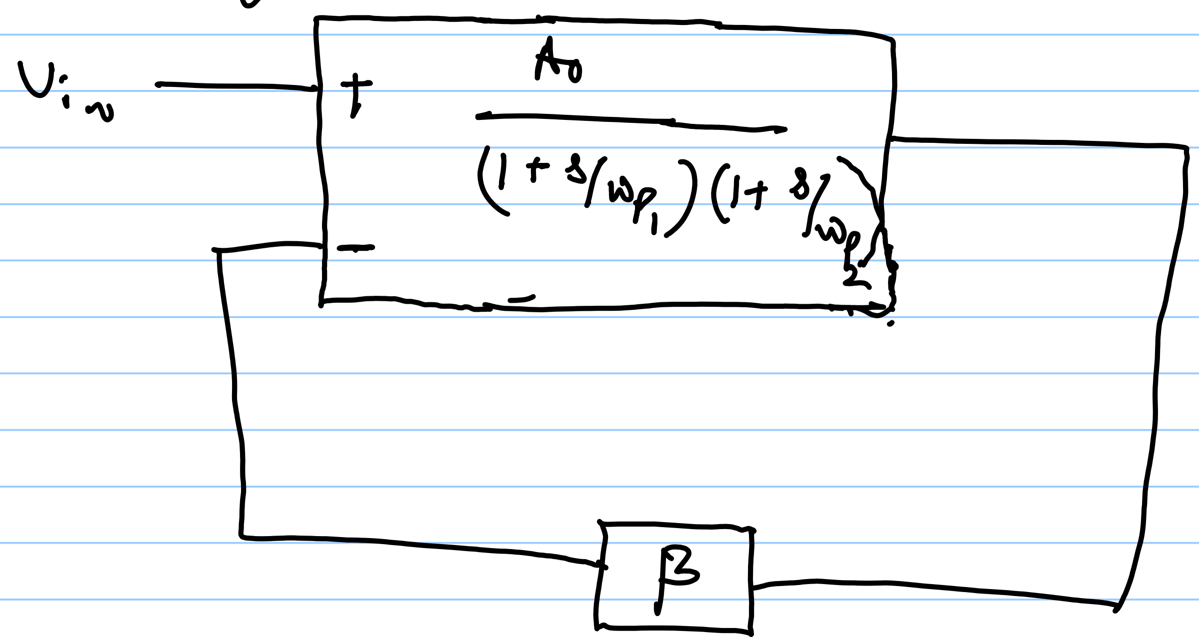
EE 2019

Note Title 2/21/2019

2nd order system in negative FB



$$H(s) = \frac{A(s)}{1+\beta A(s)} = \frac{\frac{A_0}{(1+s/\omega_{p_1})(1+s/\omega_{p_2})}}{1+\beta\frac{A_0}{(1+s/\omega_{p_1})(1+s/\omega_{p_2})}}$$

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$$= \frac{A_0}{(1 + s/\omega_{p_1})(1 + s/\omega_{p_2}) + \beta A_0}$$

$$= \frac{A_0}{1 + \underline{s/\omega_{p_1} + s/\omega_{p_2}} + \frac{s^2}{\omega_{p_1}\omega_{p_2}} + \beta A_0}$$

$$= \frac{A_0}{s^2/\omega_{p_1}\omega_{p_2} + s\left(\frac{1}{\omega_{p_1}} + \frac{1}{\omega_{p_2}}\right) + (1 + \beta A_0)}$$

$$= \frac{\omega_{p_1}\omega_{p_2} A_0}{s^2 + s(\omega_{p_2} + \omega_{p_1}) + (1 + \beta A_0)\omega_{p_1}\omega_{p_2}}$$

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$$1 + \beta A_0 \approx \beta A_0$$

$$= \frac{1}{\beta} \frac{\beta A_0 \omega_{p_1} \omega_{p_2}}{s^2 + s(\omega_{p_1} + \omega_{p_2}) + \underset{\geq 1}{\beta} A_0 \omega_{p_1} \omega_{p_2}}$$

$$H(s) = \frac{K \omega_n^2}{s^2 + 2\xi \omega_n s + \omega_n^2}$$

assume $\beta = 1$

$\omega_n \rightarrow$ natural frequency

$\xi \rightarrow$ damping factor

$$\omega_n^2 = A_0 \omega_{p_1} \omega_{p_2}$$

$$2\xi\omega_n = \omega_{p_1} + \omega_{p_2} \Rightarrow \xi = \frac{1}{2}\left(\frac{\omega_{p_1} + \omega_{p_2}}{\omega_n}\right)$$

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$$\xi = \frac{1}{2} \frac{(\omega_{p_1} + \omega_{p_2})}{\sqrt{A_0 \omega_{p_1} \omega_{p_2}}} =$$

$$= \frac{1}{2\sqrt{A_0}} \left\{ \sqrt{\frac{\omega_{p_1}^2 + \omega_{p_2}^2 + 2\omega_{p_1}\omega_{p_2}}{\omega_{p_1}\omega_{p_2}}} \right\}$$

$$\xi = \frac{1}{2\sqrt{A_0}} \sqrt{\left(\frac{\omega_{p_1}}{\omega_{p_2}} + \frac{\omega_{p_2}}{\omega_{p_1}} + 2 \right)}$$

if $\omega_{p_1} = \omega_{p_2} \Rightarrow \xi = \frac{1}{\sqrt{A_0}}$

$A_0 \gg 1 \Rightarrow \xi \to 0$ system is unstable

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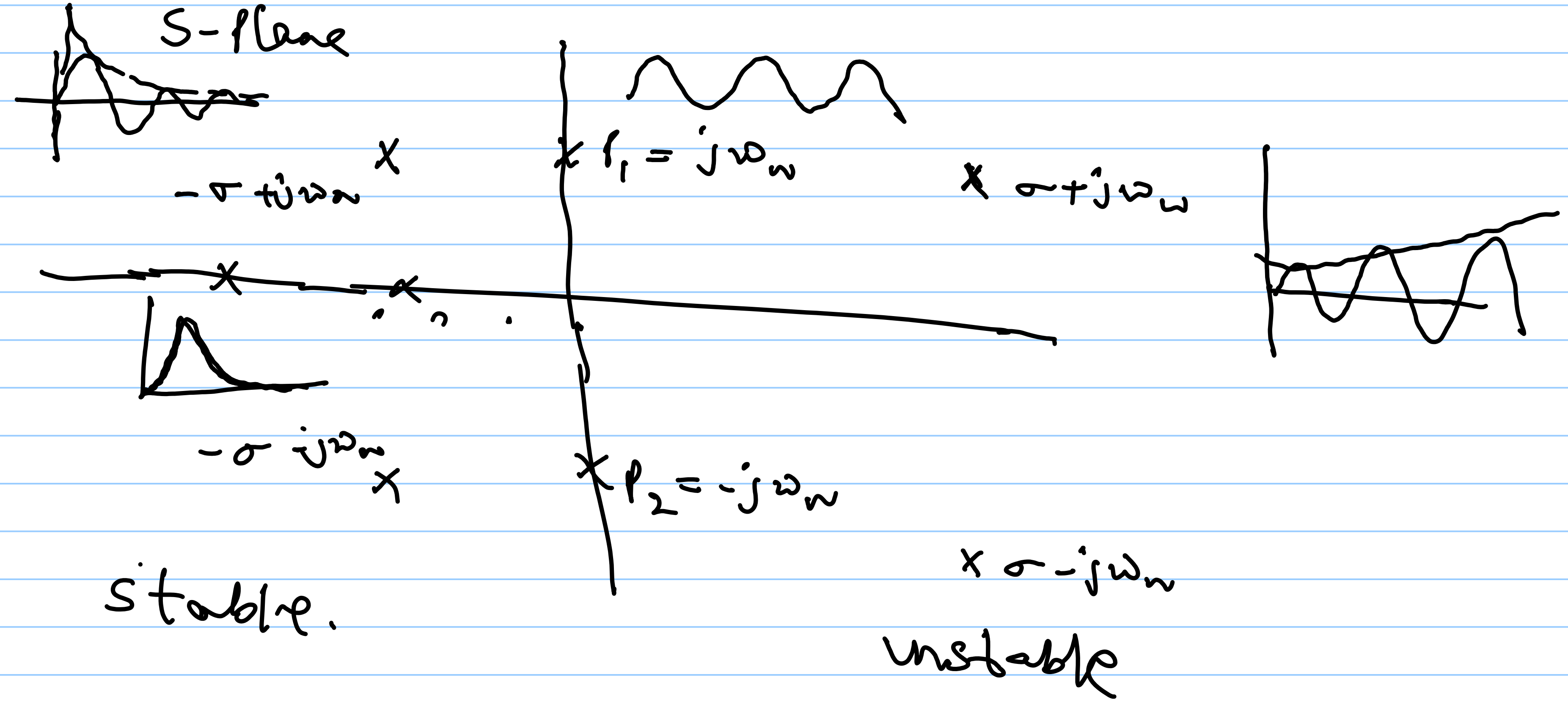


$\xi < 1$ then system under damped $\rightarrow$ oscillations

$\xi > 1$ then sytem is over damped. $\rightarrow$ no oscillations

$\xi = 1$

$$H(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n + \omega_n^2}$$

$$= \frac{\omega_n^2}{(s+\omega_n)^2}$$

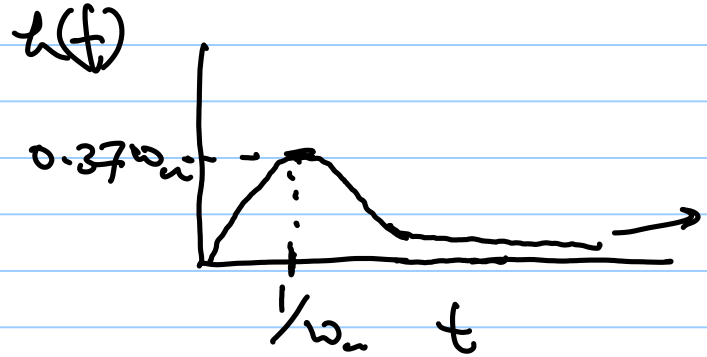
$$h(t) = \omega_n^2 t\, e^{-\omega_n t}$$

for max value $\frac{d}{dt} h(t) =$

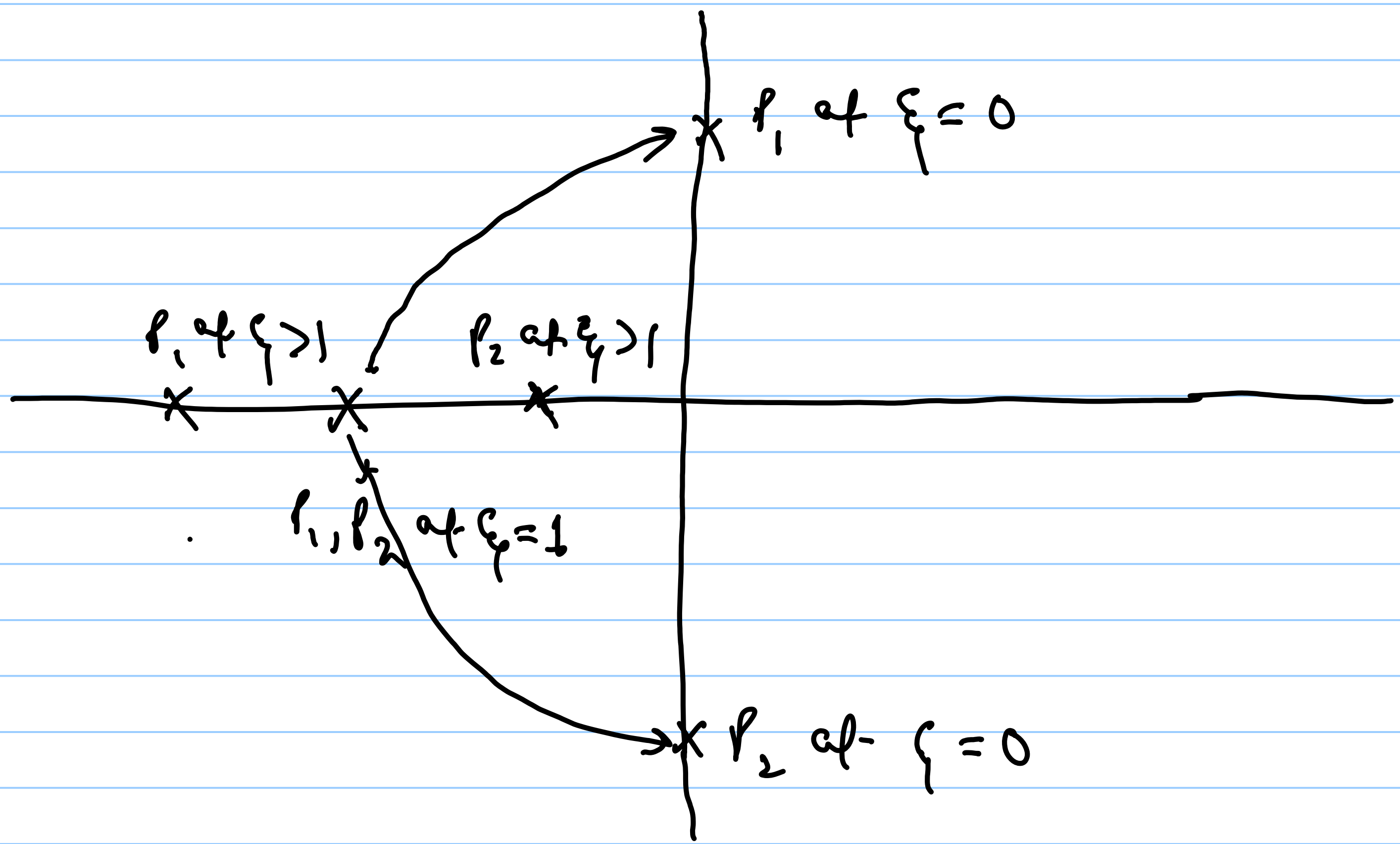
$$= \omega_n^2 t \left(-\omega_n e^{-\omega_n t}\right) + \omega_n^2 e^{-\omega_n t} = 0$$

$$e^{-\omega_n t}\left[-\omega_n t + 1\right] = 0 \Rightarrow t = \frac{1}{\omega_n}$$

$$h(t)_{max} = \frac{\omega_n}{e} \approx 0.37\,\omega_n$$



$t = \infty,\ h(t) \to 0$



p_1 at $\xi = 0$
p_1 at $\xi > 1$
p_2 at $\xi > 1$
p_1, p_2 at $\xi = 1$
p_2 at $\xi = 0$

$$\xi = \frac{1}{2\sqrt{A_0}}\sqrt{\left(\frac{\omega_{p_1}}{\omega_{p_2}} + \frac{\omega_{p_2}}{\omega_{p_1}} + 2\right)}$$

$$\omega_{p_1} = \frac{\omega_{ugb}}{A_0}$$

$$\omega_{p_2} = 2\,\omega_{ugb} \quad \text{for } PM \approx 60^\circ$$

$$\xi = \frac{1}{2\sqrt{A_0}}\sqrt{\frac{\cancel{\omega_{ugb}}}{A_0(2\cancel{\omega_{ugb}})} + \frac{2\cancel{\omega_{ugb}}}{\cancel{\omega_{ugb}}/A_0} + 2}$$

$$= \frac{1}{2\sqrt{A_0}}\sqrt{\frac{1}{2A_0} + 2A_0 + 2}$$

$$2A_0 \gg \frac{1}{2A_0} \quad \& \quad 2A_0 \gg 2$$

$$\xi = \frac{1}{\sqrt{2}} \approx 0.7$$

Phase Margin $\approx 90 \times \xi$ for $PM < 65^\circ$