EE2019-Analog Systems and Lab: Tutorial-2

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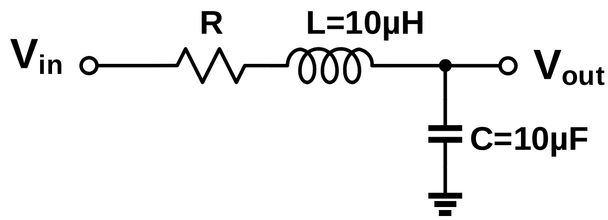
1. In the RLC circuit below,



derive the transfer function $H(s)=V_{out}(s)/V_{in}(s)$ and prove that the circuit is equivalent to a standard 2[nd] order system with transfer function:

$$H(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

a) Find the expressions for damping factor (ζ), quality factor ($Q = 1/2\zeta$), natural frequency (w_n) and poles p_1, p_2 (roots of s) and their respective frequencies, w_{p1} and w_{p2} in terms of R, L and C.

b) Considering L=10μH and C=10μF, fill the values in the following table for the corresponding values of R.

c) Show p_1 and p_2 calculated in (b) on the s-plane and comment on the movement of poles w.r.t. damping factor (ζ).

R (Ω)	ζ	Q=1/2ζ	p_1 $(\sigma + j\omega)$	p_2 $(\sigma + j\omega)$	w_{p1} (rad/s)	w_{p2} (rad/s)
0.02						
0.1						
0.4						
1						
1.4						
2						
5						
10						
20						
100						

d) Enter the circuit in the LTspice and perform following simulations for all values of R given in the table:

i. Plot AC magnitude and phase response for $V_{out}(s)/V_{in}(s)$. Comment on the behavior of AC magnitude and phase response w.r.t. damping factor, ζ.

ii. Plot the transient response by applying a unit step (0 to 1V with initial delay of 1ms and T_{rise} = 1ns) for the time span of 10ms. Comment on the effect of varying ζ on the transient response.

iii. After observing the behavior of AC and transient behavior w.r.t. ζ in (i) & (ii), it is now understood that both AC and transient response are interrelated. How can you intuitively guess the approximate value of ζ by simply looking at either AC magnitude or transient response?

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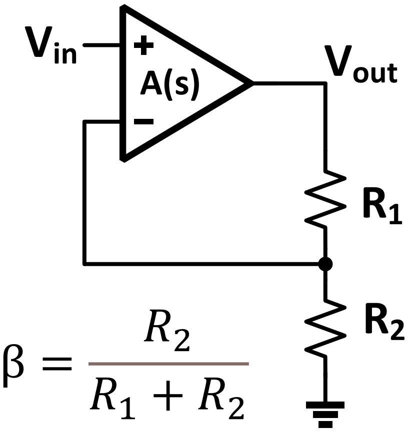
2. For the non-inverting amplifier shown in figure below, assuming op-amp is modelled as 2nd order transfer *A(s)*:



$$\beta = \frac{R_2}{R_1 + R_2}$$

$$A(s) = \frac{A_o}{\left(1 + \frac{s}{\omega_{p1}}\right)\left(1 + \frac{s}{\omega_{p2}}\right)}$$

$$A_o = 10^4$$

a) Find the loop gain transfer function, *LG(s).*

b) Find the closed loop transfer function, $H(s)=V_{out}(s)/V_{in}(s)$, poles p_{1_cl}, p_{2_cl} (roots of s) and their respective frequencies, w_{p1_cl} and w_{p2_cl} in terms of β, A_o, w_{p1} and w_{p2}.

c) Prove that, the circuit behaves similar to the RLC circuit of problem-1 for feedback factor, β=1 (i.e. R_1=0 or R_2=∞) and find the expressions for damping factor (ζ), quality factor (Q=1/2ζ), natural frequency (w_n) in terms of loop gain pole frequencies, w_{p1} and w_{p2} and DC gain, A_o.

d) For $A_o=10^4$, fill values of loop gain and closed loop parameters in the following table for the corresponding w_{p1} and w_{p2}

w_{p1} (rad/s)	w_{p2} (rad/s)	Loop Gain		Closed Loop			
		w_{ugf} (rad/s)	PM (deg.)	ζ	Q=1/2 ζ	w_{p1_cl} (rad/s)	w_{p2_cl} (rad/s)
1.00E+03	1.00E+03						
1.00E+02	1.00E+04						
2.50E+01	4.00E+04						
1.00E+01	1.00E+05						
7.07E+00	1.41E+05						
5.00E+00	2.00E+05						
2.00E+00	5.00E+05						
1.00E+00	1.00E+06						
5.00E-01	2.00E+06						
1.00E-01	1.00E+07						

e) Verify that increasing the spacing between loop gain poles, w_{p1} and w_{p2} has similar effect on damping factor as increasing R has in the RLC circuit of problem-1.

f) Show the locations of both loop gain poles p_1, p_2 and corresponding closed loop poles, p_{1_cl}, p_{2_cl} on s-plane for different values of ζ calculated in the above table.

g) Plot phase margin (PM) vs. ζ and find the range of ζ for which phase margin can be approximated as 100 times of ζ with +/-10% inaccuracy.

h) Enter the circuit in LTspice and perform following simulations for all the values of w_{p1}, w_{p2} listed in the table:

i. Plot the AC magnitude and phase response of the loop gain transfer function for all values of w_{p1}, w_{p2} and corresponding AC magnitude and phase response of the closed loop transfer function. Comment on effect of increasing and decreasing phase margin on the closed loop AC magnitude and phase response.

ii. Plot the step response by applying a unit step (0 to 1V with initial delay of 1ms and T_{rise} = 1ns) for the time span of 10ms. Comment on the effect of varying phase margin and ζ on the transient response. Find the phase margin and corresponding value of ζ for the fastest settling (output settles within 95% of the final value).

iii. Change the value of feedback factor, β from 1 to 1/10 (i.e. R_1=10R_2) and observe the effect of increasing closed loop gain ($k=1/\beta$) on phase margin, damping factor and unity gain frequency (w_{ugf}). Comment on the result.