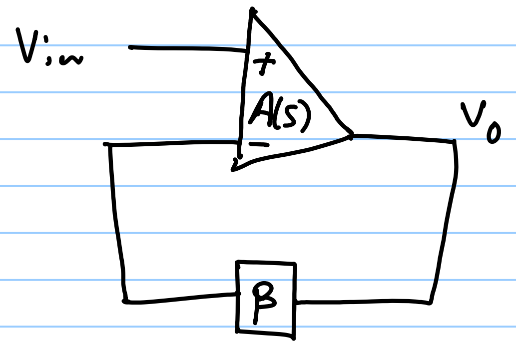
EE2019 Analog Systems & Lab

Note Title 2/28/2020

Stability of a 2^{nd} order system:



$$A(s) = \frac{A_0}{(1+s/\omega_{p_1})(1+s/\omega_{p_2})}$$

$LG(s) = -\beta A(s)$ → open loop

$$H(s) = \frac{V_o(s)}{V_{in}(s)} = \frac{A(s)}{1+\beta A(s)}$$ → closed loop

Substituting $A(s)$

$$H(s) = \frac{\dfrac{A_0}{(1+s/\omega_{p_1})(1+s/\omega_{p_2})}}{1+\beta\dfrac{A_0}{(1+s/\omega_{p_1})(1+s/\omega_{p_2})}}$$

$$= \frac{A_0}{(1+s/\omega_{p_1})(1+s/\omega_{p_2}) + \beta A_0} = \frac{A_0}{1 + s/\omega_{p_1} + s/\omega_{p_2} + \frac{s^2}{\omega_{p_1}\omega_{p_2}} + \beta A_0}$$

$$= \frac{A_0}{\frac{s^2}{\omega_{p_1}\omega_{p_2}} + \left(\frac{1}{\omega_{p_1}} + \frac{1}{\omega_{p_2}}\right)s + (1+\beta A_0)} = \frac{A_0 \; \omega_{p_1}\omega_{p_2}}{s^2 + (\omega_{p_2} + \omega_{p_1})\, s + (1+\beta A_0)\,\omega_{p_1}\omega_{p_2}}$$

$$1 + \beta A_0 \simeq \beta A_0 \quad (\beta A_0 \gg 1)$$

$$= \frac{A_0 \omega_{p_1} \omega_{p_2}}{s^2 + (\omega_{p_2} + \omega_{p_1})s + \beta A_0 \omega_{p_1} \omega_{p_2}} = \frac{1}{\beta} \frac{\beta A_0 \omega_{p_1} \omega_{p_2}}{s^2 + (\omega_{p_1} + \omega_{p_2})s + \beta A_0 \omega_{p_1} \omega_{p_2}}$$

2^n order transfer function

$$H(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

if $\beta = 1$

$$\boxed{H(s) = \frac{A_0 \omega_{p_1} \omega_{p_2}}{s^2 + (\omega_{p_1} + \omega_{p_2})s + A_0 \omega_{p_1} \omega_{p_2}}}$$

$$\omega_n = \sqrt{A_0 \omega_{p_1} \omega_{p_2}}, \quad 2\xi\omega_n = \omega_{p_1} + \omega_{p_2} \Rightarrow \xi = \frac{1}{2} \frac{\omega_{p_1} + \omega_{p_2}}{\omega_n}$$

$$\xi = \frac{1}{2} \frac{\omega_{p_1} + \omega_{p_2}}{\sqrt{A_0 \omega_{p_1} \omega_{p_2}}} = \frac{1}{2} \sqrt{\frac{(\omega_{p_1} + \omega_{p_2})^2}{A_0 \omega_{p_1} \omega_{p_2}}}$$

$$= \frac{1}{2} \sqrt{\frac{\omega_{p_1}^2 + \omega_{p_2}^2 + 2\omega_{p_1}\omega_{p_2}}{A_0 \omega_{p_1} \omega_{p_2}}} = \frac{1}{2} \sqrt{\frac{\omega_{p_1}}{A_0 \omega_{p_2}} + \frac{\omega_{p_2}}{A_0 \omega_{p_1}} + \frac{2}{A_0}}$$

$$\frac{2}{A_0} << 1 \qquad \frac{2}{A_0} \simeq 0$$

$$\frac{1}{2\sqrt{A_0}} \sqrt{\frac{\omega_{p_1}}{\omega_{p_2}} + \frac{\omega_{p_2}}{\omega_{p_1}}} = \xi$$

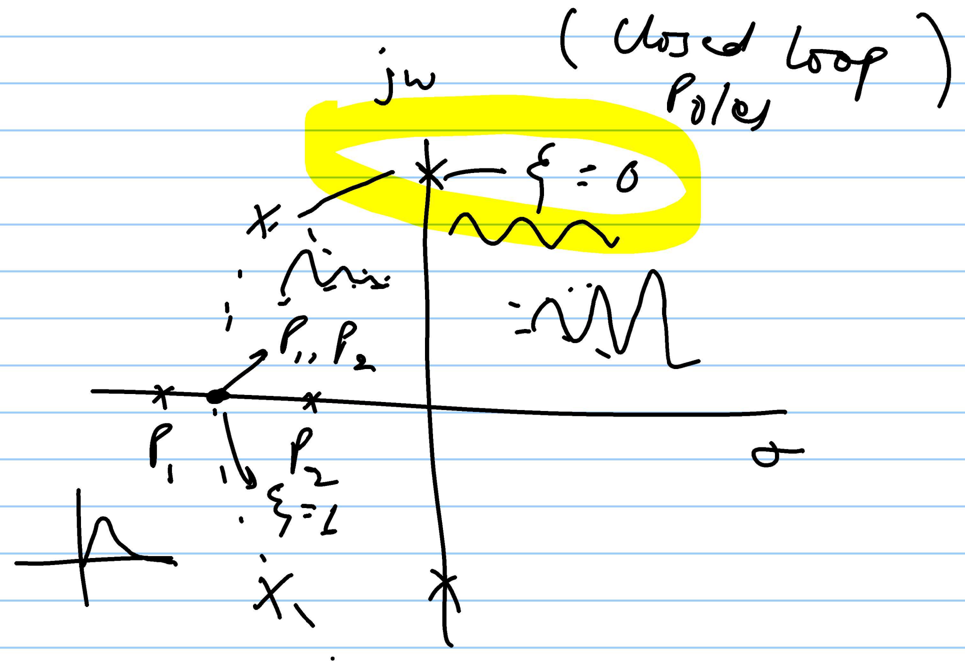
if $\omega p_1 = \omega p_2$

$$\xi = \frac{1}{\sqrt{2 A_0}} \qquad A_0 >> 1$$

$$H(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

$$s = \frac{-2\xi\omega_n \pm \sqrt{4\xi^2\omega_n^2 - 4\omega_n^2}}{2}$$

$$= -\xi\omega_n \pm \omega_n\sqrt{\xi^2 - 1}$$



$$P_1 = -\xi\omega_n + \omega_n\sqrt{\xi^2 - 1}$$

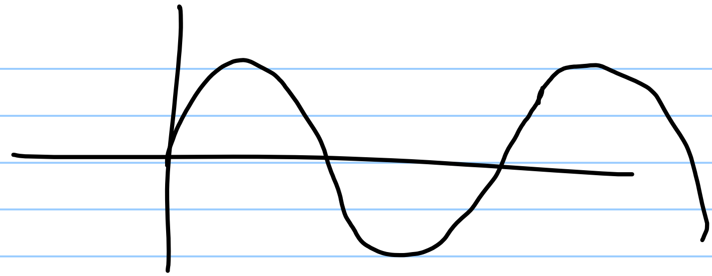
$$P_2 = -\xi\omega_n - \omega_n\sqrt{\xi^2 - 1}$$

1) $\xi = 0 \Rightarrow P_1 = +j\omega_n$

$P_2 = -j\omega_n$

2) $\xi = 1$

$P_1 = P_2 = -\omega_n$



V_{in}