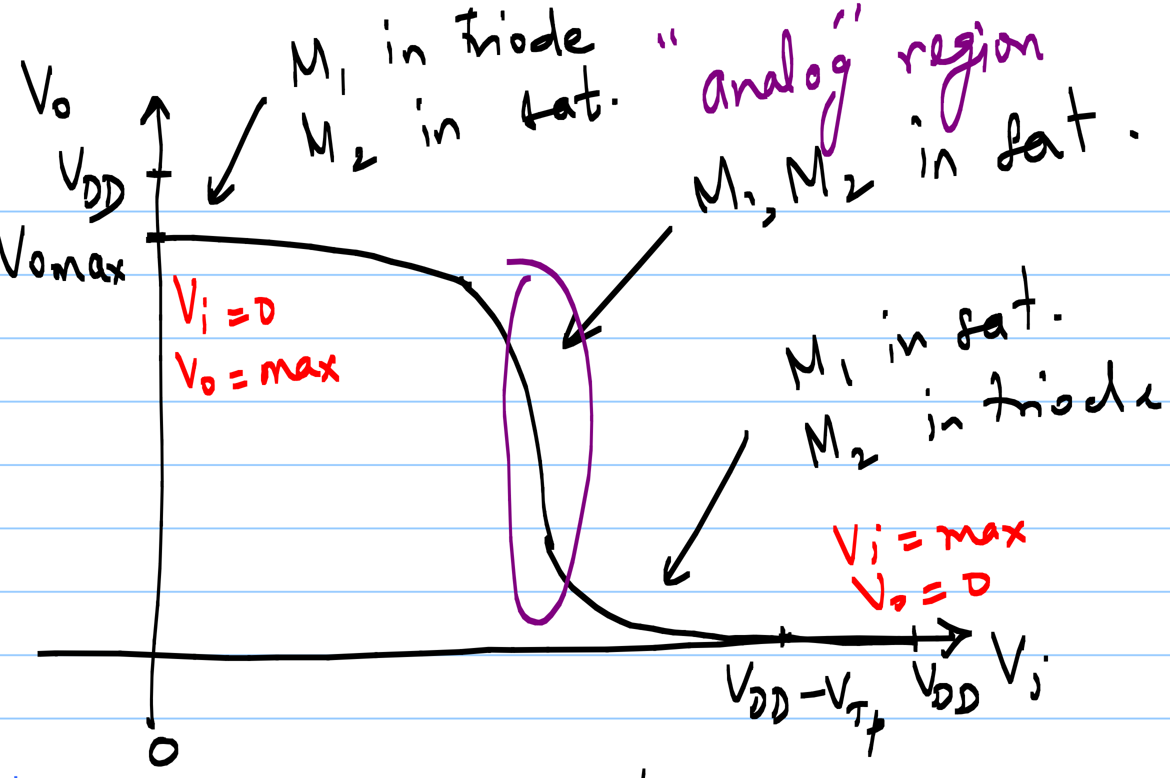
$$\frac{v_o}{v_i} = -g_{m_1}\left(r_{ds_1} \,\|\, r_{ds_2}\right)$$

V_{T_n} = threshold voltage of NMOS

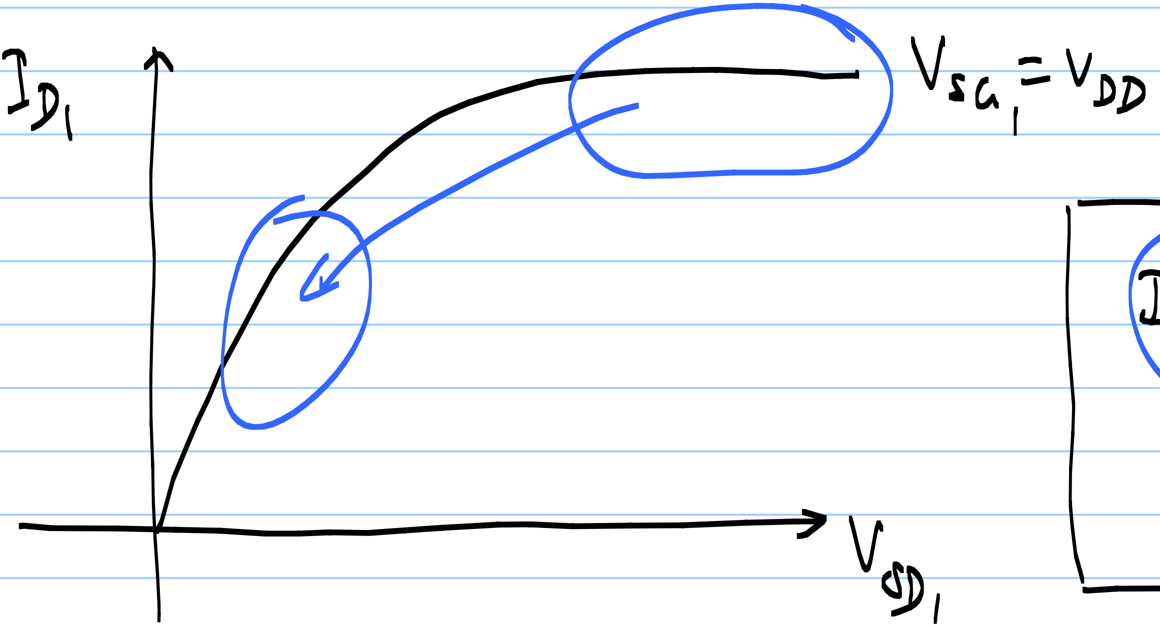
V_{T_p} = threshold voltage of PMOS



active load

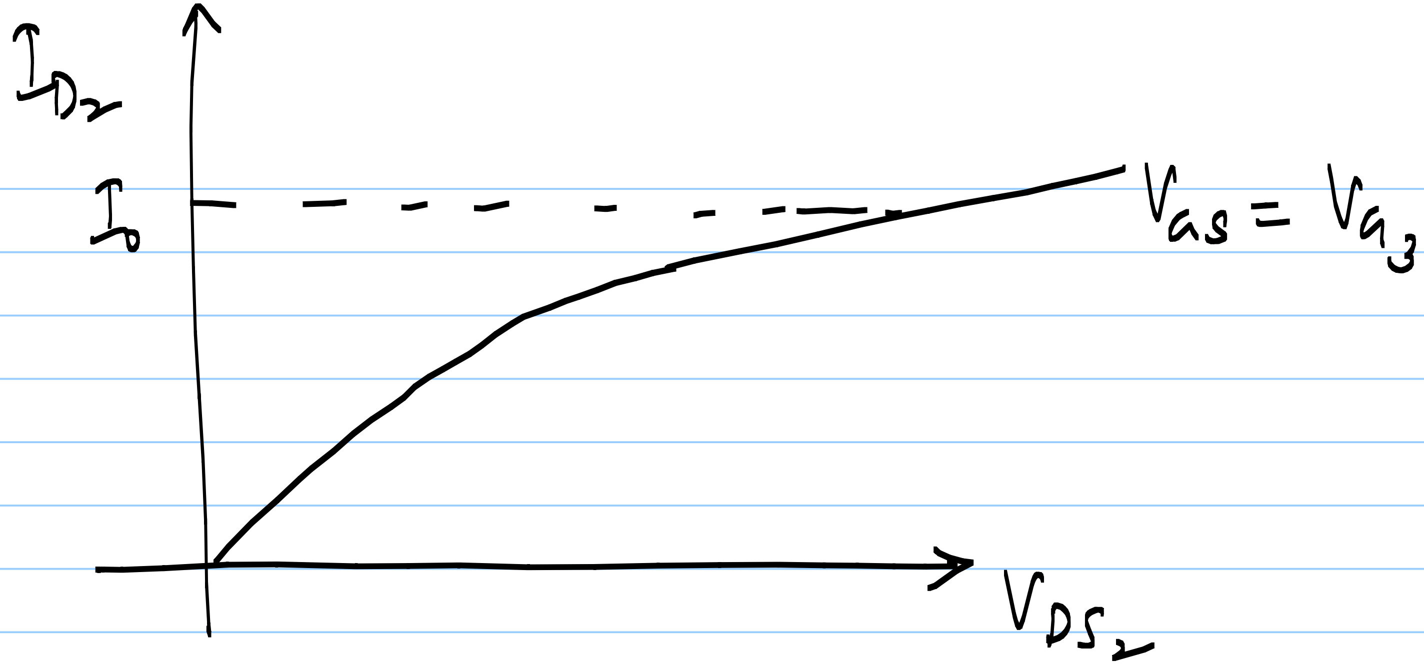


One type of digital inverter

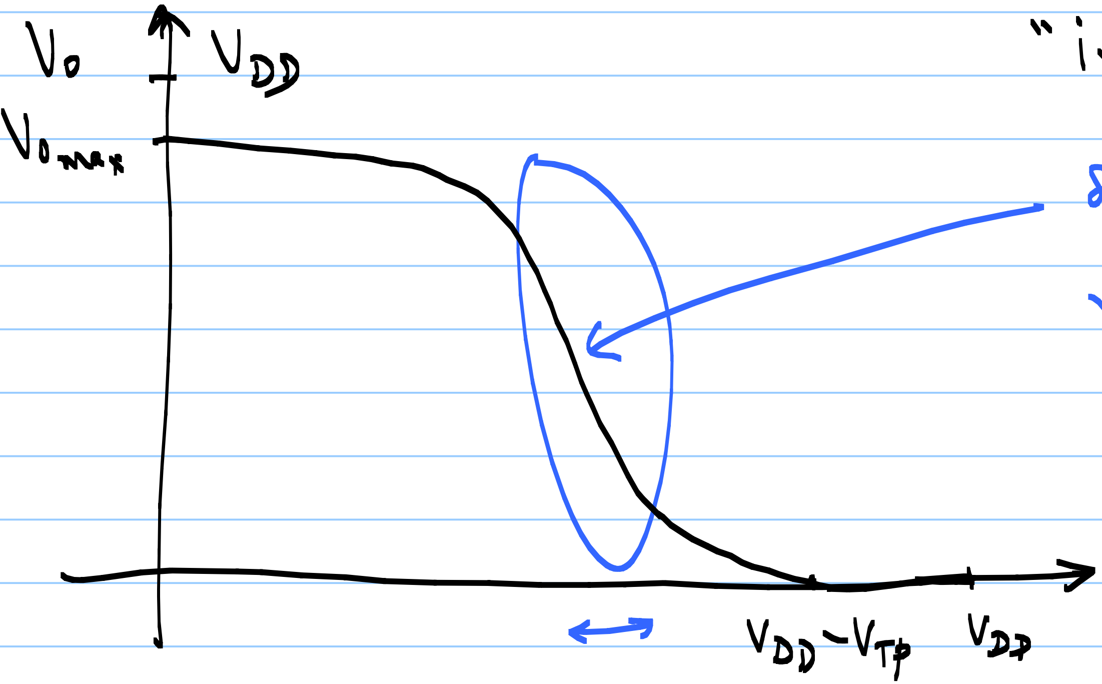


$I_{D_1} = I_{D_2}$

always!



@ $V_i = V_{DD} - V_{T_p}$: $I_{D_1} = 0$



"inverting amplifier"

slope in analog regim $= \dfrac{dV_o}{dV_i} = \dfrac{v_o}{v_i}$

$= -g_m \left(r_{ds_1} \| r_{ds_2} \right)$

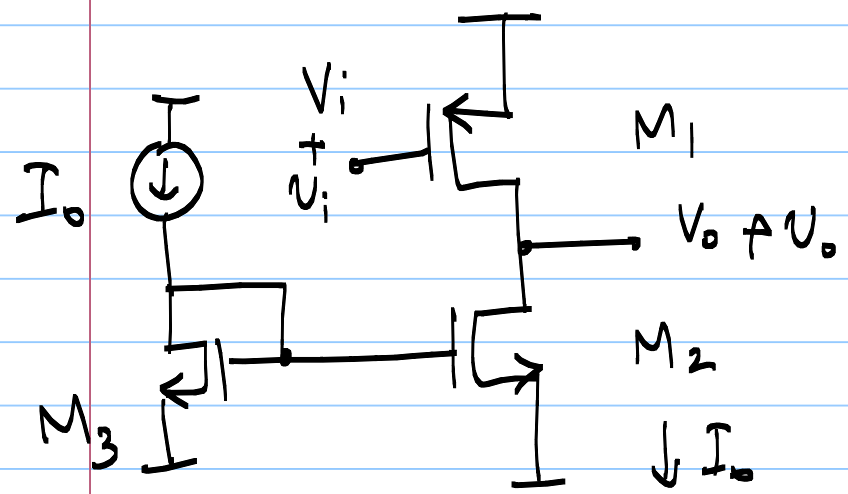
$V_{o_{max}}$

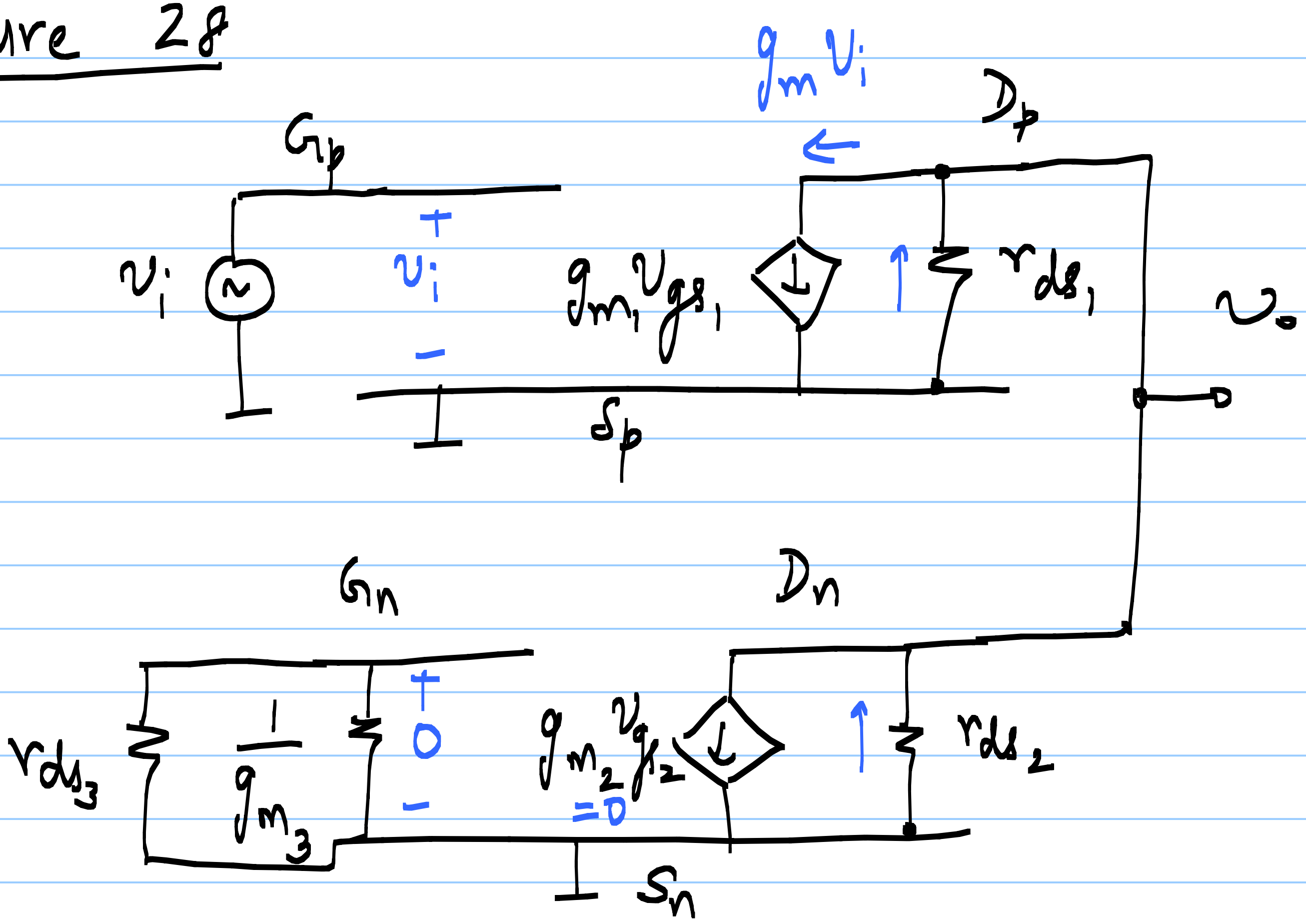
Triode I_{D_1} = Sat. I_{D_2}

In this course : normally for bias point calculations – ignore effect of λ

22|9|2020

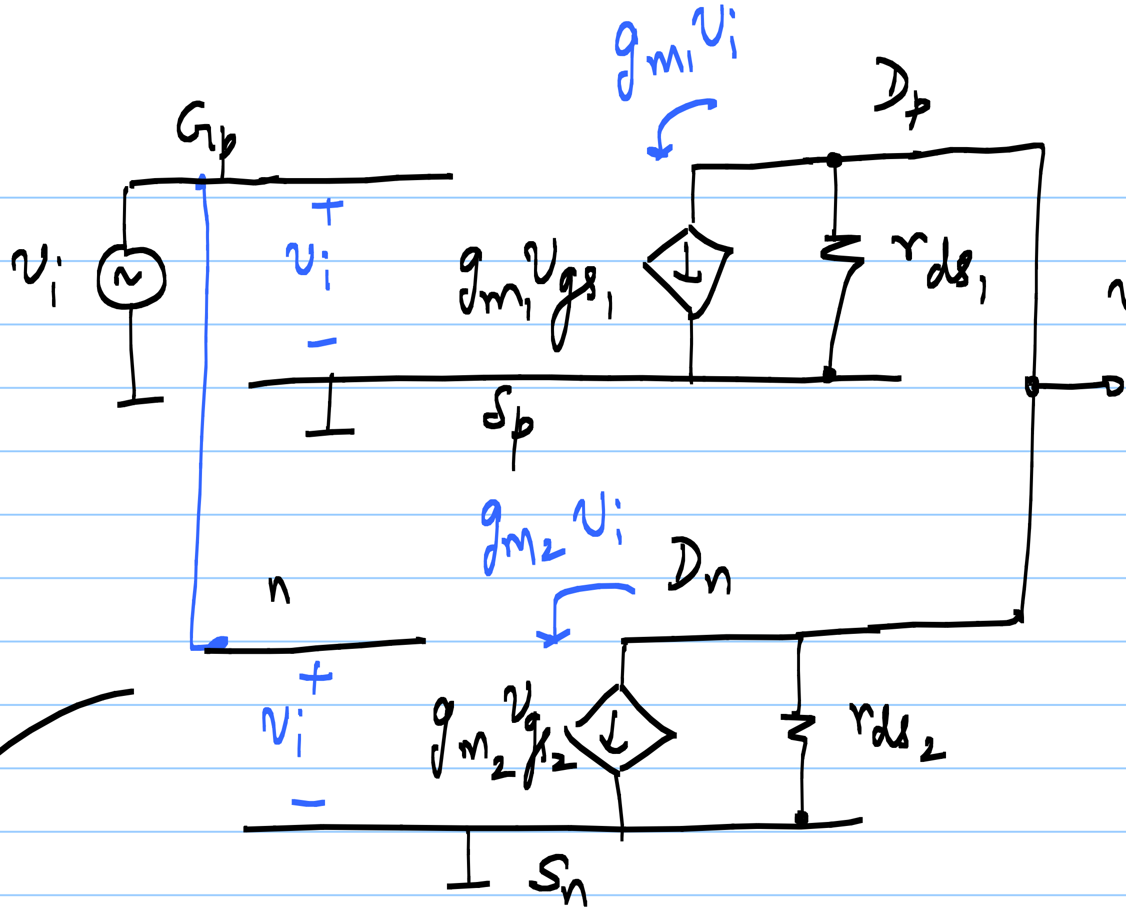
Lecture 28





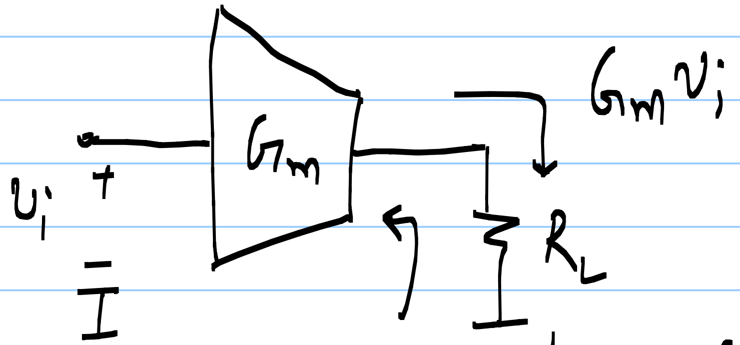
$$\frac{v_o}{v_i} = -g_{m_1}\left(r_{ds_1} \| r_{ds_2}\right)$$

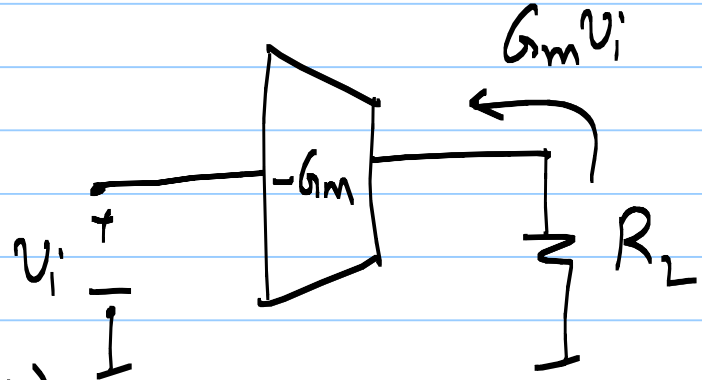
$$= \frac{-g_{m_1}}{g_{ds_1} + g_{ds_2}}$$



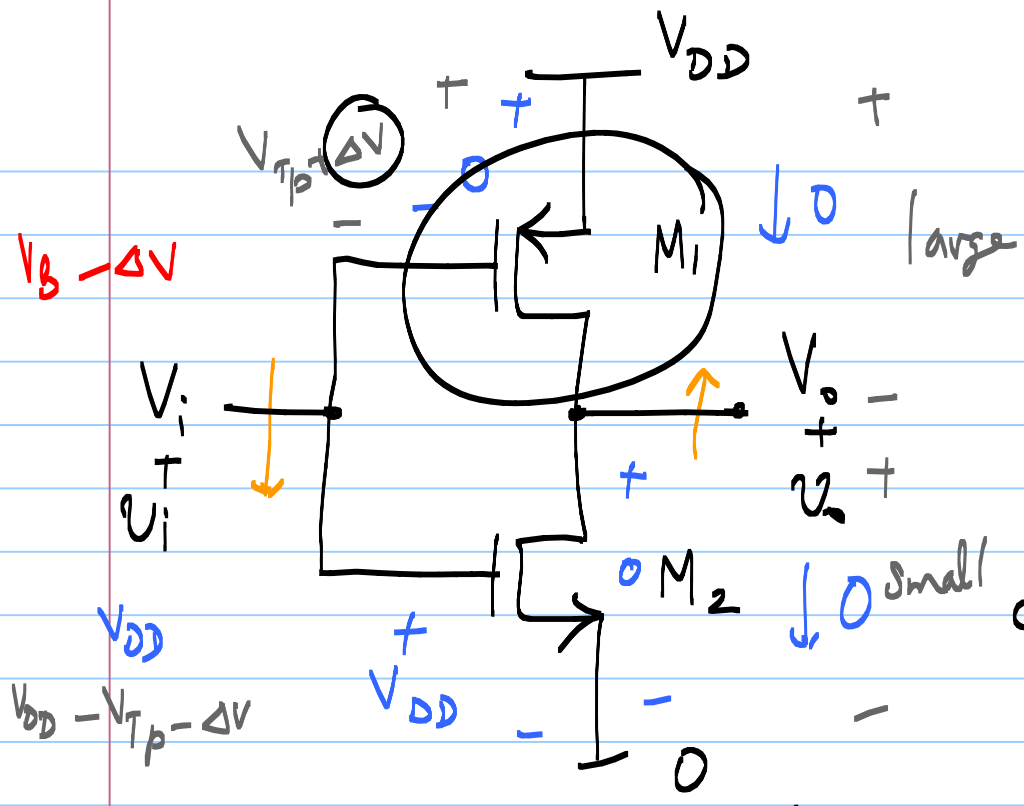
$$v_o = \frac{-(g_{m1} + g_{m2})}{g_{ds1} + g_{ds2}} v_i$$

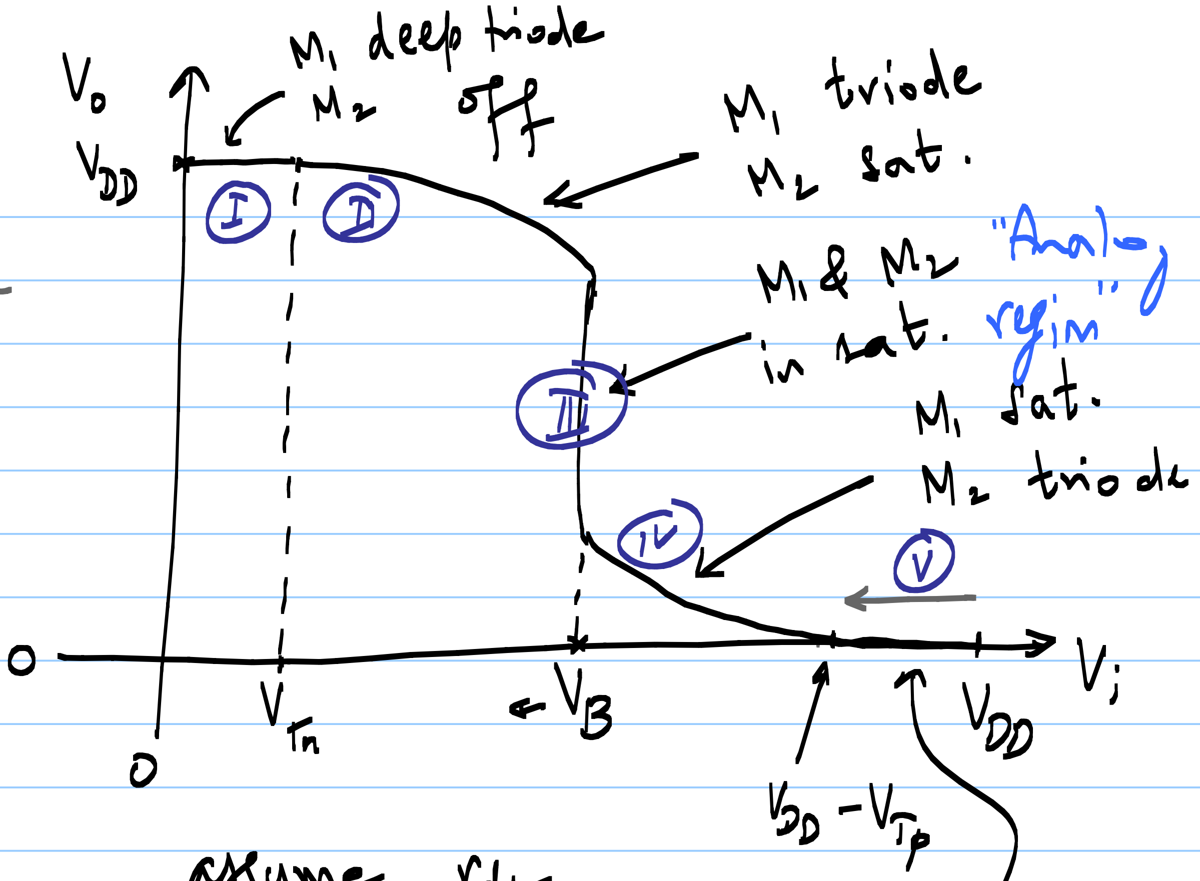
If r_{ds1} & r_{ds2} were ∞, gain $= \infty$





r_{out} = large ($\approx\infty$)

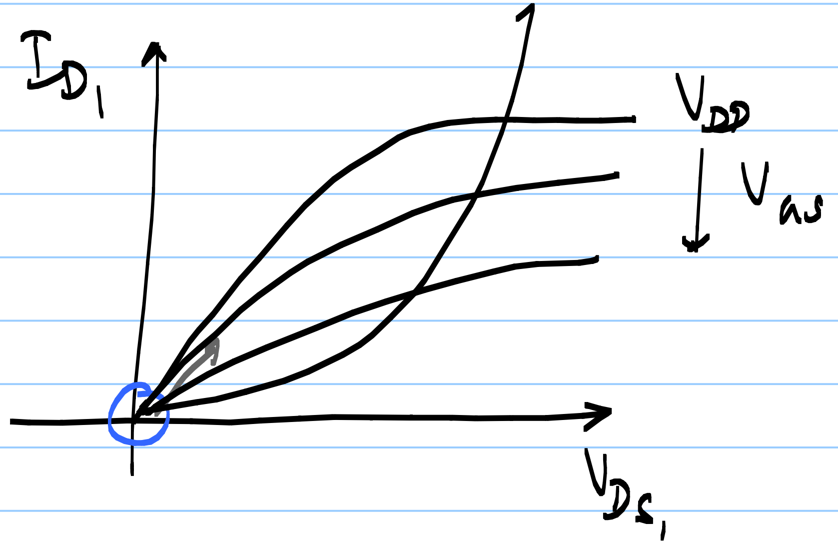




M1 off
M2 deep triode

assume $r_{ds} = \infty$

"CMOS Inverter"



$V_i = V_{DD} - V_{Tp} - \Delta V$: $I_{D_1}(sat.) = I_{D_2}(triode)$

Determine V_B :

$$I_{D_1}(sat) = I_{D_2}(sat.)$$

$$\frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_p\left[V_{DD} - V_B - V_{Tp}\right]^2 = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_n\left[V_B - V_{Tn}\right]^2$$

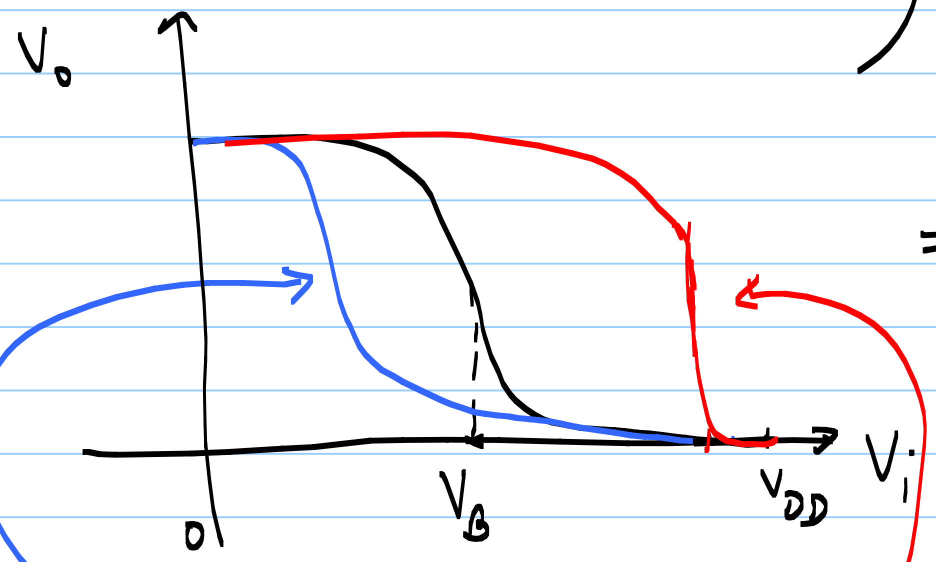
$$k_n = \mu_n\left(\frac{W}{L}\right)_n \quad ; \quad k_p = \mu_p\left(\frac{W}{L}\right)_p$$

$$V_B - V_{Tn} = \sqrt{\frac{k_p}{k_n}} \cdot \left(V_{DD} - V_B - V_{Tp}\right)$$

$$V_B\left[1 + \sqrt{\frac{k_p}{k_n}}\right] = V_{Tn} + \left(V_{DD} - V_{Tp}\right)\left[\sqrt{\frac{k_p}{k_n}}\right]$$

$$V_B = \frac{V_{T_n} + (V_{DD} - V_{T_p})\left[\sqrt{\frac{k_p}{k_n}}\right]}{1 + \sqrt{\frac{k_p}{k_n}}}$$

* $\mu_n \approx 3\,\mu_p$



1) $V_{T_n} = V_{T_p} = V_T$

We want $V_B = V_{DD}/2$

$\Rightarrow k_p = k_n \Rightarrow \left(\frac{W}{L}\right)_p = 3\left(\frac{W}{L}\right)_n$

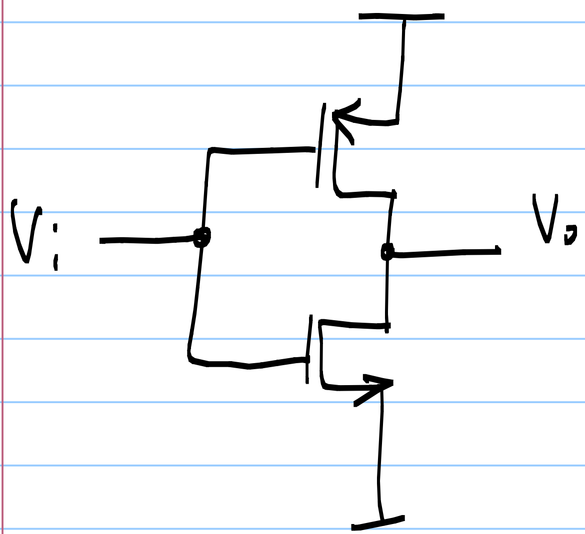
$$\left(\frac{W}{L}\right)_p = \left(\frac{\mu_n}{\mu_p}\right)\left(\frac{W}{L}\right)_n$$

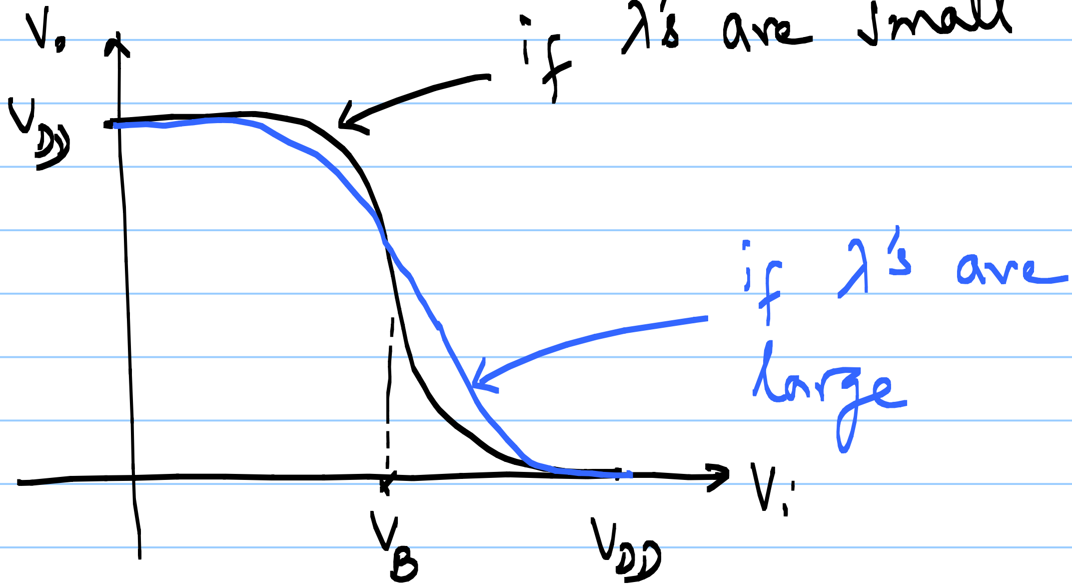
2) If $\left(\frac{W}{L}\right)_n \gg \left(\frac{W}{L}\right)_p$ $\quad V_B \approx V_{T_n}$

3) If $\left(\frac{W}{L}\right)_p \gg \left(\frac{W}{L}\right)_n$ $\quad V_B \approx V_{DD} - V_{T_p}$

23/9/2020

Lecture 29

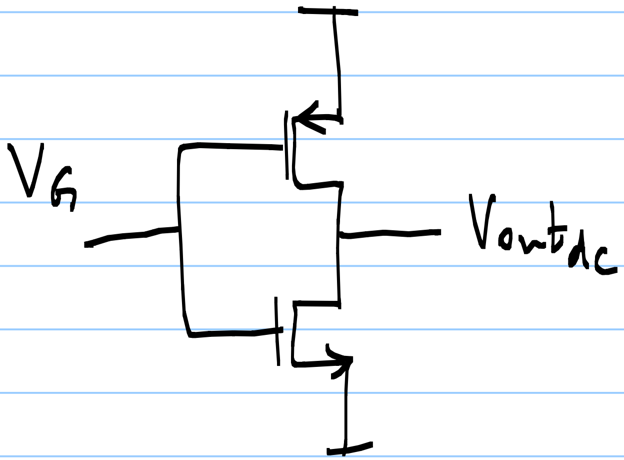


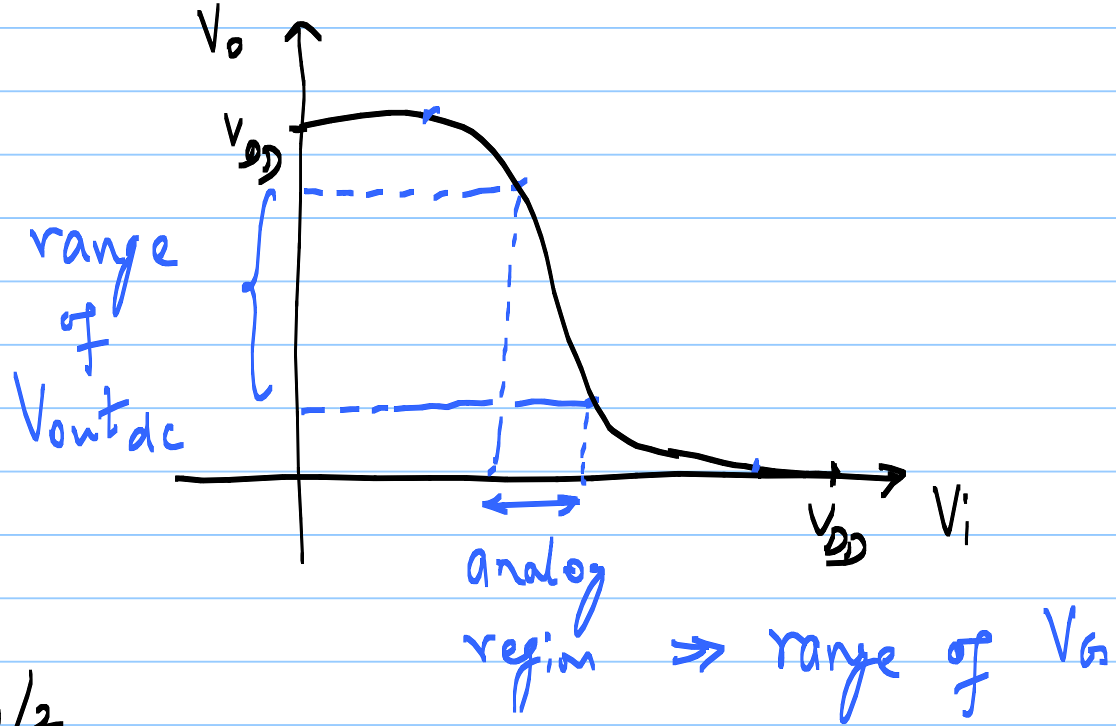


Small signal gain $\frac{v_o}{v_i} = \frac{-(g_{m_1} + g_{m_2})}{g_{ds_1} + g_{ds_2}}$

$$= -(g_{m_1} + g_{m_2})(r_{ds_1} \| r_{ds_2})$$

Biasing of CMOS inv.





* Assume $V_G \sim V_{DD}/2$

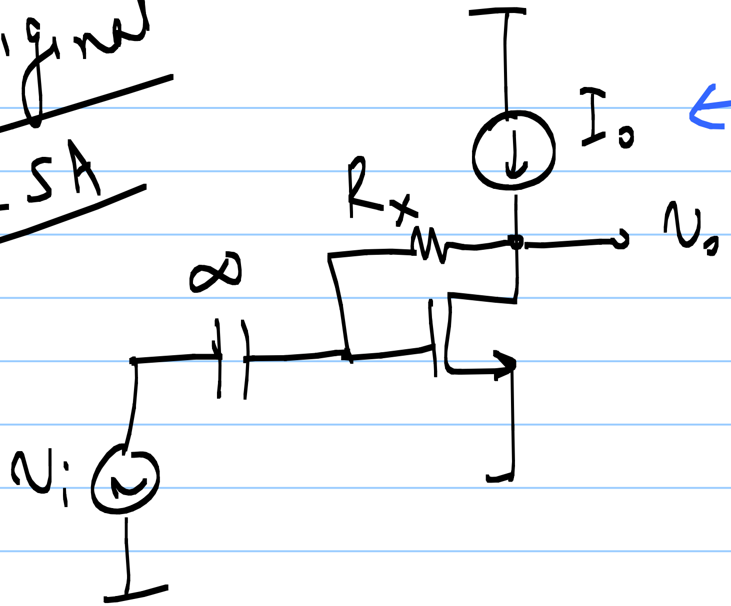
i.e. PMOS ~ NMOS except for $\mu_p \sim 3\mu_n$

$$\left(\frac{W}{L}\right)_p \sim 3\left(\frac{W}{L}\right)_n$$

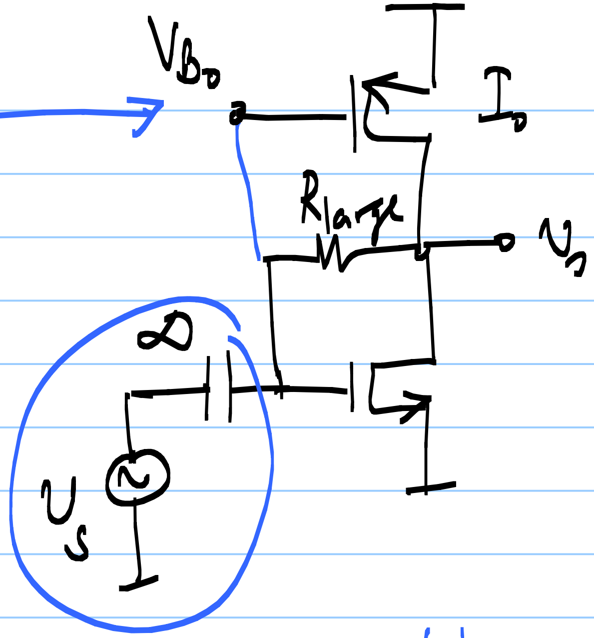
* Negative f.b. biasing

→ Drain to Gate f.b. using a current source & resistor

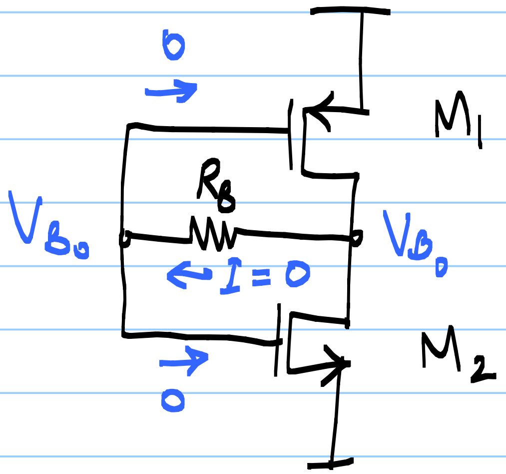
Original CSA

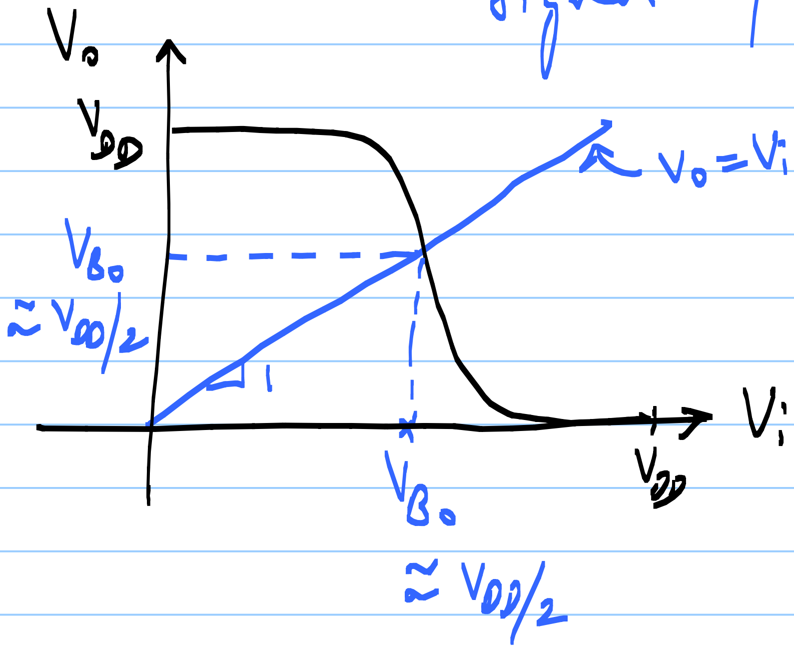


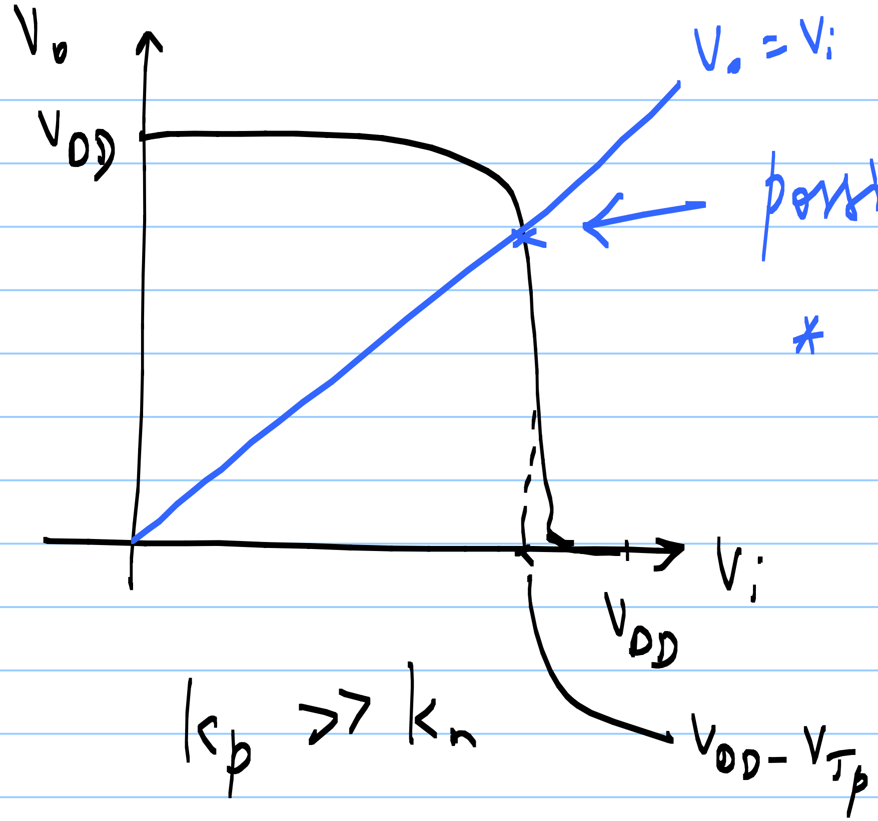
get gain from this device



signal portion



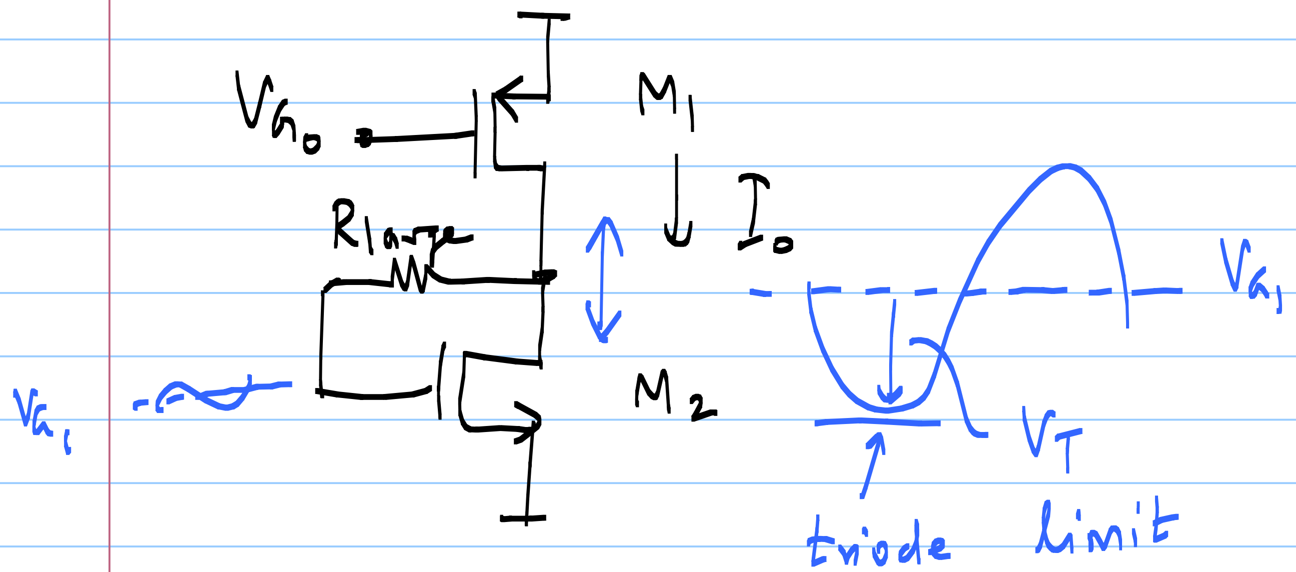




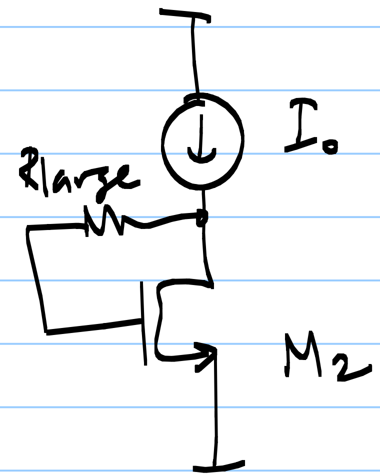
$k_p \gg k_n$

possible to end up close to region D

* any small voltage swing will push PMOS into triode



$\equiv$



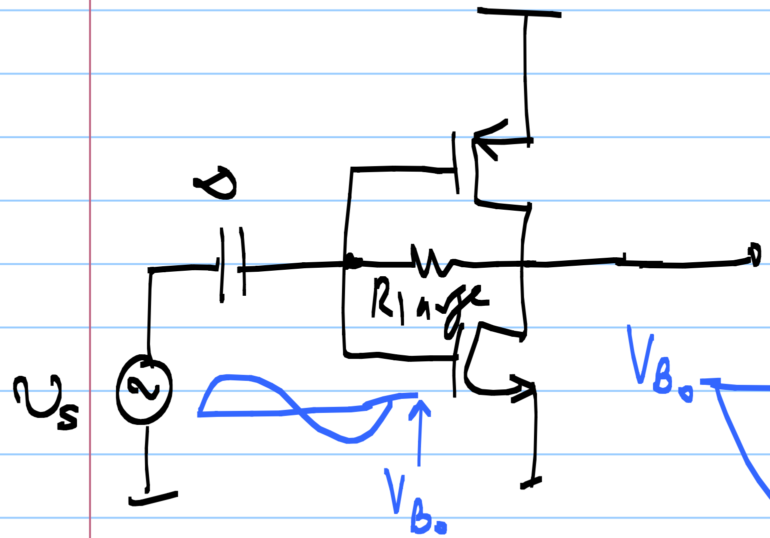
HW

swing limit calc.

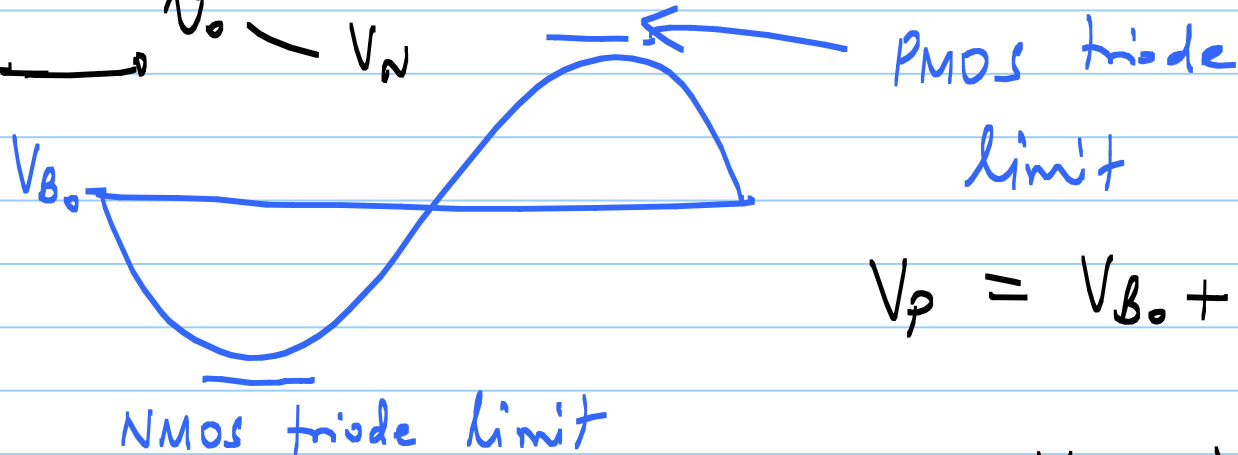
Swing limits of CMOS Inv.

PMOS triode limit :

$$V_{Dp} = V_{Gp} + V_{Tp}$$



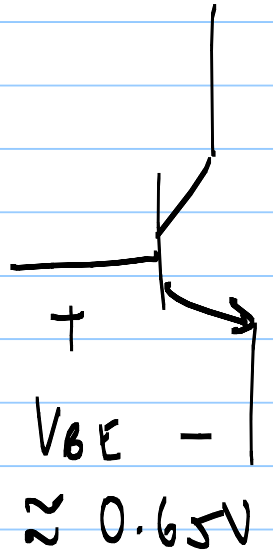
$$v_o \begin{cases} V_p \\ V_N \end{cases}$$



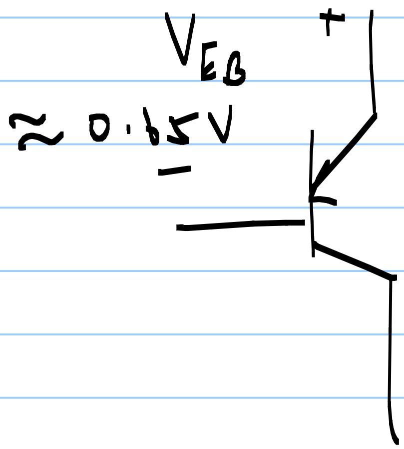
$$V_p = V_{B_0} + V_{Tp}$$

$$V_{DN} = V_{GN} - V_{TN} \Rightarrow V_N = V_{B_0} - V_{TN}$$

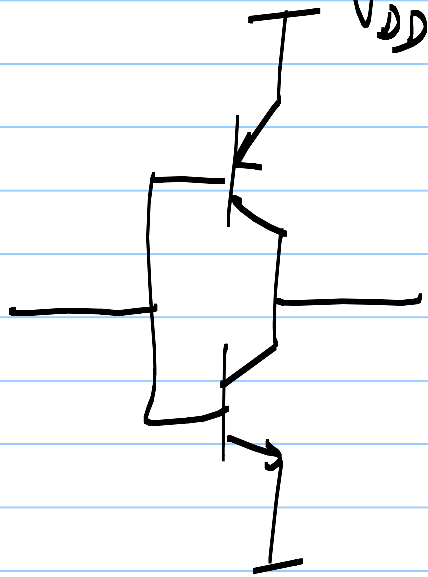
HW – accurate swing limit calculation for inv.



npn BJT



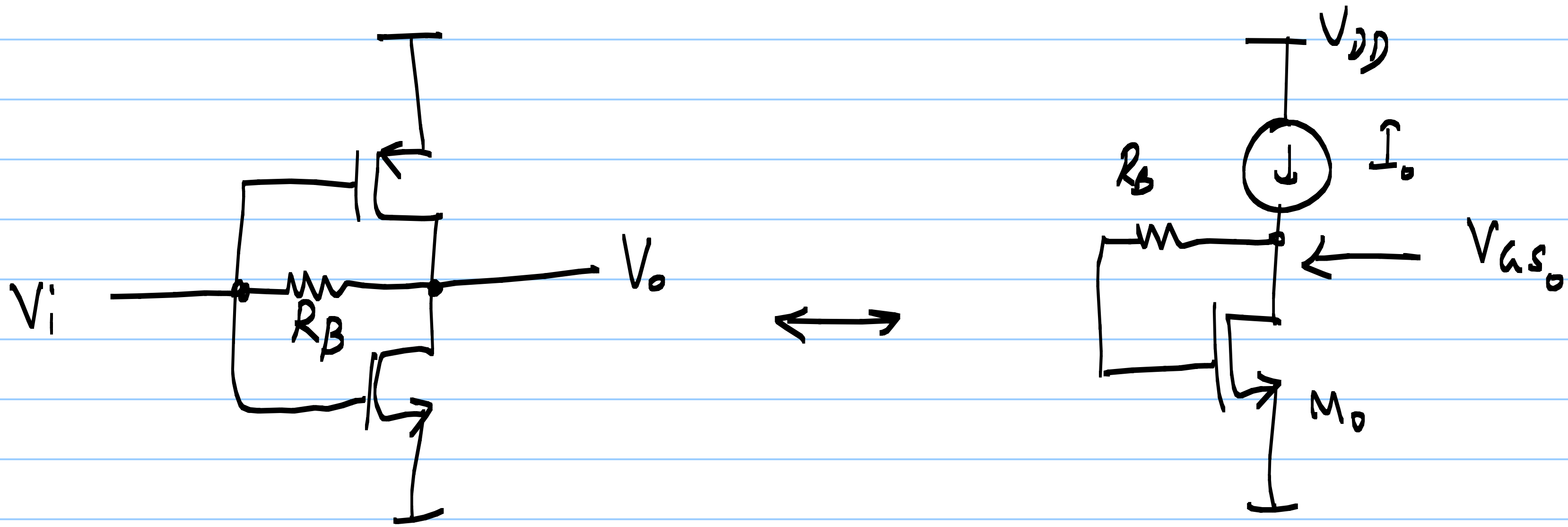
pnp BJT

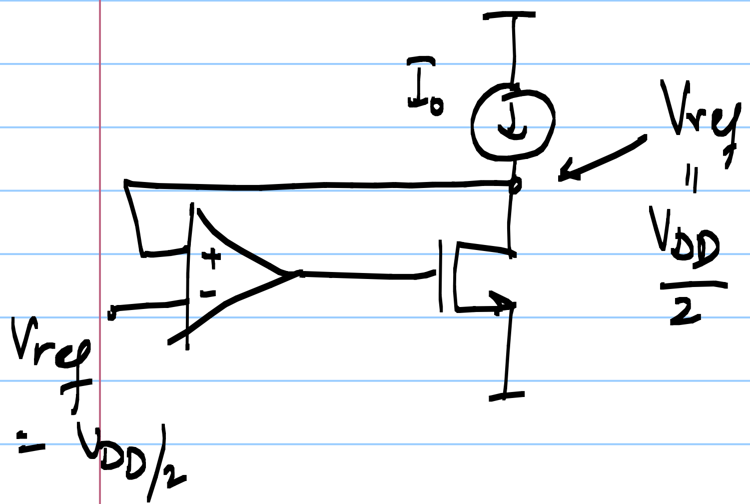


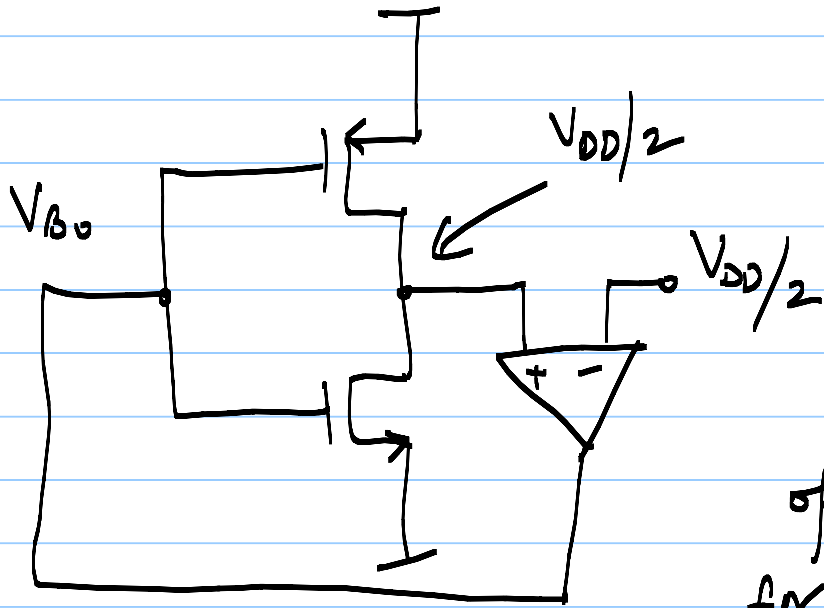
$V_{DD} \approx V_{BE_n} + V_{EB_p} \approx 1.3V$

24/9/2020

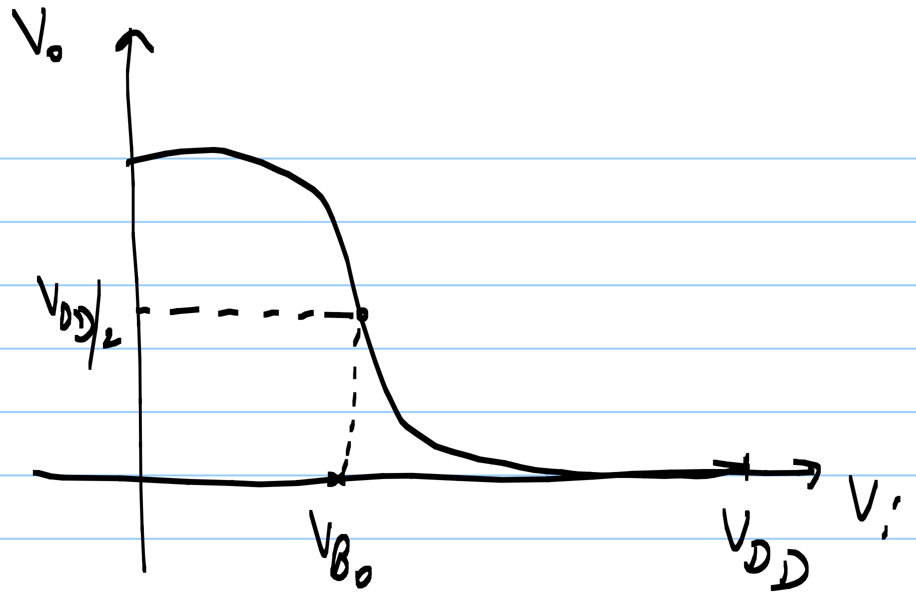
Lecture 30





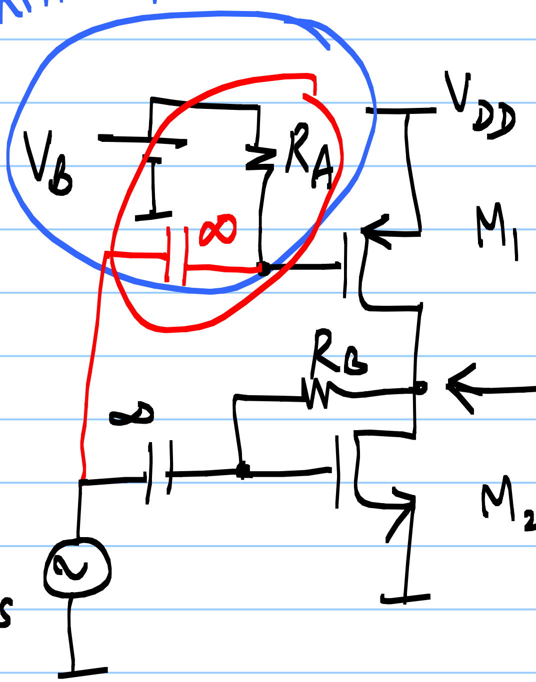


Modify the circuit to break opamp loop for AC.

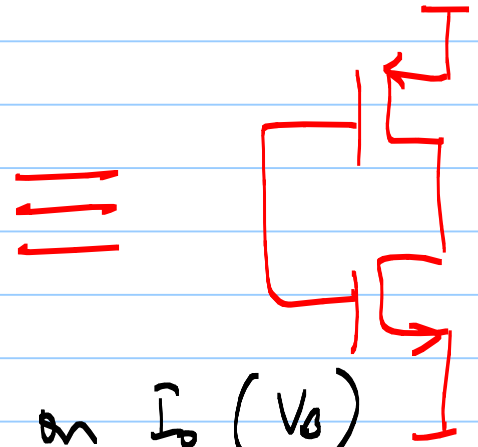


$< V_{DD}/2$

Thevenin equivalent of Bias

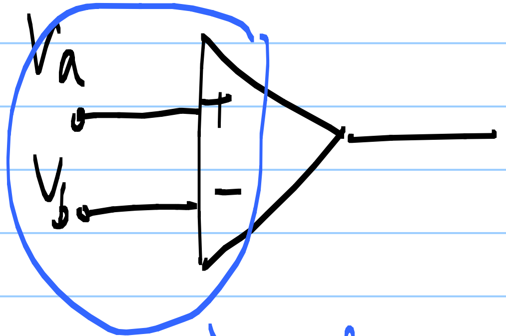


$V_{o_{dc}}$ adjusts itself based on $I_o\,(V_o)$



for Ac

Differential Amplifiers

opamp: 

$A(V_a - V_b)$; $A \to \infty$

first half of course

opamp responds only to $V_a - V_b$

$$V_a = \left(\frac{V_a+V_b}{2}\right) + \left(\frac{V_a-V_b}{2}\right)$$

$$V_b = \underbrace{\left(\frac{V_a+V_b}{2}\right)}_{\substack{\text{common-mode} \\ \text{voltage } V_{CM}}} - \underbrace{\left(\frac{V_a-V_b}{2}\right)}_{\substack{\text{differential mode} \\ \text{voltage } V_{DM}}}$$

e.g. 1) $V_a = 1\,V$, $V_b = 1.1V$

$V_{CM} = 1.05V$ $V_{DM} = -0.05V$

$V_a = V_{CM} + V_{DM}$

$V_b = V_{CM} - V_{DM}$

* opamp should amplify only V_{DM}

2) $V_a = 1.1V$, $V_b = 1V$

$V_{CM} = 1.05V$, $V_{DM} = 0.05V$

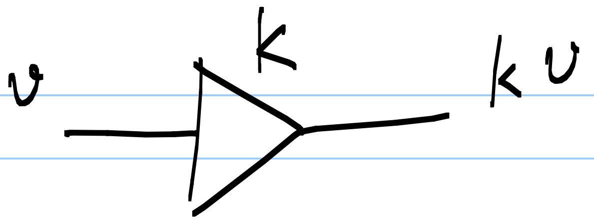
3) $V_a = 0.6V$, $V_b = 0.5V$

$V_{CM} = 0.55V$, $V_{DM} = 0.05V$

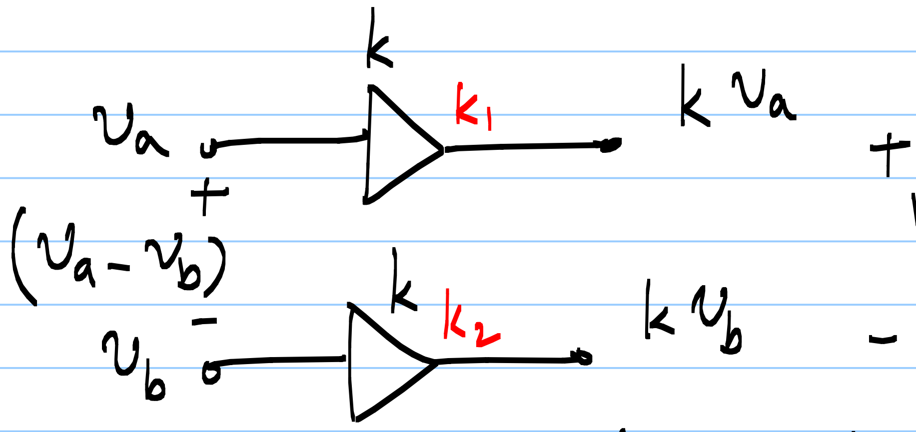
* Assume $V_a = v_a$ & $V_b = v_b$

$V_a = V_{CM} + \frac{\Delta V}{2}$ $V_b = V_{CM} - \frac{\Delta V}{2}$

We know:

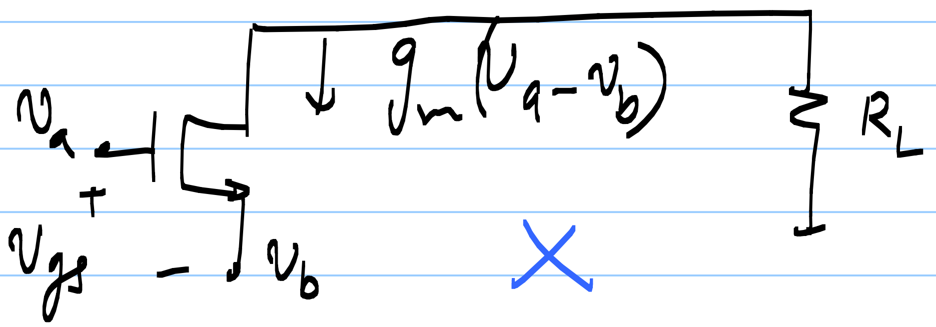


Circuit I



$V_o = k(v_a - v_b)$

* We wanted single-ended output (ignore this for now)



For circuit I, what are v_{icm}, v_{idm}, v_{ocm}, v_{odm}?

(I) has equal A_{CM} & A_{DM}

$$v_{iCM} = \frac{v_a + v_b}{2} \quad ; \quad v_{iDM} = \frac{v_a - v_b}{2}$$

bad ↓ ↓ good

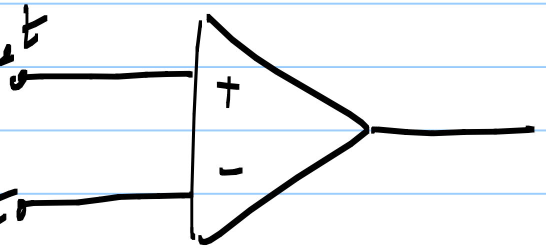
$$v_{oCM} = k\left(\frac{v_a + v_b}{2}\right) \quad ; \quad v_{oDM} = k\left(\frac{v_a - v_b}{2}\right)$$

We want large A_{DM} to A_{CM}

e.g.

$1V \sin \omega_1 t + 0.1 \sin \omega_2 t$

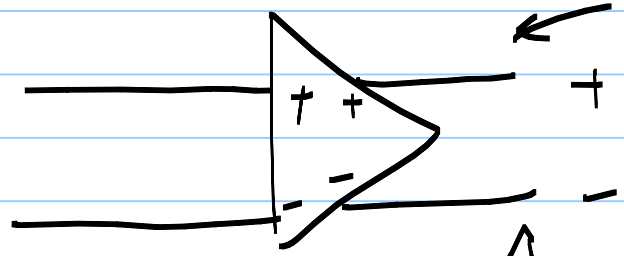
$1V \sin \omega_1 t - 0.1 \sin \omega_2 t$



$A(0.2 \sin \omega_2 t)$

0 CM

$0.1 \sin \omega_2 t + 0$



+

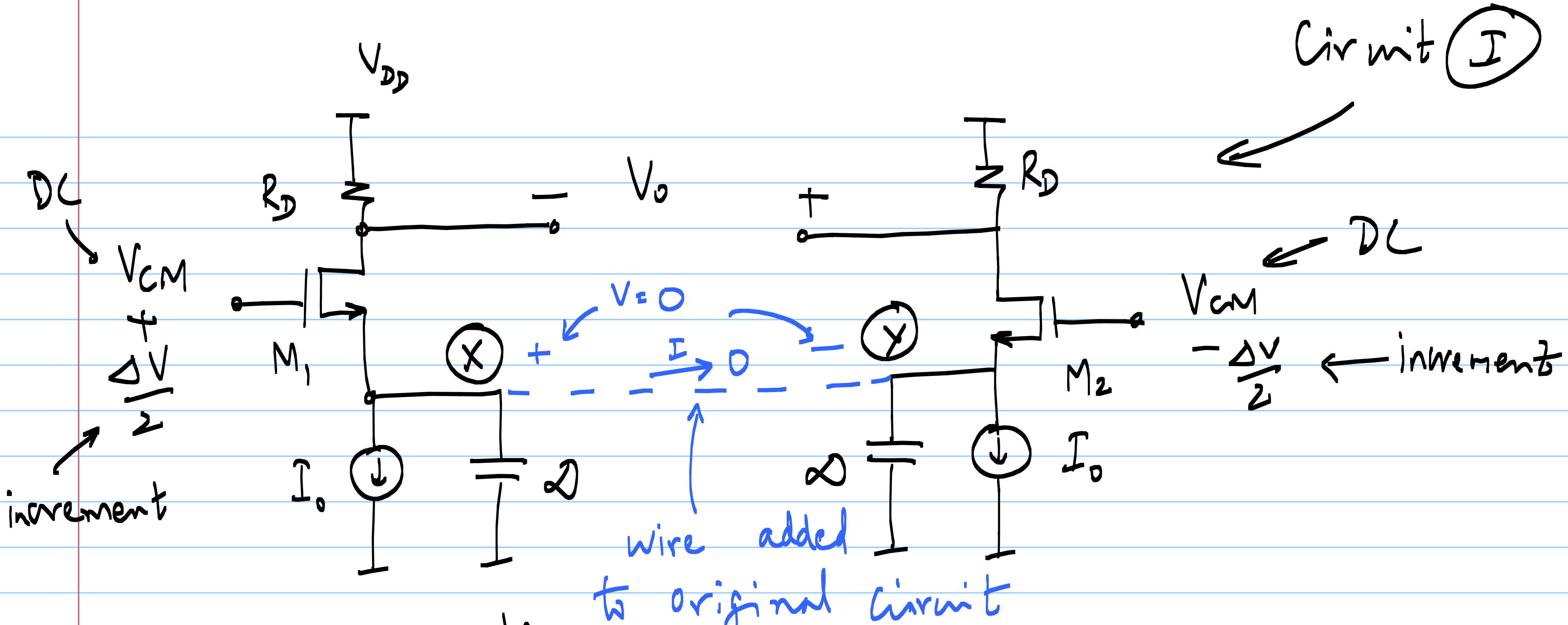
$A(0.2 \sin \omega_2 t)$

−

$-0.1 \sin \omega_2 t + 0$

0 CM

Fully Diff. opamp

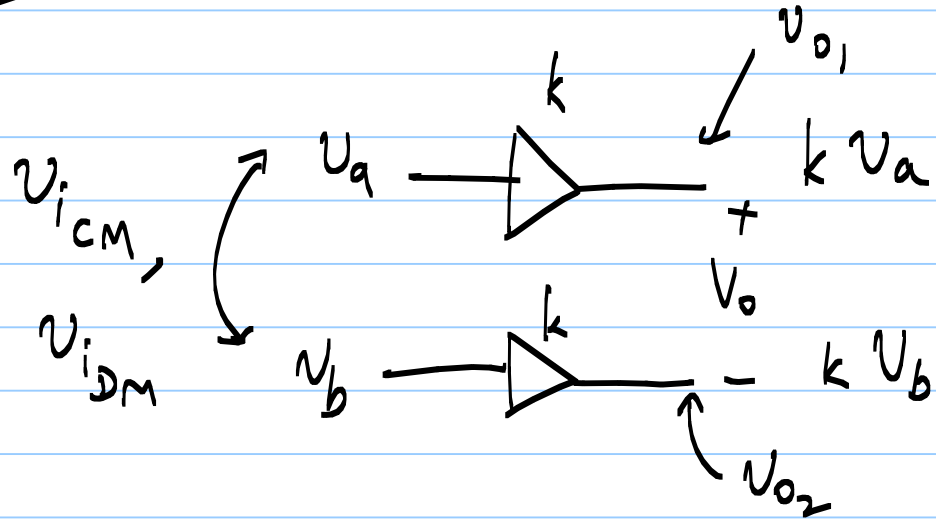


$$V_X = V_{CM} - V_{GS_1} \quad ; \quad V_Y = V_{CM} - V_{GS_2}$$

$$V_{GS_1} = V_{GS_2} \Rightarrow V_X = V_Y$$

25|9|2020

Lecture 31



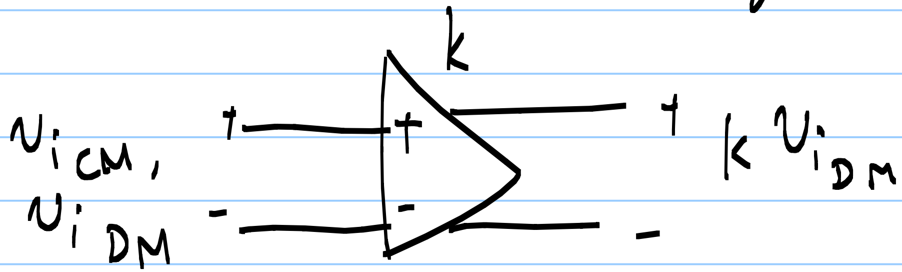
$$v_{oCM} = \frac{v_{o_1} + v_{o_2}}{2}$$

$$= \frac{k\,(v_a + v_b)}{2}$$

$$= k\, v_{iCM}$$

$$v_{oDM} = \frac{v_{o_1} - v_{o_2}}{2} = \frac{k\,(v_a - v_b)}{2} = k\, v_{iDM}$$

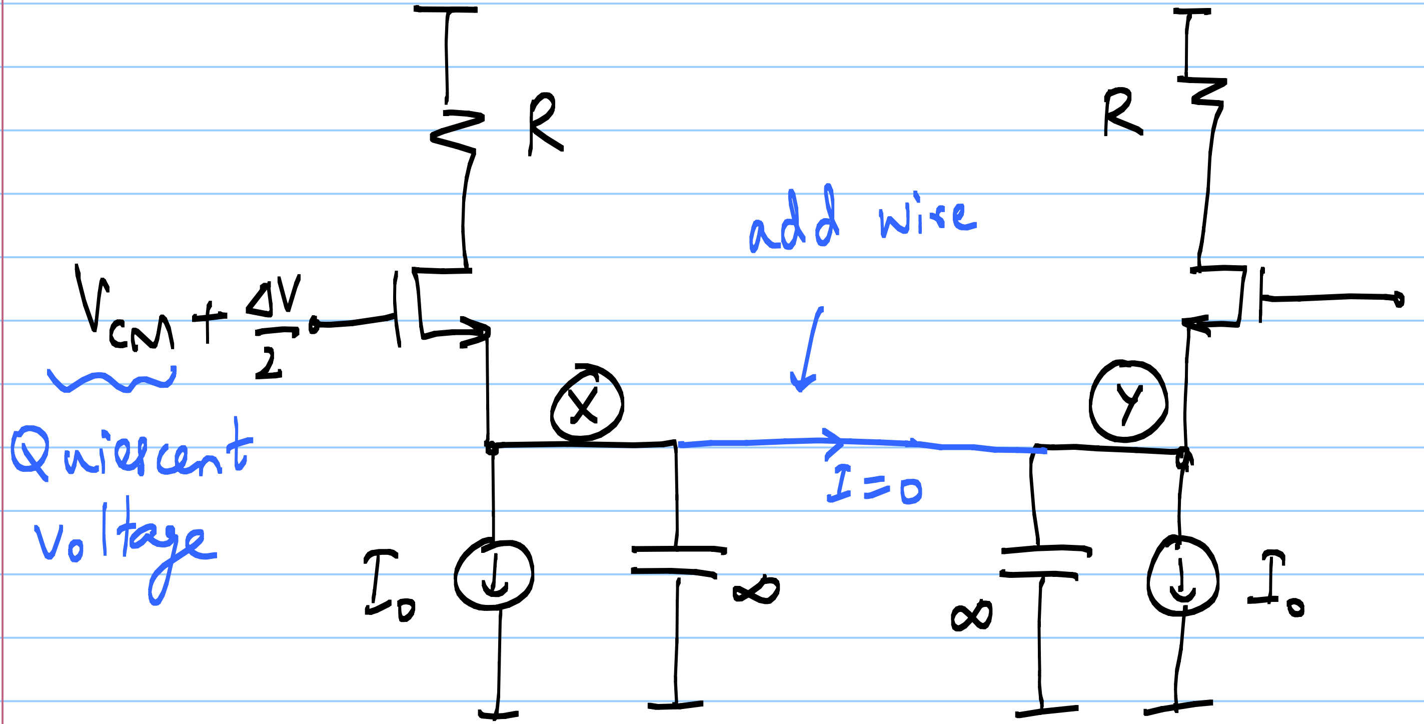
* We want to amplify only v_{iDM}



$\{ v_{oCM} = 0 \}$

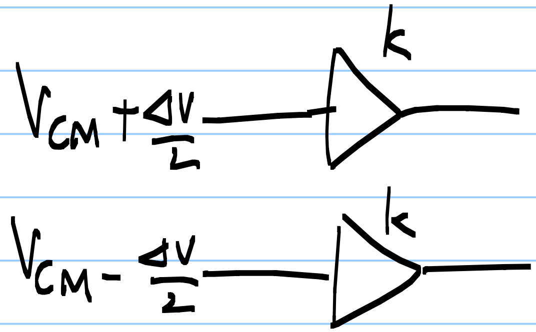
$$V_x = V_{CM} - V_{GS_1}\Big|_{I_0}$$

$$V_y = V_{CM} - V_{GS_2}\Big|_{I_0}$$



$$\boxed{V_x = V_y}$$

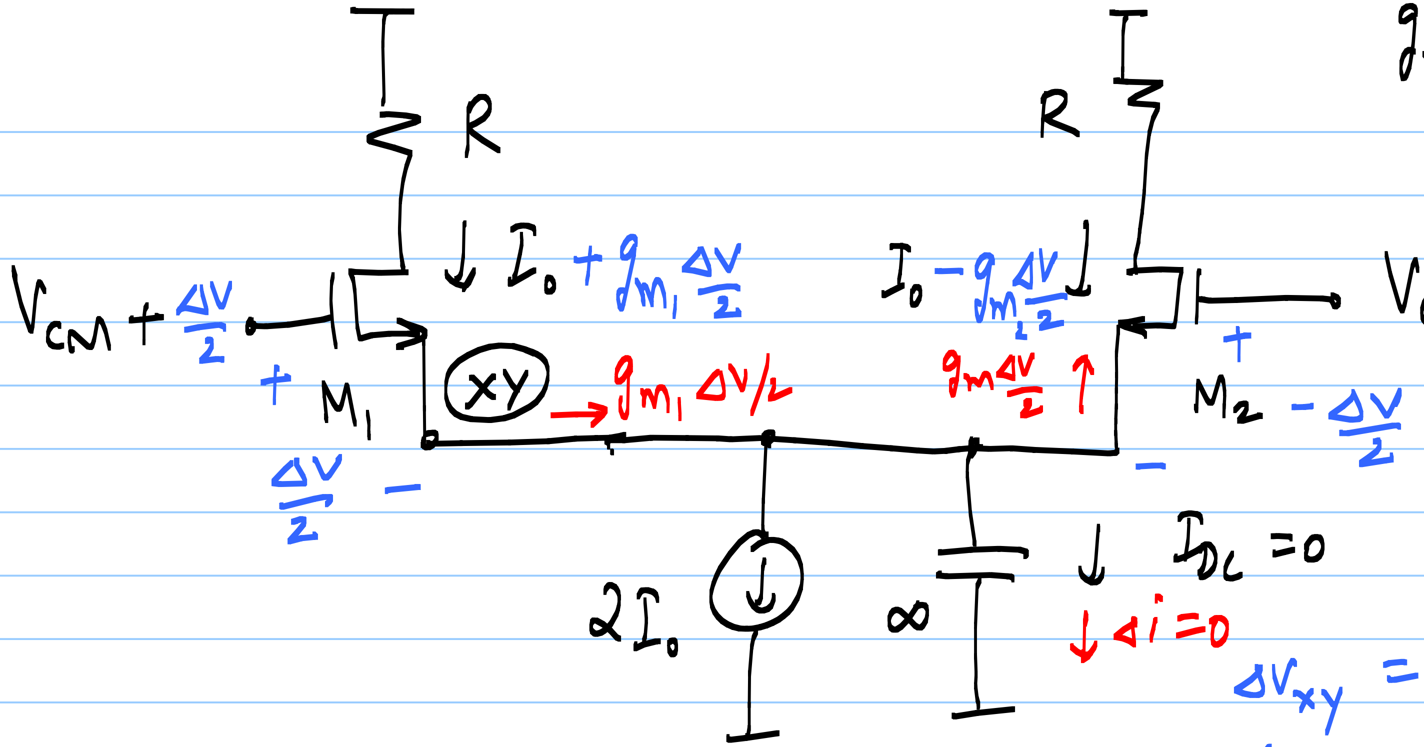
$$\Delta V_x = \Delta V_y = 0 \quad (\infty \text{ cap})$$



$$V_{iCM} = V_{CM}$$

$$V_{iDM} = \frac{\Delta V}{2}$$

$g_{m_1} = g_{m_2} = g_m$

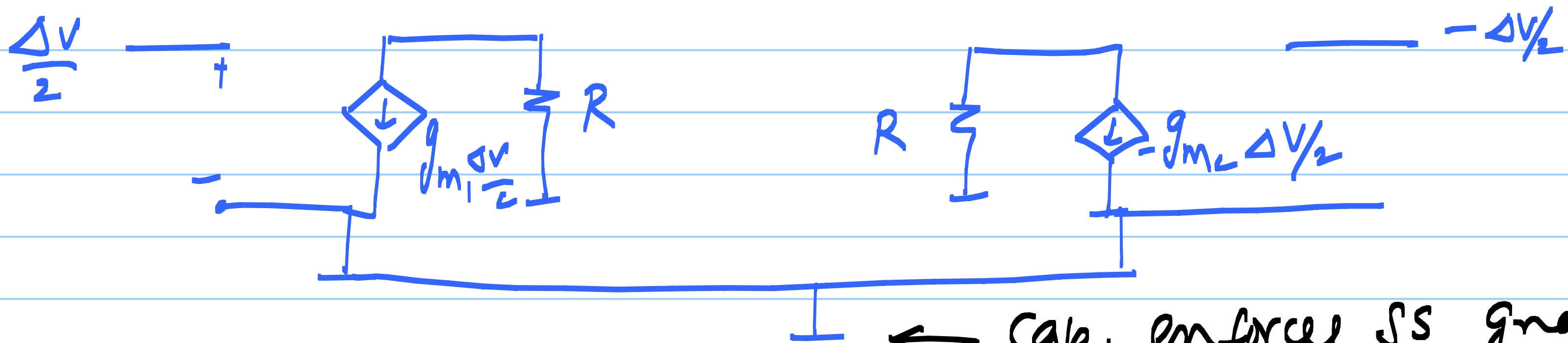


$\Delta V_{xy} = 0$ (∞ cap)

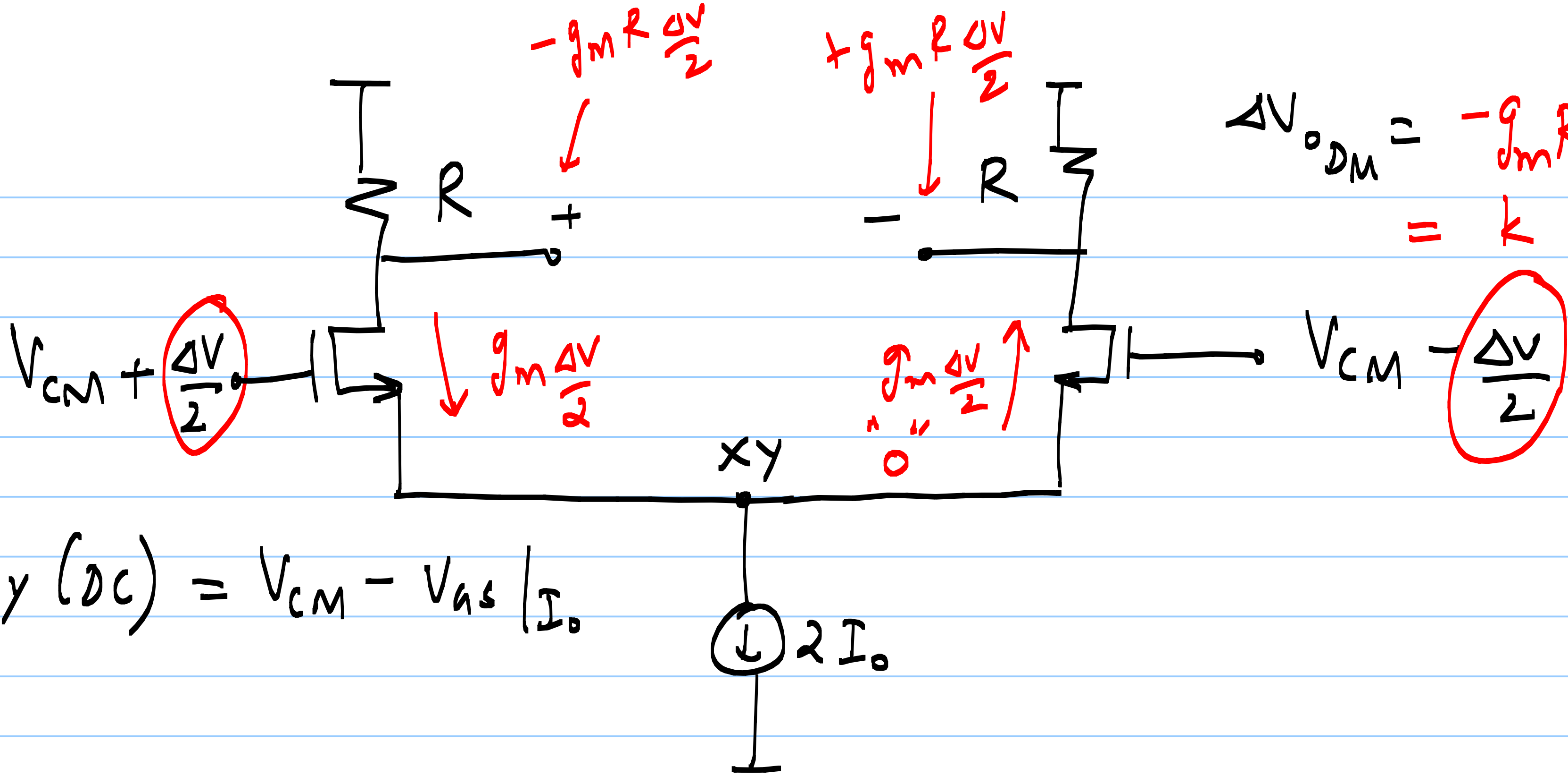
(inc. ground)

V_{xy} (DC) $= V_{CM} - V_{GS_{1,2}}\Big|_{I_0}$

Inc. eq. circuit

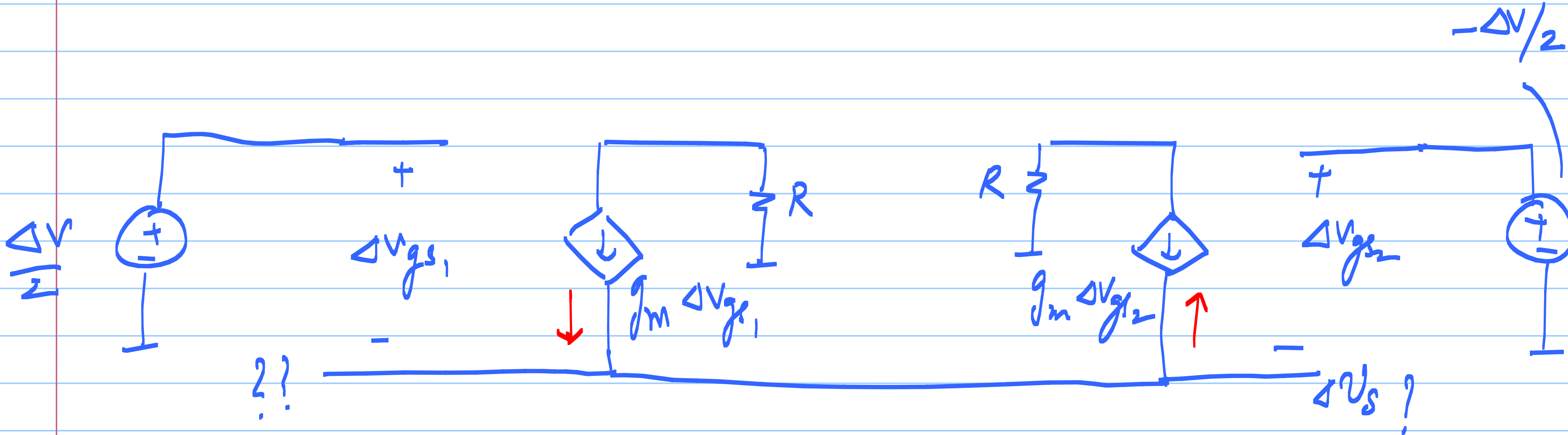


← cap. enforces ss gnd.



$$\Delta V_{oDM} = -g_m R \, \Delta V$$
$$= k \, \Delta V_{iDM}$$

$$V_{xy}(DC) = V_{CM} - V_{GS}\big|_{I_0}$$

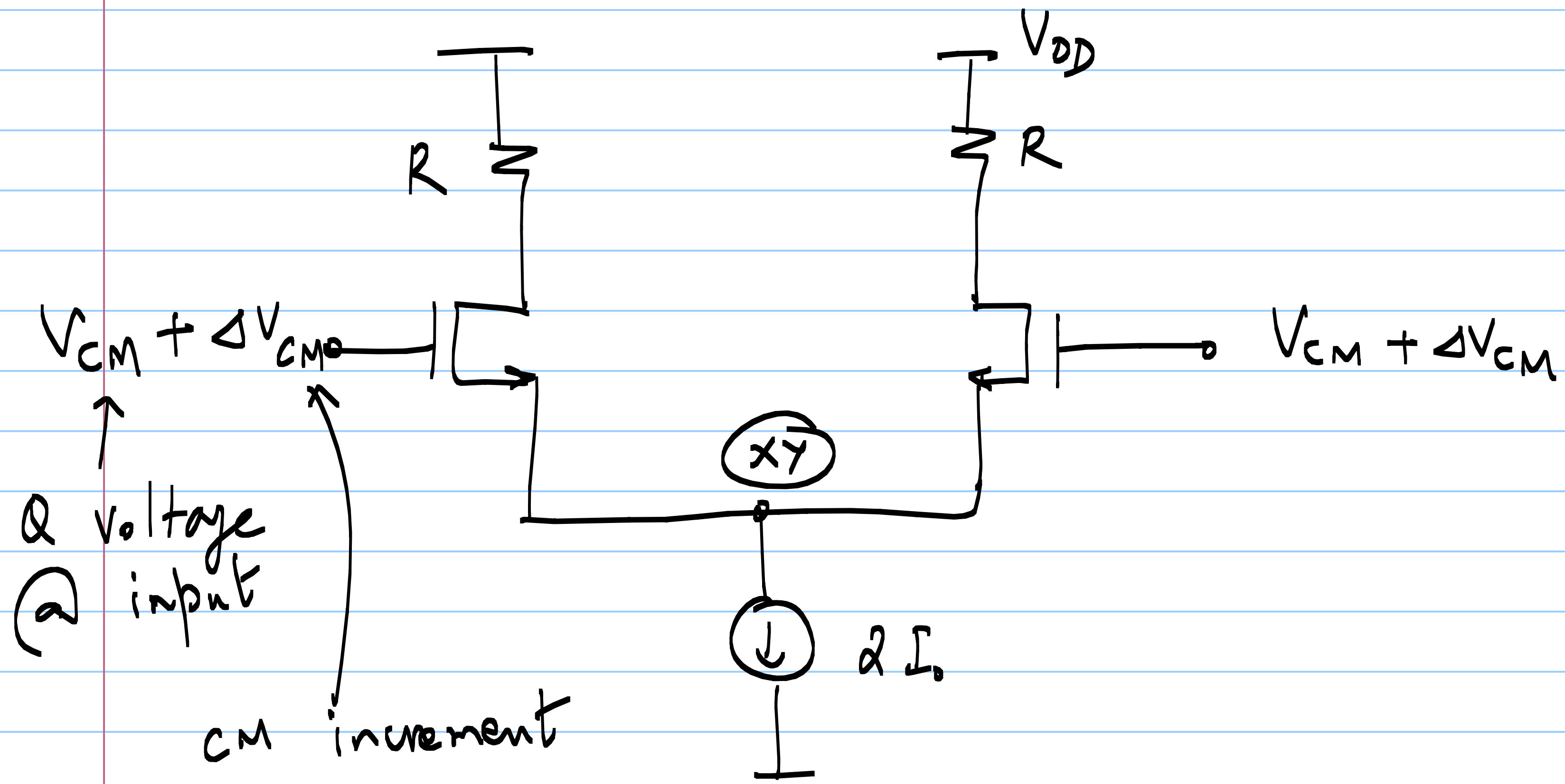


$$g_m \Delta v_{gs_1} + g_m \Delta v_{gs_2} = 0$$

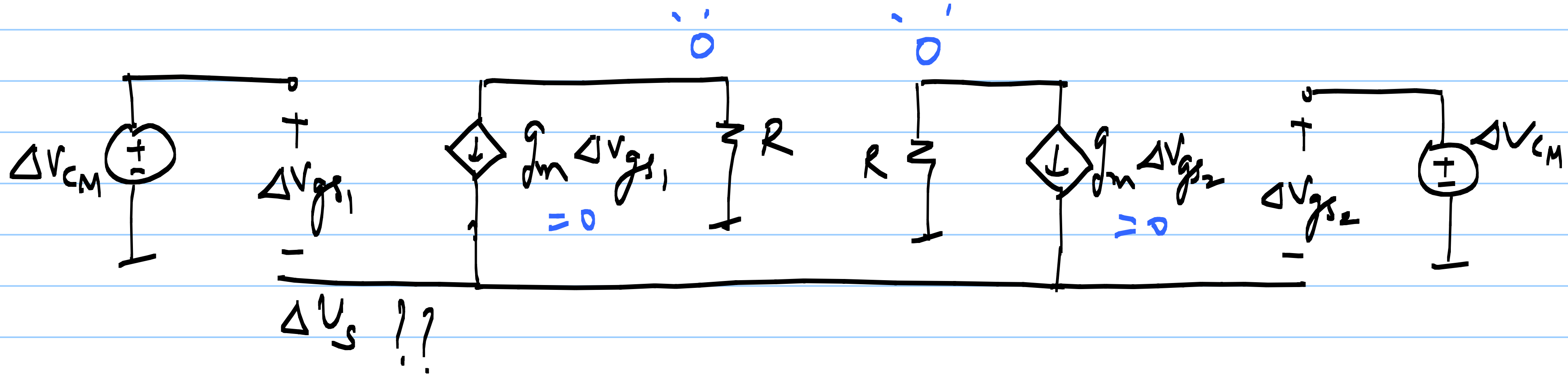
$$g_m(\Delta V_{g_1} - \Delta V_s) + g_m(\Delta V_{g_2} - \Delta V_s) = 0$$

$\Delta V_{g_1} = +\frac{\Delta V}{2}$, $\Delta V_{g_2} = -\frac{\Delta V}{2}$

$$\boxed{\Delta V_s = 0}$$



inc. eq.:



KCL @ source: $g_m \Delta V_{gs_1} + g_m \Delta V_{gs_2} = 0$

$$g_m(\Delta V_{g_1} - \Delta V_s) + g_m(\Delta V_{g_2} - \Delta V_s) = 0$$

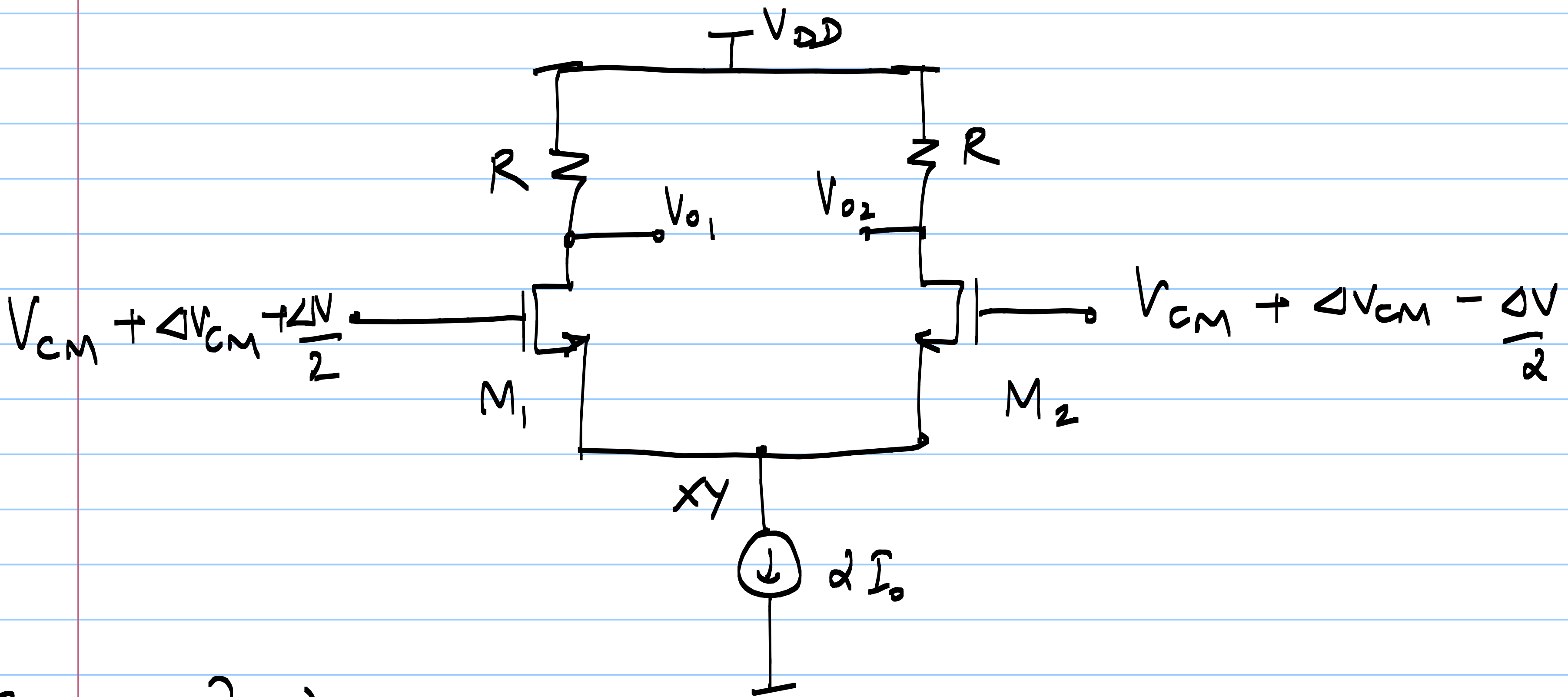
$$\Delta V_{g_1} = \Delta V_{g_2} = \Delta V_{CM}$$

$$2\Delta V_{CM} = 2\Delta V_s \qquad \boxed{\Delta V_s = \Delta V_{CM}}$$

$$\Delta V_{gs_1} = \Delta V_{gs_2} = 0$$

$\Delta V_{o_{CM}} = 0 \Rightarrow$ This circuit has 0 CM gain

"Differential Amplifier" we want



$\{ DC \}$ 1) V_{CM} : $I_{D_1} = I_{D_2} = I_o$; $V_{xy} = V_{CM} - V_{GS}\big|_{I_o}$

$V_{o_1} = V_{o_2} = V_{DD} - I_o R$;

2) ΔV_{CM} :

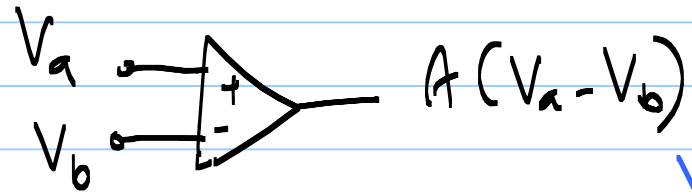
$$\Delta i_{d_1 CM} = \Delta i_{d_2 CM} = 0$$

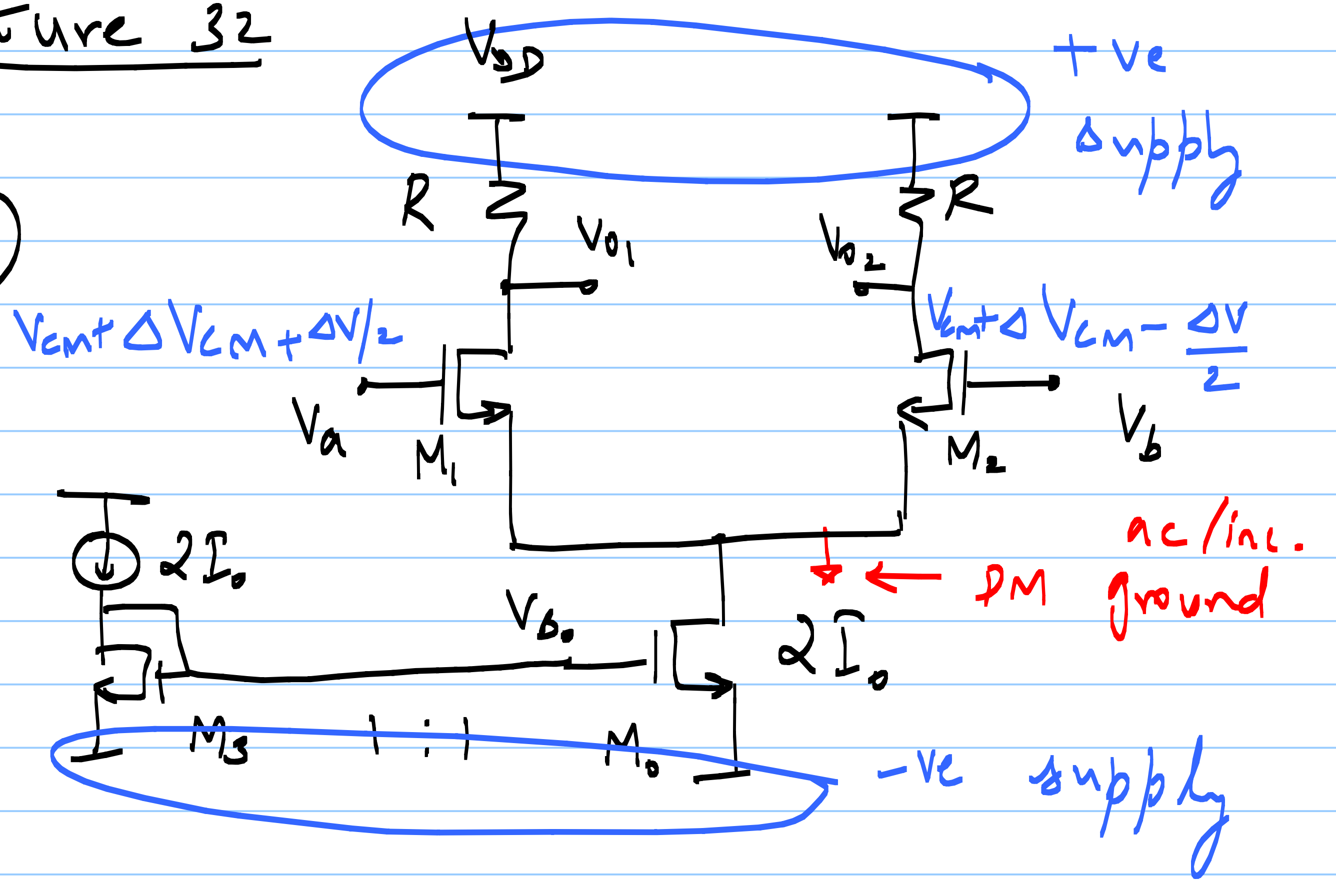
$$\Delta V_{o1_{CM}} = \Delta V_{o2\,CM} = 0$$

$$\Delta V_{xy} = \Delta V_{CM}$$

29/9/20

Lecture 32

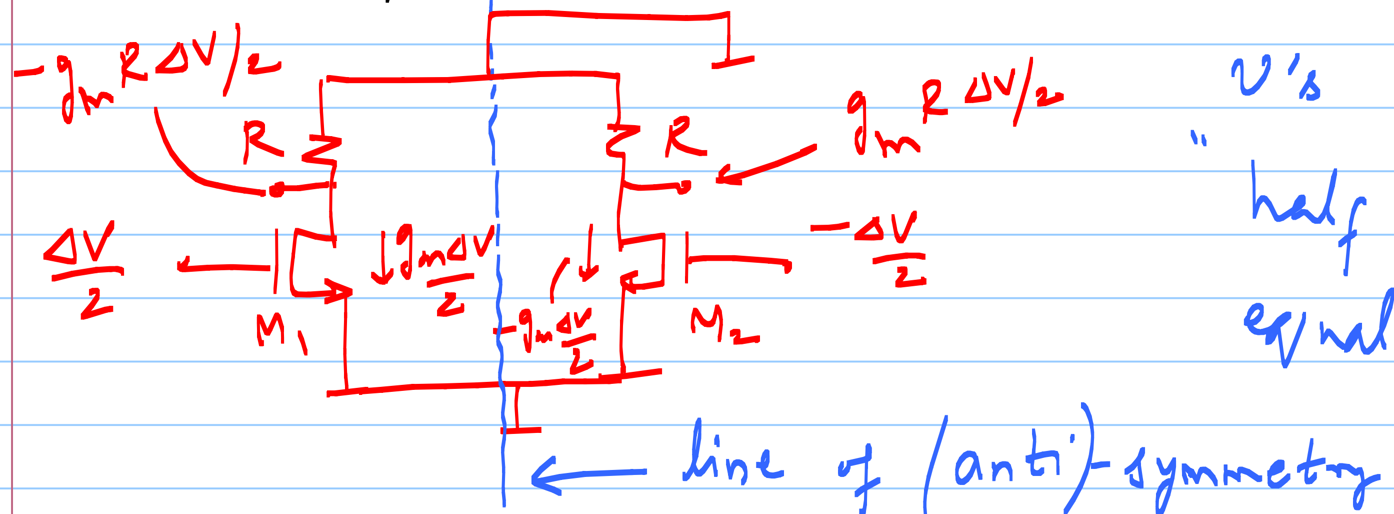




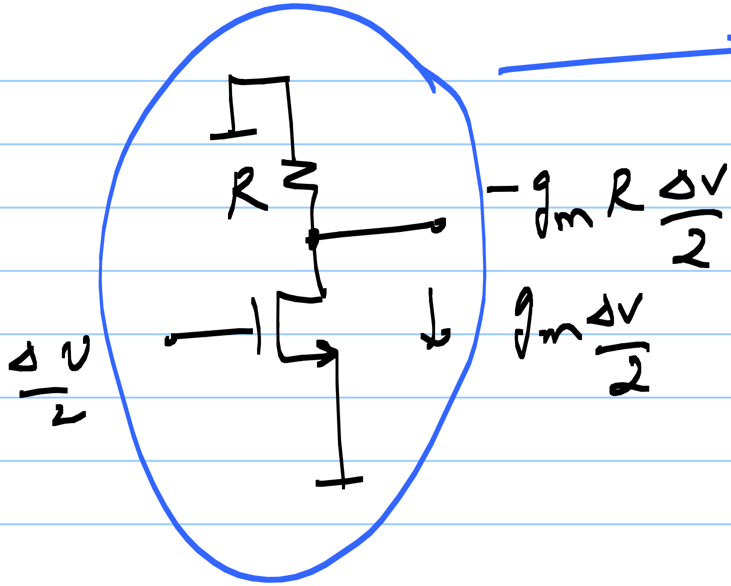
$V_{CM} = \dfrac{V_a + V_b}{2}$

$\Delta V = V_a - V_b$

DM eq. circuit

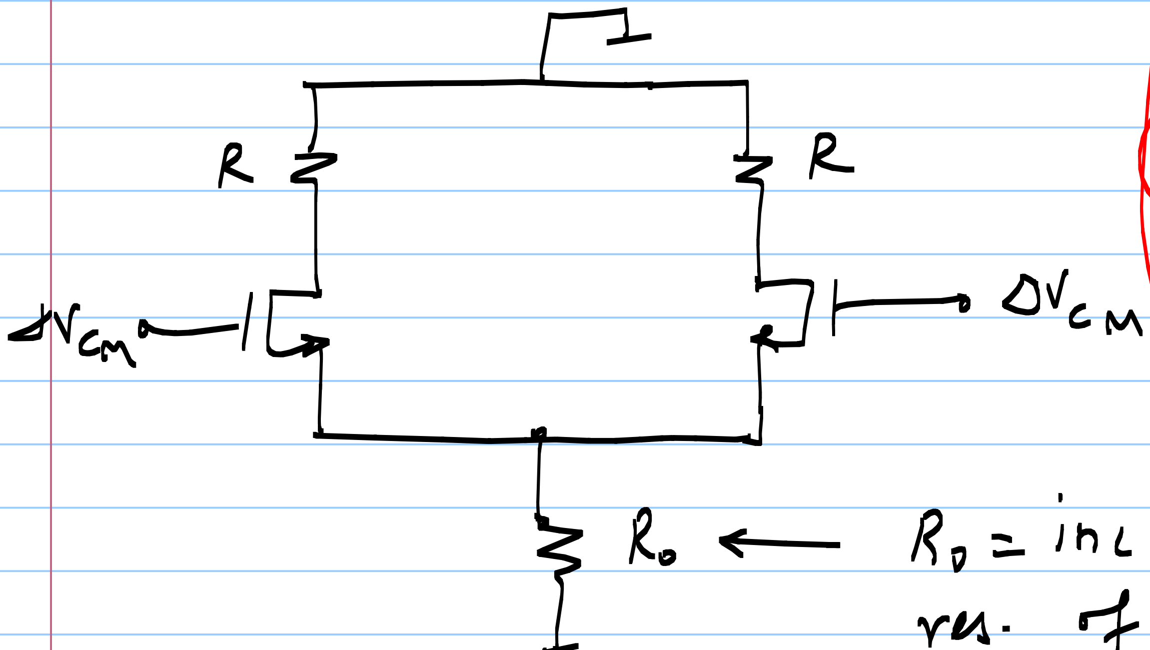


v's and i's on each "half-circuit" have equal magnitude & opposite phase



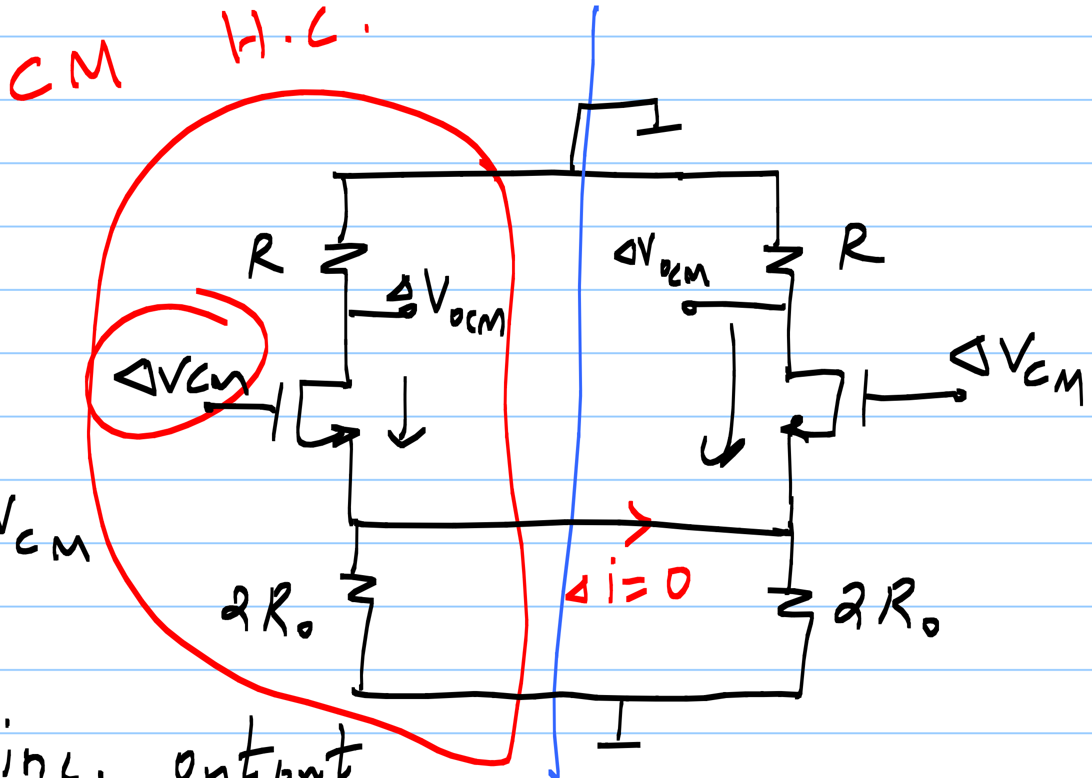
→ DM half circuit

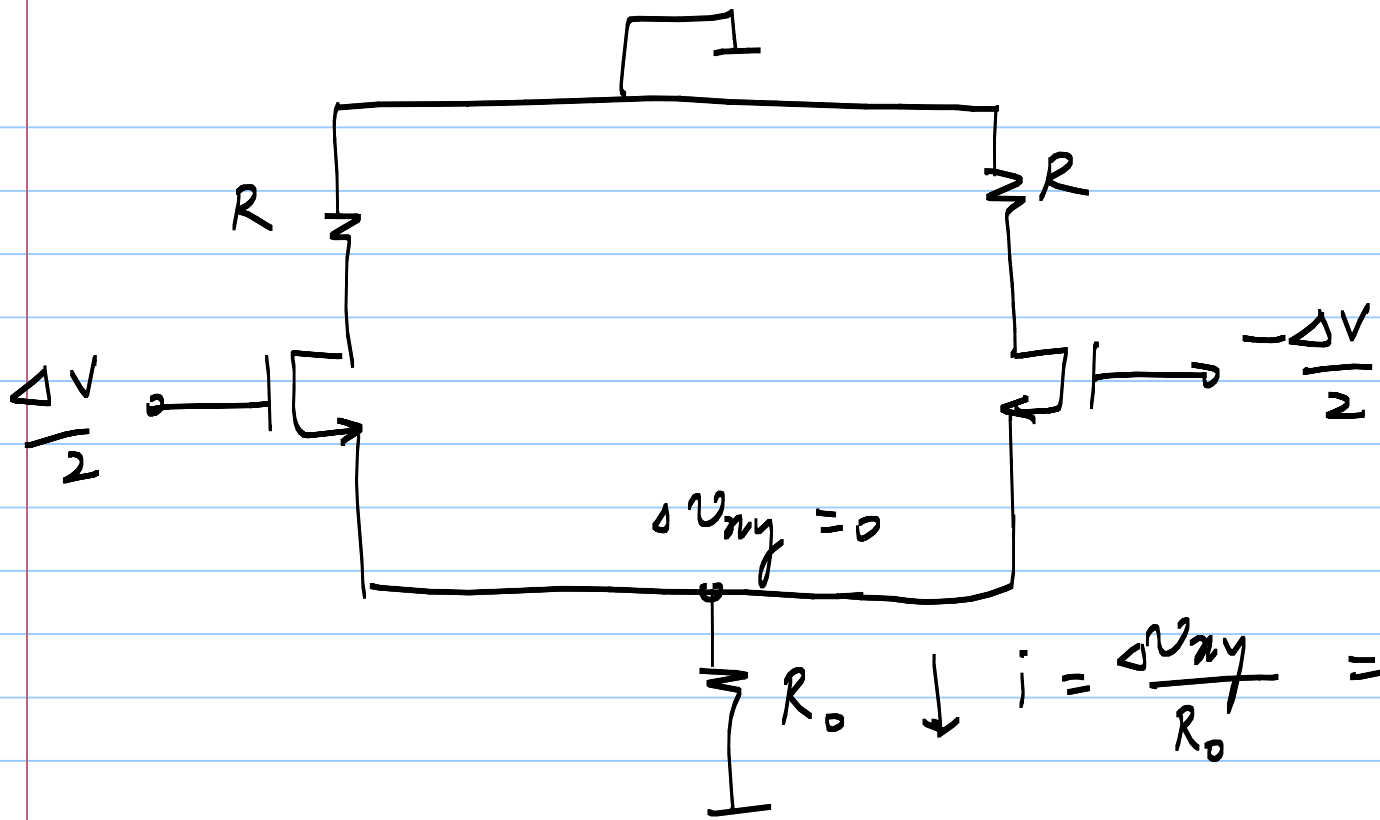
CM eq. cir.



R_o = inc. output res. of $2I_o$ current source

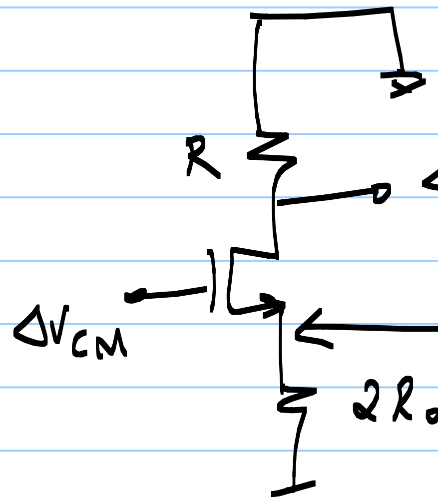
CM H.C.





$i = \frac{\Delta v_{xy}}{R_o} = 0$ is the only possible state of the DM cir.

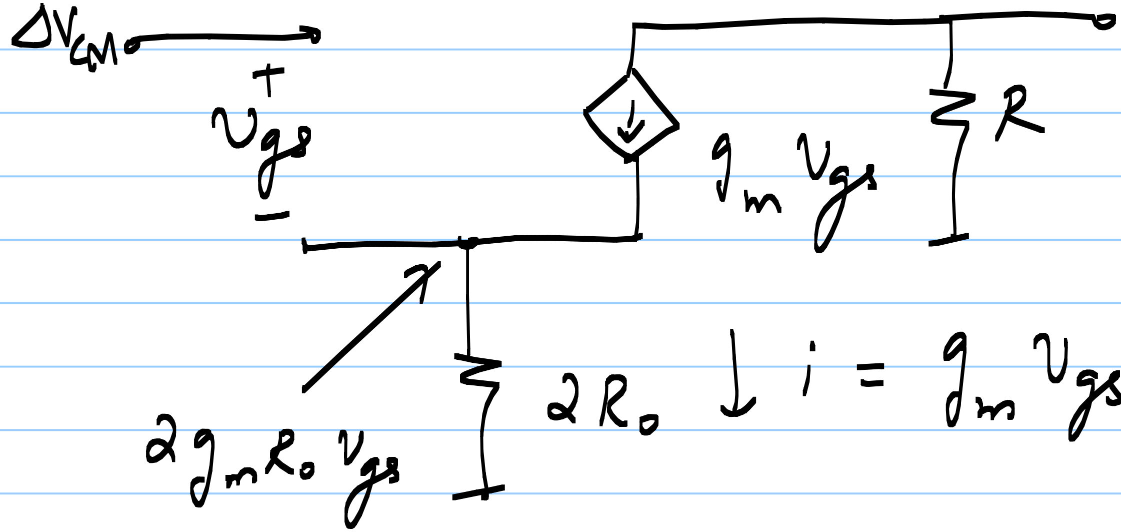
$\Delta V_{oCM} = ?$



$\Delta V_{oCM} \approx \frac{-R}{2R_o} \cdot \Delta V_{CM}$

$\approx \Delta V_{CM}$ if g_m were large

$i \approx \frac{\Delta V_{CM}}{2R_o}$



$$\Delta V_{oCM} = \frac{-g_m R}{1 + 2 g_m R_o} \Delta V_{CM}$$

$$A_{DM} = -g_m R$$

$$A_{CM} = \frac{-g_m R}{1 + 2 g_m R_o}$$

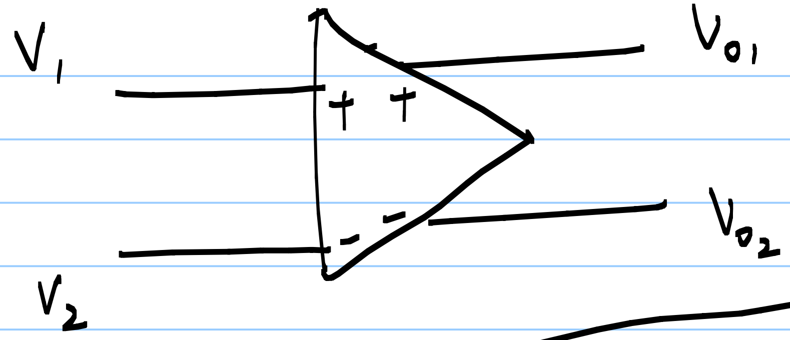
$$CMRR = \left| \frac{A_{DM}}{A_{CM}} \right|$$

Common Mode Rejection Ratio

$$CMRR_{dB} = 20 \log \left| \frac{A_{DM}}{A_{CM}} \right|$$

large CMRR = "good" differential amplifier

Diff. Amp.



DC @ (+) input

AC @ + input

e.g. $V_1 = 1V + 2mV \sin\omega t$ $+ 5mV\ (DC)$

$V_2 = 1.01V + 1mV \sin\omega t$ $+ 5mV\ (DC)$

1.01V

$$V_{iCM} = \frac{V_1 + V_2}{2} = 1.005V + \underline{1.5mV \sin\omega t} + 1\mu V \cos\omega_2 t$$

$$V_{iDM} = \frac{V_1 - V_2}{2} = \underline{-0.005V} + \underline{0.5mV \sin\omega t} + 2\mu V \cos\omega_2 t$$

$V_1 = V_{iCM} + V_{iDM}$; $V_2 = V_{iCM} - V_{iDM}$

$V_{iDC} = V_{CM} = 1.005V \rightarrow 1.01V$

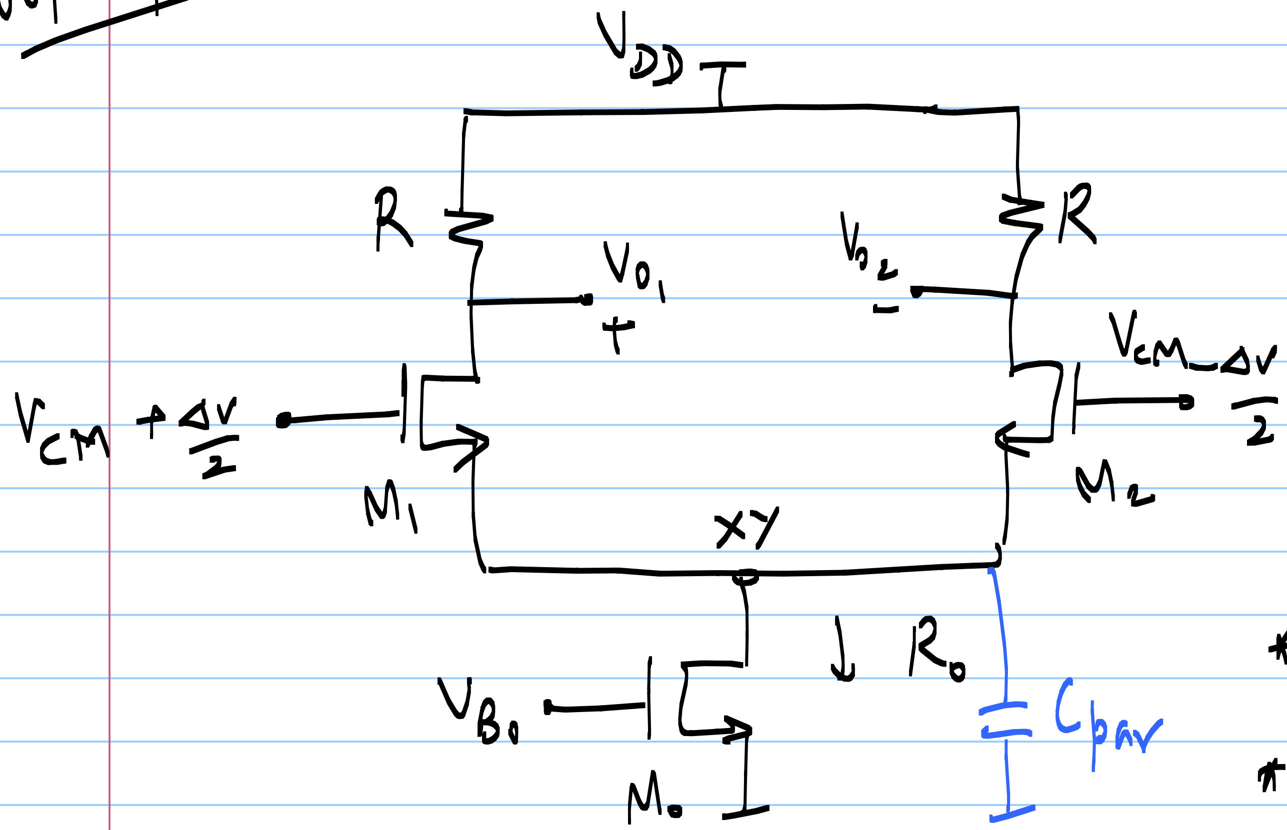
$$v_{CM} = 1.5\,mV \sin\omega t$$

$$\Delta V_{iDM} = -0.005$$

$$v_{DM} = 0.5\,mV \sin\omega t$$

06/10/2020

Lecture 33



A_{DM} is large $(-g_m R)$

A_{CM} is small (dep. on R_o)

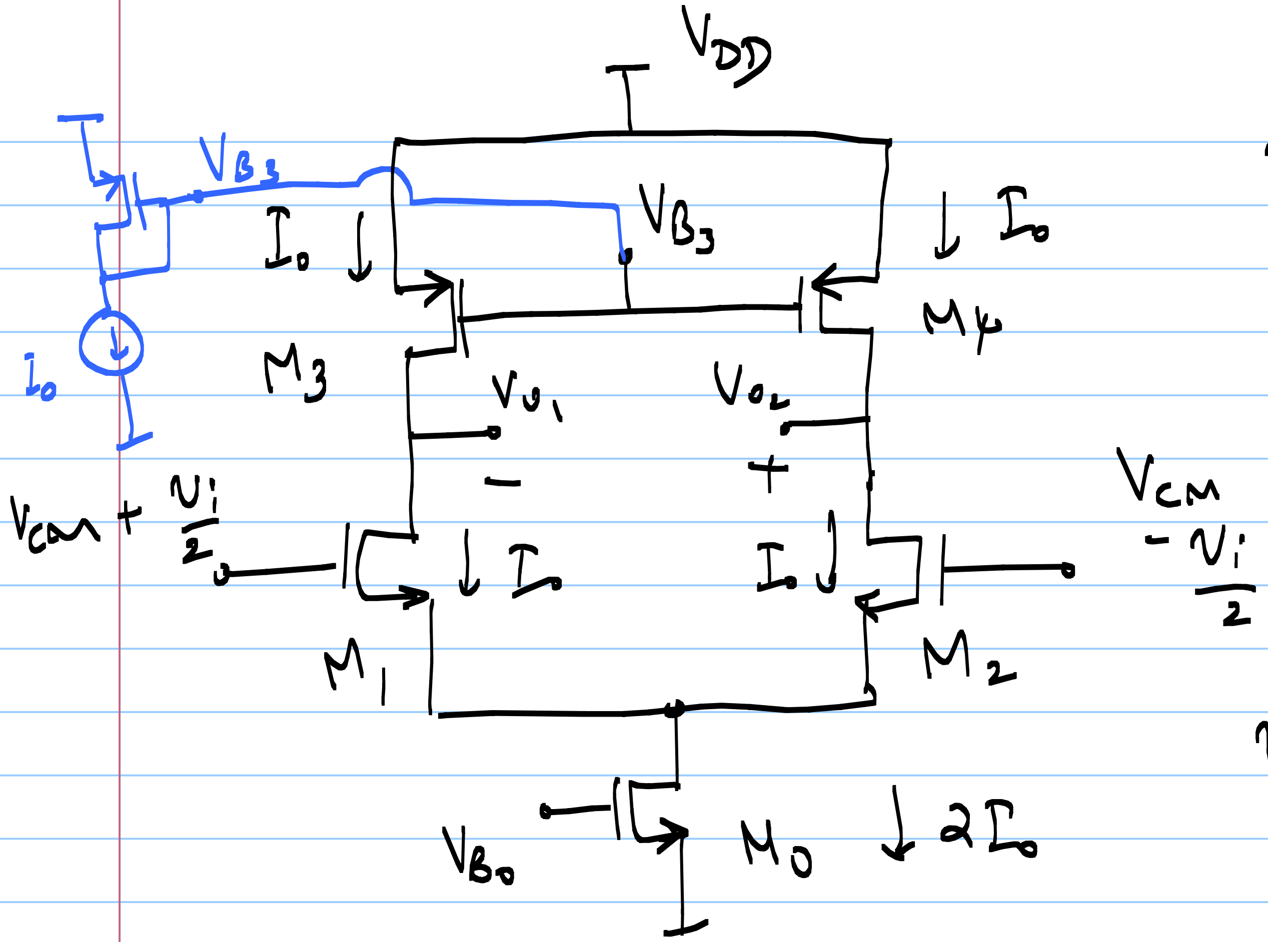
CMRR is large

we want to

* have a single ended o/p
* have more gain
* Ensure C_{par} is as low as possible

Possibilities

* SE o/p − discard V_{o_1} or V_{o_2} {not good}
* more gain − active load



$$v_{o_1} = -g_{m_1}(r_{ds_1} \| r_{ds_3}) \cdot \left(\frac{v_i}{2}\right)$$

$$v_{o_2} = -g_{m_2}(r_{ds_2} \| r_{ds_4}) \cdot \left(-\frac{v_i}{2}\right)$$

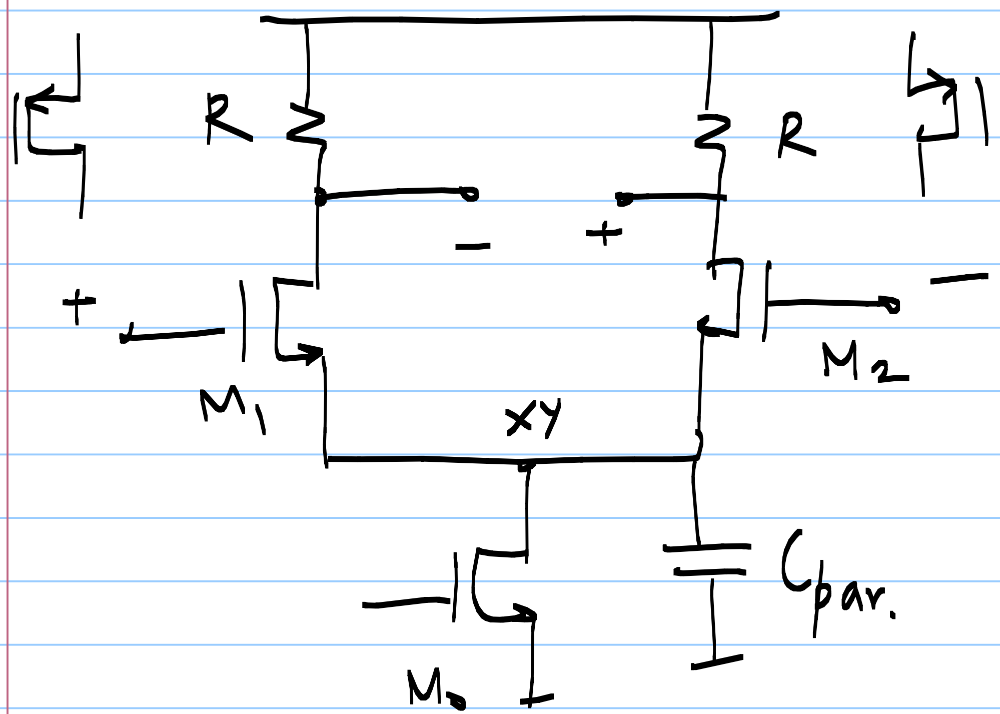
$$= +g_{m_1}(r_{ds_1} \| r_{ds_3}) \cdot \frac{v_i}{2}$$

$$v_o = v_{o_2} - v_{o_1}$$

$$= \underbrace{+g_{m_1}(r_{ds_1} \| r_{ds_3})}_{\text{gain similar to CSA with active load}} \cdot v_i$$

* Set V_{B_3} so that

$$I_{D_3} = I_{D_4} = I_o$$



* C_{par} = "parasitic" cap (undesired cap)

* Normally C_{par} @ xy dominated by device cap. (M_0, M_1 & M_2) (<u>Not</u> M_3 & M_4)

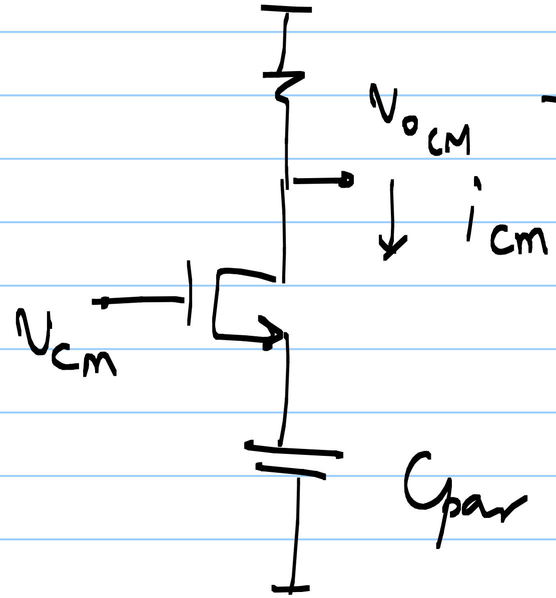
* C_{par} does not affect DM performance significantly

* C_{par} affect CM performance

e.g. R_o $(r_{ds_o}) = \infty$

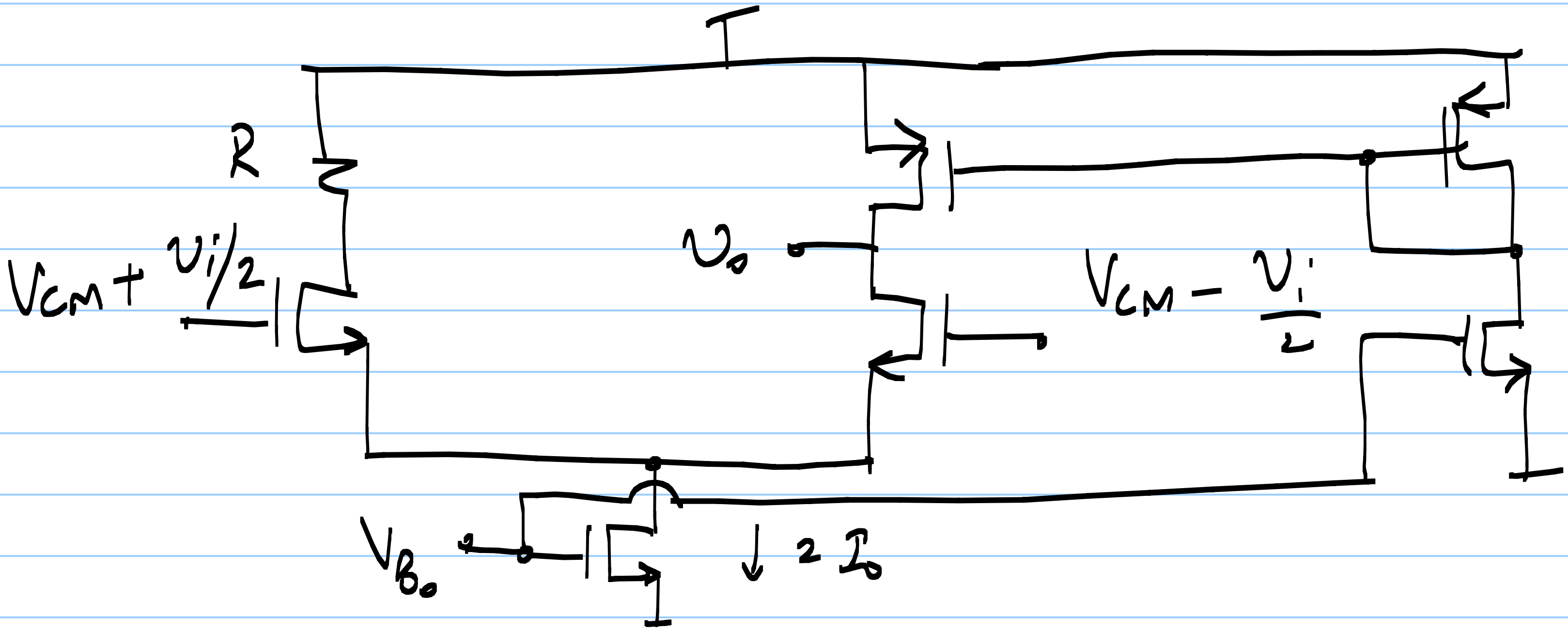
original $A_{cm} = 0$ (without C_{par})

With C_{par}:

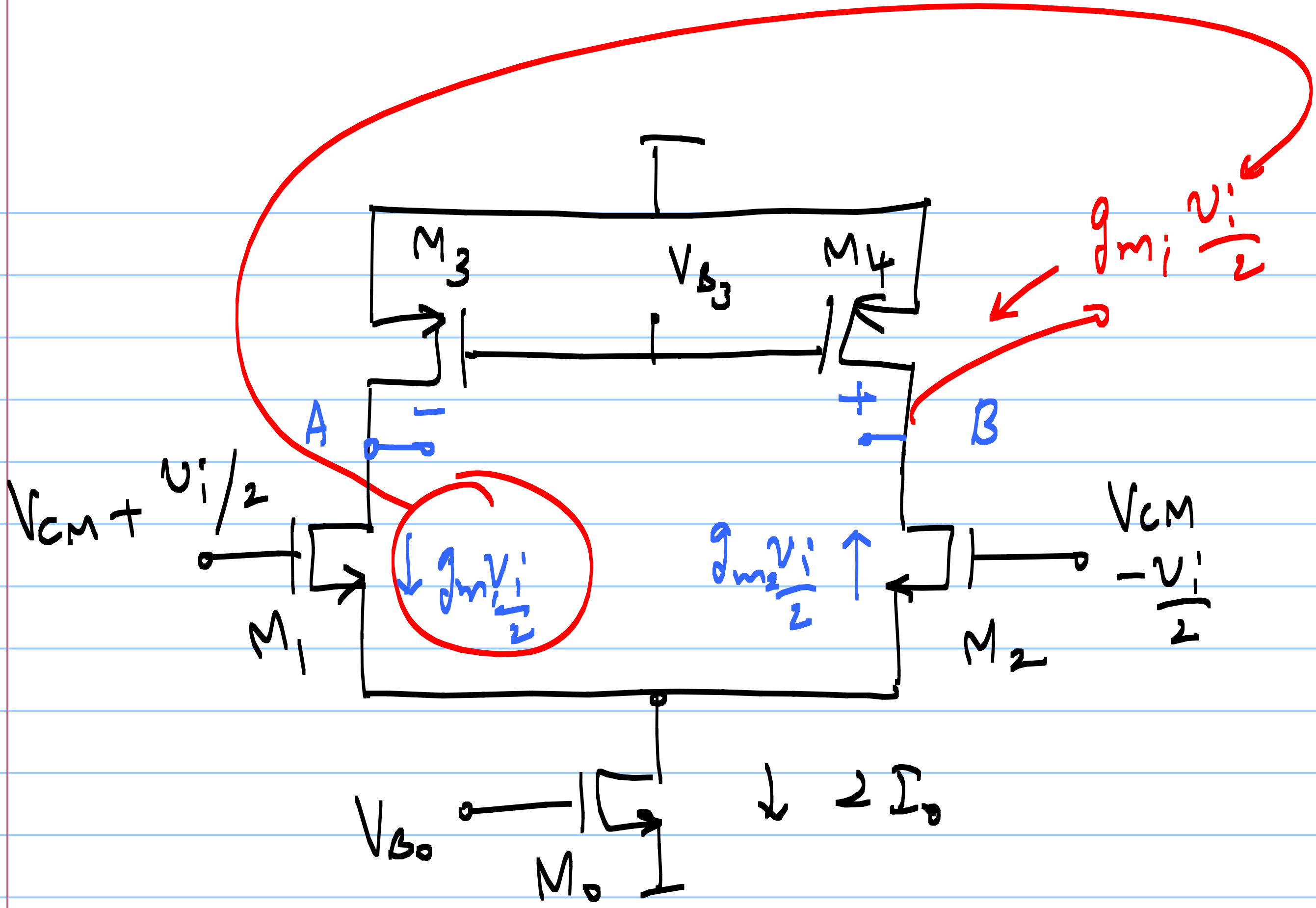


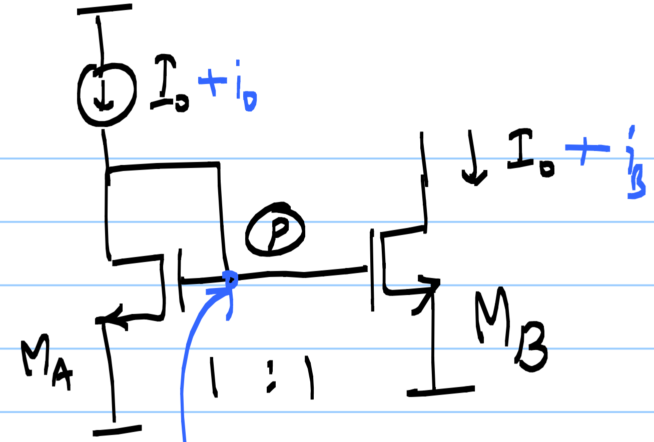
functions of freq.

CMRR is degraded @ high freq.



* Only half the original gain



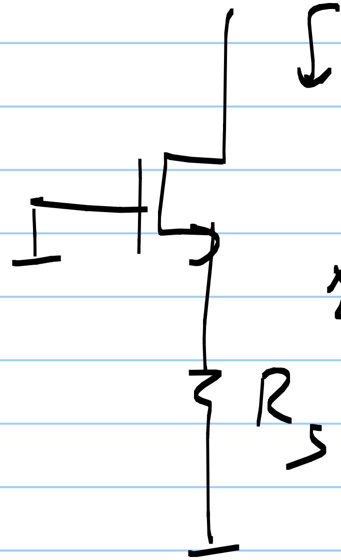


$$V_p = V_{GS_A}\Big|_{I_0 + \frac{i_0}{g_{mA}}}$$

$$I_{D_B} = I_0 + \frac{i_0 \cdot g_{mB}}{g_{mA}}$$

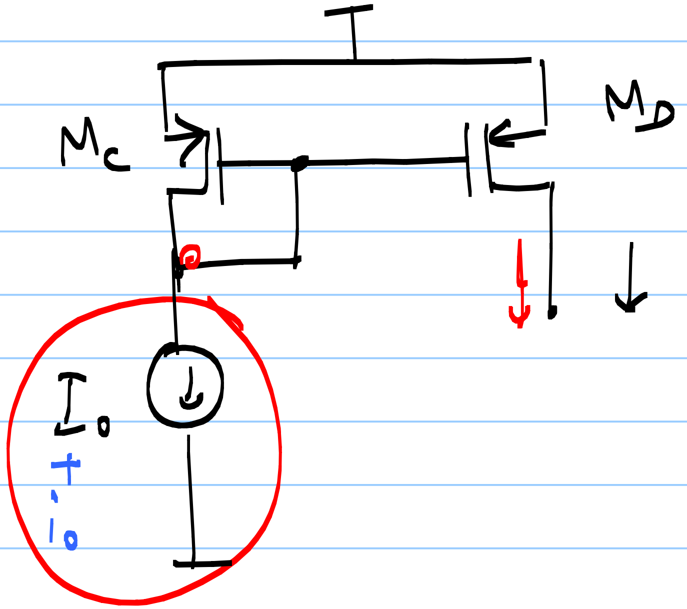
$i_B = i_0$ here (1:1 CM)

C.G.A.



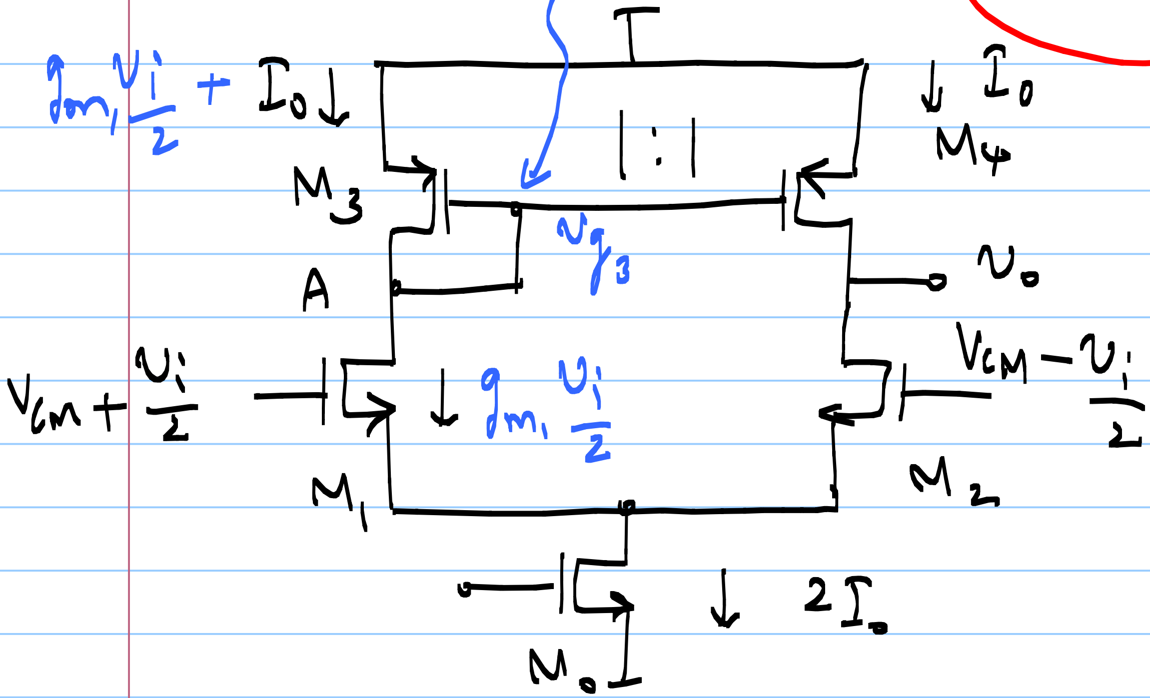
$R_{out} = g_m r_{ds} \cdot R_s$

$R_{in} = \frac{1}{g_m}$



* Repurpose M_3 & M_4 to form $M_C - M_D$ C.M.

$$V_{DD} - V_{SG_3} \Big| _{I_0} \left(- g_{m_1} r_A \frac{v_i}{2} \right) \approx - \frac{g_{m_1}}{g_{m_3}} \cdot \frac{v_i}{2} \qquad \{\text{i.e. } g_{m_1} >> g_{ds_1}, g_{ds_3}\}$$

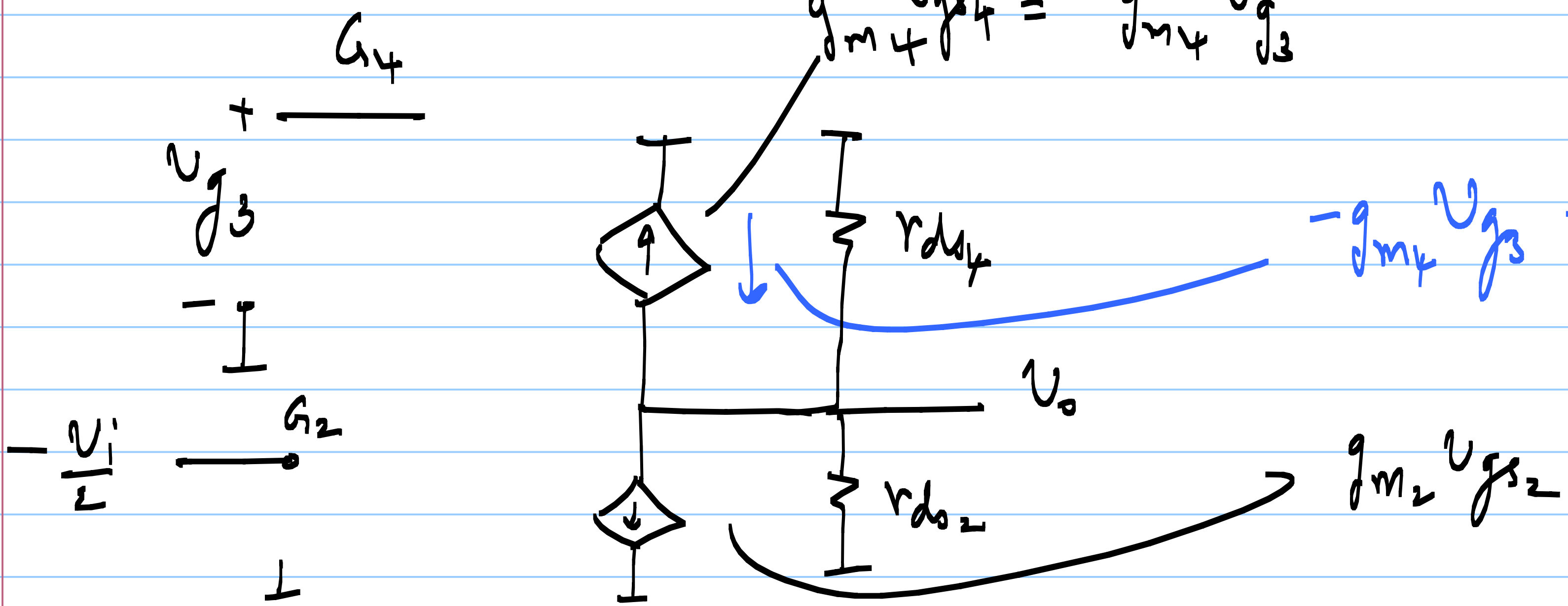


$$r_A = r_{ds_1} \,||\, \frac{1}{g_{m_3}} \,||\, r_{ds_3} \begin{pmatrix}\text{low}\\ \text{imp}\end{pmatrix}$$

$$\approx \frac{1}{g_{m_3}}$$

(originally high imp.)

$$v_{g_3} = \frac{-g_{m_1}}{g_{m_3}} \cdot \frac{v_i}{2}$$

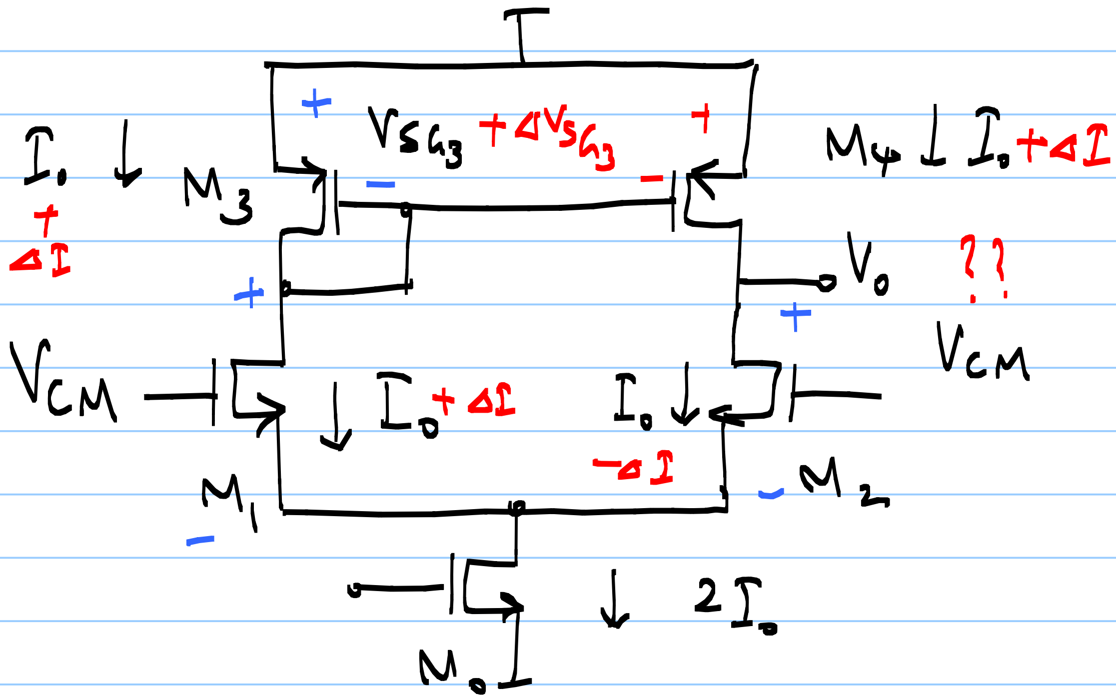


$$-g_{m_4} v_{g_3} = -\cancel{g_{m_4}} \cdot \frac{-g_{m_1}}{\cancel{g_{m_3}}} \cdot \frac{v_i}{2}$$

$$= + g_{m_1} \frac{v_i}{2}$$

7/10/20

Lecture 34



$M_1 \equiv M_2$

$M_3 \equiv M_4$

$$V_{SG_3} = V_{T_3} + \sqrt{\frac{2I_o}{\mu_p C_{ox}\left(\frac{W}{L}\right)_3}}$$

$$= V_{SG_4}$$

?? = 0

$$V_{D_1} = V_{DD} - V_{SG_3} - \Delta V_{SG_3}$$

$\Delta I = 0$; $V_{o_{CM}} = V_{DD} - V_{SG_3}$

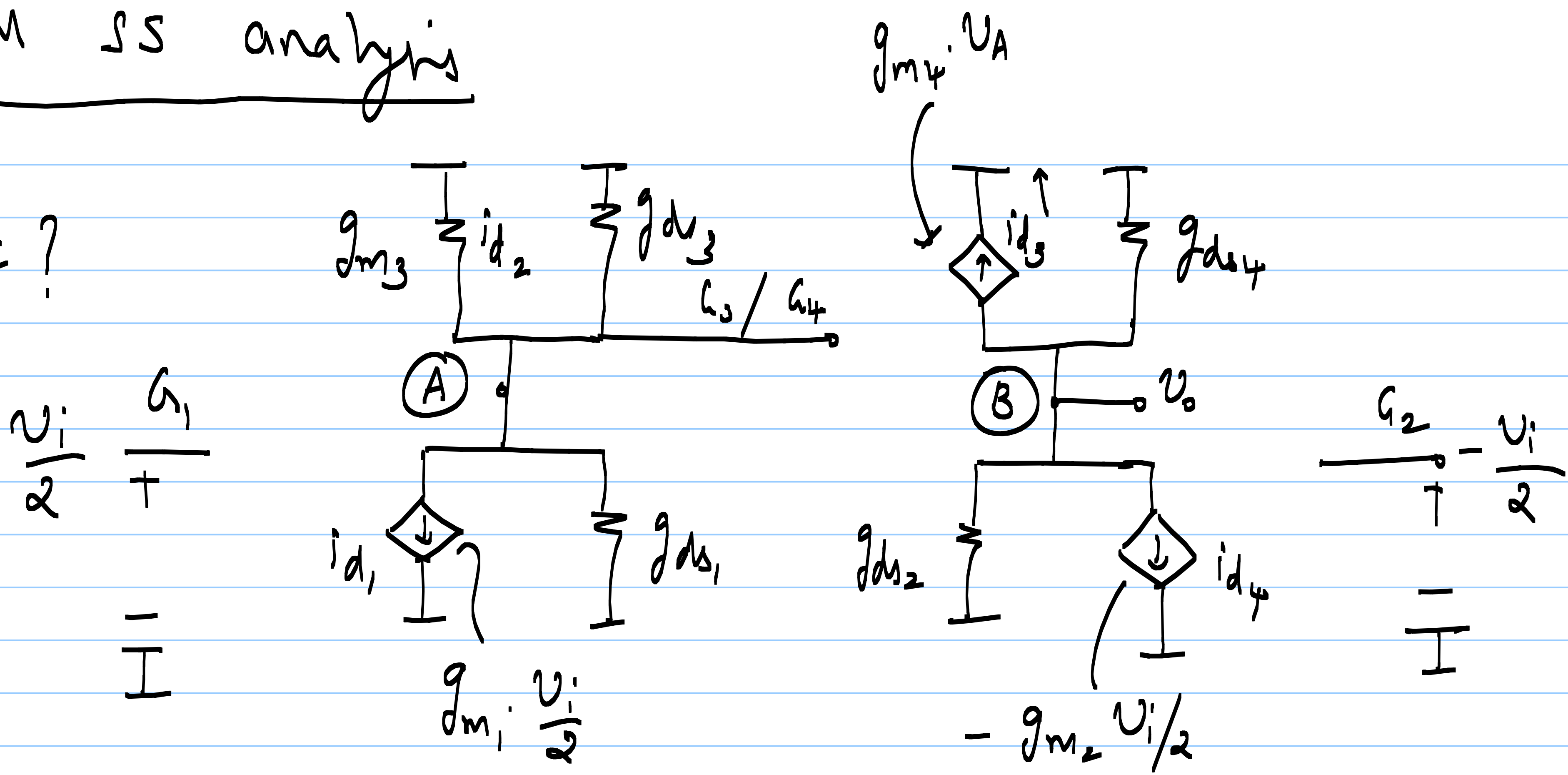
Case 2 : λ's are non-zero

$$I_{D_1} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{GS_1} - V_{T_n})^2 (1 + \lambda V_{DS_1})$$

$$I_{D_2} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_2 (V_{GS_2} - V_{T_n})^2 (1 + \lambda V_{DS_2})$$

DM ss analysis

$$\frac{v_o}{v_i} = ?$$



$$v_A = -\frac{g_{m1}}{g_{m3} + g_{ds1} + g_{ds3}} \cdot \frac{v_i}{2}$$

$$i_{d3} = g_{m4} \cdot v_A = -\frac{g_{m4}}{g_{m3} + g_{ds1} + g_{ds3}} \cdot g_{m1} \frac{v_i}{2}$$

$$\approx -g_{m_1}\frac{v_i}{2} \quad \text{if} \quad g_{m_3} \gg g_{ds_1} \;\&\; g_{ds_3}$$

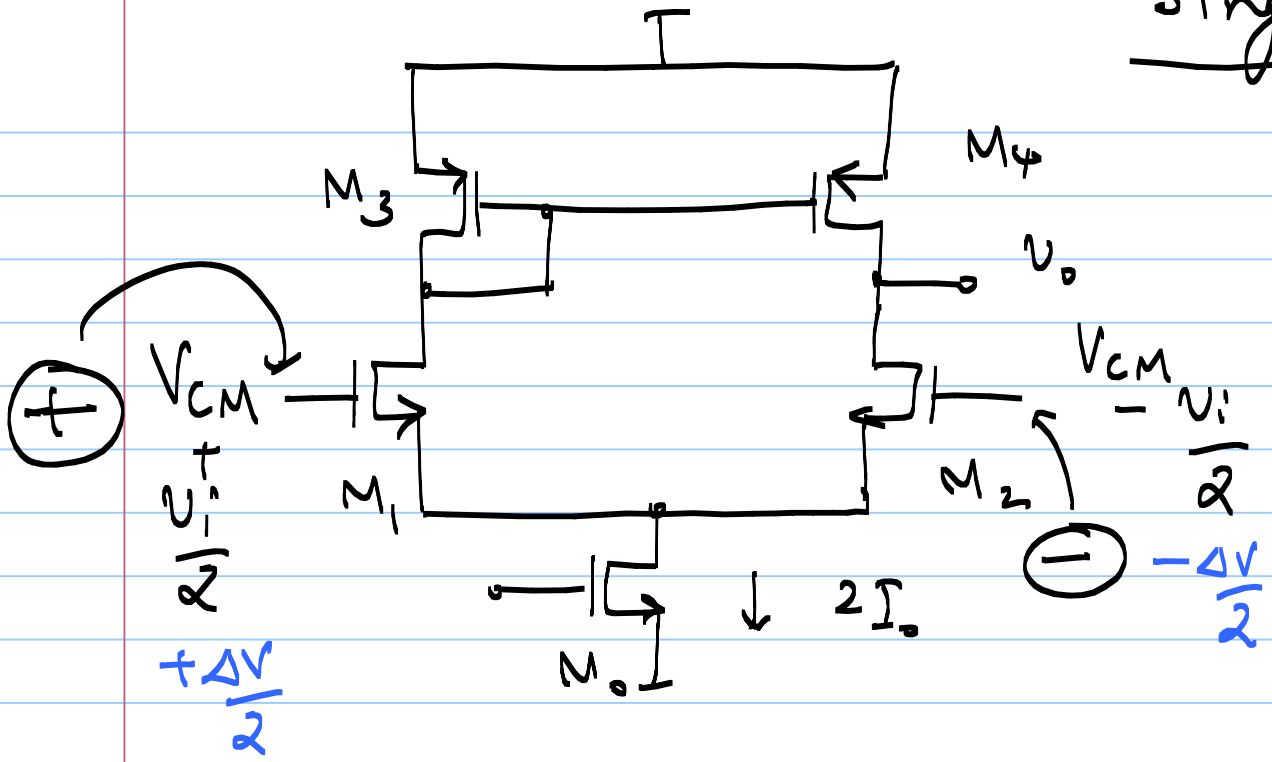
$$v_o = -\left(i_{d_3} + i_{d_4}\right) \cdot \frac{1}{g_{ds_2} + g_{ds_4}}$$

$$= -\left(-g_{m_1}\frac{v_i}{2} - g_{m_2}\frac{v_i}{2}\right) \cdot \frac{1}{g_{ds_2} + g_{ds_4}}$$

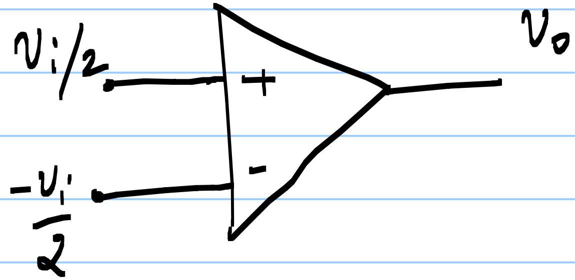
$$= \frac{g_{m_1}}{g_{ds_2} + g_{ds_4}} \cdot v_i$$

$$\frac{v_o}{v_i} = \frac{g_{m_1}}{g_{ds_2} + g_{ds_4}} = g_{m_1}\left(r_{ds_2} \,||\, r_{ds_4}\right)$$

"Single-Stage Opamp"



$\equiv$



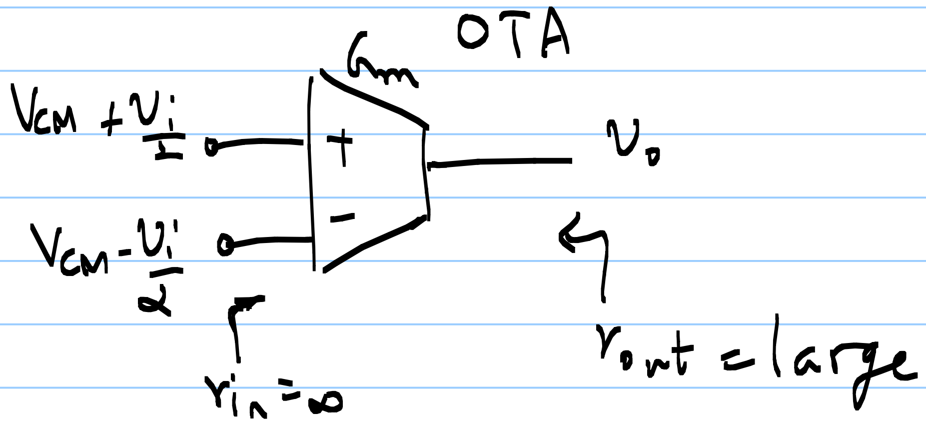
DM gain $= g_{m_1}(r_{ds_2} \| r_{ds_4})$

$r_{in} = \infty$

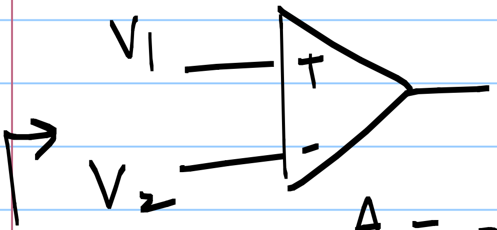
$r_{out} = r_{ds_2} \| r_{ds_4}$ (large)

$G_m = g_{m_1}$

$\equiv$



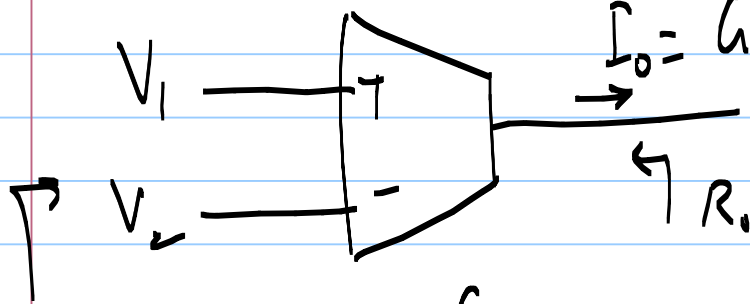
OTA $\equiv$ Operational Transconductance Amplifier



$V_o = A(V_1 - V_2)$ $\equiv$ ideal opamp

$A = \infty$

$R_{in} = \infty$ $R_{out} = 0$



$I_o = G_m(V_1 - V_2)$ $\equiv$ ideal OTA

$R_{out} = \infty$

$G_m = \infty$

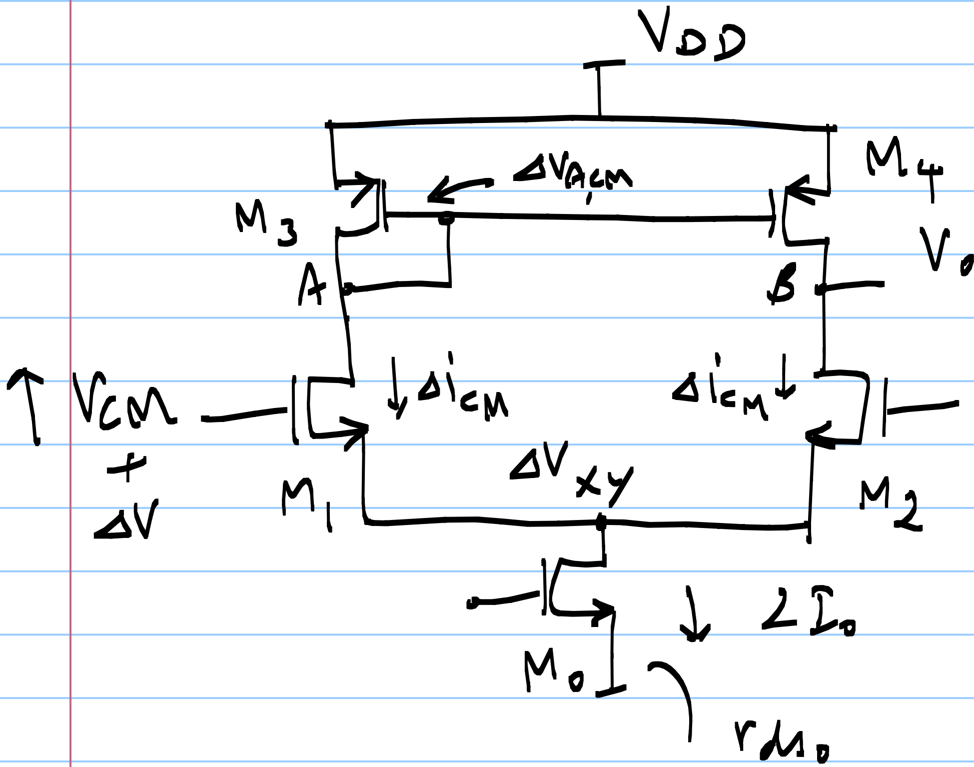
$R_{in} = \infty$

$$\text{gain} = \frac{V_o}{V_1 - V_2} = G_m R_{out} = \text{large}$$

8/10/2020

Lecture 35

"Single Stage Opamp"



$V_{oCM} = V_{DD} - V_{SG_3} + \underline{\underline{\Delta V_{oCM}}}$

$\frac{\Delta V_{oCM}}{\Delta V} = A_{CM}$ {HW}

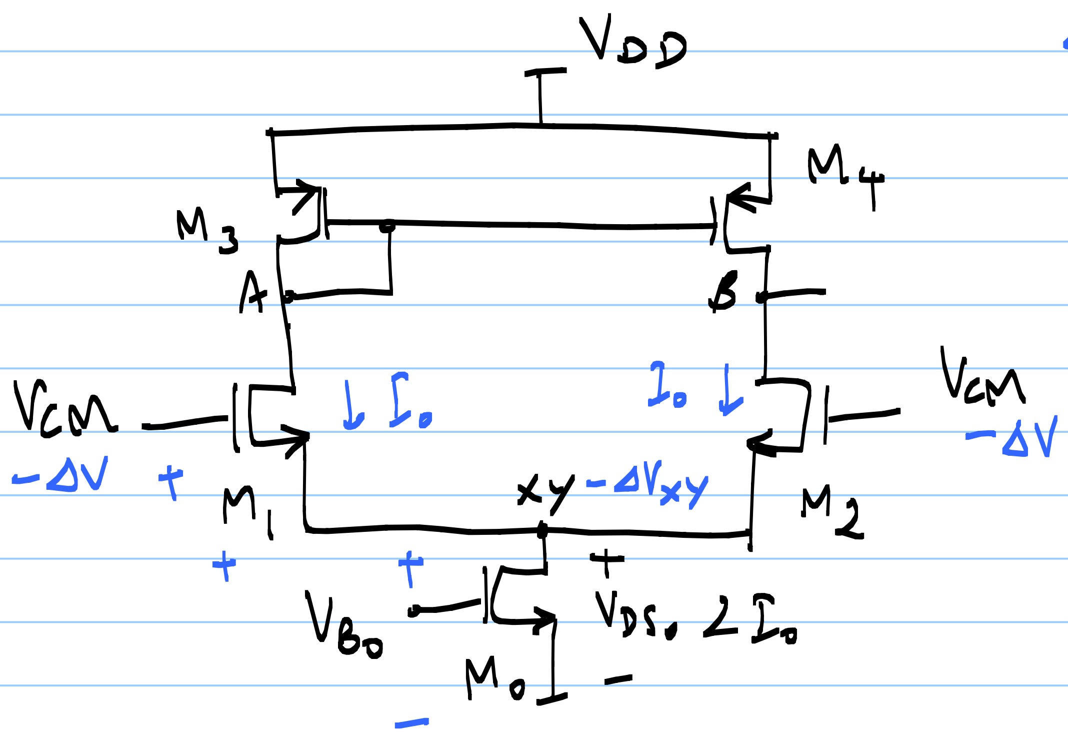
V_{CM} = input CM level → range?

V_{oCM} = Output CM level → range?

ICMR - input CM range

OCMR - output CM range

ICMR : 1) keep ↓ V_{CM}



$\Delta V_{xy} = \Delta V$ if M_o current does not change

ΔV_{xy} slightly less than ΔV if M_o has large r_{ds_o}

$\{ V_{DS_o} \downarrow \}$

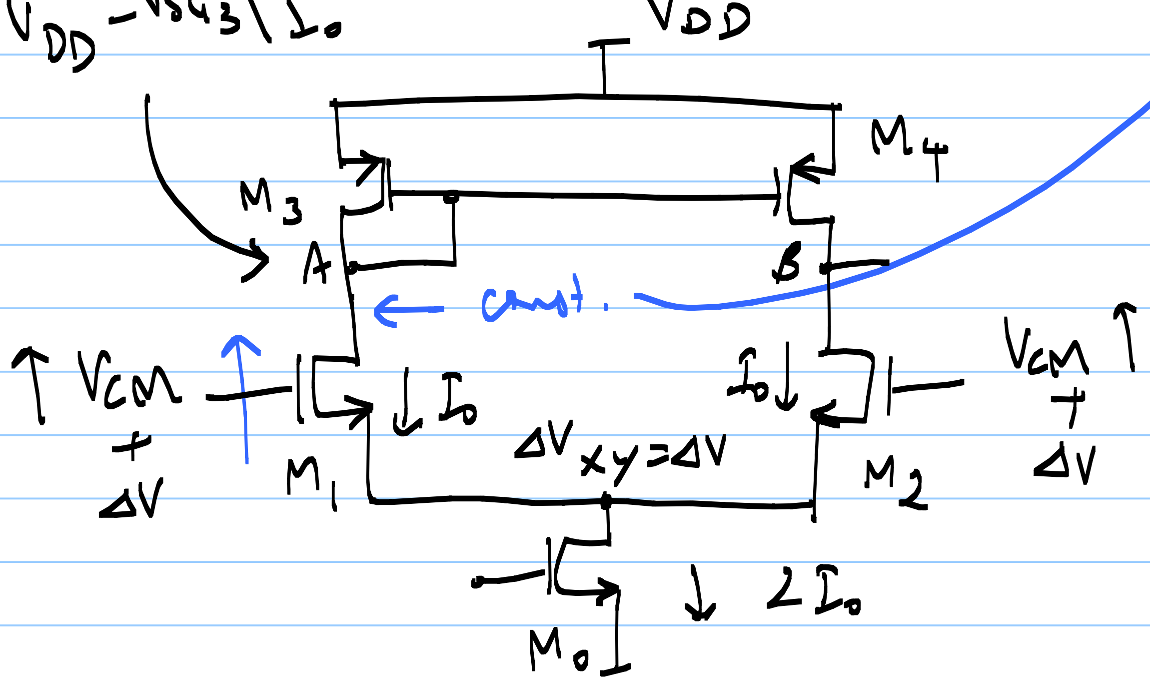
Eventually M_o will go into triode region when $V_{DS_o} = V_{B_o} - V_{T_o} = V_{Dsat_o}$

$$V_{CM_{min}} = V_{Dsat_o}\Big|_{2I_o} + V_{GS_1}\Big|_{I_o}$$

2) $V_{CM} \uparrow \rightarrow V_{CM_{max}}$

M_o moves away from triode boundary

$V_A = V_{DD} - V_{SG_3}\big|_{I_o}$ = constant

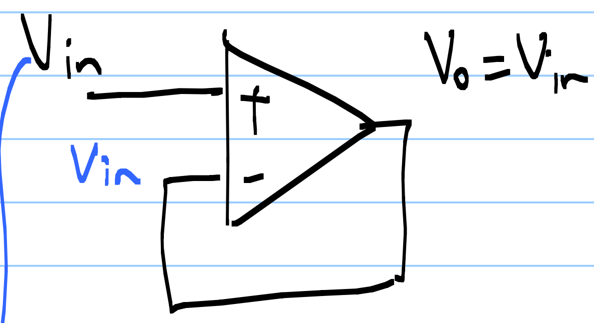


Moving M_1 & M_2 towards triode boundary

M_1 & M_2 will go into triode!

$$V_{D_1} = V_{G_1} - V_{T_1}$$

(DC)

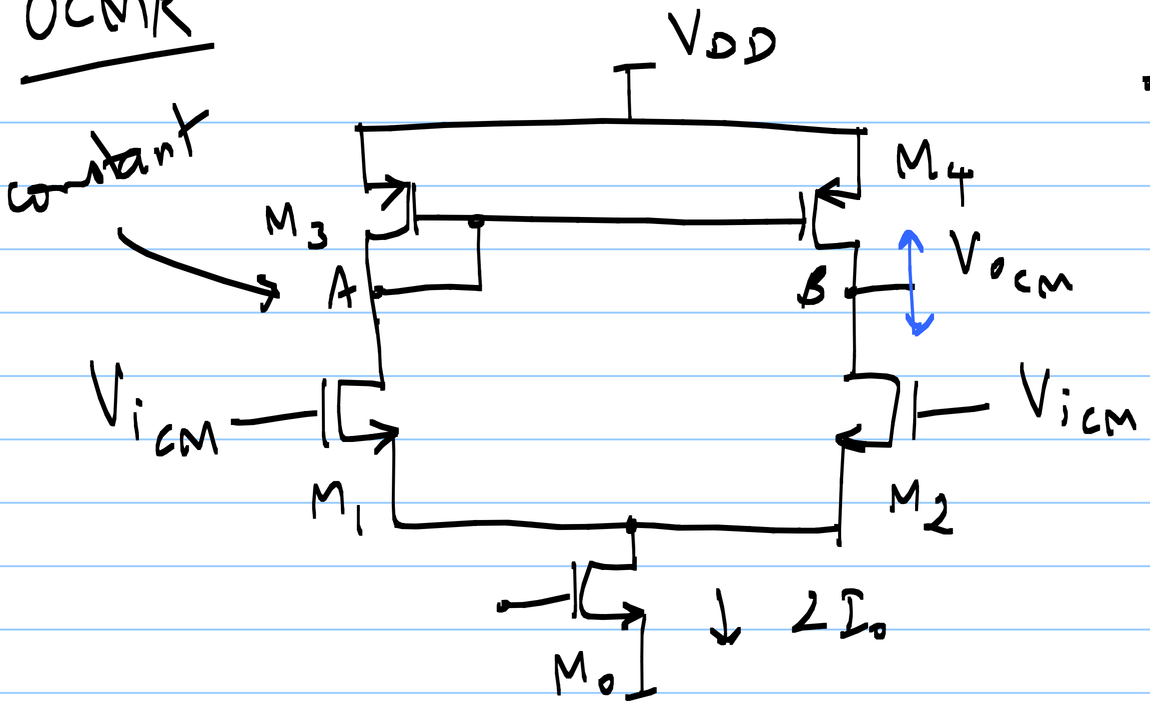


$$V_{DD} - V_{SG_3}\big|_{I_o} = V_{CM_{max}} - V_{T_1}$$

$$V_{CM_{max}} = V_{DD} - V_{SG_3}\big|_{I_o} + V_{T_1}$$

$$ICMR = \{ V_{CM_{min}}, V_{CM_{max}} \}$$

OCMR



* V_{ocm} is set by f.b.

1) $V_{ocm_{max}}$ = voltage at which M_4 goes into triode

$$V_{D_4} = V_{G_4} + V_{T_4}$$

$$V_{ocm_{max}} = V_A + V_{T_4}$$

$$V_A = V_{DD} - V_{SG_3}\big|_{I_0}$$

$$V_{ocm_{max}} = V_{DD} - \underbrace{V_{SG_3}\big|_{I_0}}_{V_{T_3} + V_{SD_{sat_3}}} + V_{T_4}$$

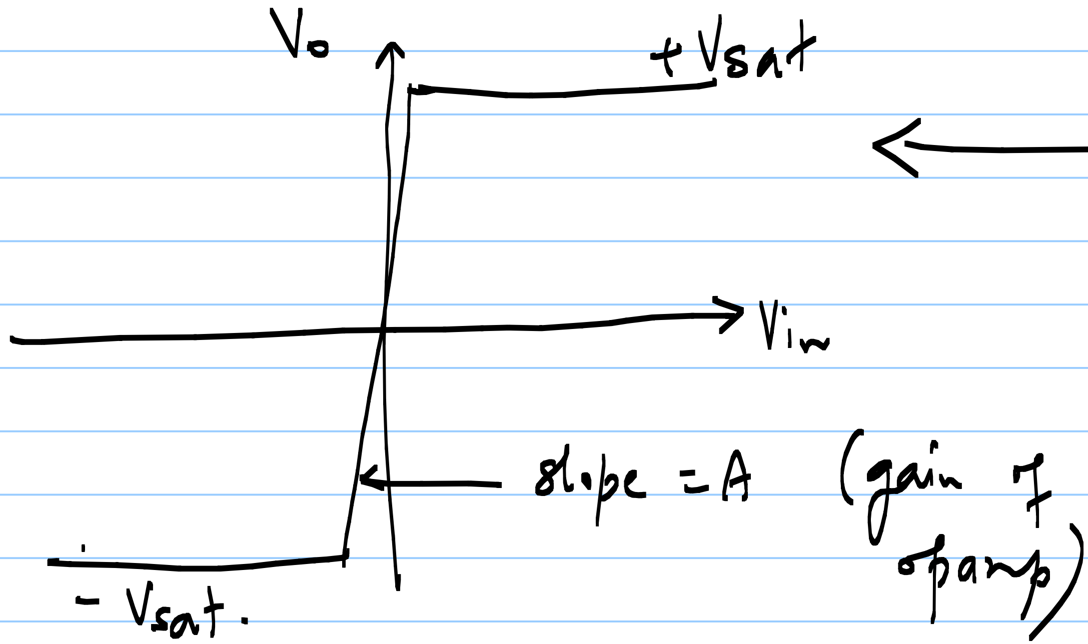
$$V_{ocm_{max}} = V_{DD} - V_{SD_{sat_4}}$$

2) $\downarrow V_{ocm} \rightarrow V_{ocm_{min.}}$ = voltage at which M_2 goes into triode

$$V_{D_2} = V_{G_2} - V_{T_2}$$

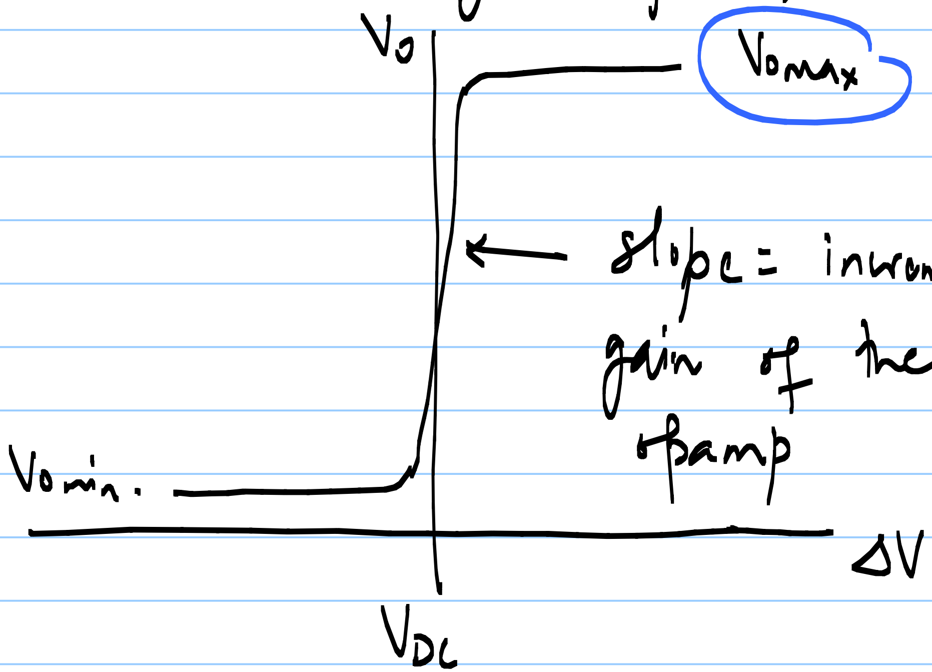
$$V_{OCM_{min.}} = \underbrace{V_{iCM}}_{\text{range of values within ICMR}} - V_{T_2}$$

$$OCMR = \{ V_{CM_{min.}}, V_{CM_{max}} \}$$

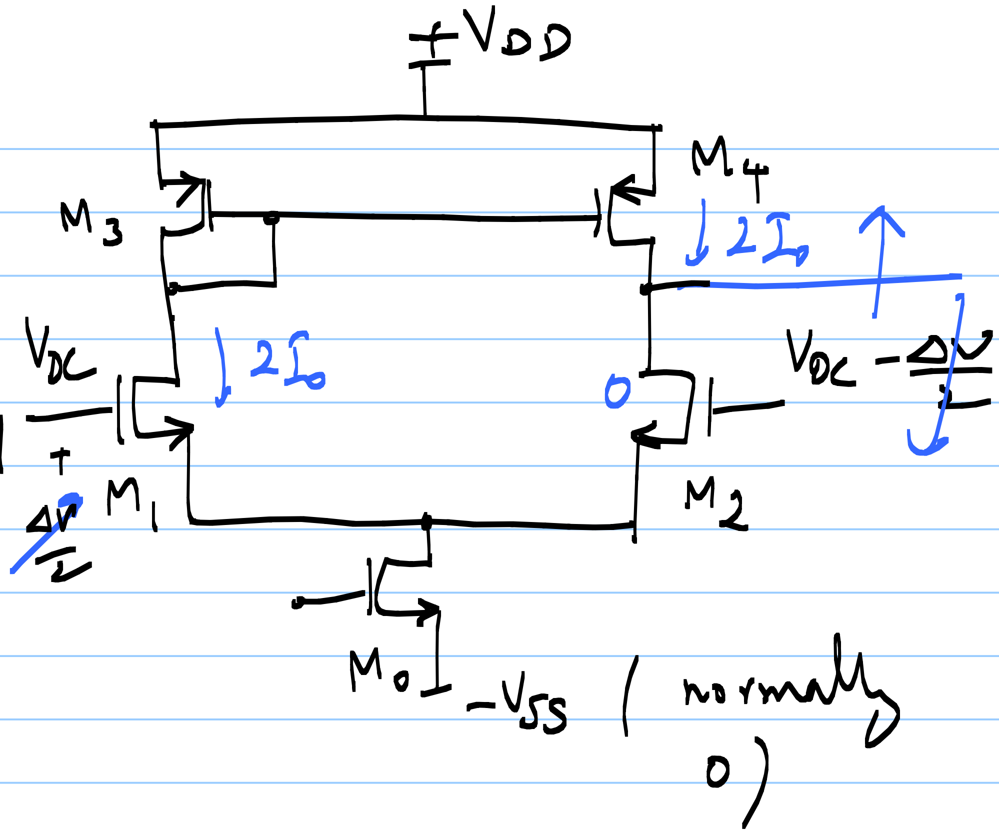


Ideal opamp characteristics

One stage opamp :

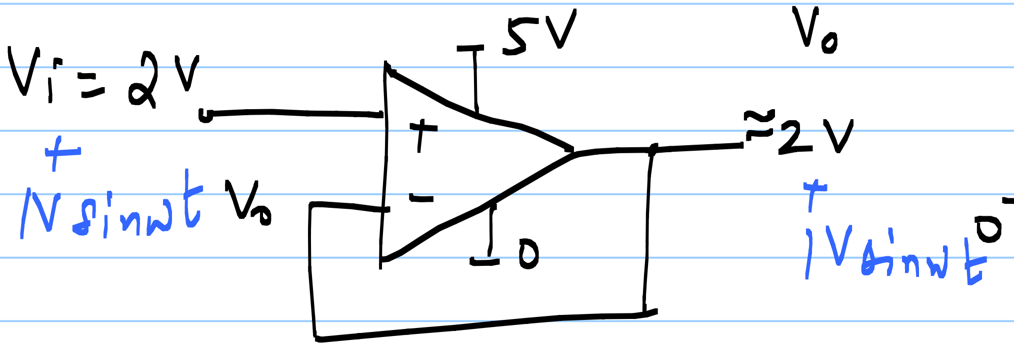


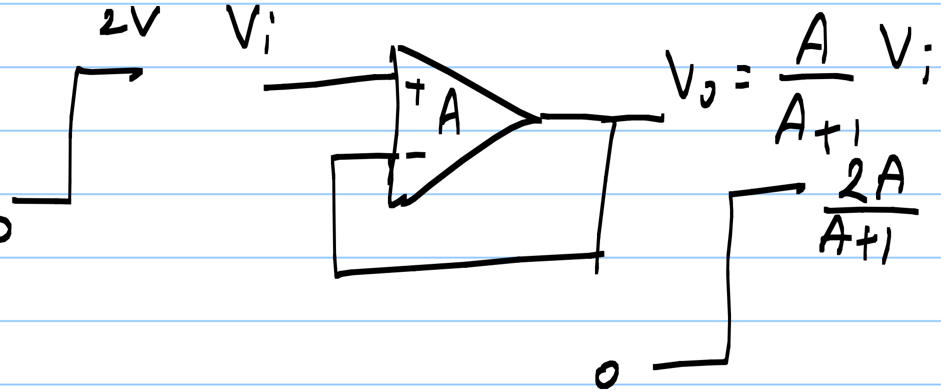
slope = incremental gain of the opamp



9/10/2020

Lecture 36





$A = 100$

here $V_o = \frac{100}{101} \cdot 2V$

$$V_{CM} = \frac{V_+ + V_-}{2} = \frac{V_i + V_o}{2} = \frac{1}{2}\left[V_i + \frac{100}{101} V_i\right]$$

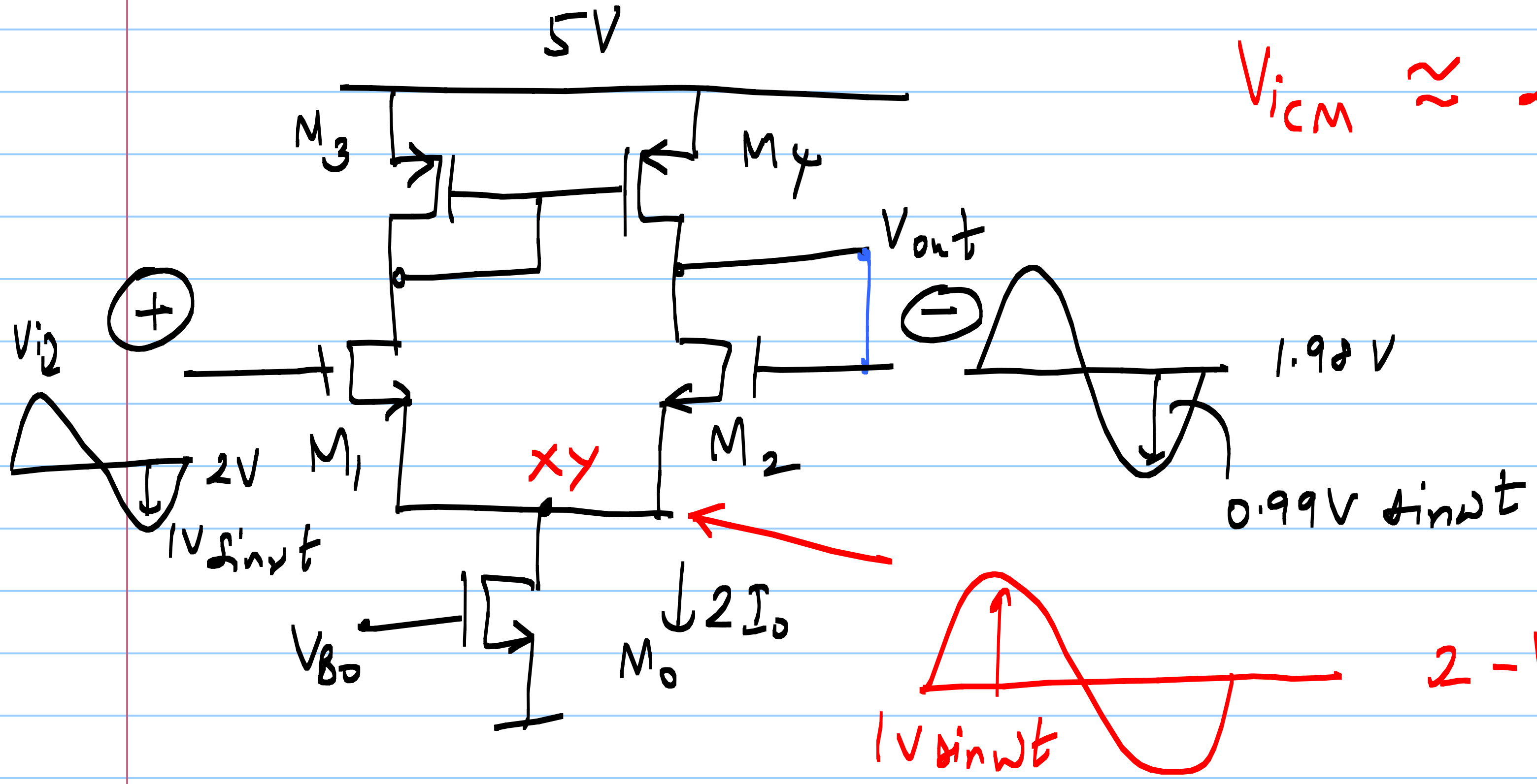
$$= \frac{201}{202} \cdot V_i \approx V_i$$

$$V_{Dm} = \frac{V_+ - V_-}{2} = \frac{V_i - V_o}{2} = \frac{1}{2}\left[V_i - \frac{100}{101} V_i\right]$$

$$V_{DM} = \frac{V_i}{2} \cdot \frac{1}{101} = \frac{V_i}{202}$$

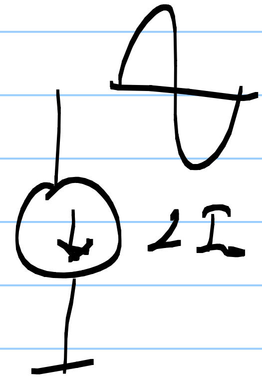
$$V_+ = V_{CM} + V_{DM} = \frac{201}{202} V_i + \frac{V_i}{202} = V_i$$

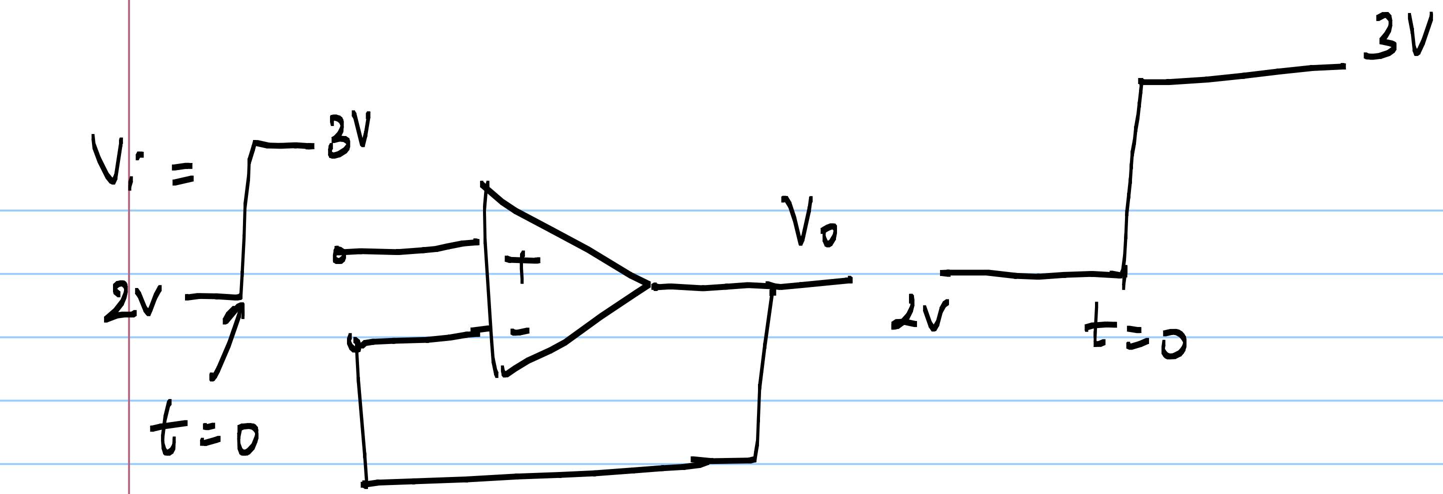
$$V_- = V_{CM} - V_{DM} = \frac{100}{101} V_i$$

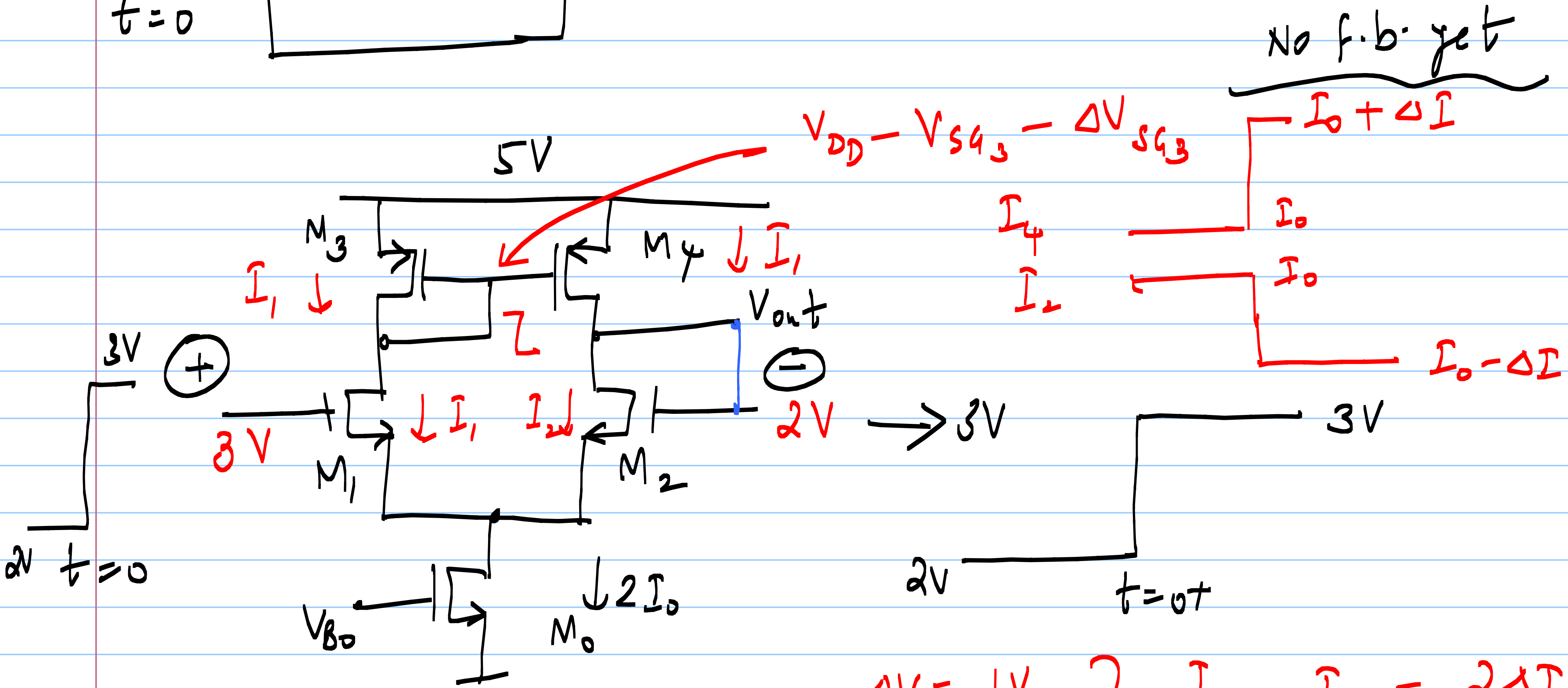


$V_{iCM} \approx 2V + 1V\sin\omega t$

$g_m, r_{ds_0} >> 1$

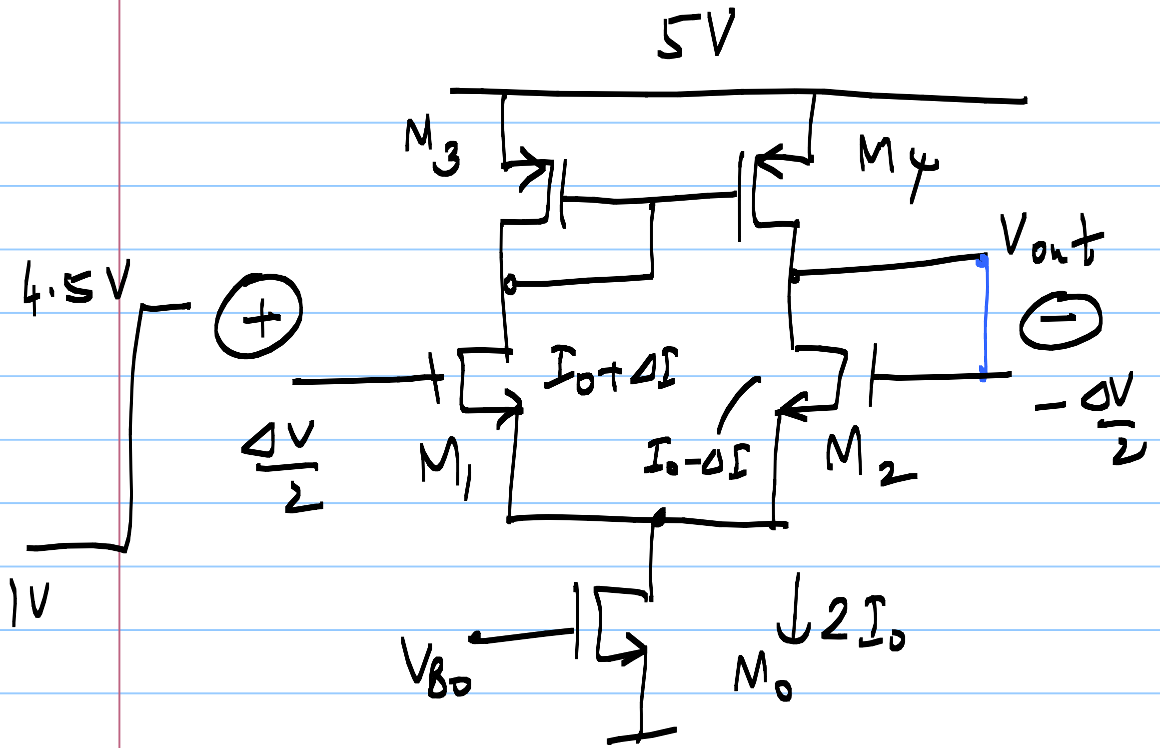






$$\Delta V = 1V$$

$$\left.\begin{array}{l} I_1 = I_0 + \Delta I \quad ; \quad I_2 = I_0 - \Delta I \end{array}\right\} \quad I_4 - I_2 = 2\Delta I$$



ΔI depends on $\frac{\Delta V}{2}$

larger $\frac{\Delta V}{2} \rightarrow$ larger ΔI

small signals: $\Delta I = g_{m_1} \frac{\Delta V}{2}$

large signals (large ΔV)

larger $\frac{\Delta V}{2} \rightarrow$ larger ΔI

still true

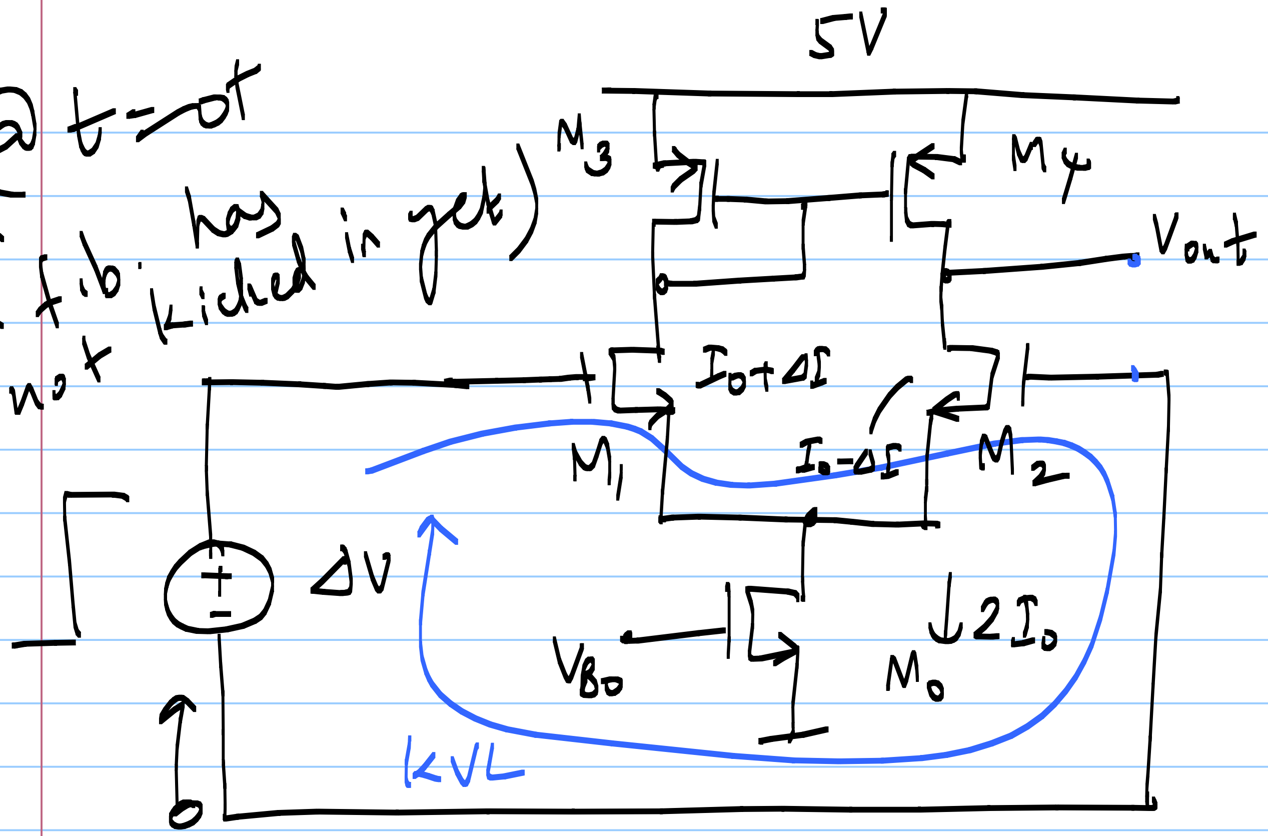
$M_2 = 0$

largest possible $\Delta I = I_0$ $\{ I_1 = 2I_0, \ I_2 = 0 \}$

Beyond this: larger $\Delta V \not\Rightarrow$ larger ΔI

@ $t = 0^+$

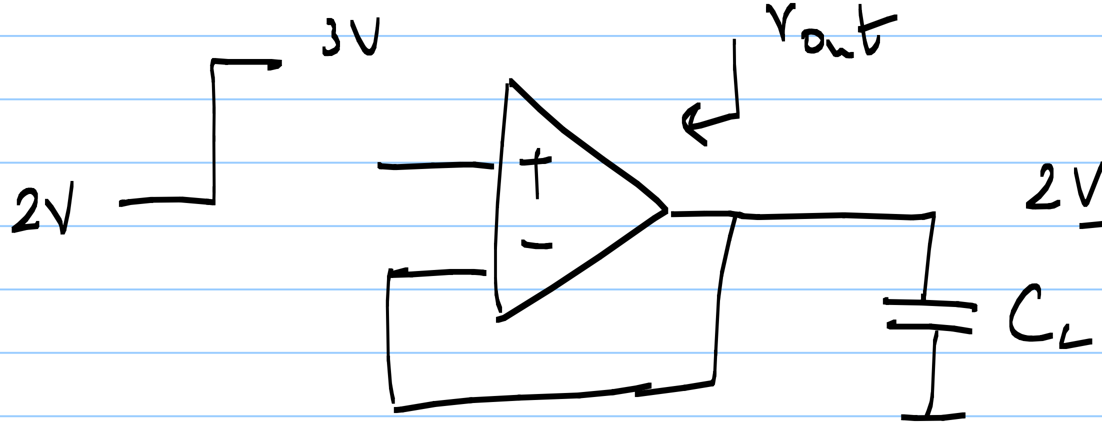
(fb has not kicked in yet)

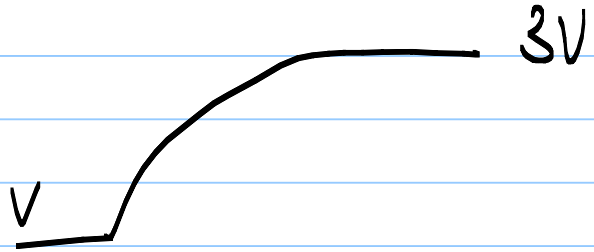


KVL

$$+\Delta V - V_{GS_1} + V_{GS_2} = 0$$

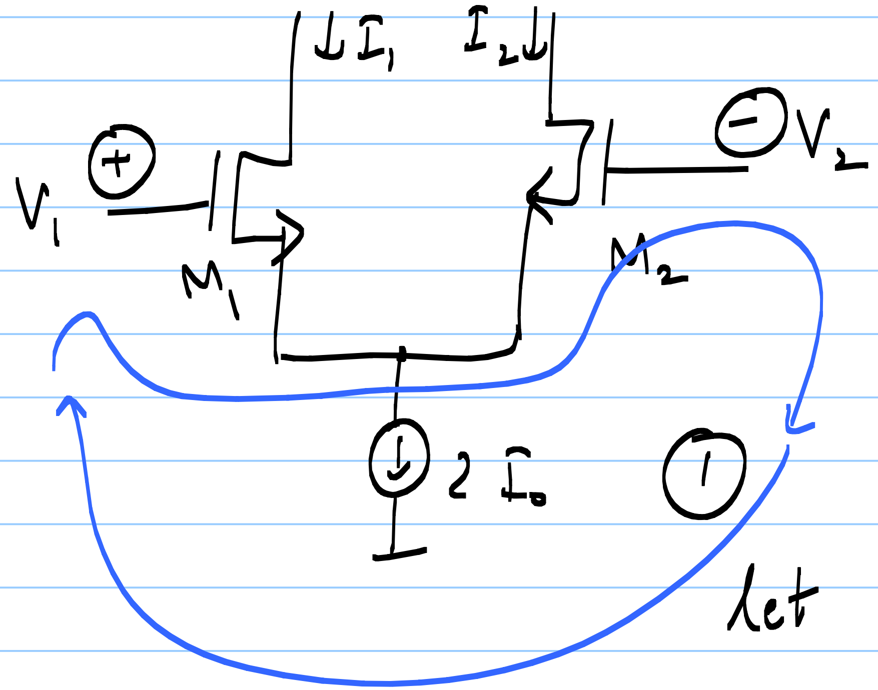
$$\Delta V = V_{GS_1} - V_{GS_2}$$





13|10|2020

Lecture 37



$V_{id} = V_1 - V_2$

$V_{id} = 0 \Rightarrow I_1 = I_2 = I_o$

$I_{od} = I_1 - I_2$

$= g_m V_{id}$ for small signals / increments

let $k' = \mu_n C_{ox}$

KVL around 1 : $V_1 - V_{GS_1} + V_{GS_2} - V_2 = 0$

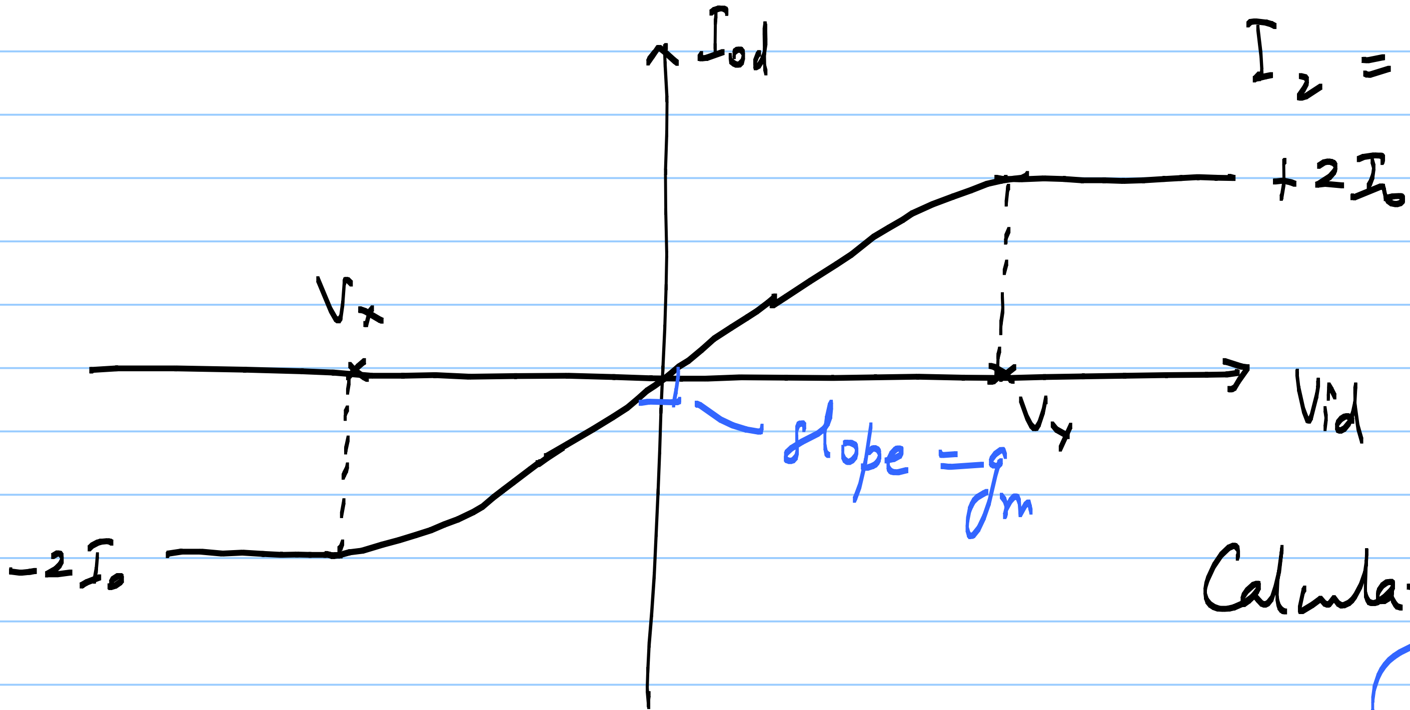
$$V_{id} = V_1 - V_2 = V_{GS_1} - V_{GS_2}$$

$$= \left[V_{T_1} + \sqrt{\frac{2I_1}{k'\left(\frac{W}{L}\right)_1}}\right] - \left[V_{T_2} + \sqrt{\frac{2I_2}{k'\left(\frac{W}{L}\right)_2}}\right]$$

$$V_{id} = \sqrt{\frac{2}{k'\left(\frac{W}{L}\right)}}\left[\sqrt{I_1} - \sqrt{I_2}\right] \quad \text{—— } (A)$$

$$I_1 + I_2 = 2I_0 \quad \text{—— } (B)$$

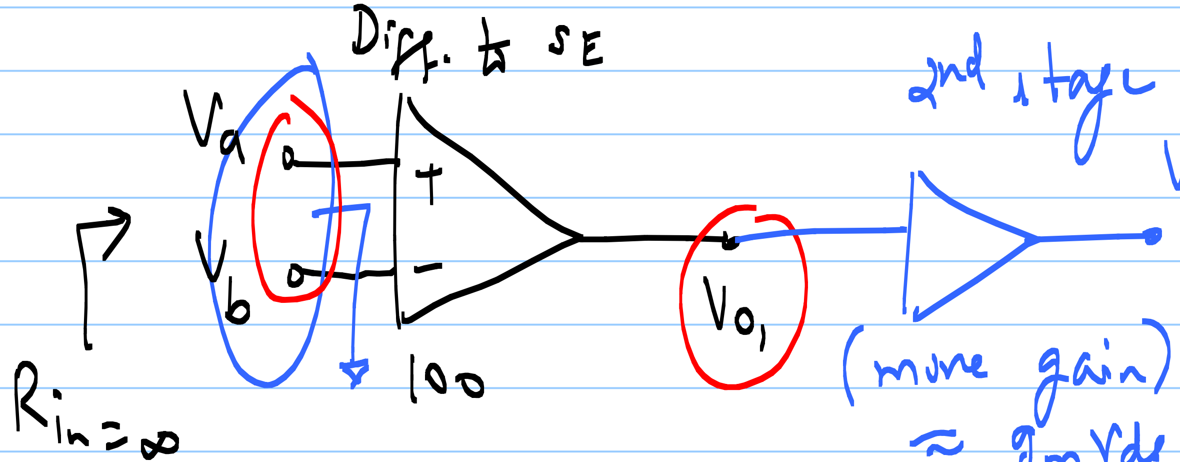
Use (A) & (B) to get $I_1 = f(V_{id})$, $I_2 = g(V_{id})$ (HW)



Calculate V_x & V_y (HW)

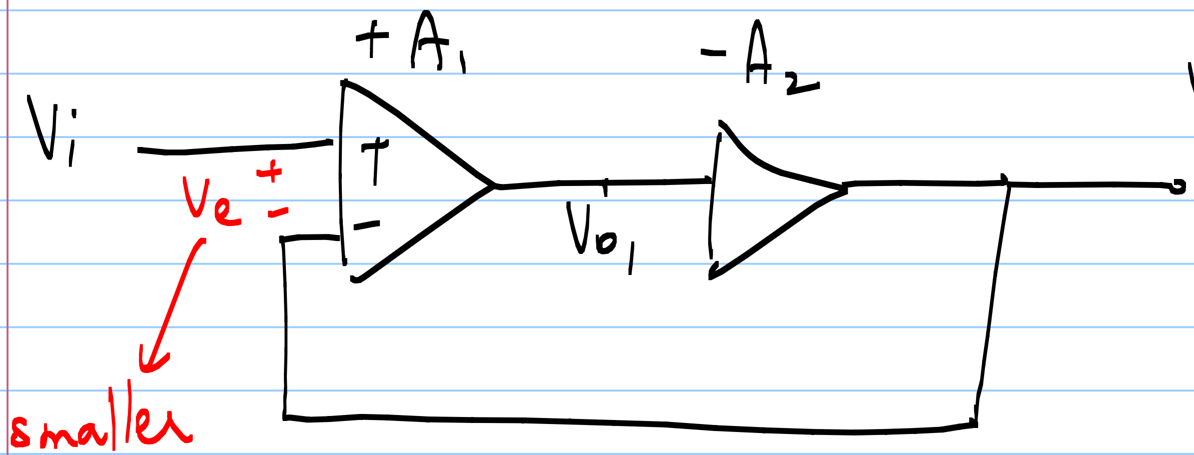
V_x & V_y : currents become $2I_0$ & 0

One stage opamp gain $\approx g_m (r_{ds_2} \| r_{ds_4})$



(more gain) $\approx g_m r_{ds} \approx 100$

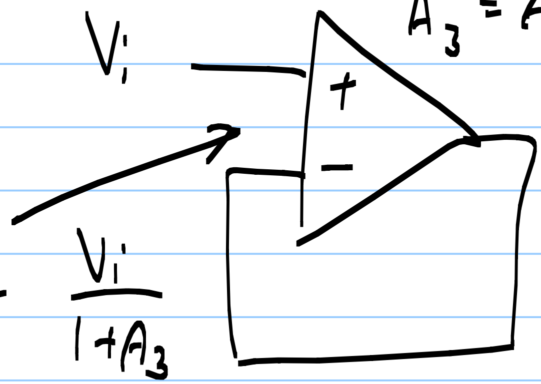
overall 2-stage gain $= 10^4$



$$V_o = \frac{1}{1 + \frac{1}{A_1 A_2}} V_i = \frac{A_1 A_2}{1 + A_1 A_2} \cdot V_i$$

$$V_e = \frac{V_i}{1 + A_1 A_2} \; ; \; V_{o_1} = \frac{A_1 V_i}{1 + A_1 A_2} = \frac{V_i}{A_2 + \frac{1}{A_1}}$$

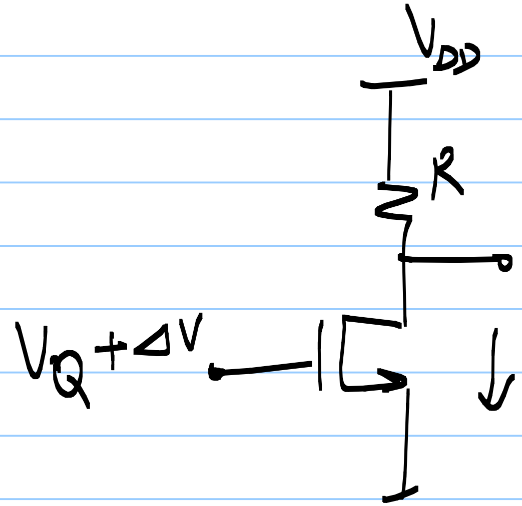
$A_3 = A_1$



$$V_o = \frac{A_3}{1+A_3} V_i$$

$$V_{e_3} = \frac{V_i}{1+A_3}$$

$$= \frac{V_i}{1+A_1}$$

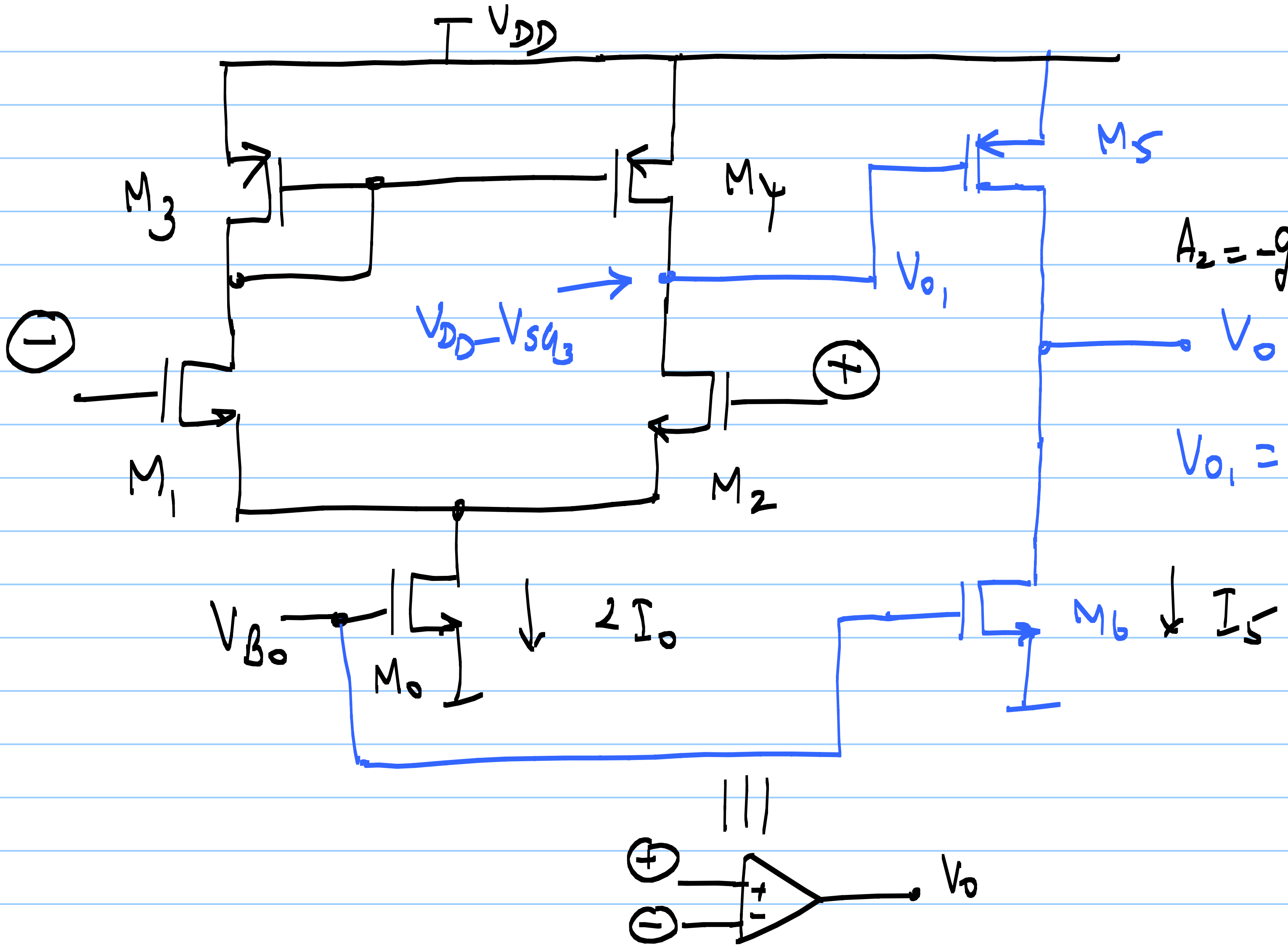


$$g_m = f(V_Q \text{ or } I_Q)$$

$V_{DD} - I_Q R - g_m R \Delta V$

$\downarrow I_Q + g_m \Delta V$

2-stage opamp:

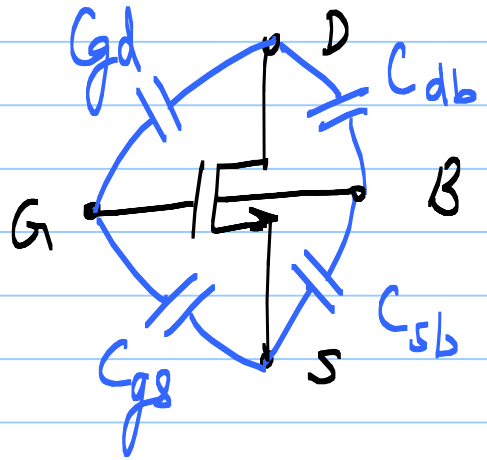


$$A_1 = g_{m_1}(r_{ds_2} \| r_{ds_4})$$

$$A_2 = -g_{m_5}(r_{ds_5} \| r_{ds_6})$$

$$V_{o_1} = -g_{m_1}(r_{ds_2} \| r_{ds_4})$$

* Every MOSFET has a speed limitation/delay

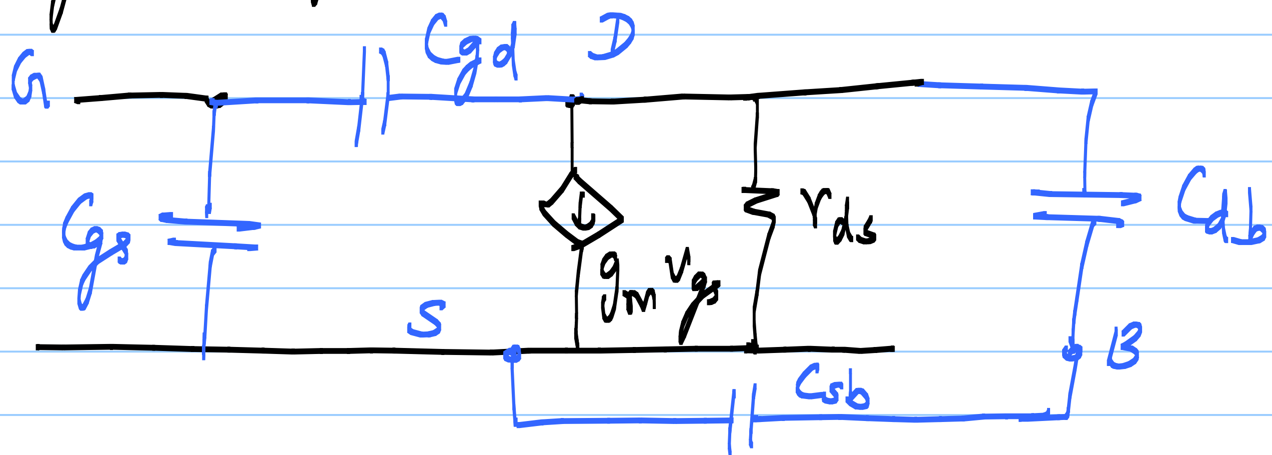


bulk/substrate terminal {normally signal ground}

B = gnd for NMOS

B = Vdd for PMOS

AC small signal eq. cir.:



smallest cap.

* In sat. : $C_{gs} \gg C_{gd}, C_{db}, C_{sb}$

largest cap.

* $C_{gs} \approx \frac{2}{3} W \cdot L \cdot C_{ox}$ in saturation

fF

μm

fF/μm²

$V_1 = 3V$ $V_2 = 4V$

$V_{CM} = 3.5V$

$V_{DM} = -0.5V$

14|10|2020

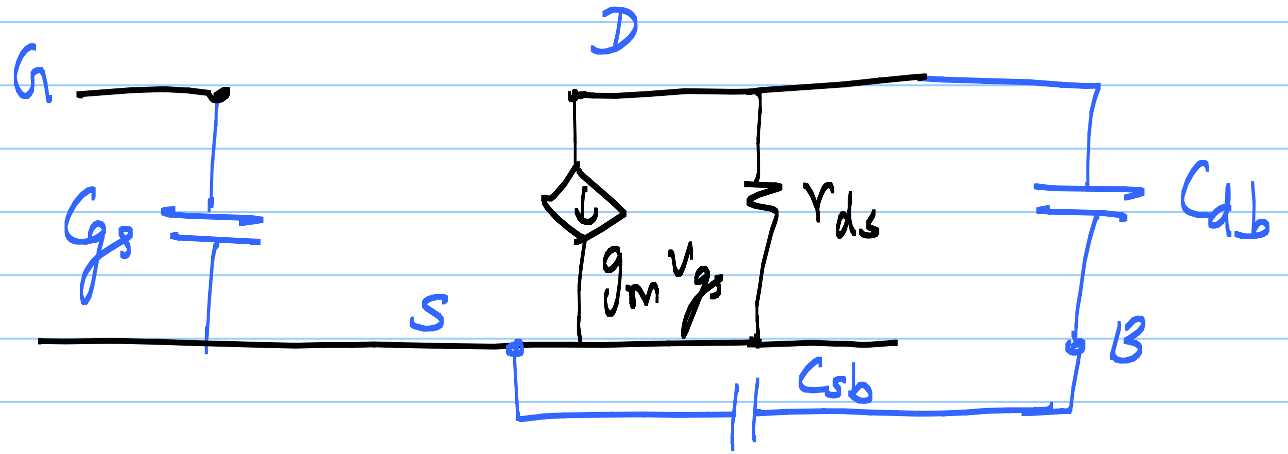
Lecture 38

* MOSFET caps — C_{gs}, C_{gd}, C_{db}, C_{sb}

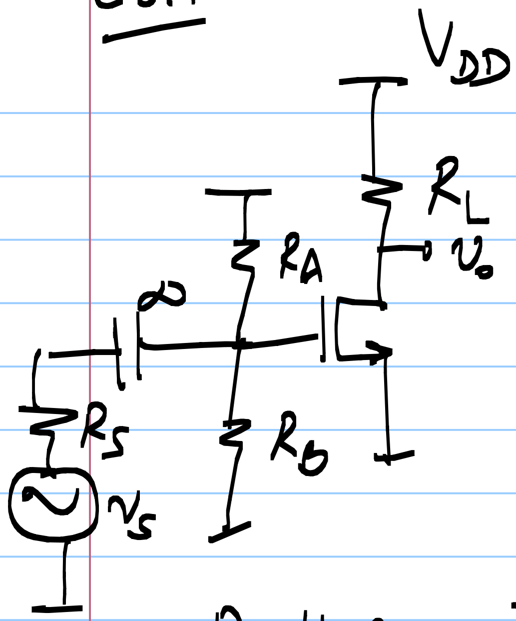
largest (C_{gs})   smallest (C_{gd})

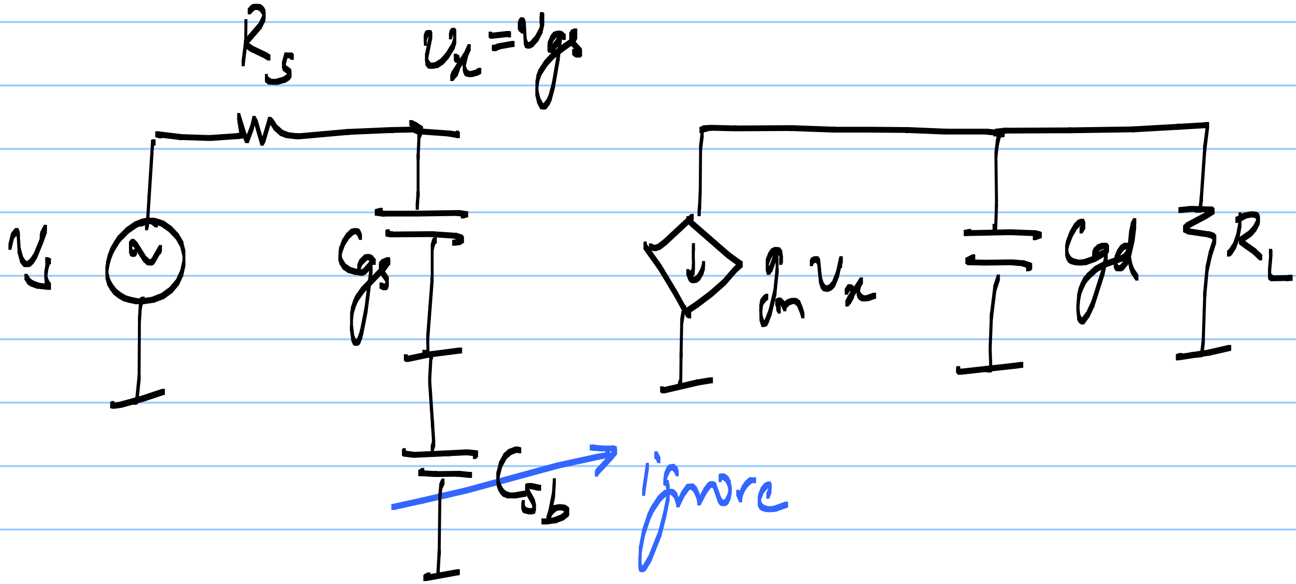
$\frac{2}{3} WL C_{ox}$

1) Assume C_{gd} is negligible



CSA





$$R_A \| R_B \gg R_S$$

$$v_x(s) = \frac{1/sC_{gs}}{R_S + 1/sC_{gs}} \cdot v_s(s)$$

$$v_o(s) = -g_m \cdot \frac{1}{G_L + sC_{db}} \cdot v_x(s)$$

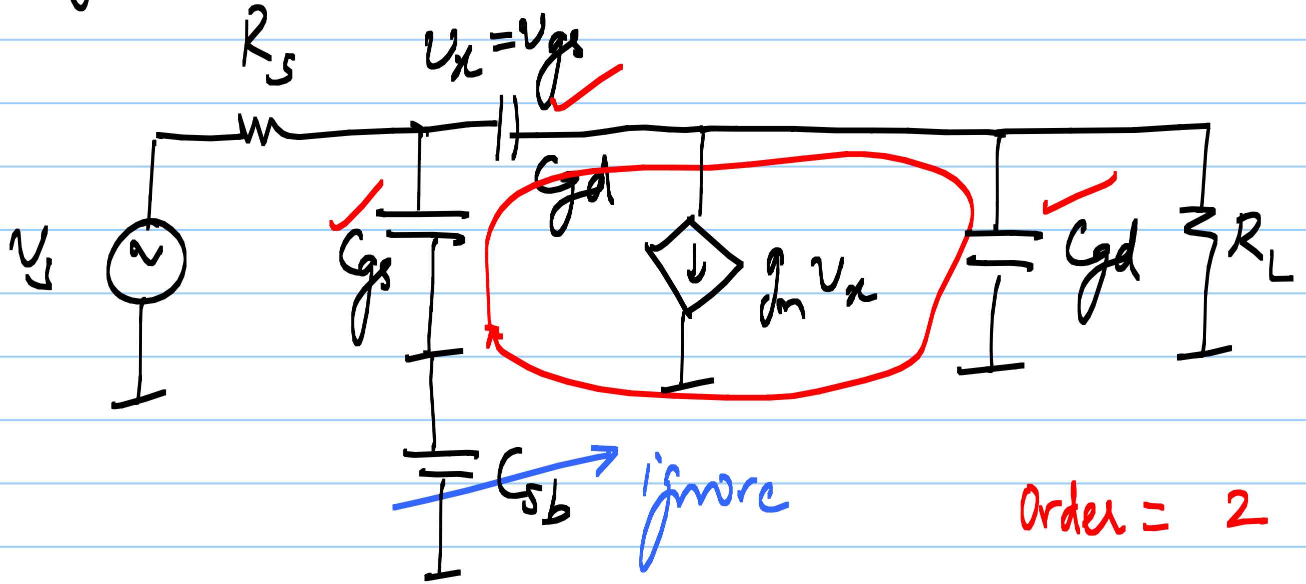
$$\frac{v_o}{v_s}(s) = \frac{1}{1 + sC_{gs}R_s} \cdot \frac{R_L}{1 + sC_{db}R_L} \cdot -g_m$$

$$\frac{v_o}{v_s}(s) = (-g_m R_L) \cdot \frac{1}{(1+sC_{gs}R_s)(1+sC_{db}R_L)}$$

input pole $p_1 = -\frac{1}{R_s C_{gs}}$

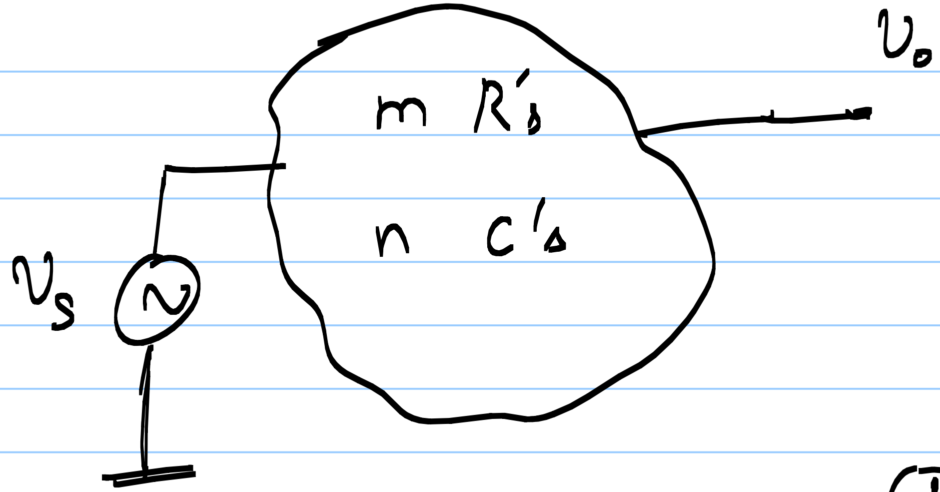
output pole $p_2 = -\frac{1}{R_L C_{db}}$

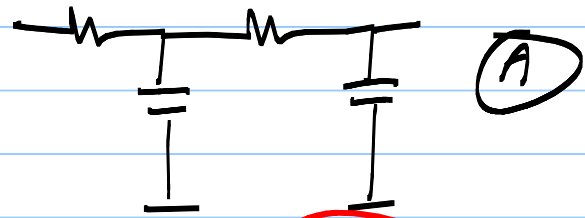
2) <u>With C_{gd}</u>:



Order = 2 $\underbrace{\{D(s)\}}_{}$ quadratic

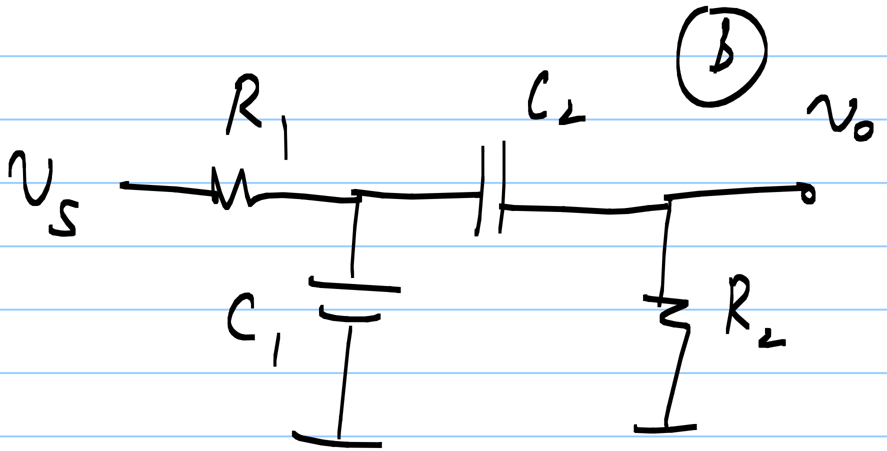
2 poles + 1 zero



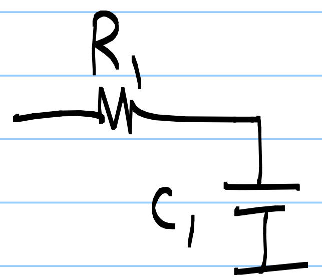


$$\frac{v_o}{v_s} = \frac{N(s)}{D(s)}$$

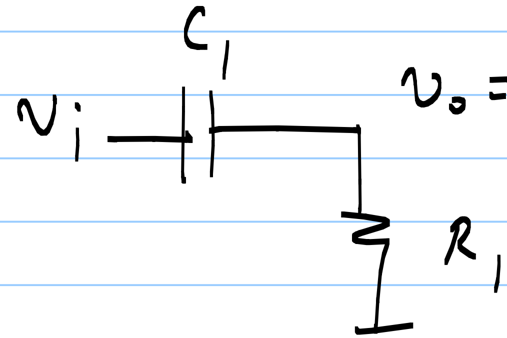
order = degree of $D(s)$



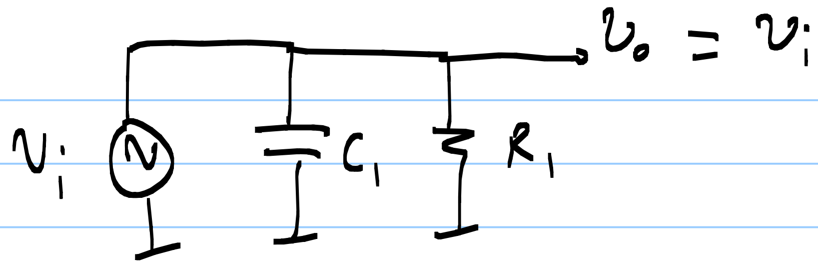
1 cap. ⇒ order = 1



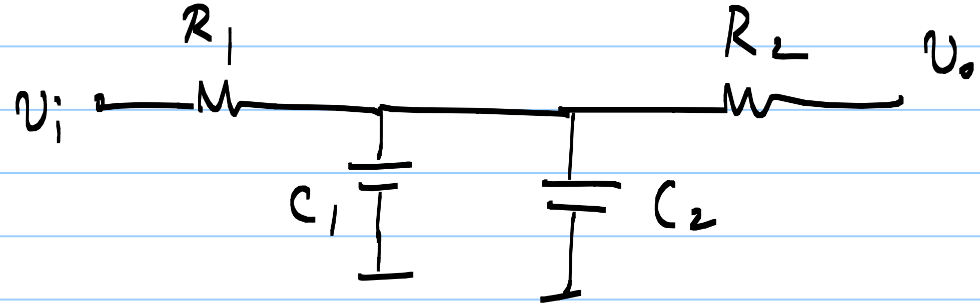
or



$$v_o = \frac{sC_1R_1}{1+sC_1R_1}$$

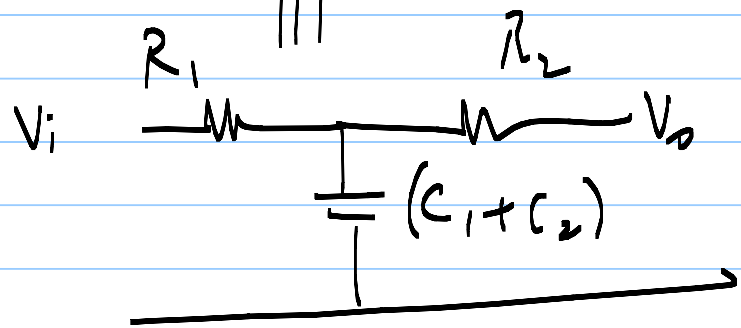


order = 0



order = 1

$\equiv$



Order = # of capacitors − # of all-capacitor loops − # of capacitor-voltage source loops

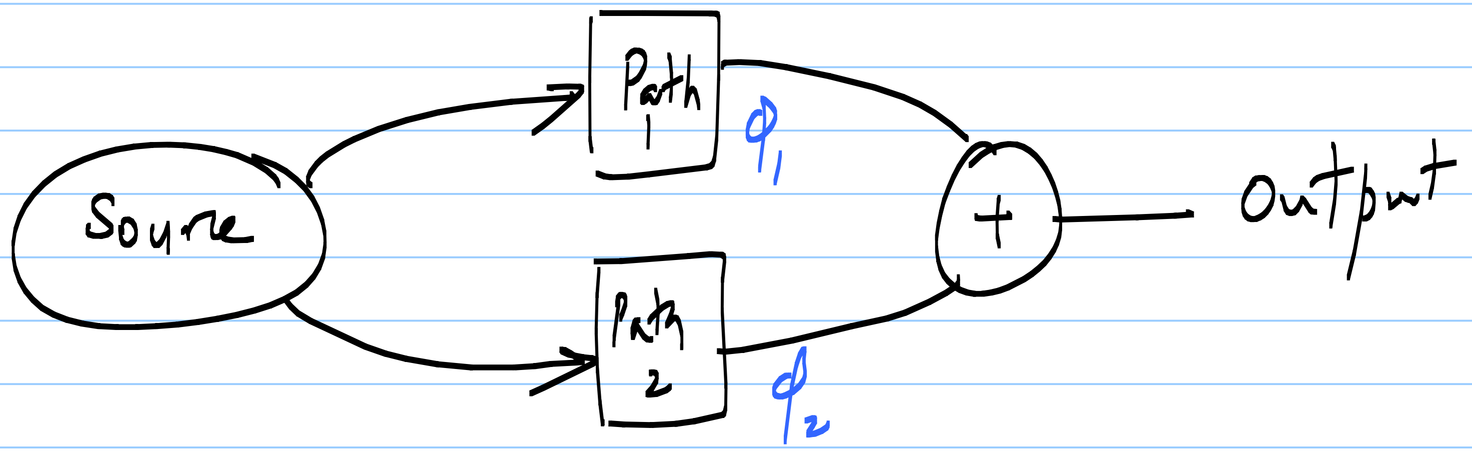
= Degree of $D(s)$

$N(s)$ degree = # of zeroes

$$\frac{v_o}{v_s}(s) = \frac{N(s)}{D(s)}$$

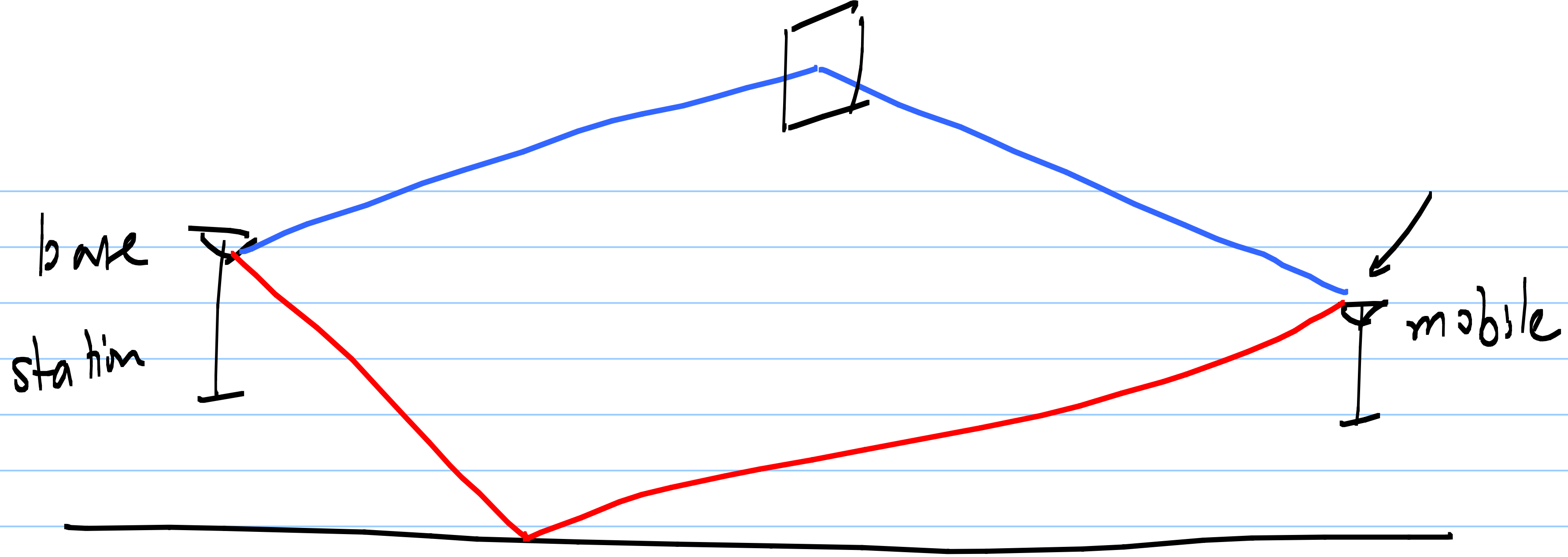
@ zero frequency ('s), $v_o(s) = 0$

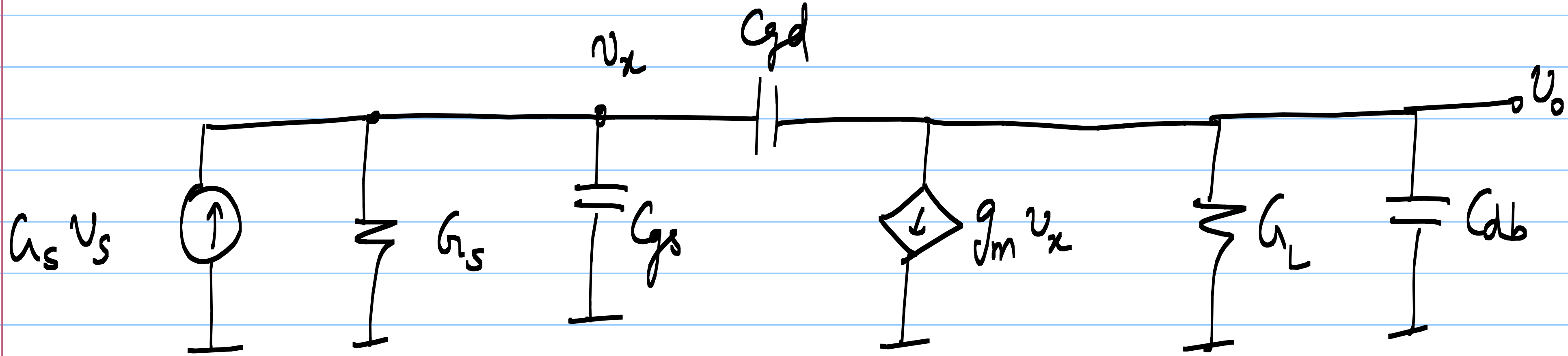
independent of v_s



Zero ⇒ * multiple paths from input to output

and * phase shifts along paths are different





KCL @ input:

$$v_x sC_{gs} + v_x G_s + (v_x - v_o) sC_{gd} = v_s G_s$$

$$\Rightarrow \quad v_x \left[G_s + s\,(C_{gs} + C_{gd}) \right] = v_s G_s + v_o\, s C_{gd}$$

$$v_x = \frac{v_s G_s + v_o\, s C_{gd}}{G_s + s\,(C_{gs} + C_{gd})}$$

KCL @ output :

$$(v_o - v_x)\, s C_{gd} + v_o \left(G_L + s C_{db} \right) + g_m v_x = 0$$

HW $\rightarrow$ $\dfrac{v_o}{v_s}(s) = \,?$

$$\frac{v_o}{v_s} = \left(\begin{array}{c}\text{Low freq.}\\ \text{gain}\end{array}\right) \frac{\text{Zeroes} \quad (N(s))}{\text{poles} \quad (D(s))}$$

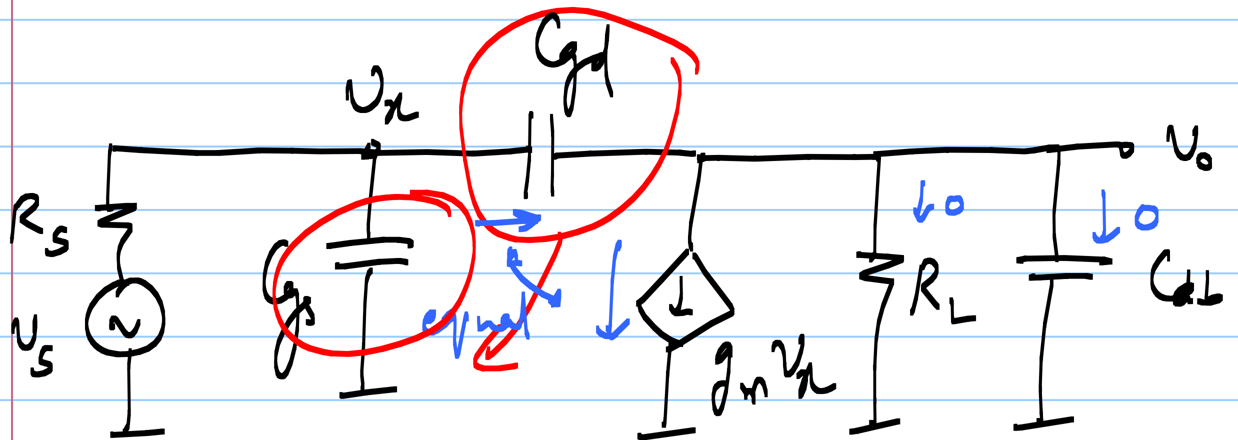
$$= \left(-\frac{g_m}{G_L}\right) \frac{1^{st} \text{ order } N(s)}{2^{nd} \text{ order } D(s)}$$

15/10/2020

Lecture 39

2) CSA with C_{gd} considered :

$$\frac{V_o}{V_s}(s) = \left(-\frac{g_m}{G_L}\right)\frac{\left[1-\dfrac{sC_{gd}}{g_m}\right]}{\left[\dfrac{s^2}{G_L G_s}\left(C_{gs}C_{gd}+C_{gs}C_{db}+C_{gd}C_{db}\right) + \dfrac{s}{G_L G_s}\left(G_L\left(C_{gs}+C_{gd}\right)+G_s\left(C_{gd}+C_{db}\right)+g_m C_{gd}\right)+1\right]}$$



@ zero freq.

$\Rightarrow v_o = 0$

$$D(s) = \frac{s^2}{G_L G_s}\left[C_{gs}\left(C_{gd} + C_{db}\right)\right]$$

(ignore $C_{gd} C_{db}$)

(ignore C_{gd})

$$+ \frac{s}{G_L G_s}\left[G_L C_{gs} + G_s\left(C_{gd} + C_{db}\right) + g_m C_{gd}\right]$$

$$+$$

$$1$$

$$D(s) = \frac{s^2}{G_L G_s}\left[C_{gs}(C_{gd} + C_{db})\right]$$

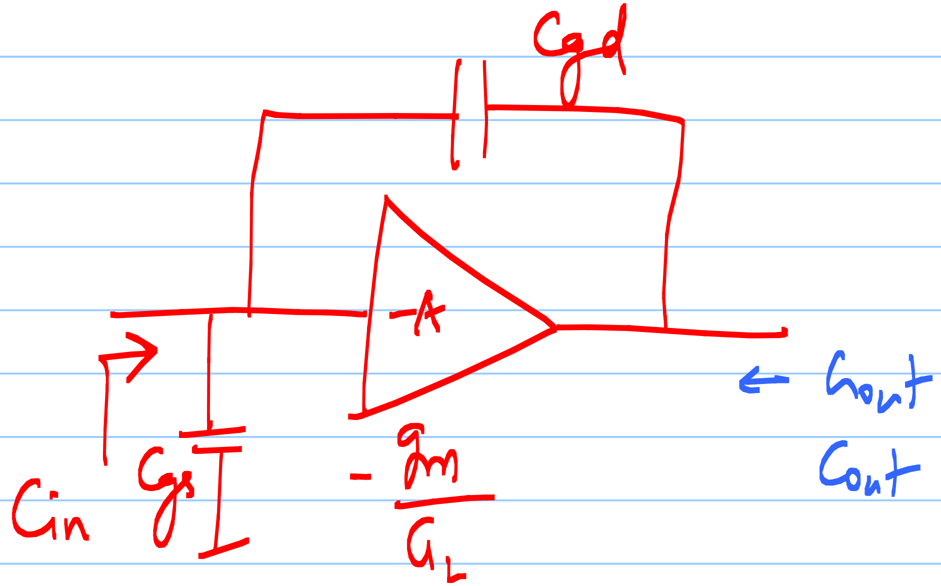
$$+ \frac{s}{G_s}\left[C_{gs} + \frac{g_m}{G_L} C_{gd} + \cancel{\frac{G_s}{G_L}(C_{gd} + C_{db})}\right]$$

$$+ 1$$

due to Miller Effect

ignore

Miller Effect



$$C_{in} \approx (1 + A)\, C_{gd} + C_{gs}$$

$$\approx \frac{g_m}{G_L} C_{gd} + C_{gs}$$

If $D(s) = \left(1+\frac{s}{p_1}\right)\left(1+\frac{s}{p_2}\right)$

$$= \frac{s^2}{p_1 p_2} + s\left(\frac{1}{p_1}+\frac{1}{p_2}\right) + 1$$

$$p_1 >> p_2 \Rightarrow \frac{1}{p_2} >> \frac{1}{p_1}$$

$$D(s) \approx \frac{s^2}{p_1 p_2} + \frac{s}{p_2} + 1$$

$$\boxed{p_2 = \frac{G_S}{C_{gs} + C_{gd}\cdot\frac{g_m}{G_L}}}$$

Compare this with case without C_{gd}

$$p_{20} = \frac{G_S}{C_{gs}}$$

$$p_1 = p_1 p_2 \cdot \frac{1}{p_2}$$

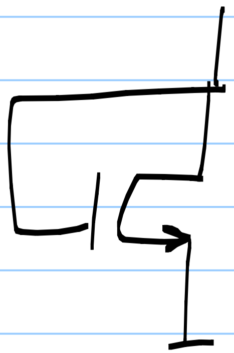
$$= \frac{G_L \; G_S}{C_{gs}\left(C_{gd} + C_{db}\right)} \cdot \frac{C_{gs} + C_{gd} \cdot \frac{g_m}{G_L}}{G_S}$$

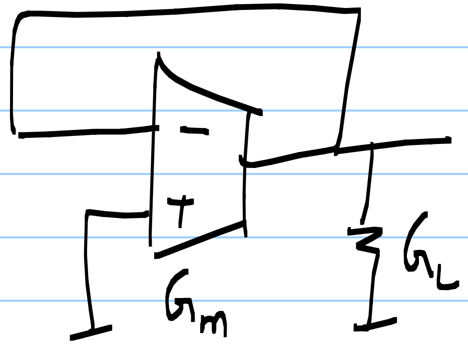
$$\boxed{p_1 = \frac{G_L\left(C_{gs} + C_{gd} \cdot \frac{g_m}{G_L}\right)}{C_{gs}\left(C_{gd} + C_{db}\right)}}$$

Compare with case without C_{gd}

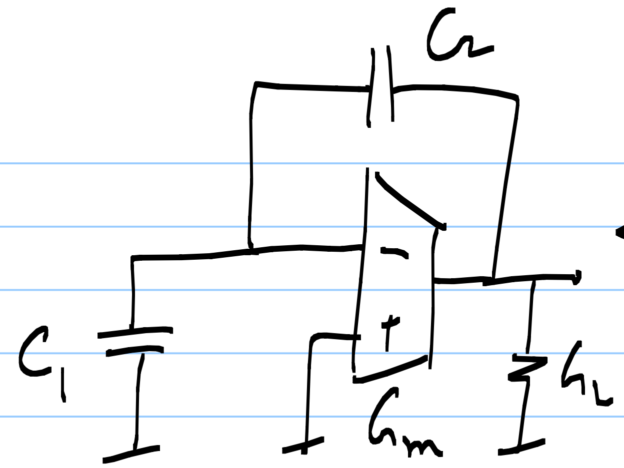
$$p_{1_0} = \frac{G_L}{C_{db}}$$

$\downarrow G_{out}' = g_m$



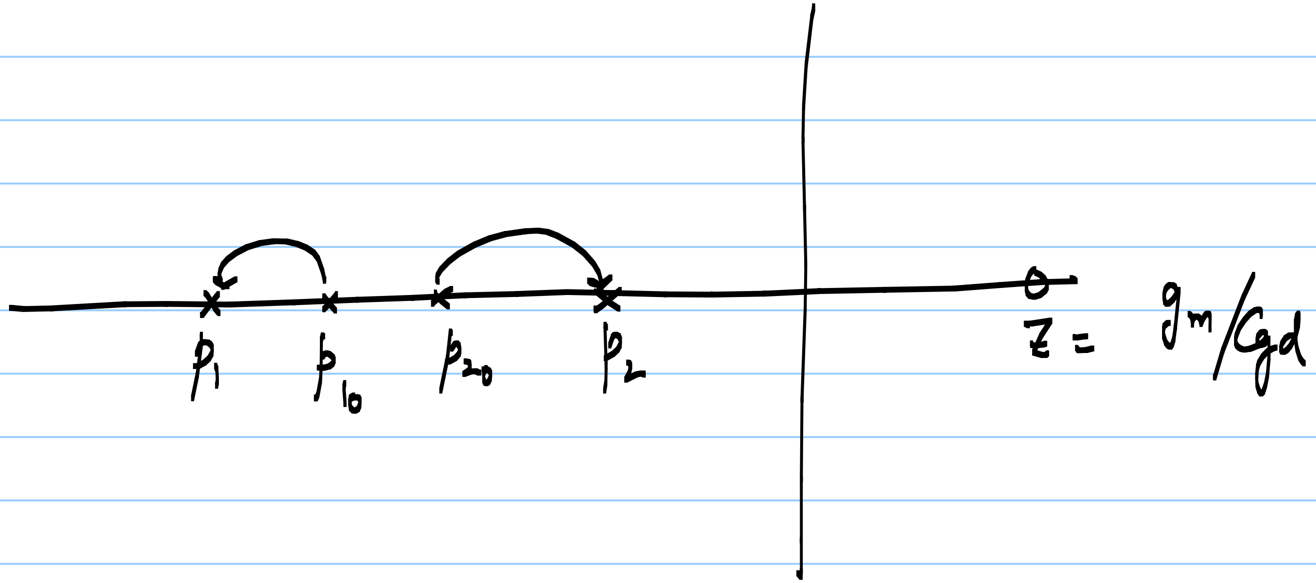


$\leftarrow G_{out} = G_m + G_L$



← G_{out} = ?

HW

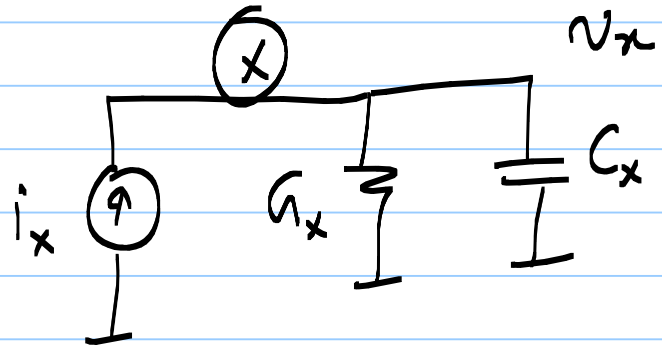


* Every node of an amplifier will have some parasitic cap.
  → made up of device cap. of transistors connected to that node

* pole associated with each node

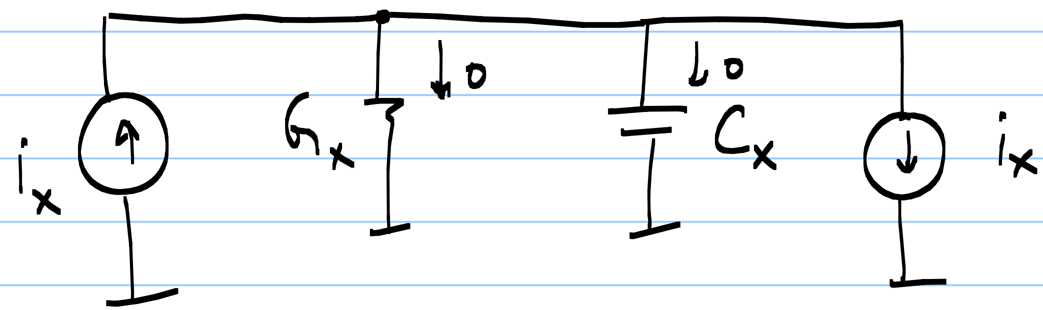
e.g.

$$v_x = \frac{i_x}{Y_x} = \frac{i_x}{G_x + j\omega C_x}$$



pole @ $\frac{G_x}{C_x}$

v_x has no freq. response



* More gain → cascade amplifier stages

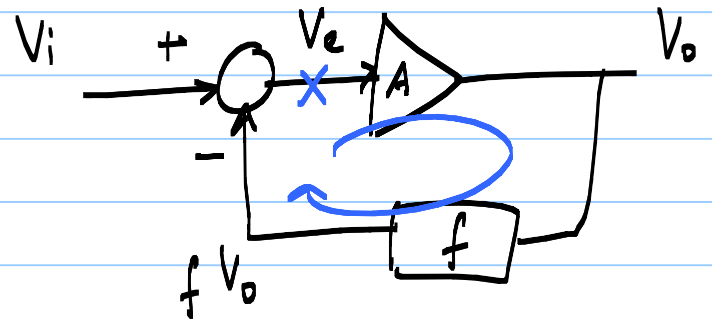
→ more # of nodes → more # of poles
(and maybe zeroes)

* More # of poles → possibility of instability
when placed in <u>feedback</u>.

16|10|2020

Lecture 40

Feedback Systems



$$CLG = \frac{V_o}{V_i} = \frac{1}{f}\frac{Af}{1+Af}$$

$$\approx \frac{1}{f} \quad \text{if } \underbrace{Af}_{\text{loop gain (LG)}} \text{ is large}$$

If $A = A(s)$, f is freq.-indep.

$$CLG = \underbrace{\frac{1}{f}}\frac{A(s)\cdot f}{1+A(s)f} = CLG(s)$$

1) 1^{st} order: $A(s) = \dfrac{A_o}{1+s/\omega_p}$ ⟵ "DC" gain

(single pole amp.)

@ low freq. $A(s) \simeq A_o \Rightarrow LG \simeq A_o f \Rightarrow CLG \simeq \frac{1}{f}$

Assume $CLG \approx \frac{1}{f}$ is valid till $|LG| \sim 1$

$$CLG(s) = \frac{1}{f} \frac{A(s) \cdot f}{1 + A(s) \cdot f}$$

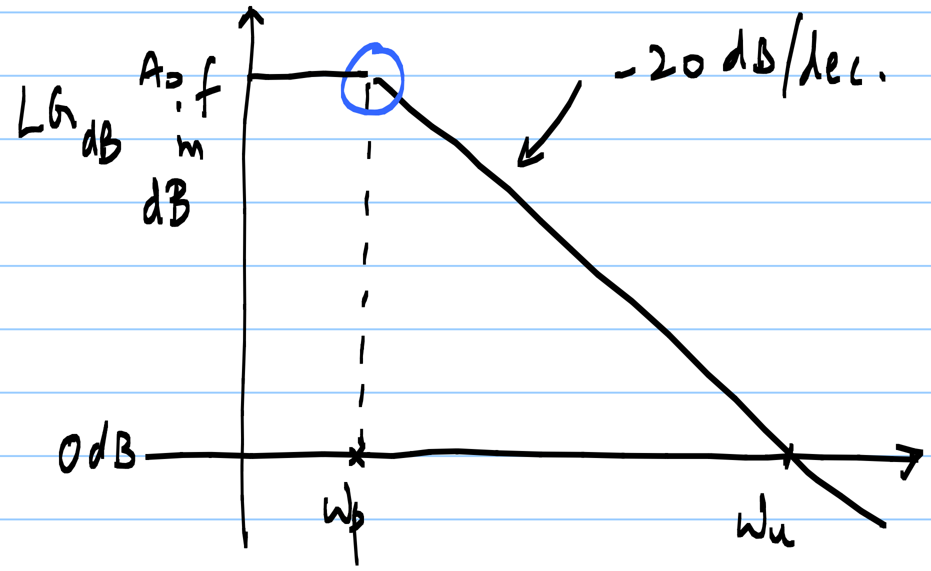
$$= \frac{1}{f} \frac{\frac{A_0 f}{1 + s/\omega_p}}{\left(1 + \frac{A_0 f}{1 + s/\omega_p}\right)}$$

$$= \frac{1}{f} \frac{A_0 f}{(1 + A_0 f + s/\omega_p)}$$

$$= \frac{1}{f} \cdot \frac{A_0 f}{1 + A_0 f} \cdot \frac{1}{1 + \frac{s}{\omega_p (1 + A_0 f)}}$$

CLG pole is @ $\omega_p (1 + A_0 f)$

$$\text{CLG BW} = \omega_p(1 + A_o f) \approx \omega_p A_o f$$



$$= A_o f \cdot \omega_p \approx \text{CLG BW}$$

Note:

1) Single-pole system in closed loop has LHP poles

2) Unconditionally stable

2) <u>2nd order system</u> : $A(s) = \dfrac{A_o}{\left(1+\frac{s}{\omega_p}\right)^2}$

$$CLG(s) = \frac{1}{f} \cdot \frac{A_o f}{1+A_o f} \cdot \frac{1}{\underbrace{1 + \frac{2s}{A_o f\, \omega_p} + \frac{s^2}{A_o f\, \omega_p^2}}}$$

* LHP poles
* Unconditionally stable

general 2nd order system has

$$D(s) = 1 + \frac{s}{Q \cdot \omega_o} + \frac{s^2}{\omega_o^2}$$

compare

Quality factor → Q

(or) $D(s) = s^2 + 2\zeta\omega_n s + \omega_n^2$

ζ ← damping factor

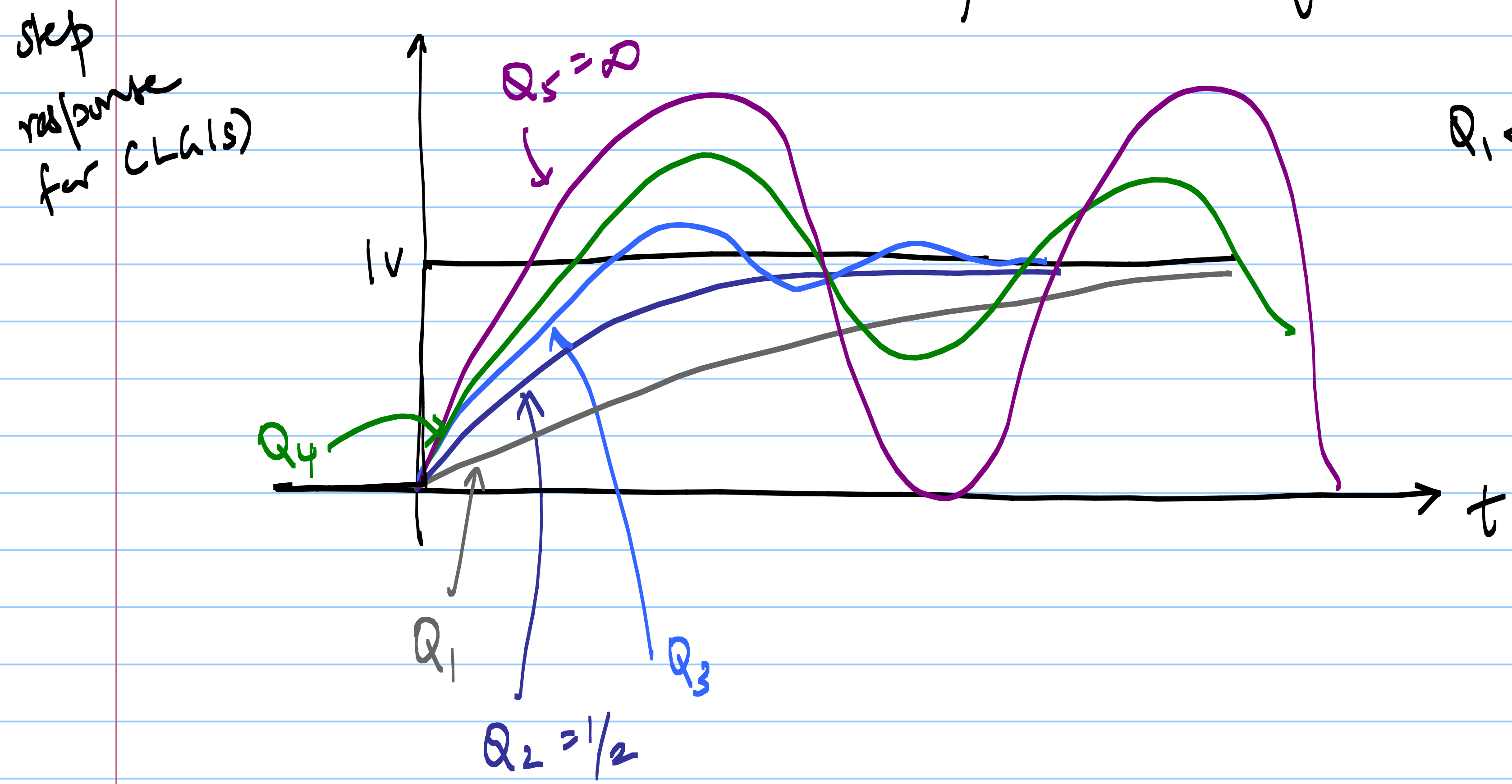
roots are $\dfrac{s}{\omega_o} = -\dfrac{1}{2Q} \pm j\underbrace{\sqrt{1-\dfrac{1}{4Q^2}}}$

small Q → 2 real LHP poles

$Q = 1/2$ → 2 equal poles

$Q > 1/2$ → pair of complex conjugate poles

$Q = \infty$ → poles on $j\omega$ axis



$Q_1 < Q_2 < Q_3 < Q_4 < Q_5$

Here : $\omega_o = \omega_p \sqrt{A_o f}$

$$Q = \frac{\sqrt{A_o f}}{2}$$

No ringing $\Rightarrow$ $Q \le 1/2$

But : $A_o f$ = large due to large LA requirement

3) <u>3rd order system</u> : $A(s) = \dfrac{A_o}{\left(1 + s/\omega_p\right)^3}$

$$= \frac{A_o}{1 + \frac{3s}{\omega_p} + \frac{3s^2}{\omega_p^2} + \frac{s^3}{\omega_p^3}}$$

$$CLA(s) = \frac{1}{f} \cdot \frac{A_o f}{1 + A_o f} \cdot \frac{1}{1 + \left(\frac{1}{1+A_o f}\right)\left[\frac{3s}{\omega_p} + \frac{3s^2}{\omega_p^2} + \frac{s^3}{\omega_p^3}\right]} \leftarrow D(s)$$

$$D(s) = 1 + \frac{3s}{\omega_p(1+A_0f)} + \frac{3s^2}{\omega_p^2(1+A_0f)} + \frac{s^3}{\omega_p^3(1+A_0f)}$$

$$x = \frac{s}{\omega_p}$$

$$D(x) = 1 + \frac{3x}{1+A_0f} + \frac{3x^2}{1+A_0f} + \frac{x^3}{1+A_0f}$$

$$= \left(\frac{1}{1+A_0f}\right)\Big[\underbrace{(1+A_0f) + 3x + 3x^2 + x^3}\Big]$$

we want roots f

$$(1+x)^3 = -A_0f$$

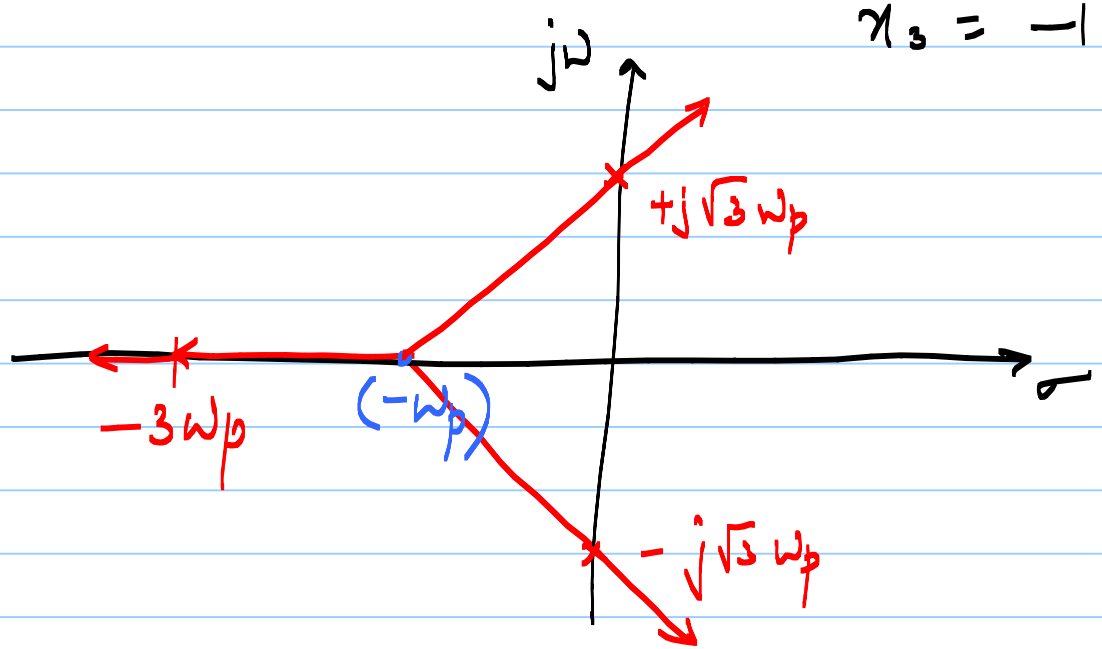
$$x = -1 + \underbrace{(-A_0f)^{1/3}}$$

3 roots

e.g. 1) $A_o f = 0 \rightarrow$ all 3 roots @ -1

2) $A_o f = 8 \rightarrow$

$$\left.\begin{aligned} x_1 &= -1 - 2 = -3 \\ x_2 &= -1 - 2e^{-j2\pi/3} \\ x_3 &= -1 - 2e^{+j2\pi/3} \end{aligned}\right) \quad \begin{aligned} s_1 &= -3\omega_p \\ s_2 &= +j\sqrt{3}\,\omega_p \\ s_3 &= -j\sqrt{3}\,\omega_p \end{aligned}$$



If $A_o f > 8$

↳ complex conjugate roots move into RHP

Unstable for $A_o f > 8$

20/10/2020

Lecture 41

Summary:

1st order → low gain
→ unconditionally stable

2nd order → larger gain
→ technically stable, but ringing in step response

3rd order → very large gain
→ unstable even for small LG_{dc}

4th order → very very large gain
→ highly unstable (guess)

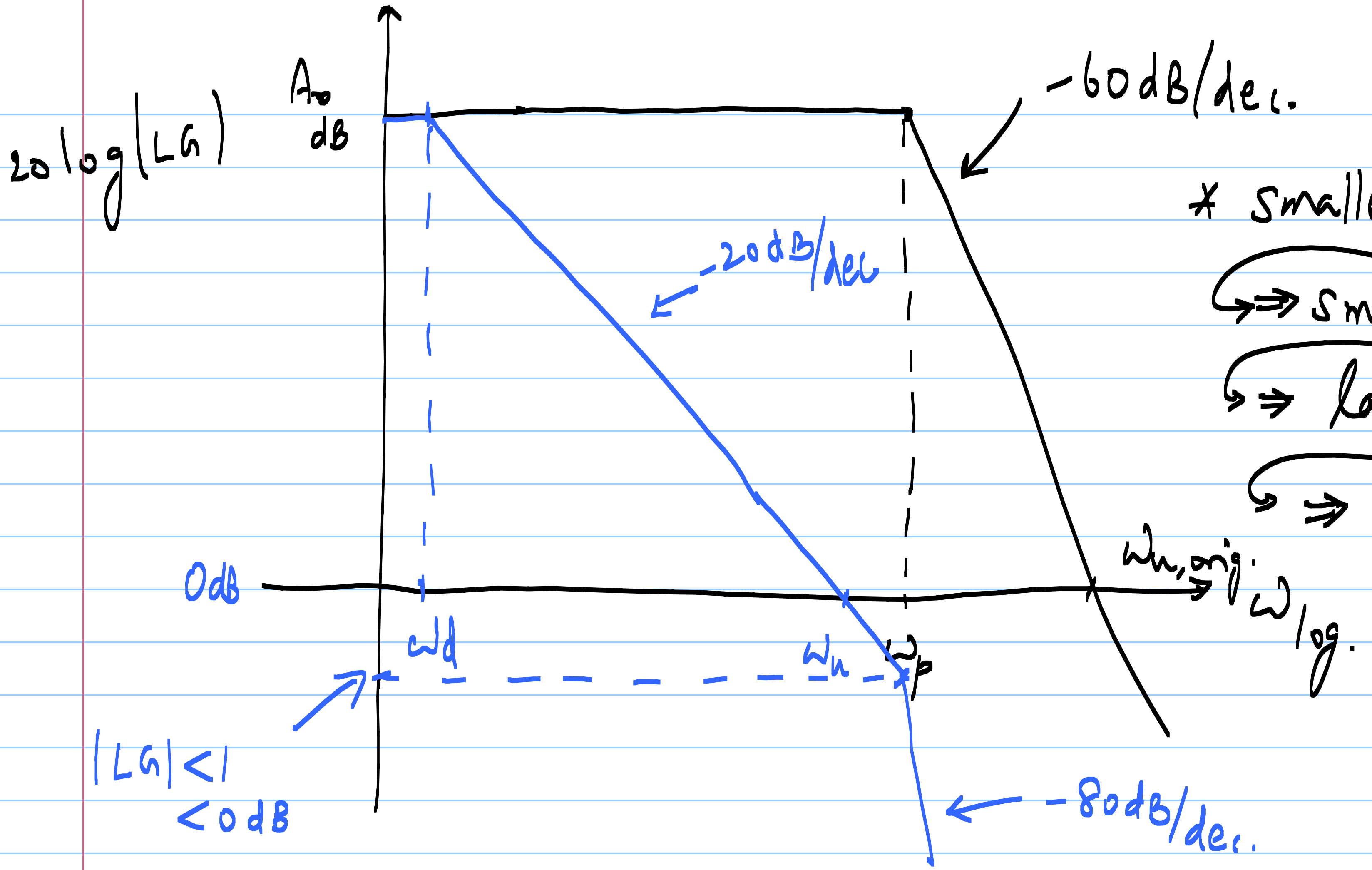
Solution: Make a higher order system look like a 1st order system from the point of view of stability.

Take 3rd order system as an example

$$\frac{A_0}{(1 + s/\omega_p)^3} \xrightarrow[\text{pole @ } \omega_d]{\text{add a}} \frac{A_0}{\left(1+\frac{s}{\omega_d}\right)\left(1+\frac{s}{\omega_p}\right)^3}$$

* In reality, you may choose to move one ω_p pole to ω_d.

* $\omega_d \ll \omega_p$ $\qquad \omega_d$ = "dominant" pole



* smaller ω_d ⇒ smaller ω_u ⇒ larger $(\omega_p - \omega_u)$ ⇒ system stability improves

Improving stability = "Frequency Compensation"

This technique = "Dominant-pole compensation"

* $LG = -1$

$$|LG| = 1 \quad \& \quad \angle LG = -180^\circ$$

* Avoid $\angle LG > 1$ when $\angle LG = -180^\circ$

* Measures of Stability :

1) Gain Margin $= 0\,dB - |LG(j\omega)| \Big|_{\angle LG(j\omega) = -180^\circ}$

2) Phase Margin $= \angle LG(j\omega) \Big|_{|LG| = 0\,dB} - (-180^\circ)$

$$= 180^\circ + \angle LG(j\omega) \Big|_{|LG| = 0\,dB}$$

* We normally want high GM & PM

21/10/2020

Lecture 42

Example : $\omega_d = \dfrac{\omega_p}{1000}$

original limit for stability : $A_o f = 8$

$$LG(s) = \frac{A_o f}{\left(1+\frac{s}{\omega_p}\right)^3\left(1+\frac{1000s}{\omega_p}\right)}$$

we need to find new limit on $A_o f$

$$LG(s) = \frac{X(s)}{Y(s)}$$

$$CLG(s) = \frac{1}{f}\cdot\frac{LG(s)}{1+LG(s)} = \frac{1}{f}\frac{X(s)/Y(s)}{1+X(s)/Y(s)} = \frac{N(s)}{D(s)}$$

roots of $LG(s) = -1$

$|LG(j\omega_o)| = 1$ & $\angle LG(j\omega_o) = -\pi$ $\leftarrow$ apply this first

$$-3\tan^{-1}\left(\frac{w_o}{w_p}\right) - \underbrace{\tan^{-1}\left(\frac{1000\,w_o}{w_p}\right)}_{} = -\pi$$

@ $w_o \to$ wd give $-\pi/2$ phase shift

$$\Rightarrow \quad -3\tan^{-1}\left(\frac{w_o}{w_p}\right) = -\pi/2$$

each w_p pole gives $-\pi/6$ (-30°)

$$\frac{w_o}{w_p} \approx \frac{1}{\sqrt{3}}$$

$$\boxed{w_o = \frac{w_p}{\sqrt{3}}}$$

* Apply magnitude condition: $|LG(jw_o)| = 1$

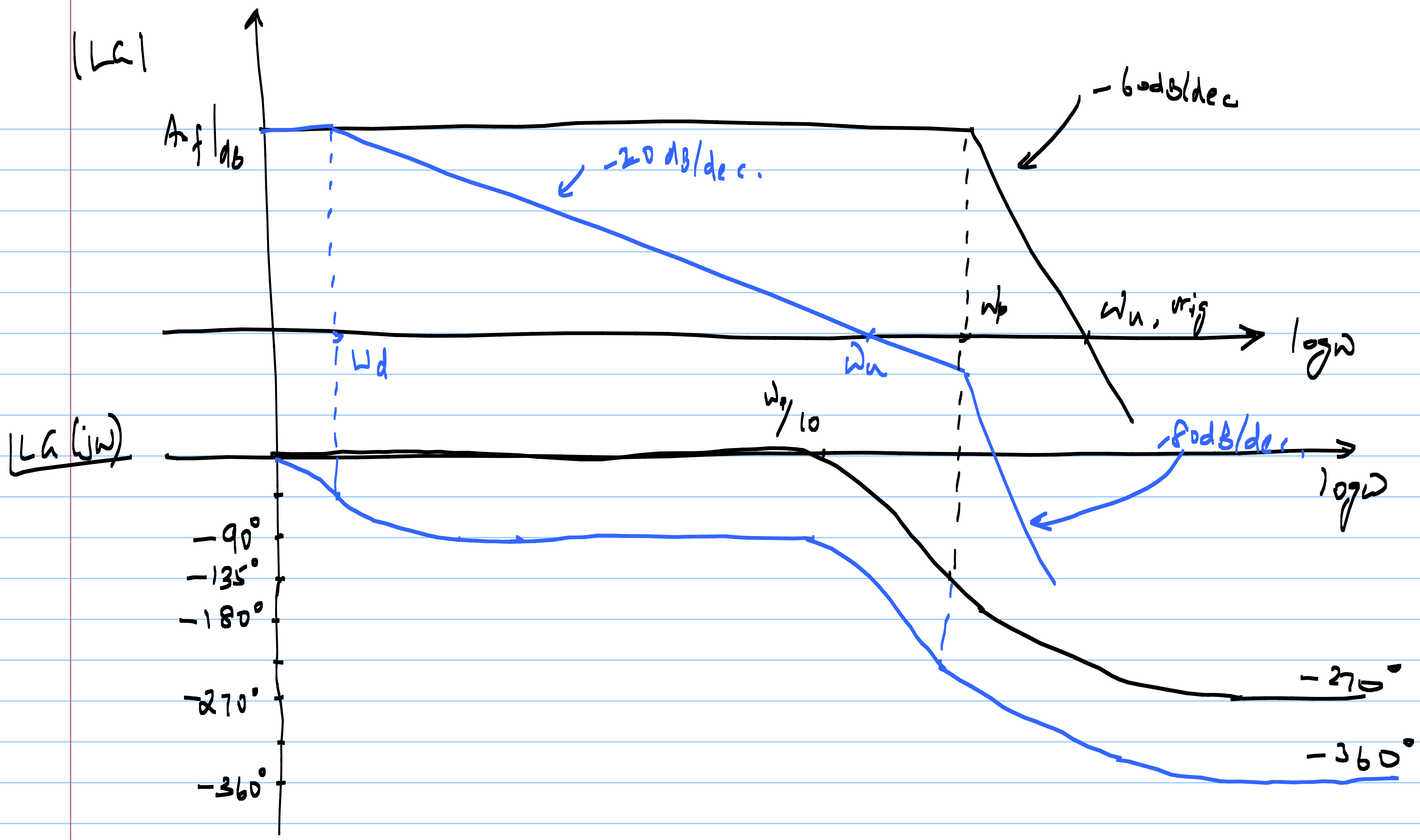
$$\left| \frac{A_o f}{\left(1+ j/\sqrt{3}\right)^3 \left(1 + j\frac{1000}{\sqrt{3}}\right)} \right| = 1$$

$$\frac{A_o f}{\left(\sqrt{1+1/3}\right)^3 \left(\frac{1000}{\sqrt{3}}\right)} = 1 \quad \Rightarrow \quad A_o f \approx 890$$

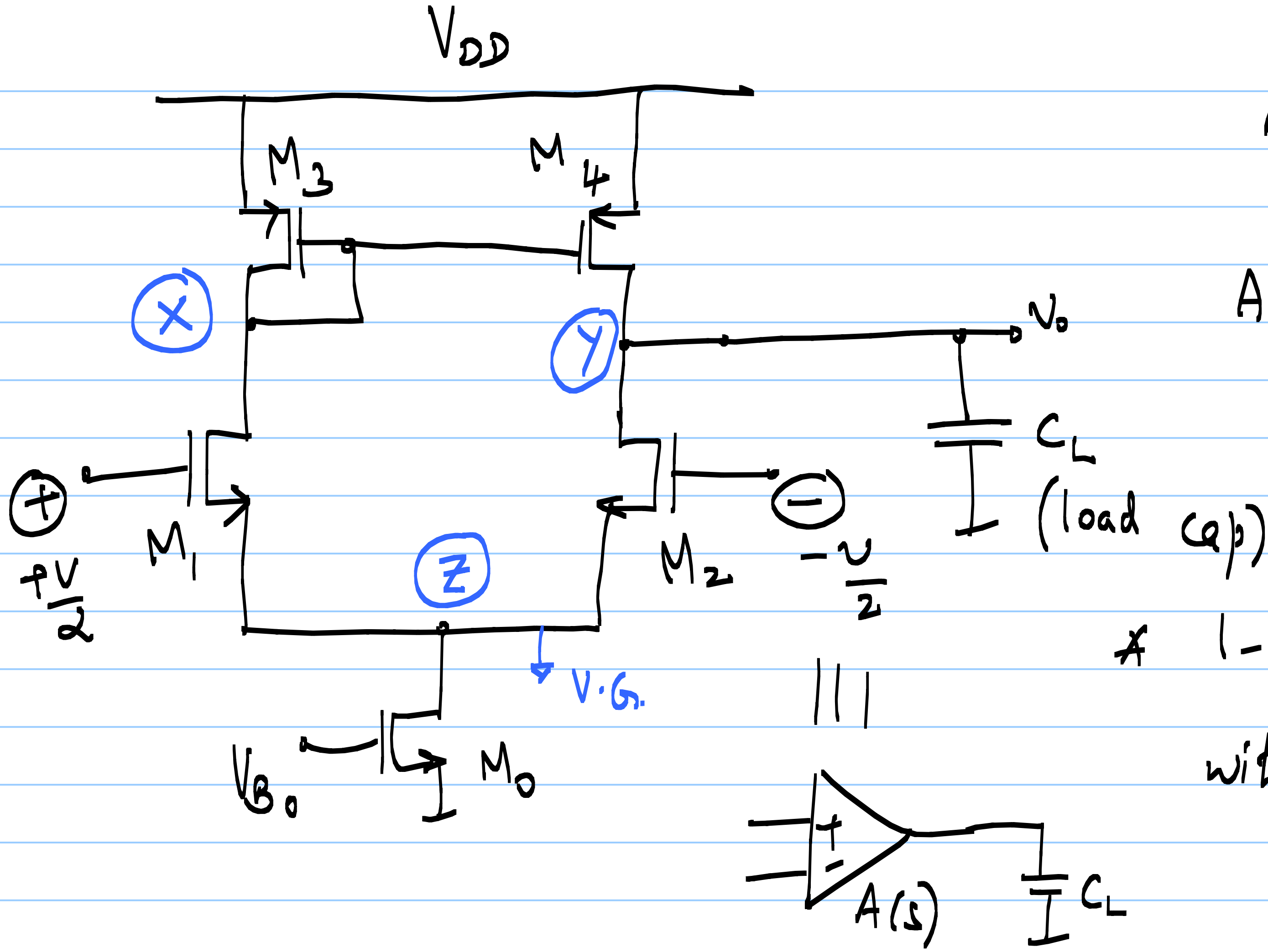
* Worst-case scenario for stability: largest possible $A_o f$

$\Rightarrow$ largest possible f

$\Rightarrow$ $f_{max} = 1$ i.e. unity gain amplifier



|LG|
$A_f|_{dB}$
-60dB/dec
-20 dB/dec.
ω_d
ω_u
ω_p
$\omega_{u,orig}$
log ω
$\omega_p/10$
∠G(jω)
-80dB/dec.
log ω
-90°
-135°
-180°
-270°
-360°
-270°
-360°



$$A_0 = g_{m_1}\left(r_{ds_2} \,\|\, r_{ds_4}\right)$$

$$A(s) = g_{m_1}\left(r_{ds_2} \,\|\, r_{ds_4} \,\|\, \frac{1}{sC_L}\right)$$

$$= A_0 \frac{1}{1+\frac{s}{\omega_p}}$$

* 1-pole system without parasitics

$$A(s) = \frac{g_{m_1}}{g_{ds_2}+g_{ds_4}+sC_L} = \left[\frac{g_{m_1}}{(g_{ds_2}+g_{ds_4})}\right] \cdot \frac{1}{1+\frac{sC_L}{g_{ds_2}+g_{ds_4}}}$$

$$\omega_p = \frac{g_{ds_2} + g_{ds_4}}{C_L} = \frac{1}{(r_{ds_2} \| r_{ds_4}) \cdot C_L}$$

* Ignore C_{gd}

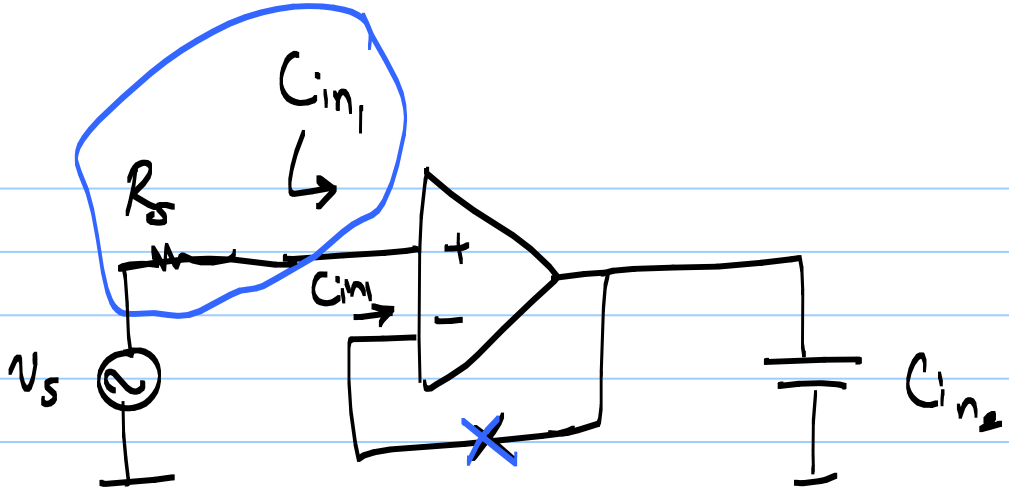
$$C_x = C_{gs_3} + C_{gs_4} + C_{db_3} + C_{db_1} \approx 2 C_{gs_3}$$

$$C_y = C_L + C_{db_2} + C_{db_4} \approx C_L$$

$$C_z = C_{db_0} + C_{gs_1} + C_{gs_2} + C_{sb_1} + C_{sb_2}$$

$\leftarrow$ No DM current through C_z

* 2-pole system when parasitic caps are taken into account



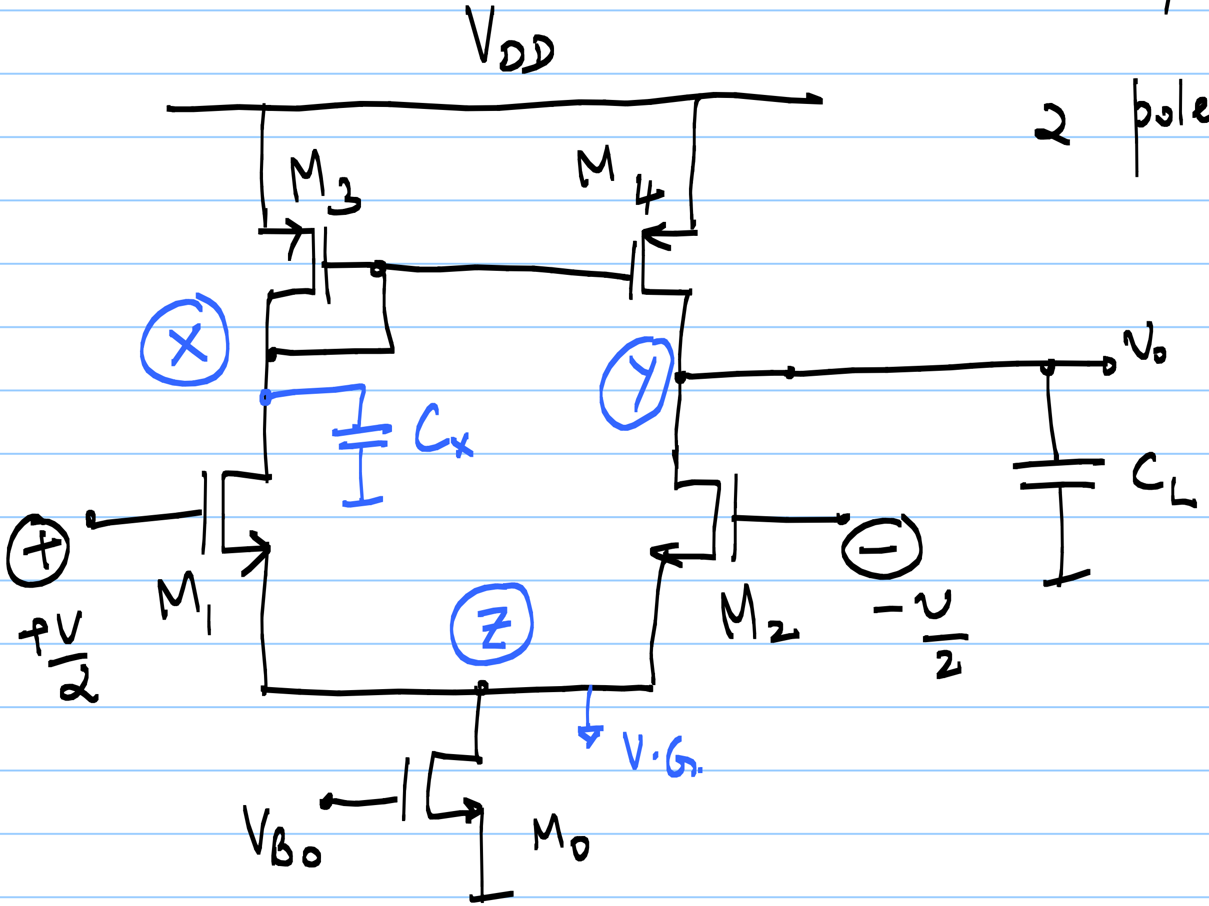
$$C_L = C_{in_1} + C_{in_2}$$

22/10/2020

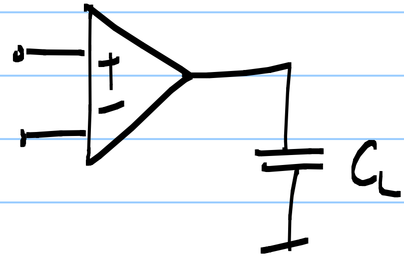
Lecture 43

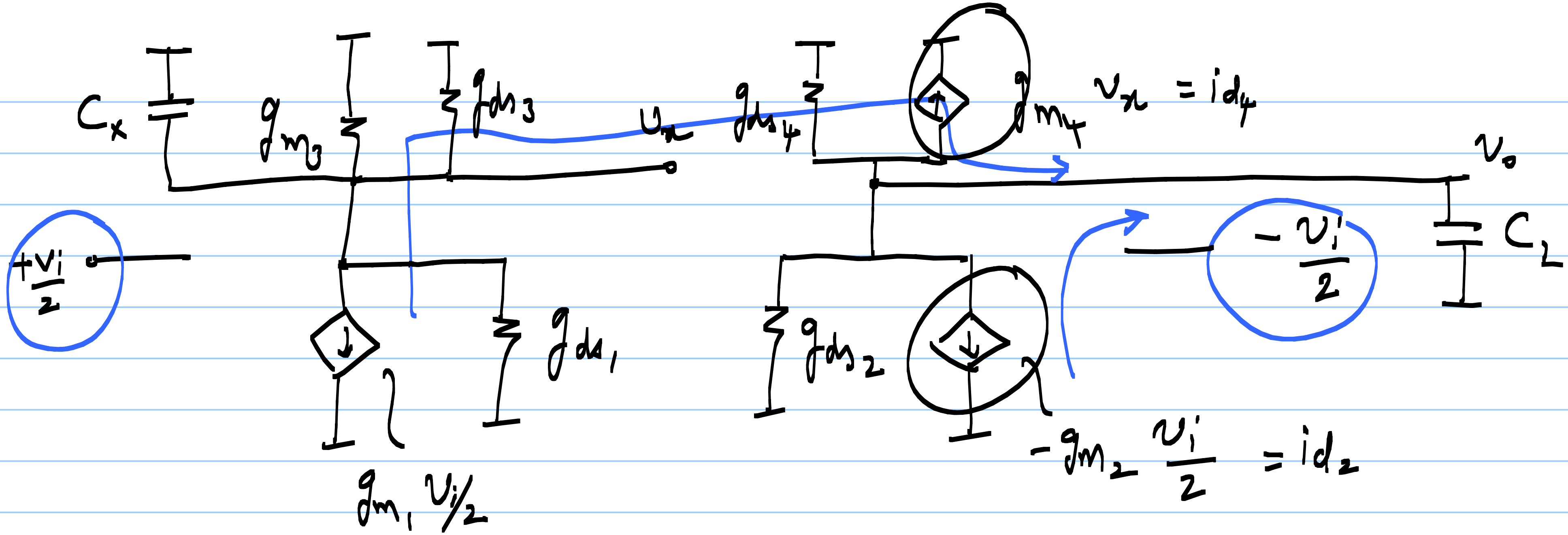
parasitic cap. @ X & Y

2 poles in A(s)



$\equiv$





* all g_m's $\gg$ all g_{ds}'s

$$v_x = -\frac{+g_{m_1} v_i/2}{g_{m_3} + g_{ds_2} + g_{ds_1} + sC_x} \approx \frac{-g_{m_1} v_i/2}{g_{m_3} + sC_x}$$

$$v_x \approx \frac{-g_{m_1}}{g_{m_3}} \cdot \frac{1}{1 + sC_x/g_{m_3}} \cdot \frac{v_i}{2}$$

$$i_{d_4} = g_{m_4} \cdot v_x$$

$$= \frac{-g_{m_1}}{1 + \frac{sC_x}{g_{m_3}}} \cdot \frac{v_i}{2}$$

$$i_{d_2} = -g_{m_2} \, v_i/2$$

$$v_o = -\frac{(i_{d_2} + i_{d_4})}{g_{ds_2} + g_{ds_4} + sC_L}$$

$$v_o = -\frac{-g_{m_2} v_i/2 - \frac{g_{m_1}}{1 + sC_x/g_{m_3}} \cdot v_i/2}{g_{ds_2} + g_{ds_4} + sC_L}$$

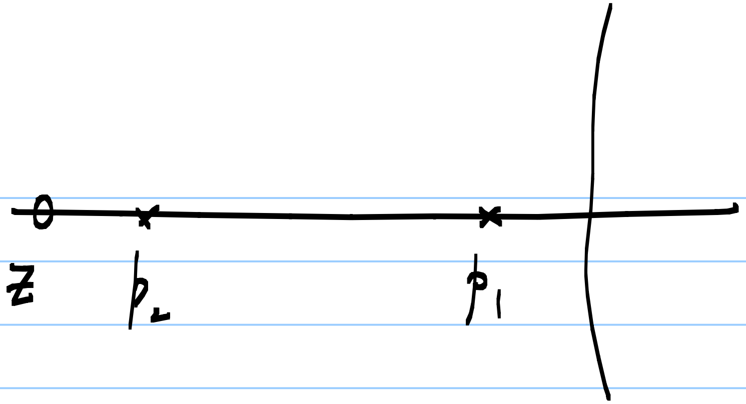
$$= \frac{g_{m_1} v_i}{g_{ds_2} + g_{ds_4}} \cdot \frac{\frac{1}{2} + \frac{1/2}{1 + sC_x/g_{m_3}}}{1 + \frac{sC_L}{g_{ds_2} + g_{ds_4}}}$$

$$\boxed{\frac{v_o}{v_i} = \frac{g_{m_1}}{g_{ds_2} + g_{ds_4}} \cdot \frac{\left(1 + sC_x/2g_{m_3}\right)}{\left(1 + sC_x/g_{m_3}\right)\left(1 + sC_L/(g_{ds_2} + g_{ds_4})\right)}}$$

* $A_o = \dfrac{g_{m_1}}{g_{ds_2} + g_{ds_4}}$

* 2 poles → @ X , Y

* 1 zero @ $2g_{m_3}/C_x$

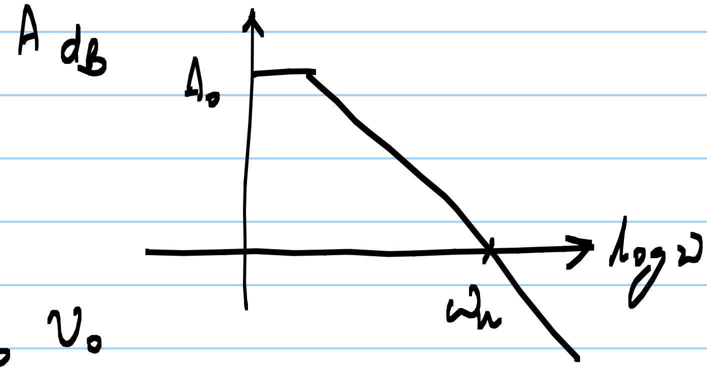


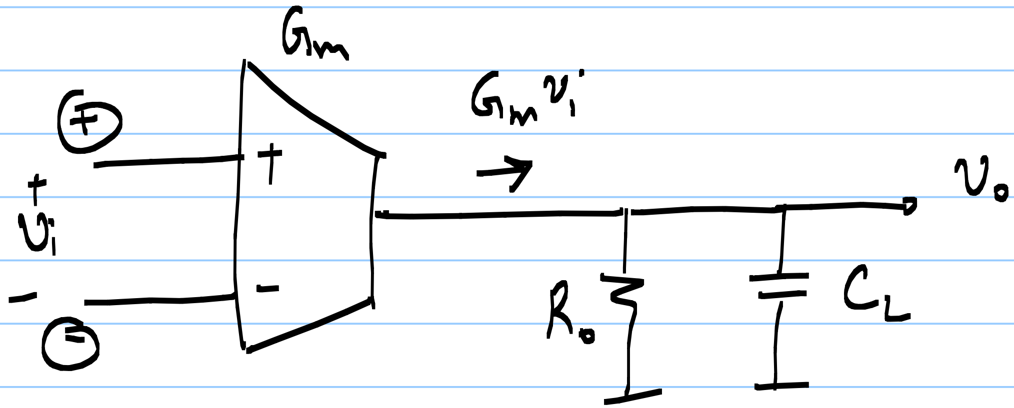
$$p_1 = \frac{g_{ds2} + g_{dsp}}{C_L}$$

$$p_2 = \frac{g_{m3}}{C_x}$$

$$z = 2 p_2 = \frac{2 g_{m3}}{C_x}$$

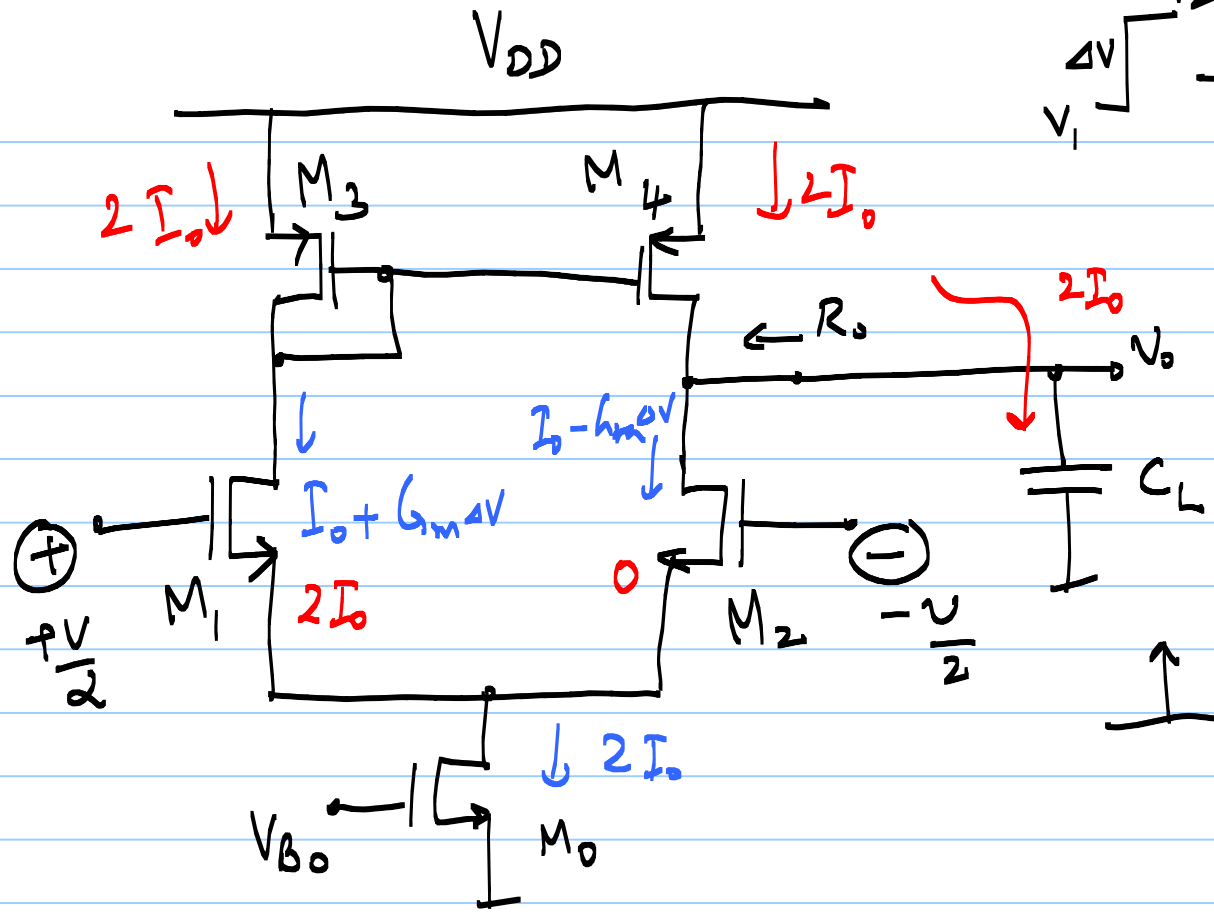
Ideal 1-stage opamp :

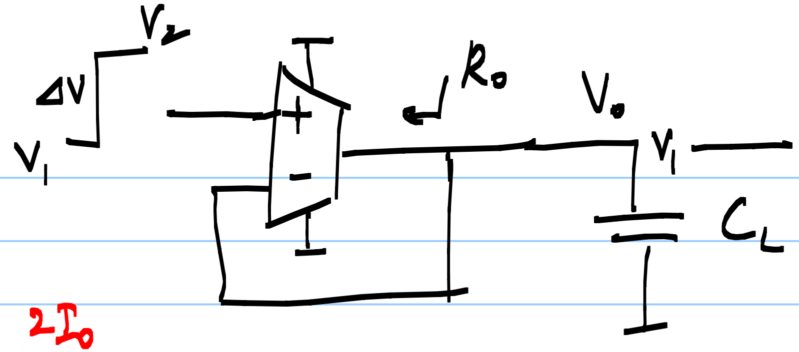


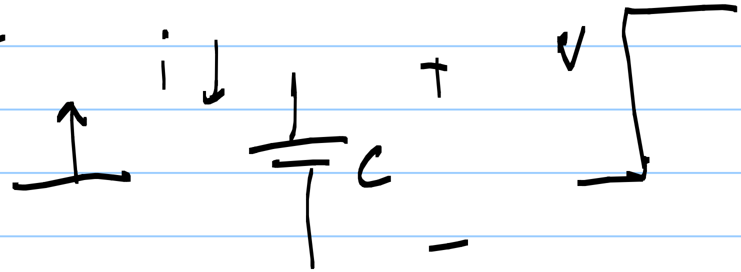


$$\omega_u = \frac{G_m}{C_L} = \frac{g_{m1}}{C_L}$$

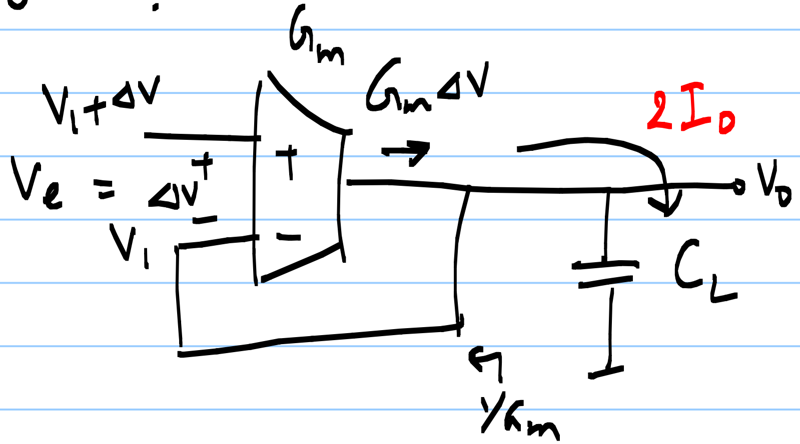
$G_m = g_{m1}$; $R_o = r_{ds2} \| r_{ds4}$; $A_o = G_m R_o = g_{m1}(r_{ds2} \| r_{ds4})$





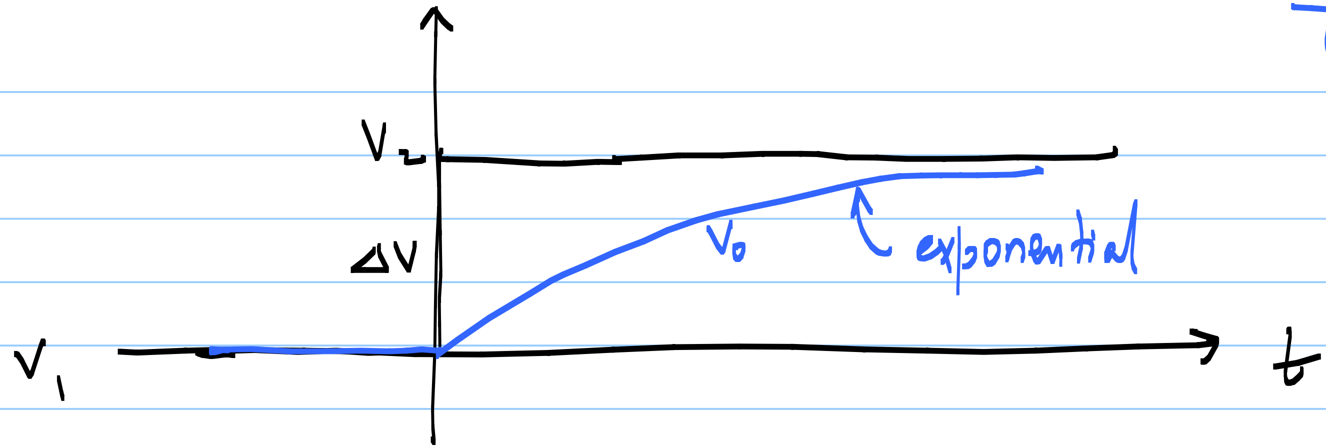


@ $t = 0^+$:



$V_e(t)$ approaches 0 as C_L charges towards $V_1 + \Delta V$

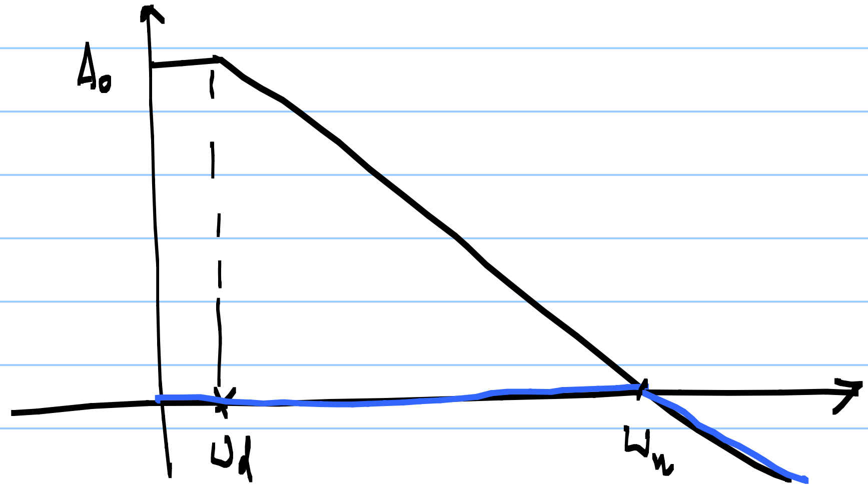
$$\tau = \frac{1}{\omega_u} = \frac{C_L}{G_m}$$



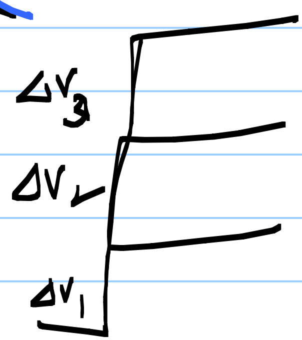
open loop

opamp

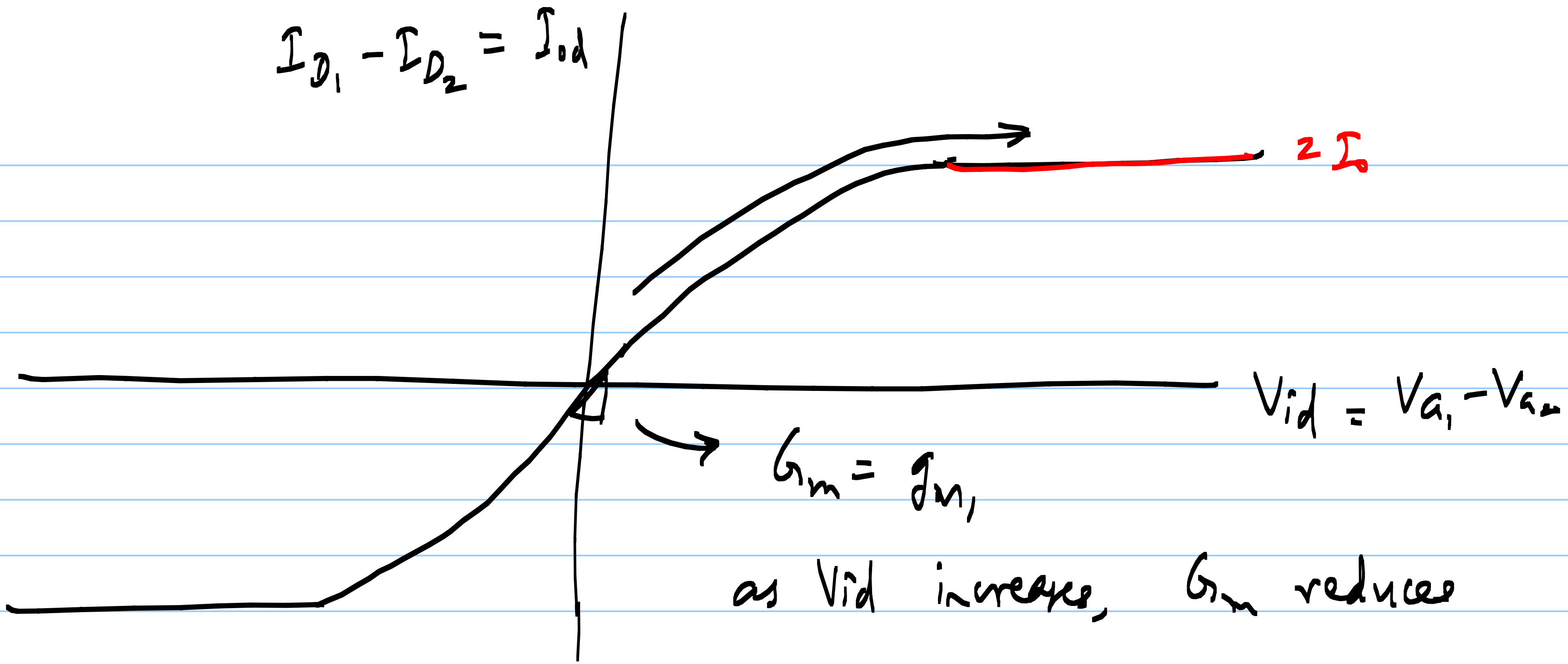
closed loop



Now start $\uparrow \Delta V$

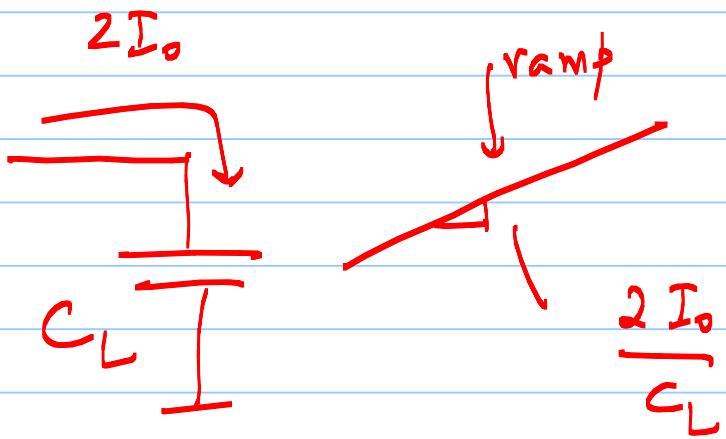


$$i_o \Big|_{t=0^+} = G_m \cdot \Delta V$$



$G_m = g_{m_1}$

as V_{id} increases, G_m reduces

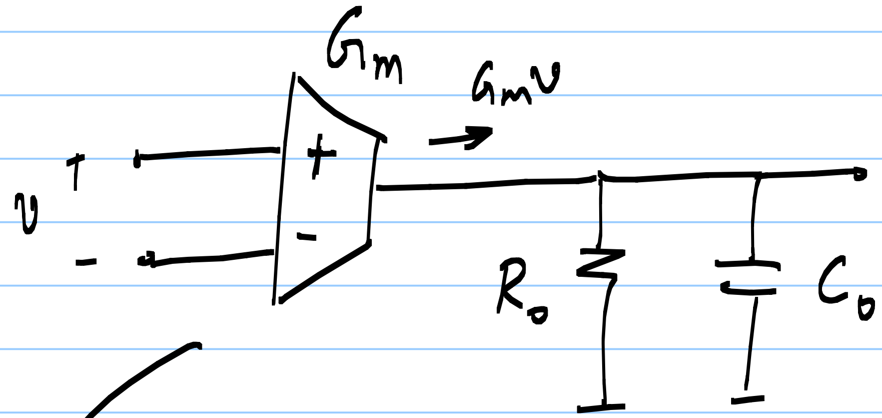


"slew"

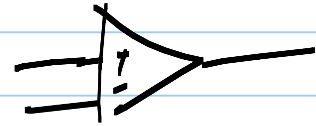
$$\text{slew rate} = \pm \frac{2I_o}{C_L}$$

23/10/2020

Lecture 44



$\equiv$

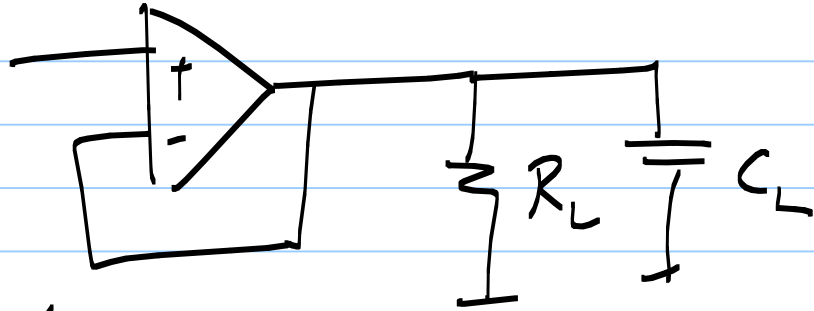


$R_o = (r_{ds_2} \| r_{ds_4})$

$A_o = G_m R_o$

$\omega_d = \dfrac{1}{R_o C_o}$

$\omega_u = \dfrac{G_m}{C_o}$



$A_o' = G_m (R_o \| R_L)$ ✗ ; $\omega_d' = \dfrac{1}{(R_o \| R_L)(C_o + C_L)}$

$\omega_u = G_m / (C_o + C_L)$ ✓

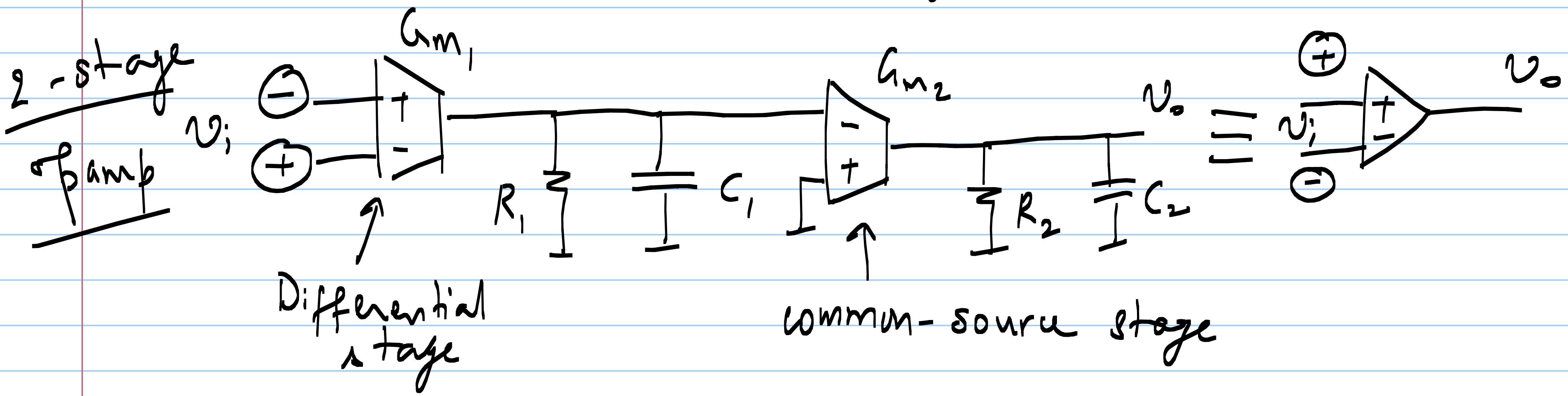
$$R_o \| R_L = R_L \| r_{ds_2} \| r_{ds_4} \approx R_L$$

$$A_o = g_{m_1} (r_{ds_2} \| r_{ds_4})$$

$$A_o' = g_{m_1} R_L$$ extremely small compared to A_o

steady state V_e can be large

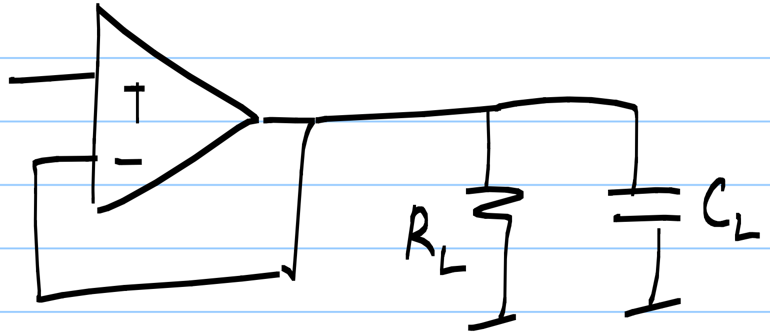
$\Rightarrow$ need a 2-stage opamp



$$A_0 = A_1 \cdot A_2$$

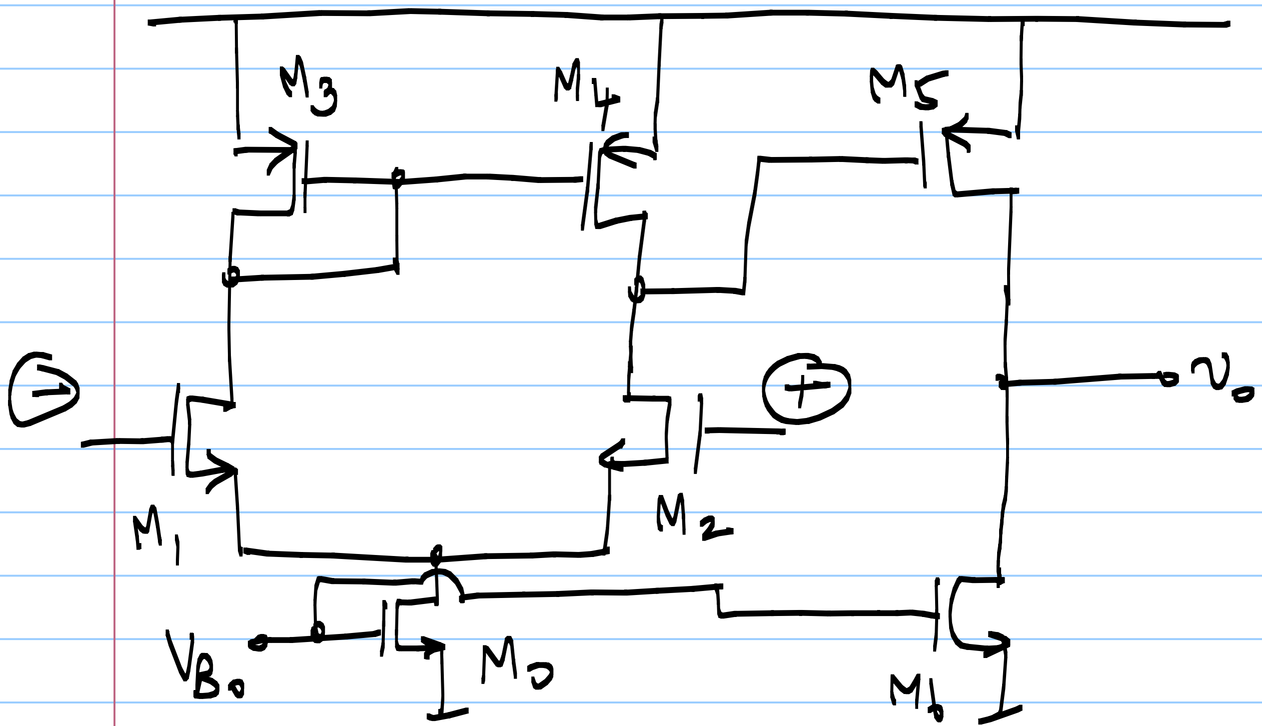
$$= G_{m_1} R_1 \cdot G_{m_2} R_2$$

resistive load



$$A_0' = G_{m_1} R_1 \cdot G_{m_2} (R_2 || R_L)$$

$$\simeq \underbrace{G_{m_1} R_1}_{\text{DC gain from 1st stage is preserved}} \cdot G_{m_2} R_L$$



$G_{m_1} = g_{m_1}$

$G_{m_2} = g_{m_5}$

$R_1 = r_{ds_2} || r_{ds_4}$

$R_2 = r_{ds_5} || r_{ds_6}$

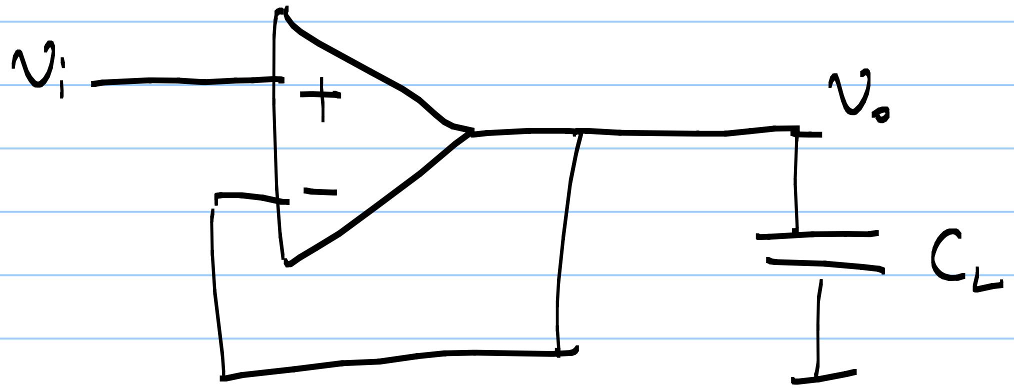
$C_1 = C_{gs_5} + C_{db_4} + C_{db_2} \approx C_{gs_5}$

$C_2 = C_{db_5} + C_{db_6}$

check to see if these are valid

* If opamp is driving C_L

$$C_2 = C_L + C_{db_5} + C_{db_6} \approx C_L$$



$A_1 = A_1(s)$

$A_2 = A_2(s)$

$A(s) = A_1(s)\, A_2(s)$

↳ 2 poles

* How to choose ω_d?

use PM, but: PM $\geq 60°$

$PM_{min.} = 50°$ etc.

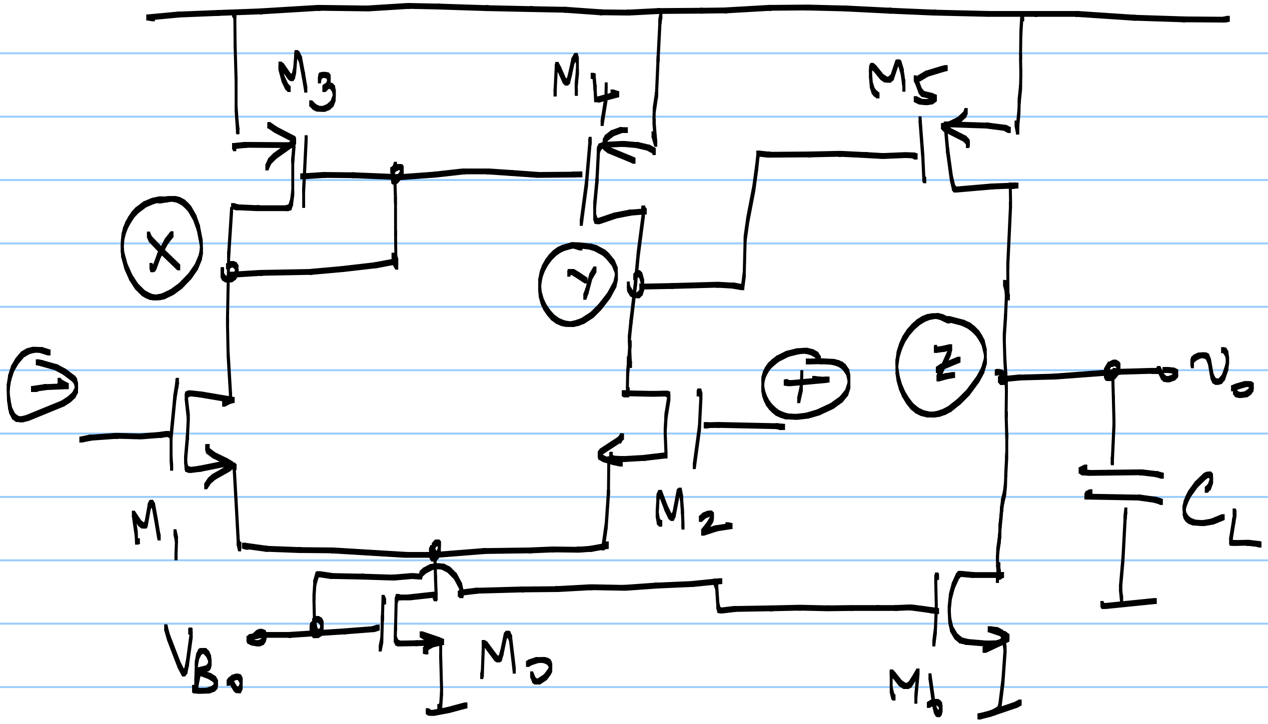
↳ sets $\omega_{d_{max}}$

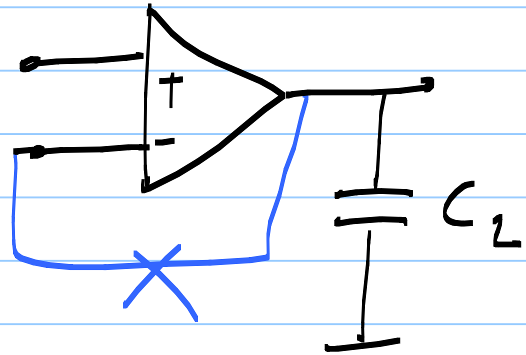
* As $\omega_d \downarrow \Rightarrow \omega_u \downarrow$

→ use BW spec. on closed loop amplifier

27/10/2020

Lecture 45





3 poles @ X, Y, Z

1 zero @ $2p_x$ (z_x)

$C_x \approx 2C_{gs_3}$; $C_y \approx C_{gs_5}$; $C_z \approx C_L$

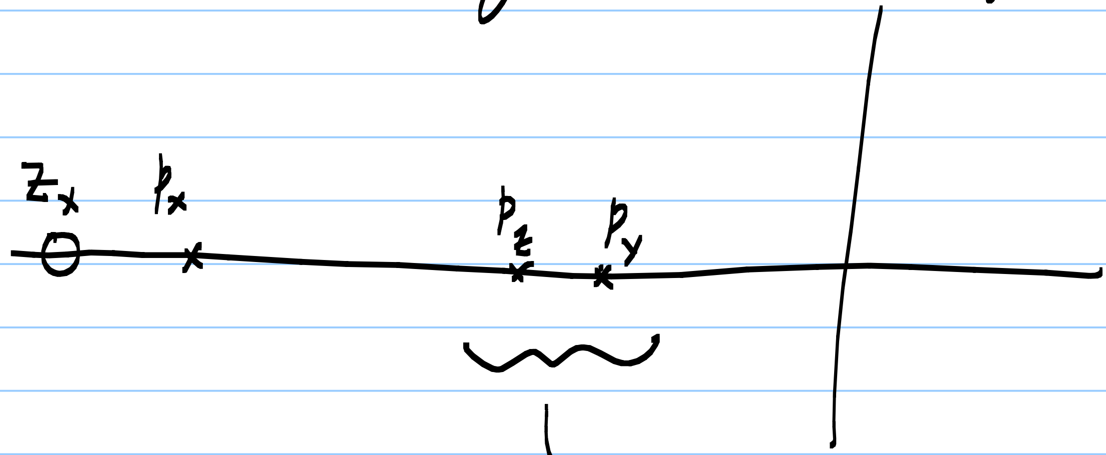
$G_x \approx g_{m_3}$; $G_y = g_{ds_2} + g_{ds_4}$; $G_z = g_{ds_5} + g_{ds_1}$

H/W

$p_x \approx \dfrac{g_{m_3}}{2C_{gs_3}}$; $p_y \approx \dfrac{g_{ds_2} + g_{ds_4}}{C_{gs_5}}$; $p_z \approx \dfrac{g_{ds_5} + g_{ds_1}}{C_L}$

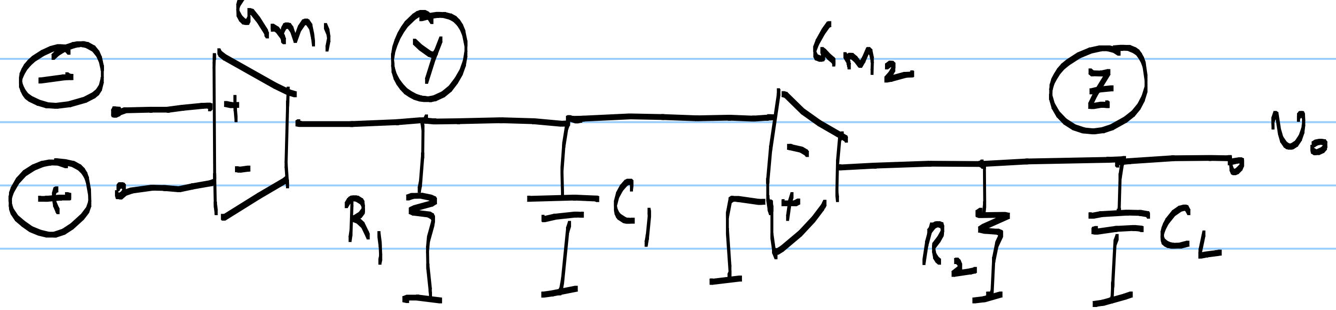
$p_x \gg p_y, p_z$; so, ignore p_x & $Z_x = 2p_x$

So $\rightarrow$ effectively a 2-pole system (p_y & p_z) with high DC gain



$\rightarrow$ high Q, low ζ, ringing in step response

<u>problem</u>

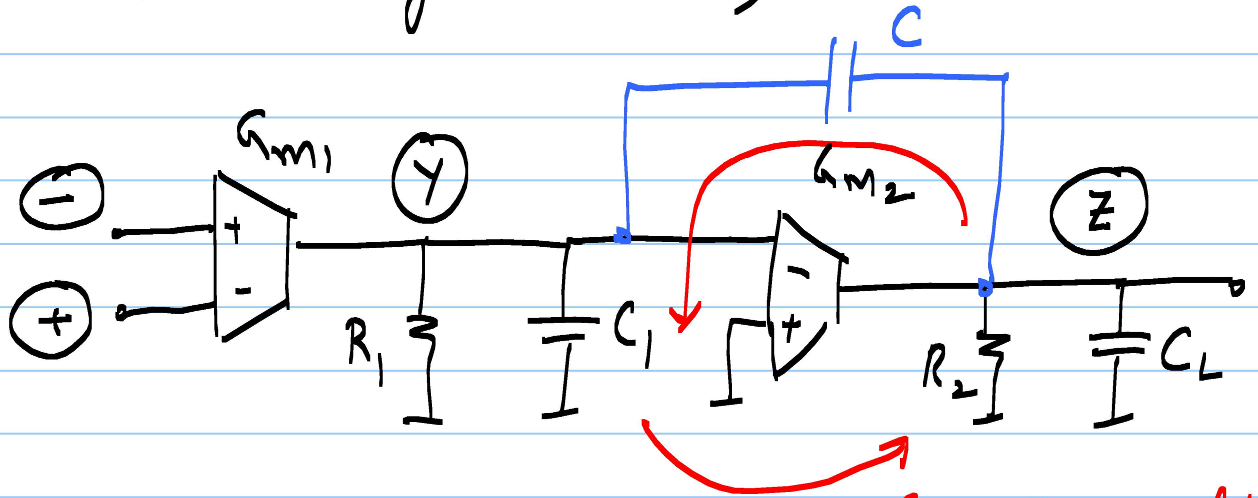


Block level view of 2-stage opamp

$G_{m_1} = g_{m_1}$; $G_{m_2} = g_{m_5}$

$R_1 = \dfrac{1}{g_{ds_2} + g_{ds_4}}$; $R_2 = \dfrac{1}{g_{ds_5} + g_{ds_6}}$

$C_1 = C_{gs_5}$; $C_2 = C_L$



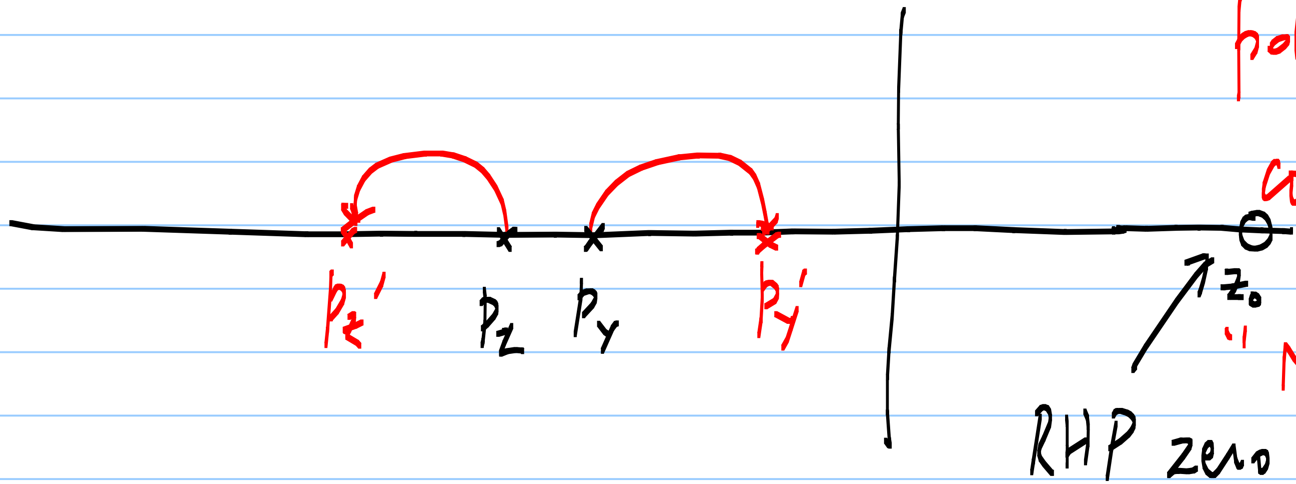
G_Z reduces due to AC feedback through C_f

gain $= G_{m_2} R_2$ (high)

C_1' $\approx C_1 + \underbrace{G_{m_2} R_2\, C}$ → "Miller Capacitance"

$$p_y' \approx \frac{1}{R_1 C_1'} << p_y$$

Dominant pole



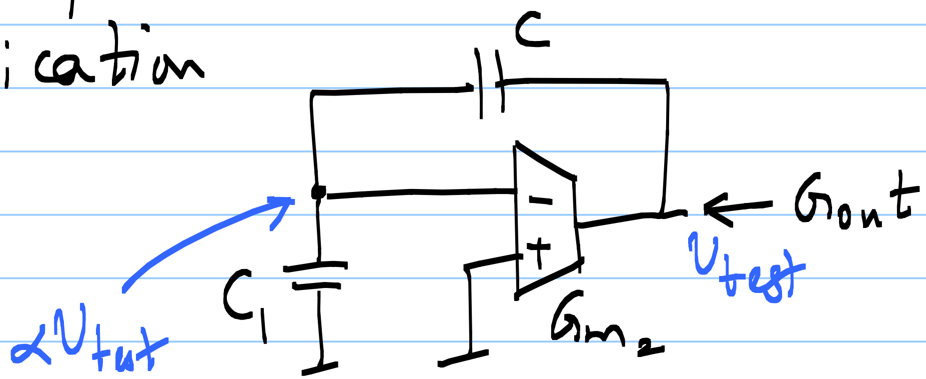
"pole splitting" compensation

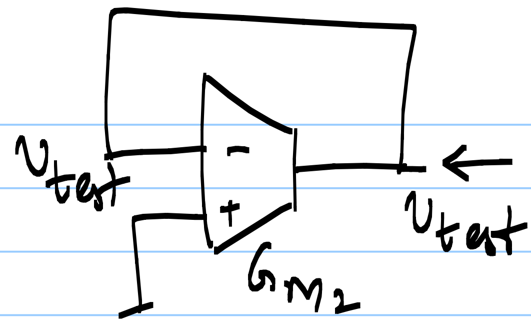
or

"Miller" Compensation.

required value of C is small due to Miller multiplication

HW: G_{out} of →

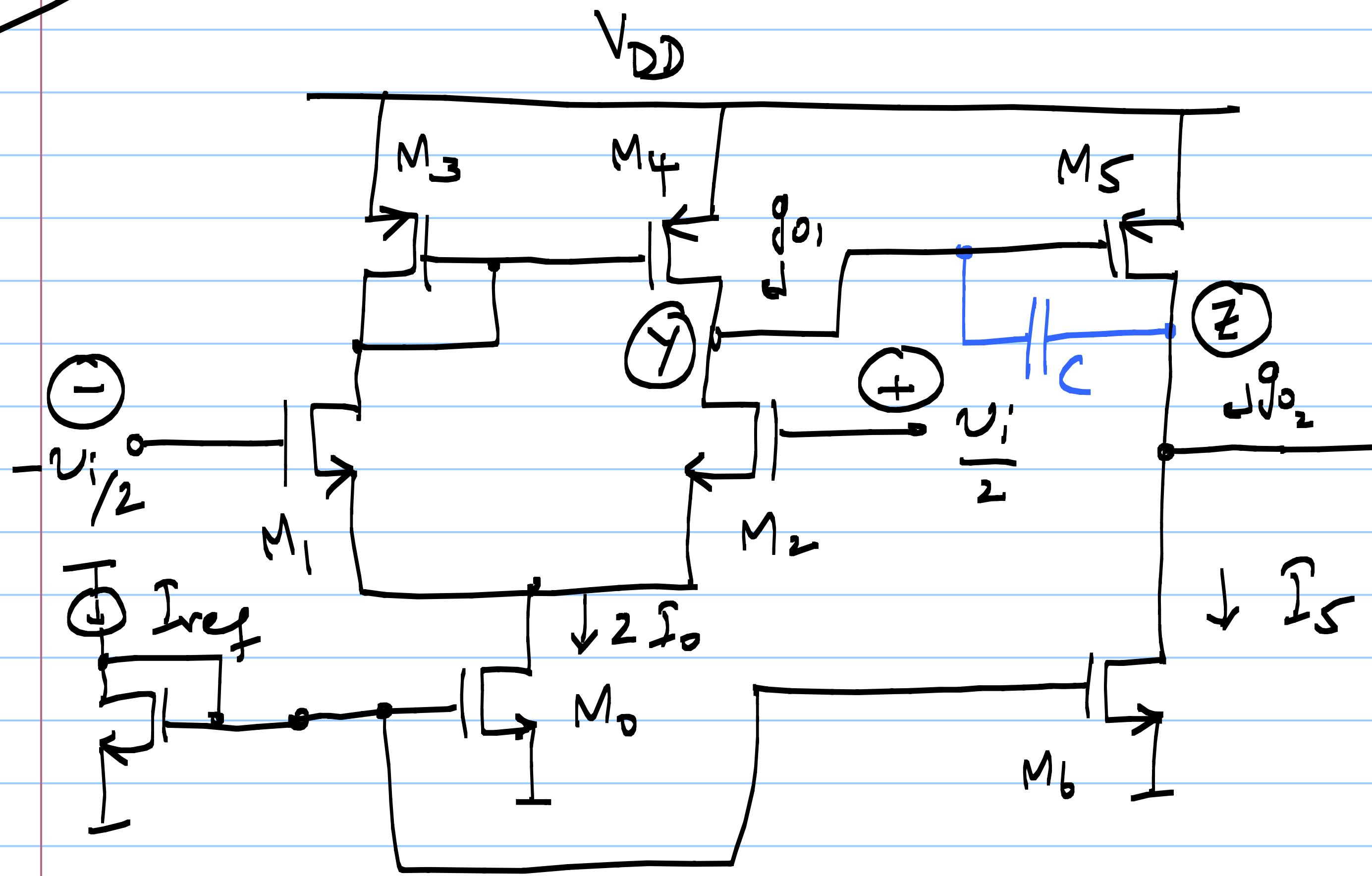




$G_{out} = G_{m_2}$

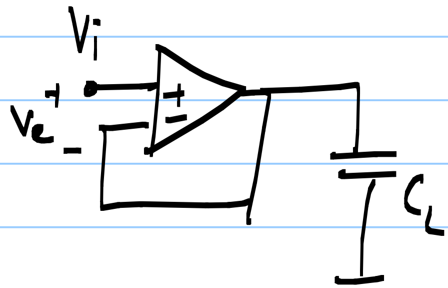
28/10/2020

Lecture 46



$p_y < p_z$

add C: $p_y \to p_y'$; add C: $p_z \to p_z'$



$g_{o_1} = g_{ds_2} + g_{ds_4}$

$g_{o_2} = g_{ds_5} + g_{ds_6}$

$$A_1 = \frac{g_{m_1}}{g_{o_1}} \; ; \quad A_2 = \frac{g_{m5}}{g_{o_2}}$$

$$p_y' \approx \frac{g_{o_1}}{A_2 \cdot C} \quad \text{dominant pole}$$

DC

$$V_y = V_{DD} - V_{SG_3}\Big|_{I_o} = V_{DD} - V_{SG_5}\Big|_{I_5}$$

$$V_{SG_3}\Big|_{I_o} = V_{SG_5}\Big|_{I_5}$$

$$V_{SG} = V_T + \underbrace{\sqrt{\frac{2 I_D}{\mu_p C_{ox}\left(\frac{W}{L}\right)}}}_{V_{D,sat}}$$

$$L_3 \equiv L_4 \equiv L_5$$

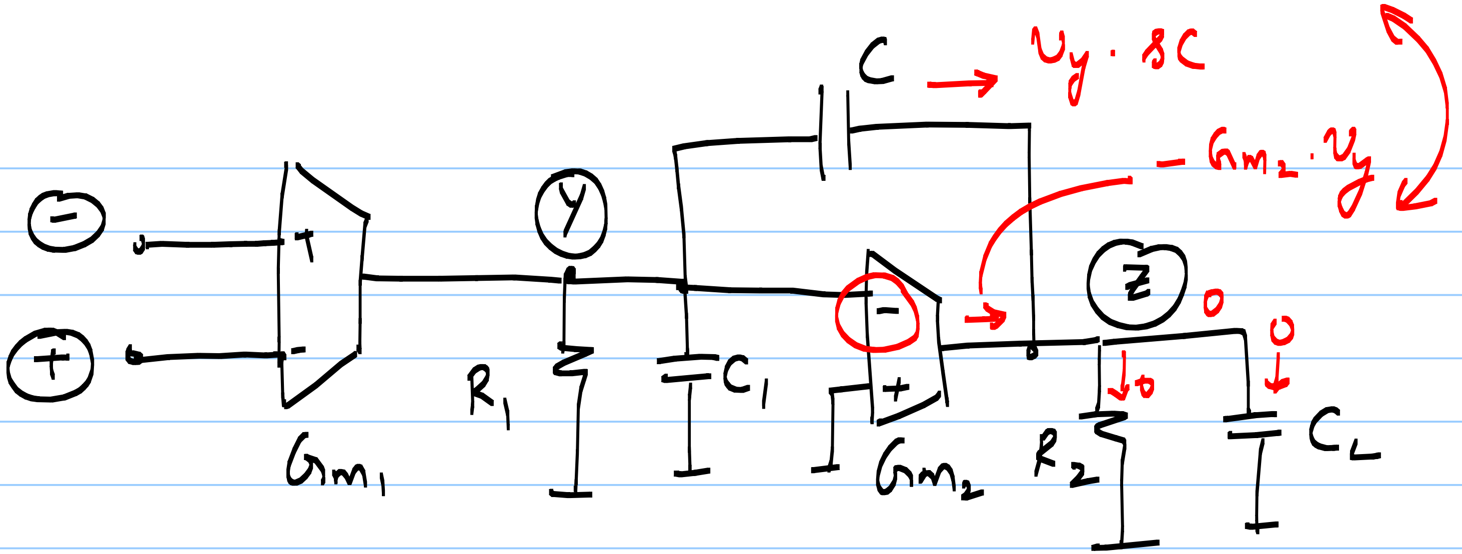
Ensure by Design:

$$V_{Dsat_3}\Big|_{I_o} = V_{Dsat_5}\Big|_{I_5}$$

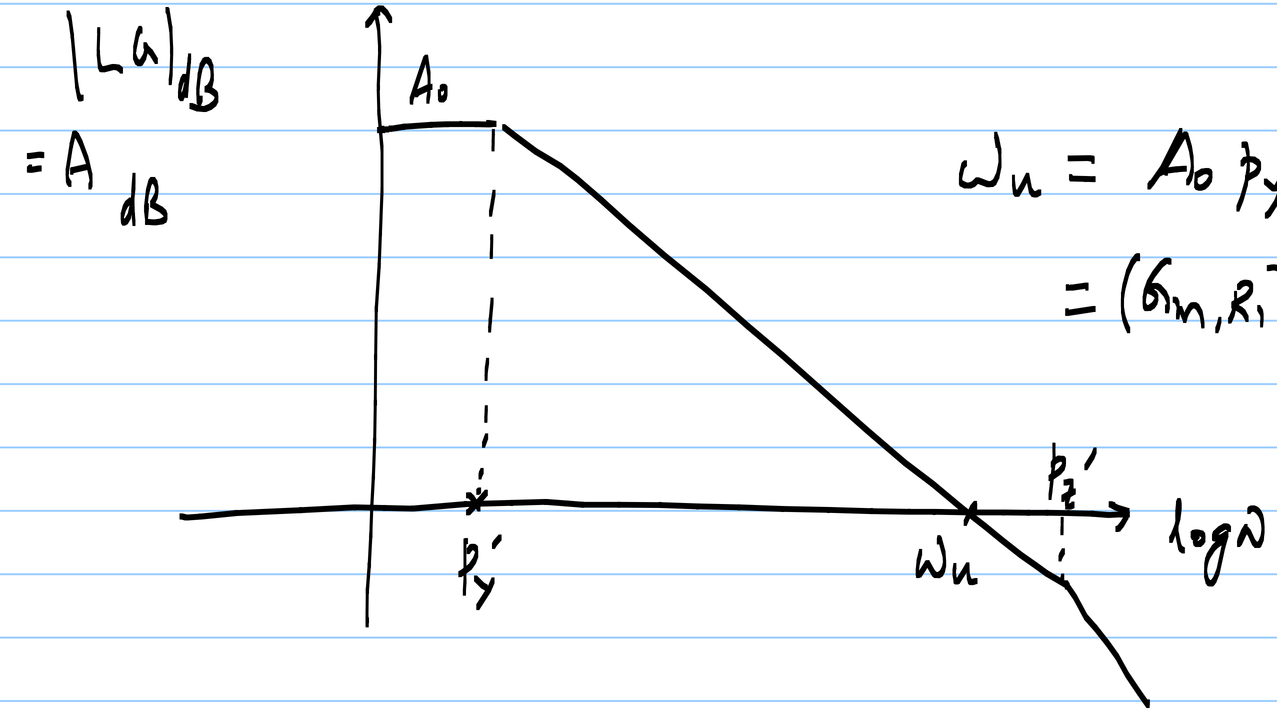
$$\sqrt{\frac{2 I_o}{\mu_p C_{ox}\left(\frac{W}{L}\right)_3}} = \sqrt{\frac{2 I_5}{\mu_p C_{ox}\left(\frac{W}{L}\right)_5}} \Rightarrow \boxed{\frac{I_{D_3}}{(W/L)_3} = \frac{I_{D5}}{(W/L)_5}}$$

Same "Current Density"

$$\omega_z = \frac{+G_{m_2}}{C}$$



$$p_y' \approx \frac{1}{R_1(G_{m_2} R_2 C)} \quad ; \quad p_z' \approx \frac{(\qquad)}{(\qquad)}$$



$$\omega_u = A_0 \, p_y'$$

$$= (G_{m_1} R_1)(G_{m_2} R_2) \cdot \frac{1}{R_1 \cdot G_{m_2} R_2 C}$$

$$\boxed{\omega_u = \frac{G_{m_1}}{C}}$$

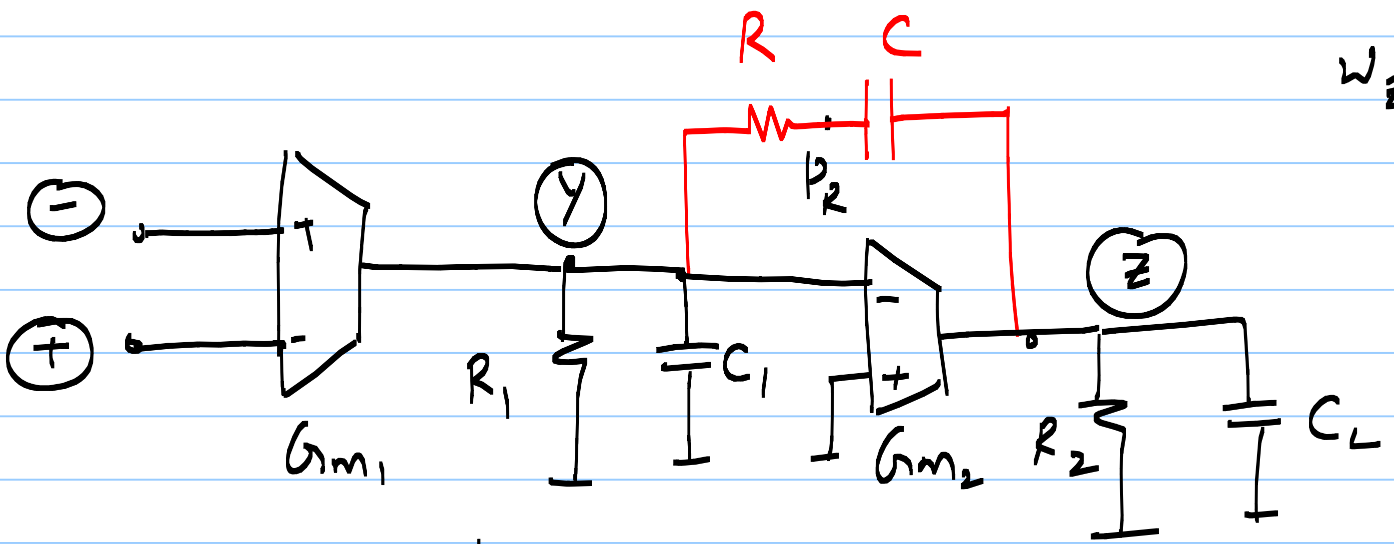
$$\omega_z = + \frac{Gm_2}{C} \quad (\text{RHP zero})$$

* Design condition: $Gm_2 \gg Gm_1$ i.e. $g_{m5} \gg g_{m1}$

so that $\omega_z \gg \omega_u$

RC - pole splitting compensation

* want to move ω_z

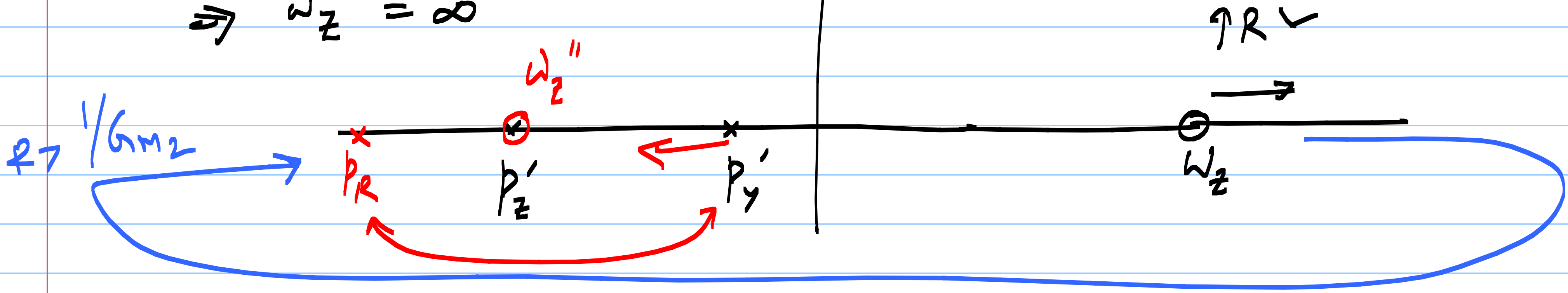


* poles will move slightly
* 3^{rd} pole added p_R

$$\omega_z = \frac{1}{\left(\frac{C}{Gm_2} - RC\right)} = \frac{1}{C\left(\frac{1}{Gm_2} - R\right)}$$

1) set $R = \frac{1}{G_{m_2}}$

$\Rightarrow \omega_z' = \infty$



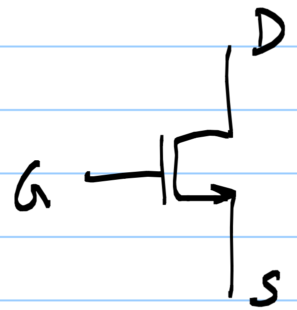
2) $R > \frac{1}{G_{m_2}} \Rightarrow$ LHP zero

choose R to set $\omega_z'' = p_z'$

29/10/2020

Lecture 47

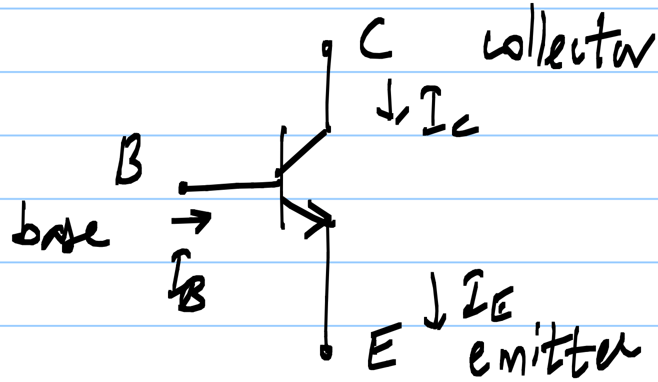
Bipolar Junction Transistor (BJT)



NMOS

cause - V_{GS}

effect - I_D

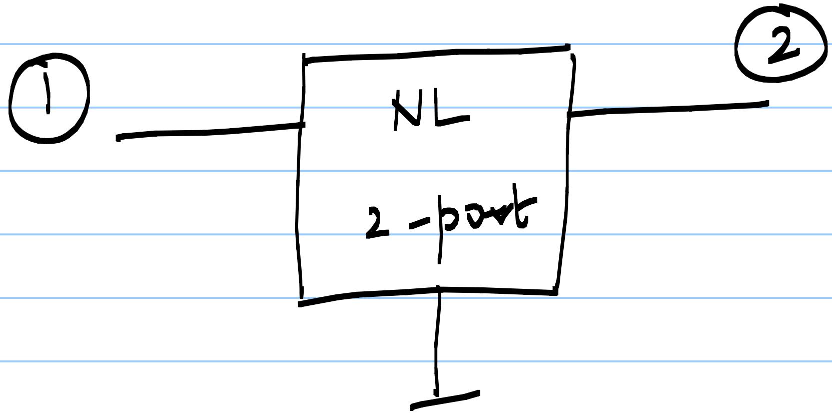


NPN transistor

cause - V_{BE}

effect - I_C

V_{BE} voltage controls I_C



$$[y] = \begin{bmatrix} 0 & 0 \\ y_{21} & 0 \end{bmatrix}$$

y_{21} as large as possible

I_1 = constant

$I_2 = g(V_1)$

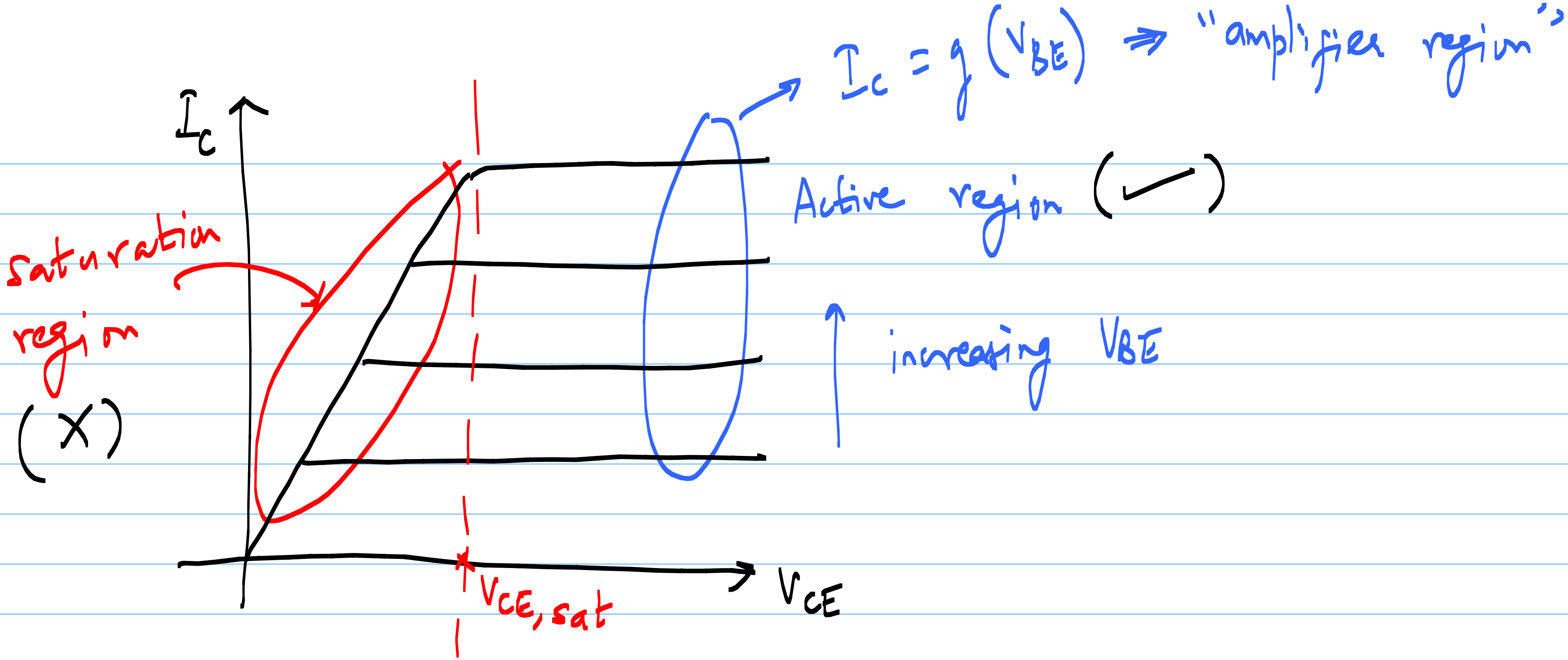
BJT also shares these characteristics

$V_{BE_{nom}} \approx 0.65V$

$$I_c = I_s \left[\exp\left[\frac{V_{BE}}{V_t}\right] - 1 \right]$$

I_B = very small

$$V_t = \frac{kT}{q} \approx 25mV$$



When $V_{CE} > V_{CE,sat} \Rightarrow$ good amplifier

$$V_{BE,on} \approx 0.65\,V, \qquad V_t \approx 25\,mV$$

* $$I_c \approx I_s \exp\left(\frac{V_{BE}}{V_t}\right)$$

* $$V_{BE} = V_t \ln\left(\frac{I_c}{I_s}\right)$$

* $$I_B = \frac{I_c}{\beta}$$

β = current gain of BJT

$\sim$ 50 – 200, typically

* $$I_E = I_c + I_B = (\beta+1)\, I_B = \left(\frac{\beta+1}{\beta}\right) I_c$$

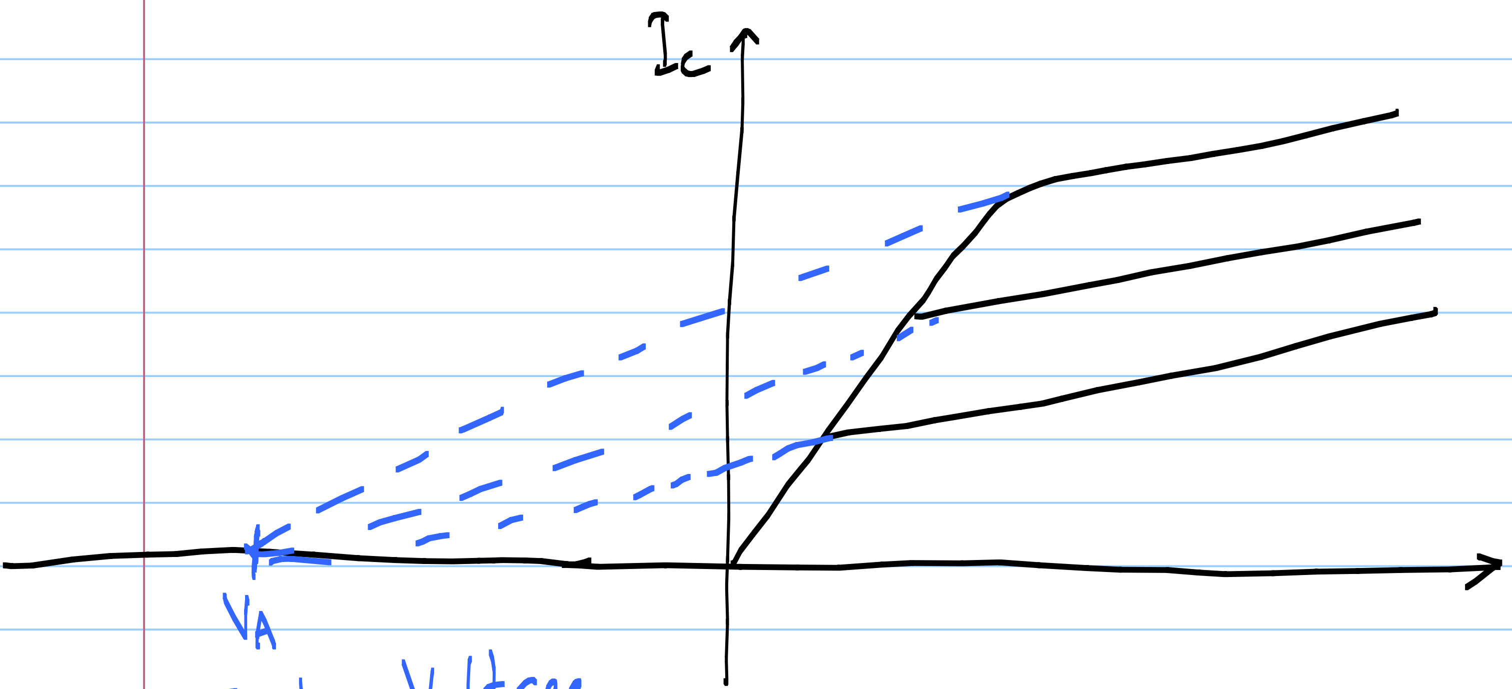
* Small signal parameters:

$$y_{11} = \frac{\partial I_B}{\partial V_{BE}} = \frac{1}{\beta}\frac{\partial I_c}{\partial V_{BE}} = \frac{I_c}{\beta V_t} = \frac{g_m}{\beta}$$

$$y_{12} = \frac{\partial I_B}{\partial V_{CE}} = 0$$

$$y_{21} = \frac{\partial I_c}{\partial V_{BE}} = \frac{I_c}{V_t} = g_m$$

$$y_{22} = \frac{\partial I_C}{\partial V_{CE}} = 0 \quad \left\{ \begin{array}{l} \text{very small in a} \\ \text{real device} \end{array} \right\}$$

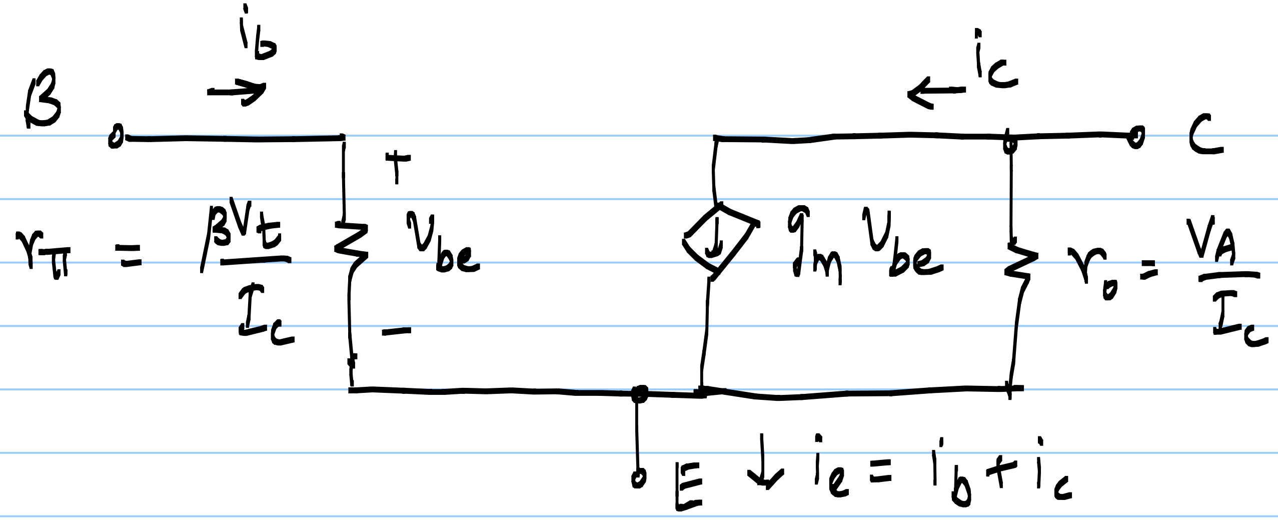


Early Voltage

$$I_C = \left[I_S \exp\left(\frac{V_{BE}}{V_t}\right)\right]\left[1 + \frac{V_{CE}}{V_A}\right]$$

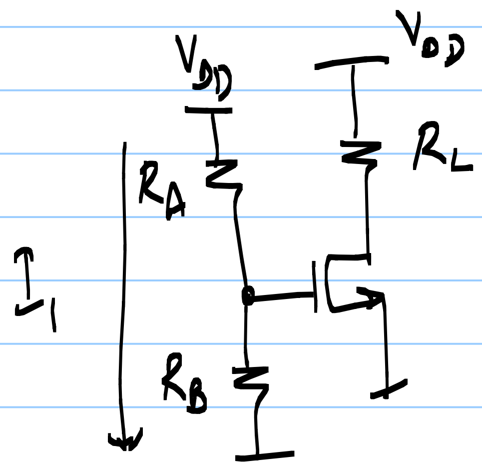
$V_A = -\infty$ for an ideal device with $y_{22} = 0$

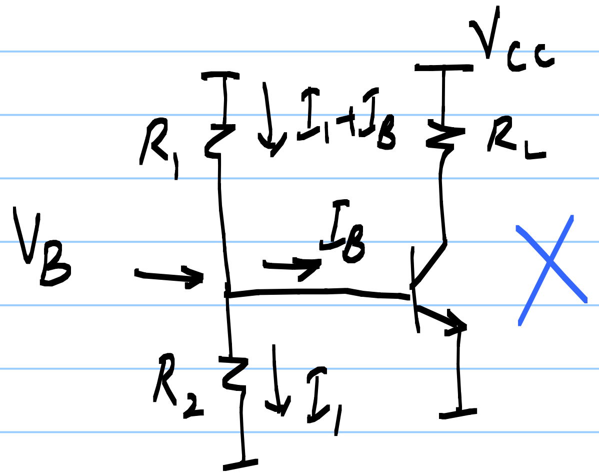
actual $y_{22} = \dfrac{I_C}{V_A}$



other small-signal representations exist

BJT Amplifiers



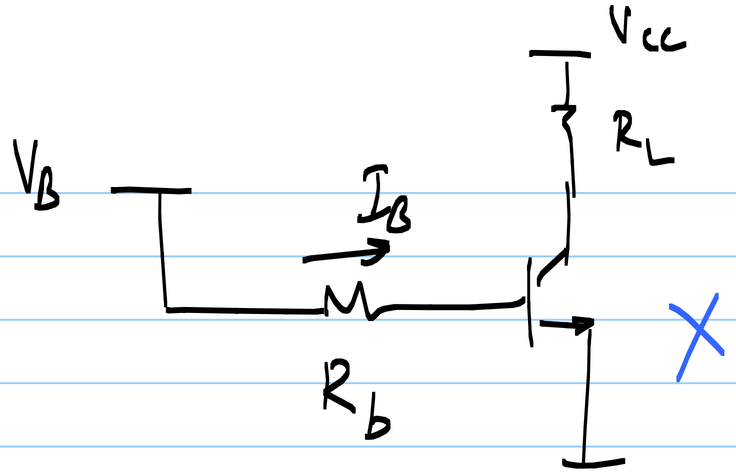


V_B will not be equal to desired bias value due to additional drop across R_1

choose R_A, R_B so that

$R_A \| R_B \gg R_s$

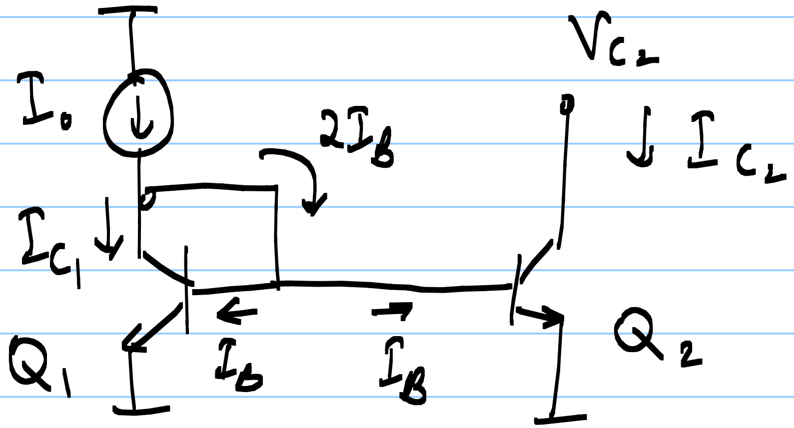
i.e. $V_B = f(I_B)$
$= f(I_c)$



BJT current mirror

o)

$Q_1 = Q_2$



$$I_{c_1} = I_o - 2I_B$$

$$I_B = \frac{I_{c_1}}{\beta}$$

$$I_{c_1} = \frac{I_o \cdot \beta}{\beta + 2}$$

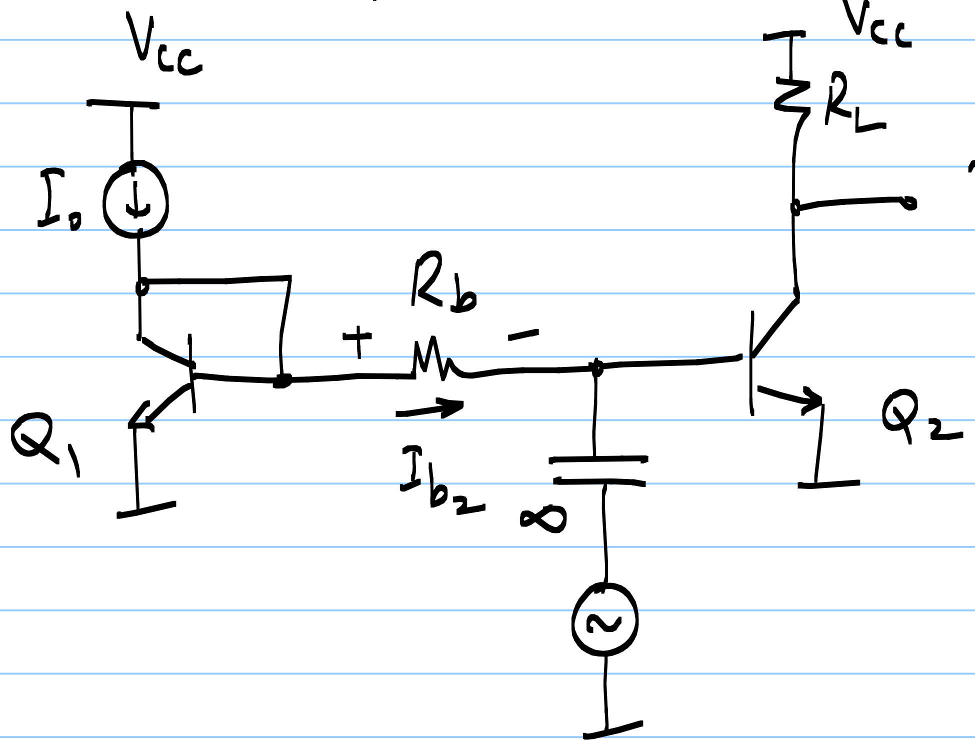
$$V_{BE_1} = V_{BE_2} \Rightarrow I_{c_1} = I_{c_2} \Rightarrow I_{B_1} = I_{B_2} = I_B$$

when $V_{c_2} > V_{CE_{sat}}$

3/11/20

Lecture 48

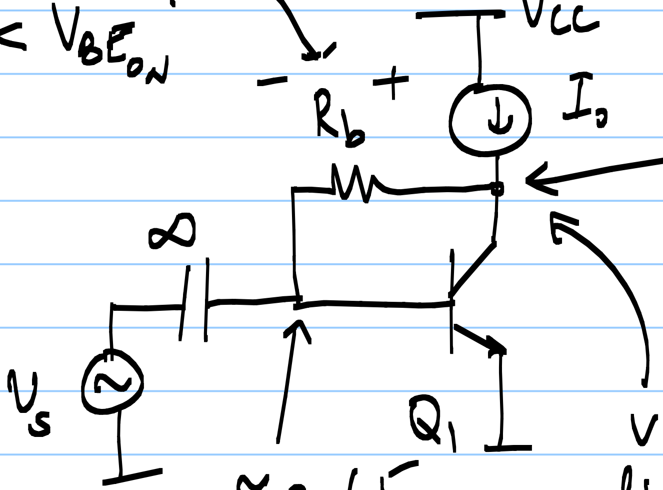
1)



Not a good way of biasing.

1.5)

$<< V_{BE_{on}}$



≈ 0.65

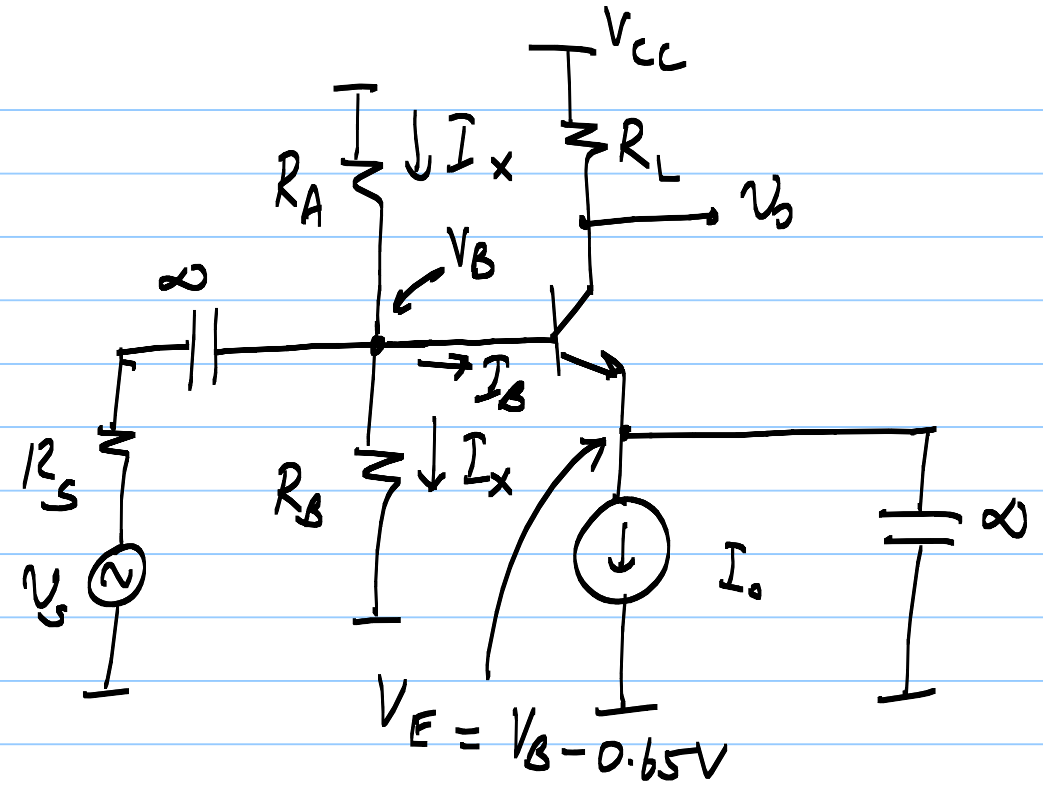
very little swing allowed

$V_{CE} \approx 0.65$

$V_{CE_{sat}} \approx 0.2 \rightarrow V_{CB} \approx 0.45\text{ V}$

$V_{CB} = 0$ if $V_{CE} = 0.65\text{ V}$

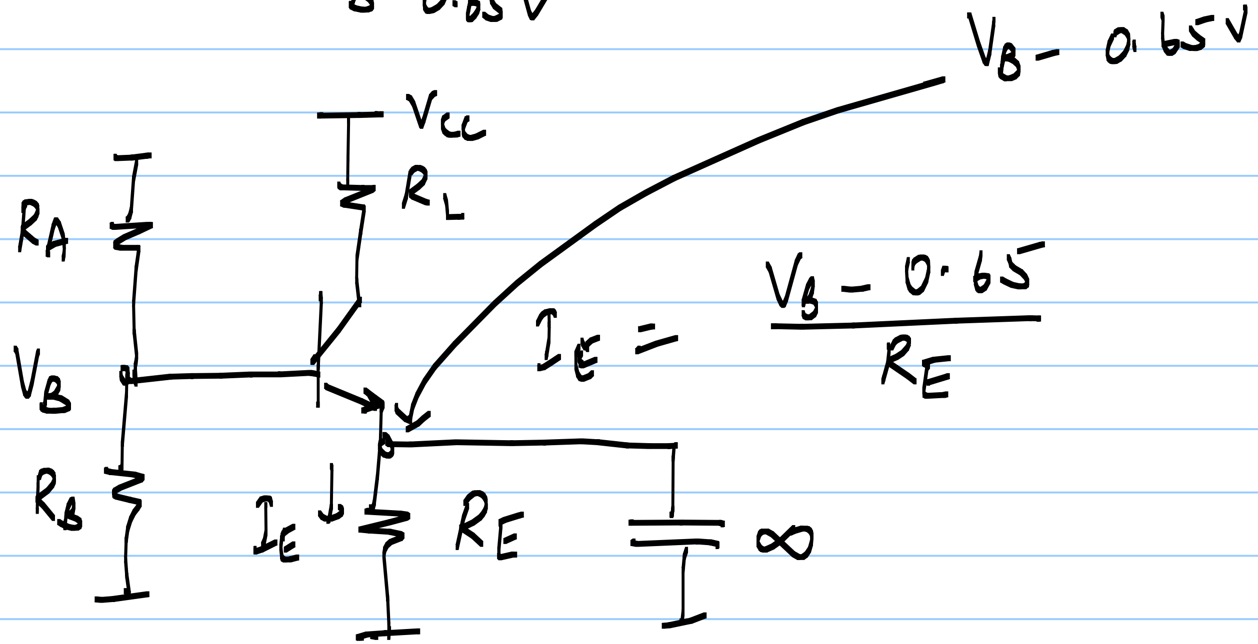
2)



$I_x \gg I_B$

"Common Emitter Amplifier"

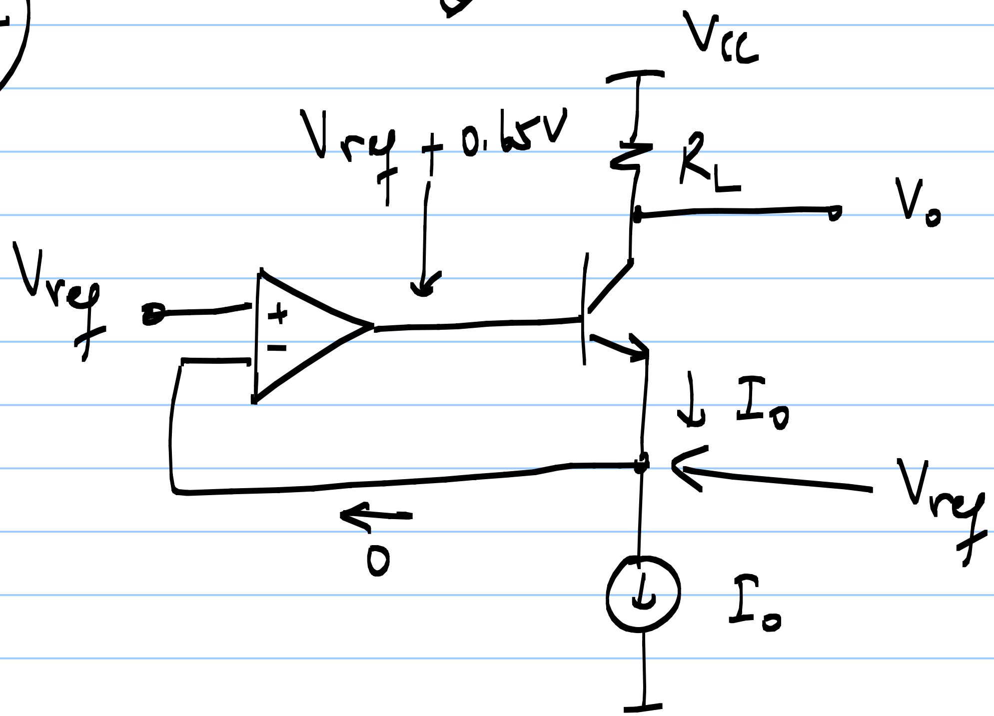
2.5)



$$I_E = \frac{V_B - 0.65}{R_E}$$

Circuits (3) & (4) – HW { require opamps }

4)



Swing Limits

1) Saturation limit

$$V_{CE_{sat}} = 0.2\,V$$

e.g. for circuit (2):

$$V_C - V_E = 0.2\,V$$

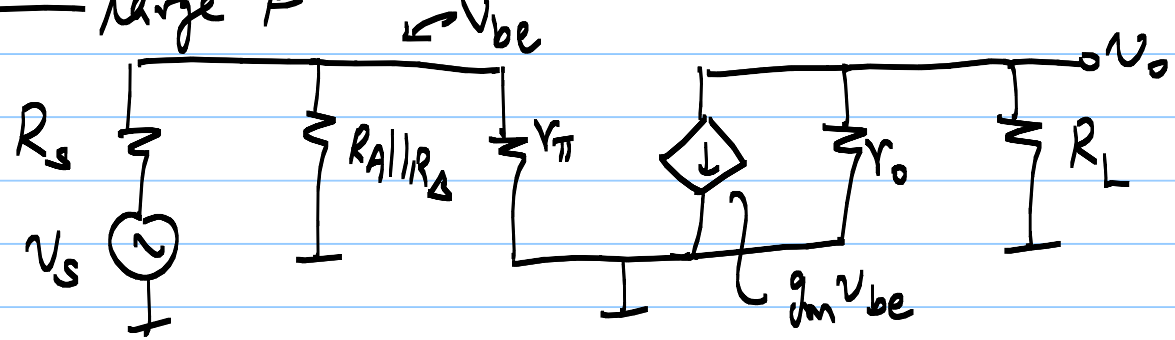
$$V_{CC} - I_o R_L + \underbrace{A}_{-g_m R_L} V_A \sin\omega t - \left(V_B - 0.65\,V\right) = 0.2\,V$$

A = gain of CE amplifier

$r_\pi >> R_s$

$R_A || R_b >> R_s$

large β



$R_L << r_o$

$(r_o >> R_L)$

$$v_{be} \simeq v_s$$

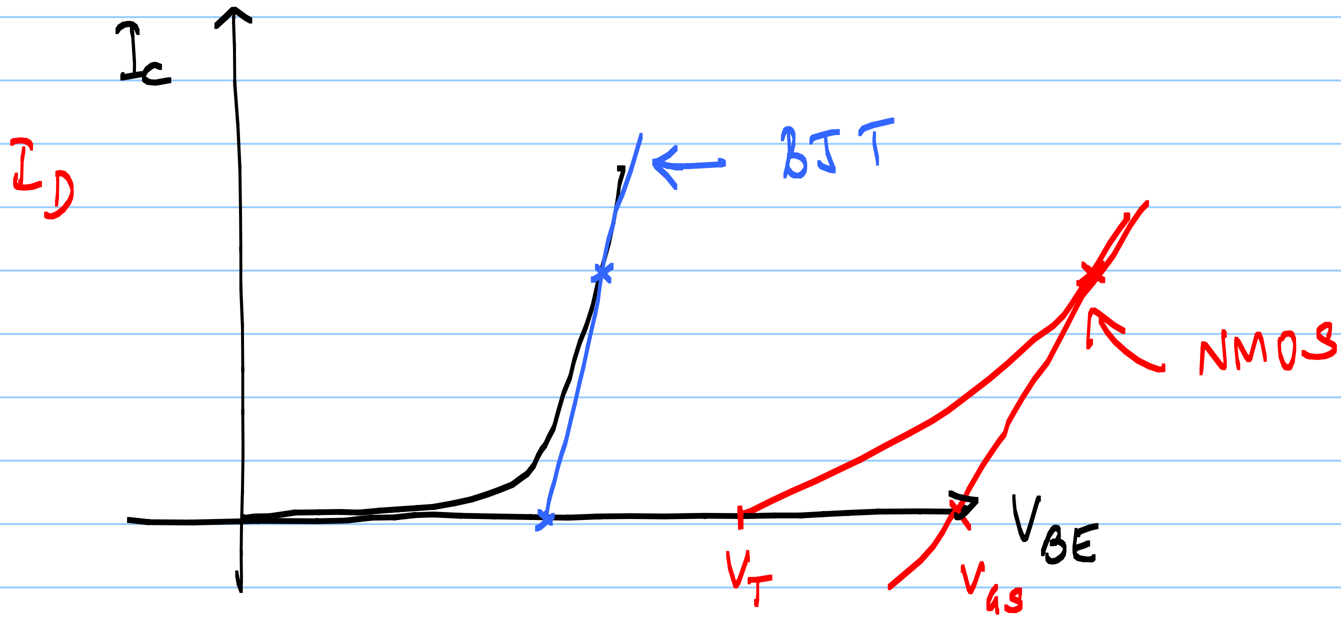
$$v_o = -g_m R_L v_s$$

$$A = \frac{v_o}{v_s} = -g_m R_L$$

2) <u>Cutoff limit</u>

$$I_c = 0$$

$$I_o + g_m V_A \sin\omega t = 0 \Rightarrow V_A = \frac{I_o}{g_m}$$



4/11/20

Lecture 49

Bandgap Reference

* Create V_{ref} that is independent of temperature
* $C, R, \mu, V_T, V_t \rightarrow$ all vary with temp.
* At least at one temp. $T_0 \rightarrow$ make $\frac{dV_{ref}}{dT} = 0$

generate $V_{ref} = \alpha_1 V_1 + \alpha_2 V_2$

$\alpha_1 V_1$: -ve Temp co

$\alpha_2 V_2$: +ve Temp co

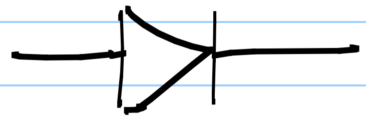
-ve Temp co: -ve $\frac{\partial V_1}{\partial T}$

+ve Temp co: +ve $\frac{\partial V_2}{\partial T}$

Temp co: temperature coefficient

@ RT : set $\left.\frac{\partial V_{ref}}{\partial T}\right|_{RT} = 0$

BE junction of a BJT



(e.g. parasitic BJT in a CMOS process)

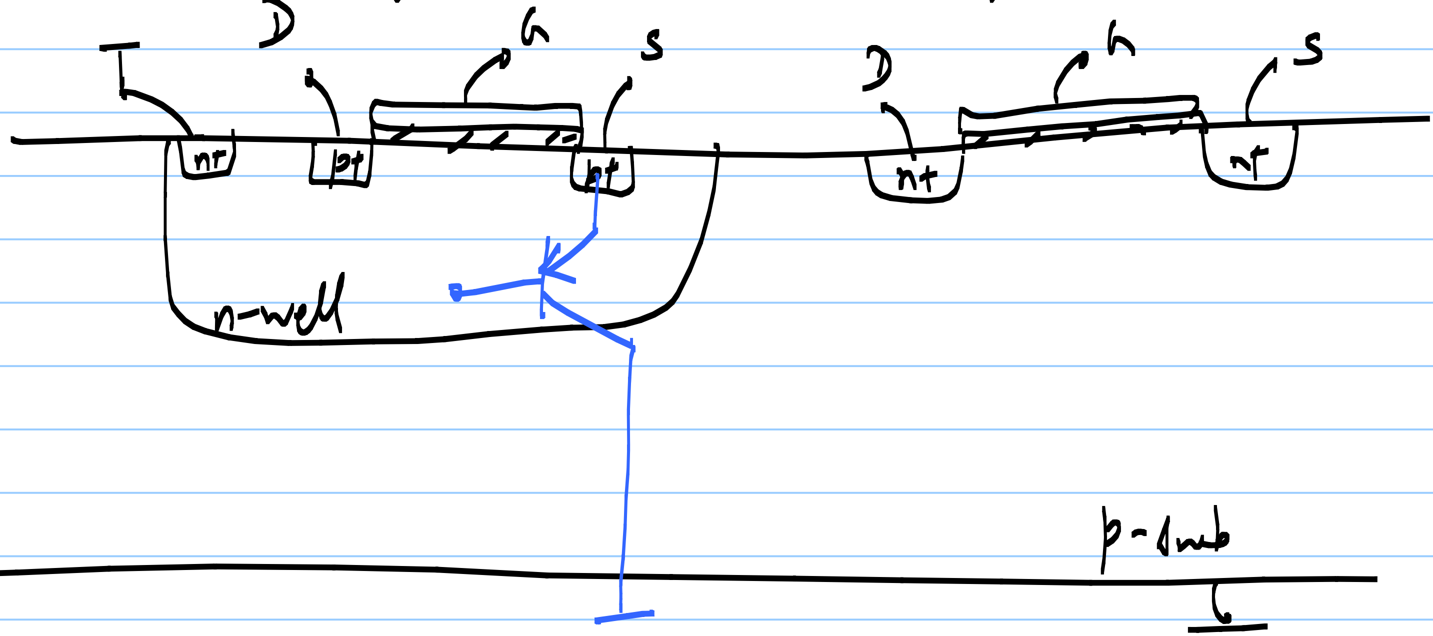
\+ V_{BE} −

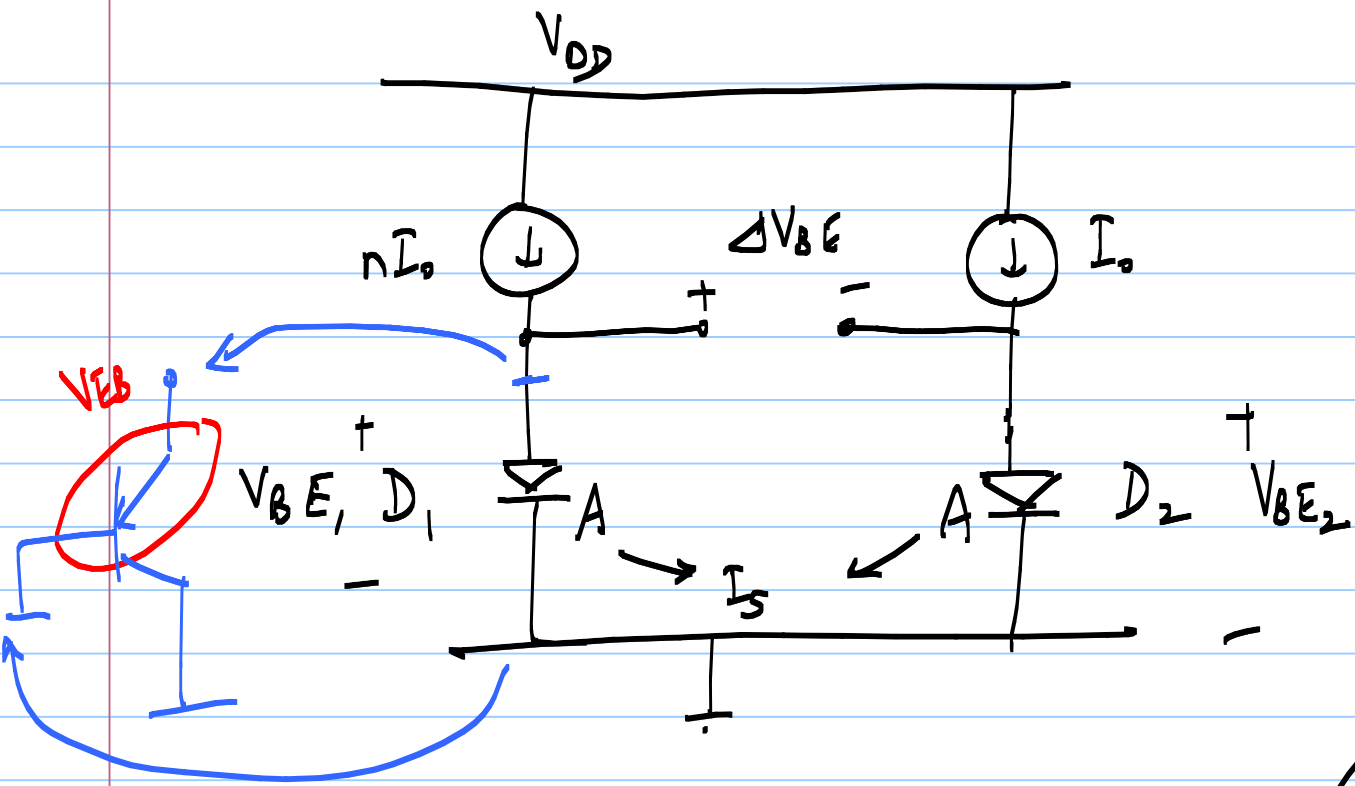
$\approx 0.65V$

CTAT
complementary to absolute temp.

$$\left.\frac{\partial V_{BE}}{\partial T}\right|_{RT} \approx -1.5 \text{ to } -2 \text{ mV/K}$$ → use to create V_1

$$V_t = \frac{kT}{q} \rightarrow \frac{\partial V_t}{\partial T} = \frac{k}{q}$$ → use to create V_2





$$\Delta V_{BE} = V_{BE_1} - V_{BE_2}$$

$$= V_t \ln\left(\frac{I_{C_1}}{I_s}\right) - V_t \ln\left(\frac{I_{C_2}}{I_s}\right)$$

$$= V_t \ln\left(\frac{nI_o}{I_s}\right) - V_t \ln\left(\frac{I_o}{I_s}\right)$$

$$\Delta V_{BE} = V_t \ln(n)$$

$$= \frac{kT}{q} \ln(n)$$

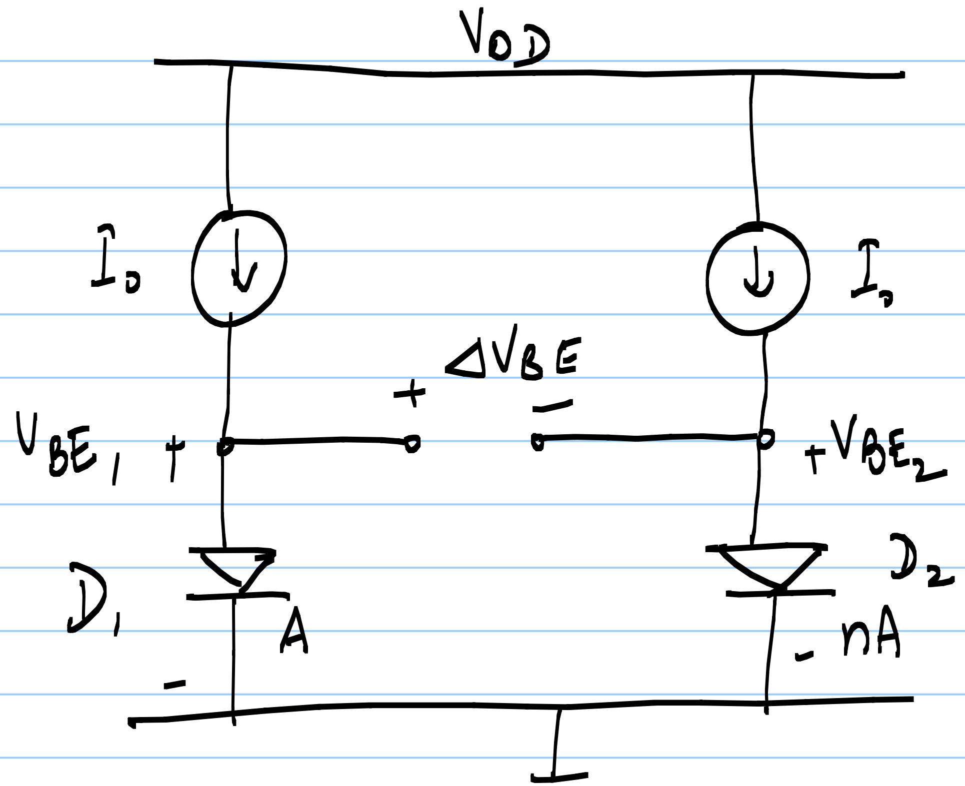
$$\propto T$$

V_2 with +ve T.C.

* exact value of I does not matter

ΔV_{BE} is called a "PTAT" voltage

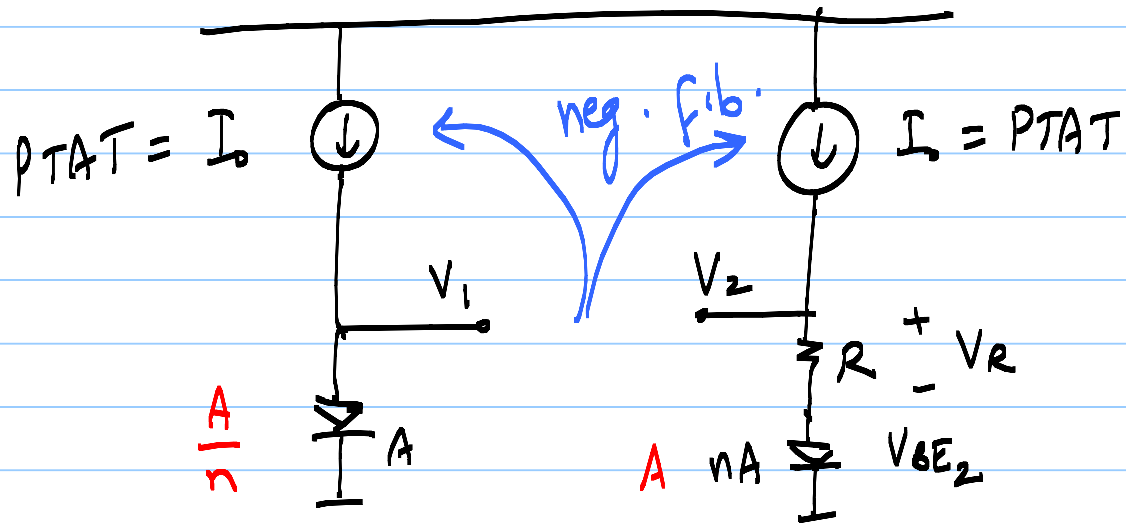
↳ proportional to absolute temperature



$\Delta V_{BE} =$

$$V_t \ln\left(\frac{I_o}{I_s/n}\right) - V_t \ln\left(\frac{I_o}{I_s}\right)$$

$$\Delta V_{BE} = V_t \ln(n)$$



force $V_1 = V_2$

$$\Rightarrow V_R = \Delta V_{BE_1} = V_t \ln(n)$$

Use an opamp in negative f.b.

V_{ref} : $V_2 = V_{BE_2} + V_R$ = $V_{BE_2} + \Delta V_{BE}$ ← set equal to V_{ref}

Set temp. coeff. @ RT of V_2 to be zero

$$\left.\frac{\partial V_{ref}}{\partial T}\right|_{300K} = 0$$

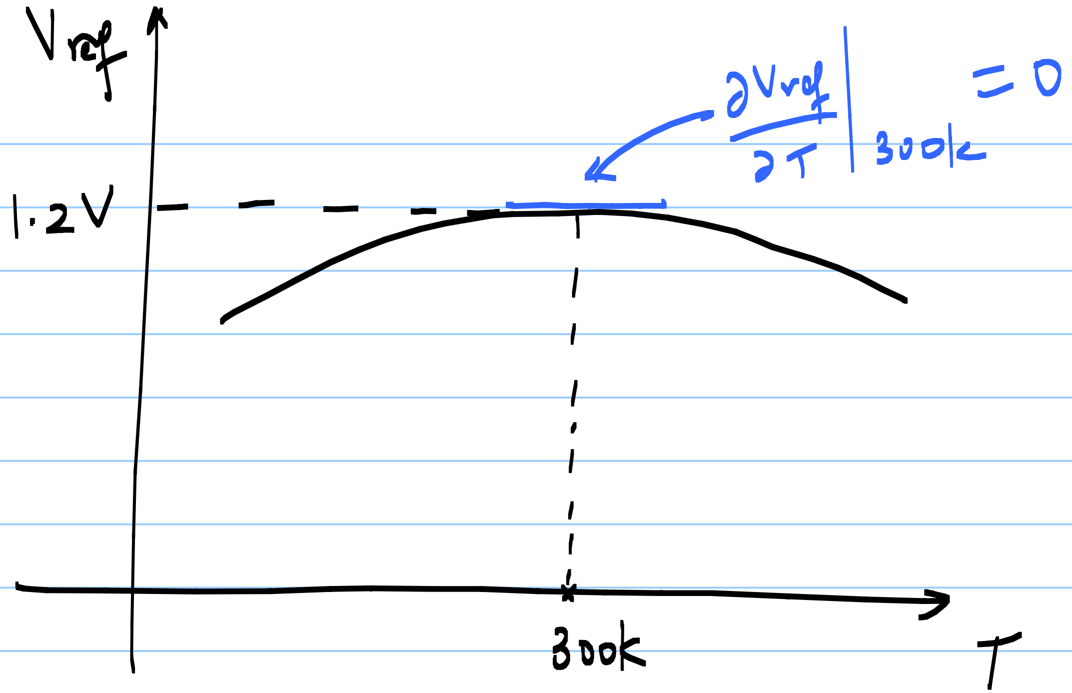
$$\left.\frac{\partial}{\partial T}\left(V_{BE_2} + \frac{kT}{q}\ln(n)\right)\right|_{RT} = 0$$

$$\left.\frac{\partial V_{BE_2}}{\partial T}\right|_{RT} + \frac{k}{q}\ln(n) = 0$$

$-1.5\,mV/K$ → $+1.5\,mV/K$

$$\ln(n) = 1.5 \times \frac{q}{k} \simeq 17.4$$

$$V_{ref} \simeq 1.2V$$

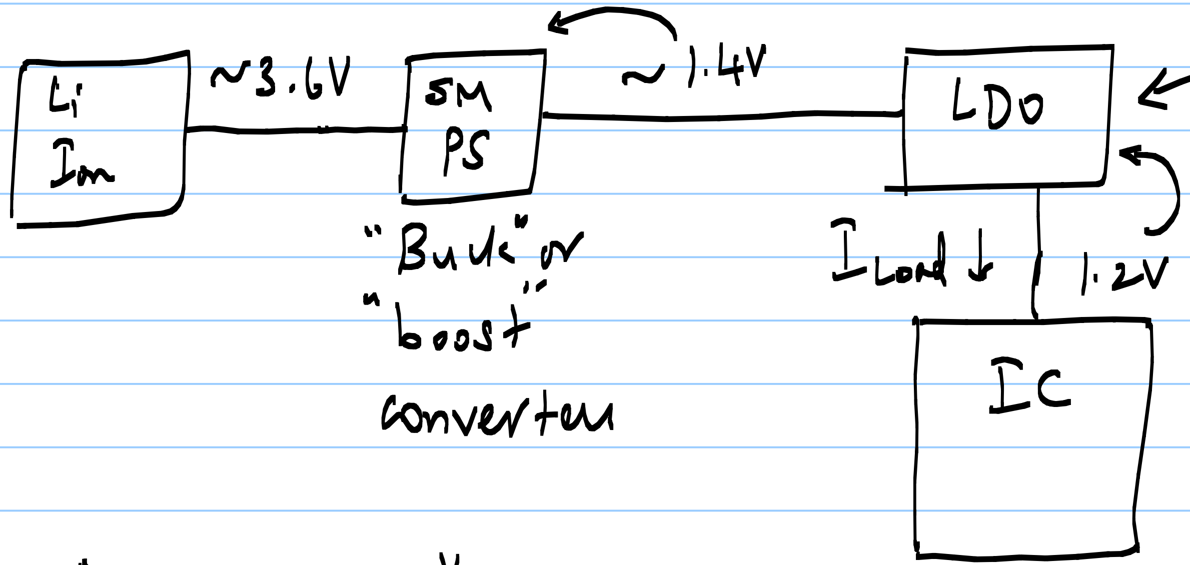


Vref
1.2V
300k
T
$\left.\frac{\partial V_{ref}}{\partial T}\right|_{300k} = 0$

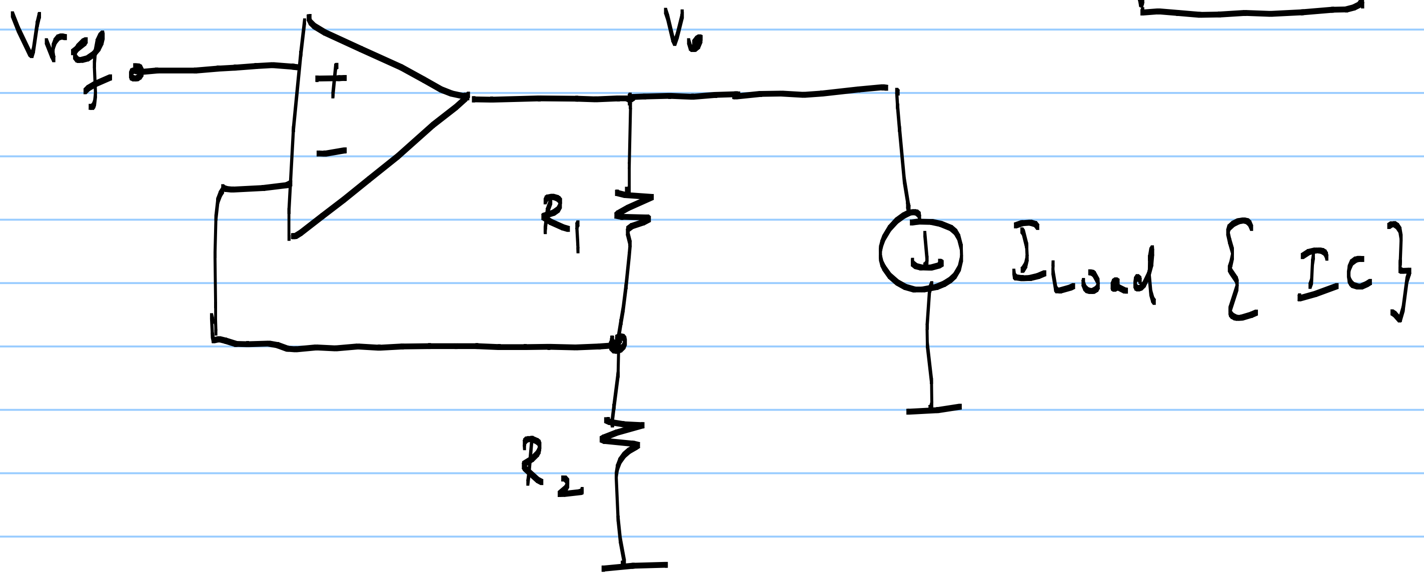
5/11/20

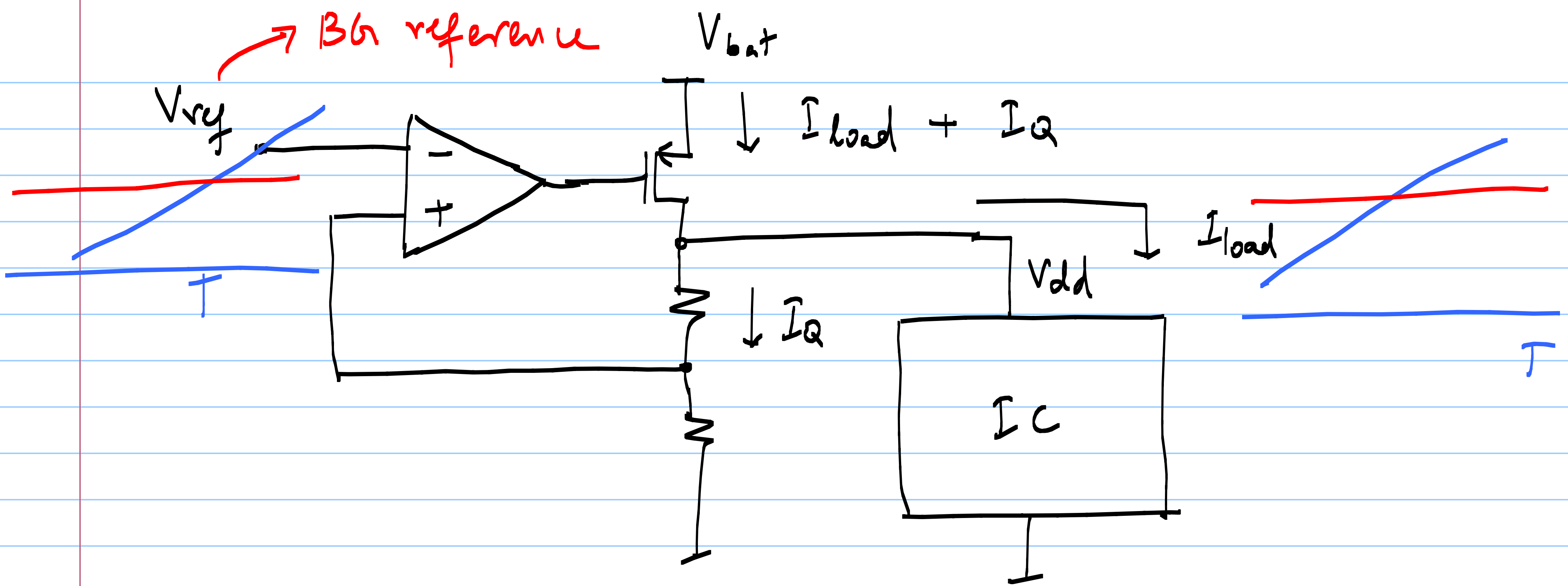
Lecture 50

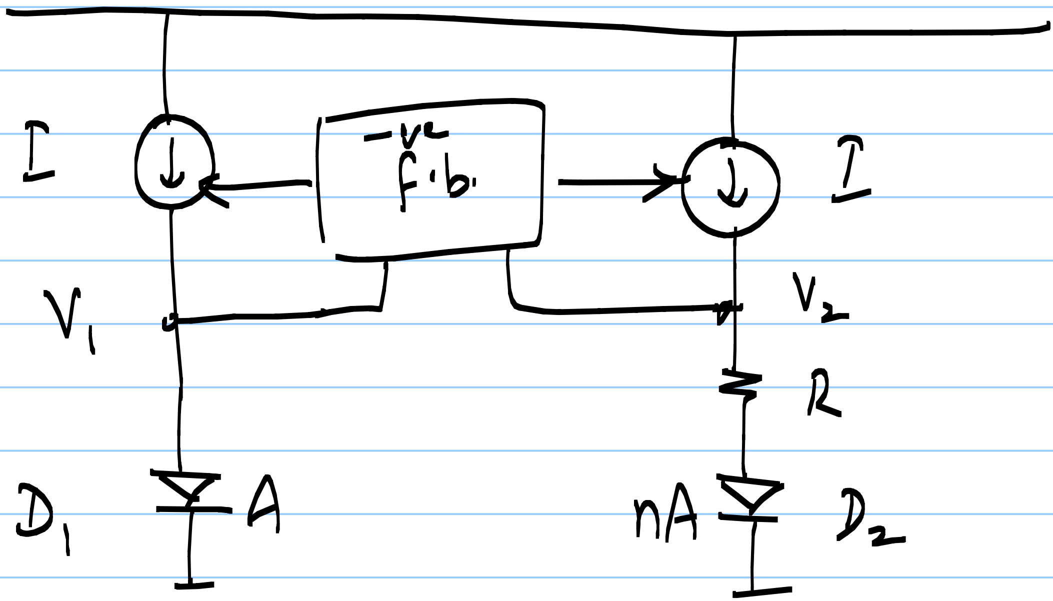
One application of BG reference — Low Dropout regulator



should be constant even if 3.6V battery voltage changes







$V_1 = V_2 = V_{ref} \sim 1.2V$

$V_1 = V_{BE_1} \sim 1.2V$

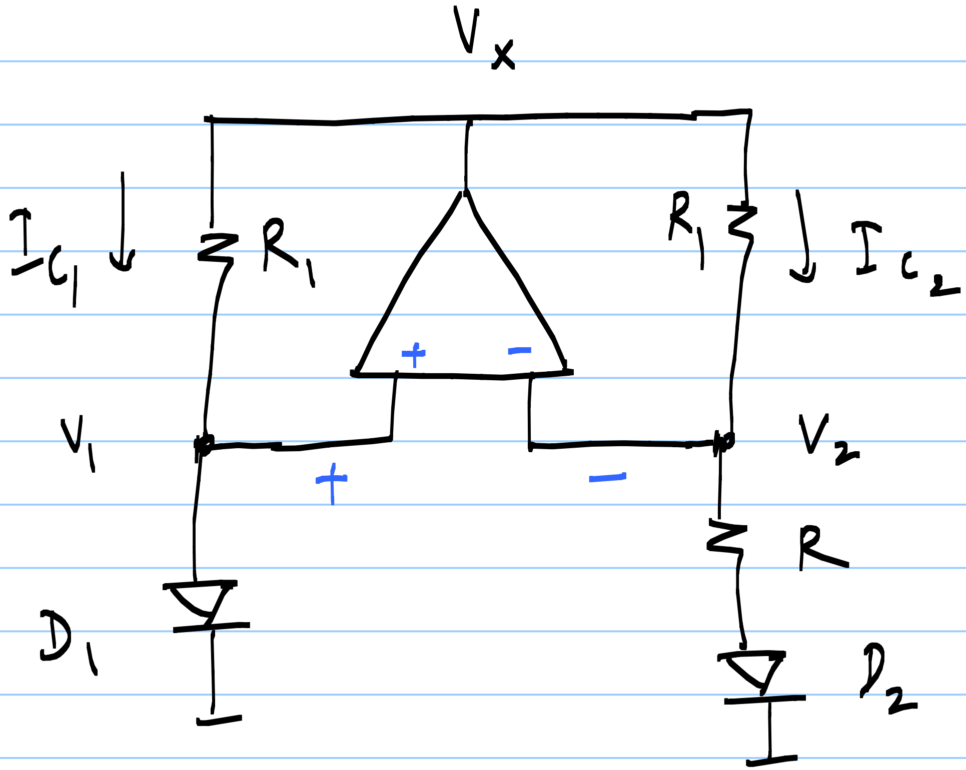
normally

$V_{BE_1} \sim 0.65V$

$$V_{BE_1} = V_t \, \overbrace{\ln\left(\frac{I_{c_1}}{I_{s_1}}\right)} \approx \frac{0.65V}{0.026V} \approx 20.xx$$

$V_{BE_1} \to 0.65V$, $V_t \to 26mV$

$\left.\begin{array}{ll} I_{c_1} & \text{large} \\ I_{s_1} & \text{small} \end{array}\right) - V_{BE} \sim 1.2V$

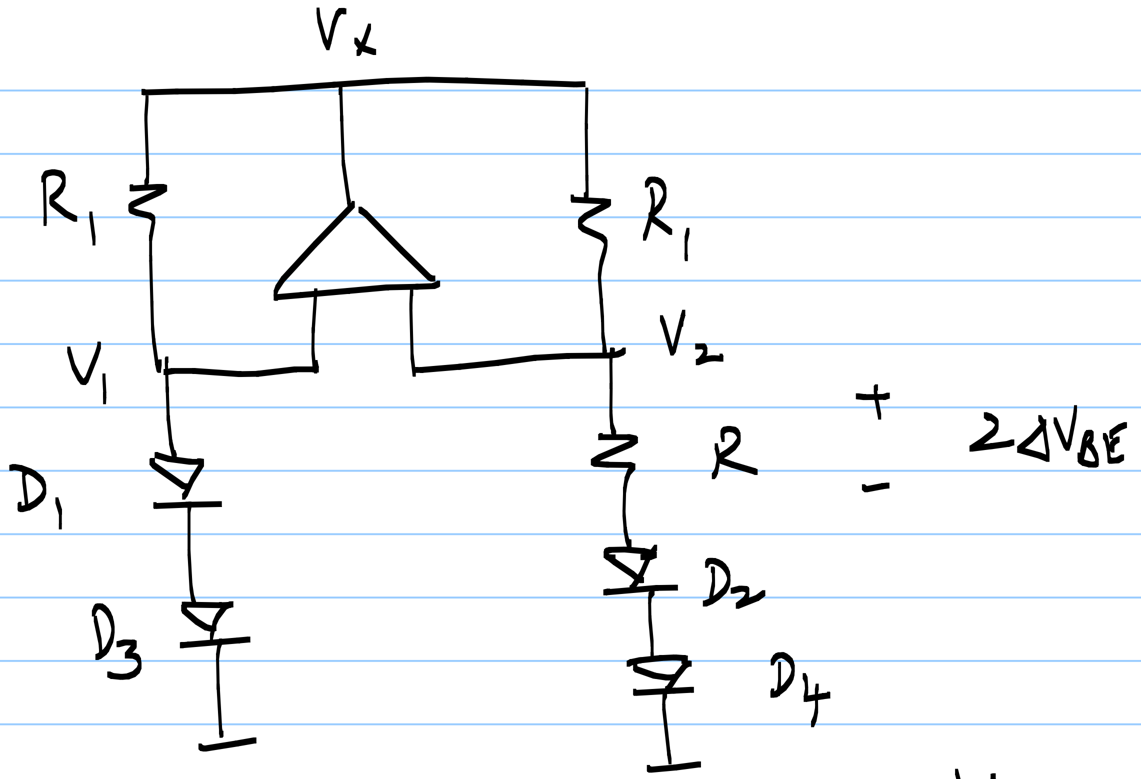


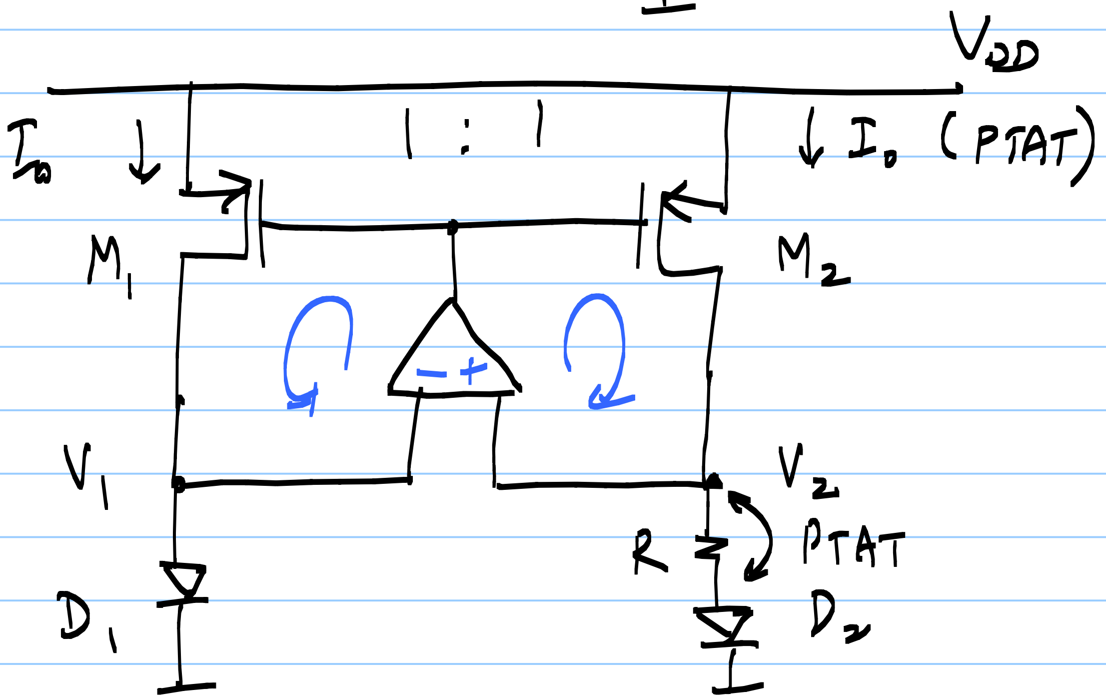
$$I_{c_1} = \frac{V_x - V_1}{R_1}$$

$$I_{c_2} = \frac{V_x - V_2}{R_1} = I_{c_1}$$

HW

* choose signs of opamp so that strength of −ve f/b. > strength of +ve f/b.



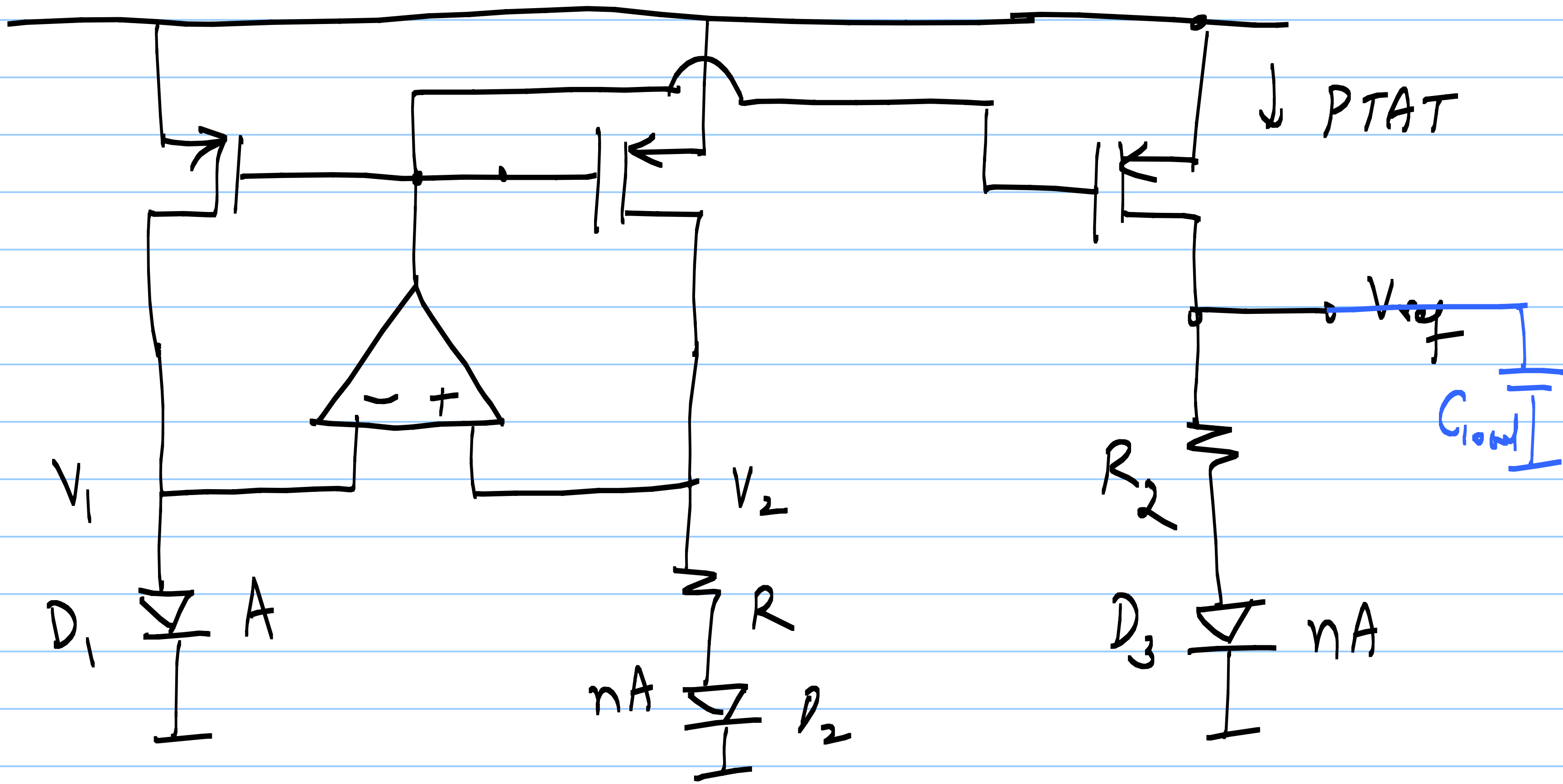


$M_1 \equiv M_2$

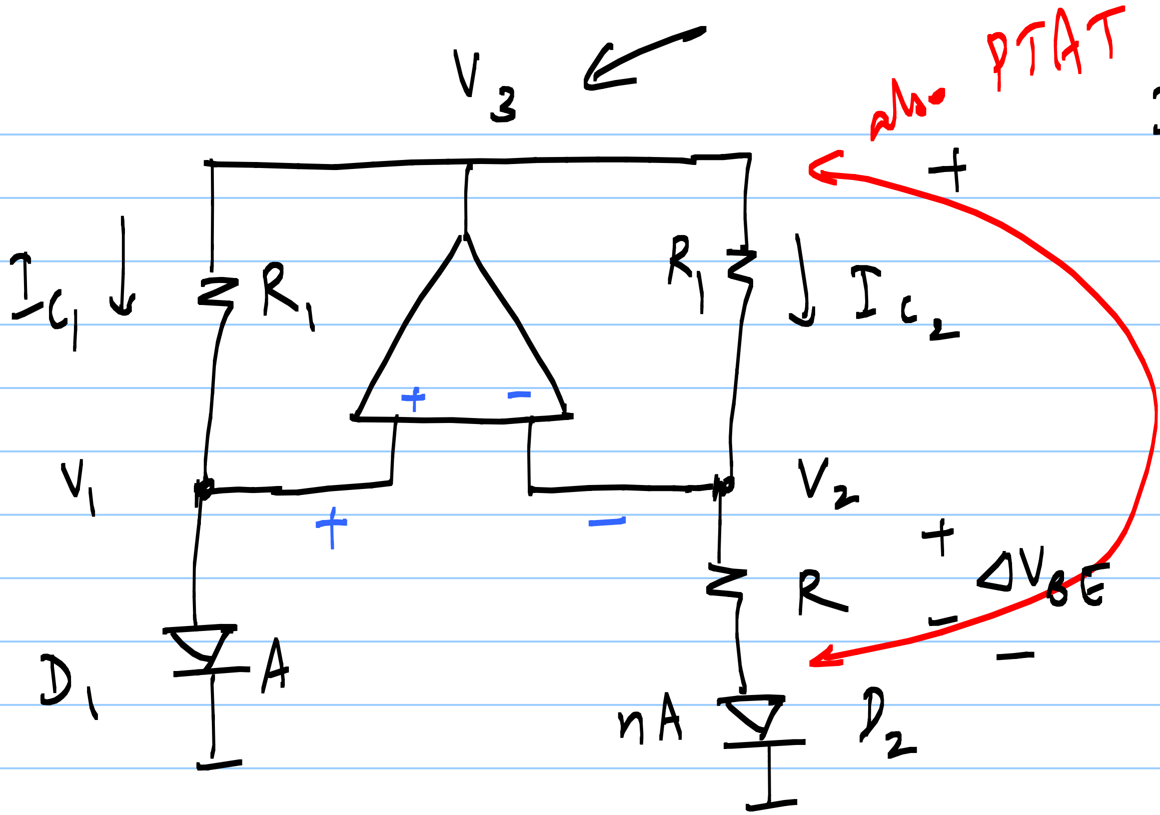
$V_{SG_1} = V_{SG_2}$

$V_{SD_1} = V_{SD_2}$

$I_{D_1} = I_{D_2} = I_0$



PTAT
V_{ref}
C_{load}
V_1
V_2
R_2
R
D_1
A
nA
D_2
D_3
nA



$$I_{C_1} = I_{C_2} = \frac{V_t \ln(n)}{R}$$

$$V_3 = V_2 + I_{C_2} R_1$$

$$= \underbrace{V_{BE_2} + I_{C_2} R} + I_{C_2} R_1$$

earlier: set this T.C. = 0

Now: Set T.C. of $V_3 = 0$ i.e. $\left.\frac{\partial V_3}{\partial T}\right|_{300K} = 0$

$$V_3 = V_{BE_2} + I_{C_2}(R + R_1)$$

$$= V_{BE_2} + \frac{V_t \ln(n)}{R}(R + R_1)$$

$$V_3 = V_{BE_2} + \left[V_t \ln(n)\right]\left[1 + \frac{R_1}{R}\right]$$

$$\left.\frac{\partial V_3}{\partial T}\right|_{300k} = 0 \Rightarrow \left[1 + \frac{R_1}{R}\right] \ln(n) = 17.2$$

* choose $\frac{R_1}{R}$ so that n is small

* $V_3 = V_{ref}$ {BG ref. voltage)

* V_{ref} depends on $\underbrace{\left(R_1/R\right)}_{\text{well controlled over "PVT" variations}}$