EE2019

Note Title 4/10/2019

Butterworth Filter

$$H(s) = \frac{1}{D(s)} = \frac{1}{1 + a_1 s + a_2 s^2 + a_3 s^3 + \cdots a_n s^n}$$

$$|H(j\omega)|^2 = \frac{1}{1 + c_2\left(\frac{\omega}{\omega_n}\right)^2 + c_4\left(\frac{\omega}{\omega_n}\right)^4 + \cdots c_{2N}\left(\frac{\omega}{\omega_n}\right)^{2N}}$$

$$c_2 = c_4 = \cdots c_{2N-2} = 0 \;\&\; c_{2N} = 1$$

$$|H(j\omega)|^2 = \frac{1}{1 + \left(\frac{\omega}{\omega_n}\right)^{2N}}$$

$$|H(j\omega)|^2 = \underbrace{H(s)}_{\text{N poles in L.H.P.}} \; \underbrace{H(-s)}_{\text{N poles in R.H.P}} = \frac{1}{1+\left(\frac{\omega}{\omega_n}\right)^{2N}}$$

$$s = j\omega \Rightarrow \omega = \frac{s}{j}$$

$$H(s)\,H(-s) = \frac{1}{1+\left(\frac{s}{j\omega_n}\right)^{2N}}$$

$$1+\left(\frac{s}{j\omega_n}\right)^{2N} = 0 \Rightarrow \left(\frac{s}{j\omega_n}\right)^{2N} = -1$$

$$-1 = \exp j(2k+1)\pi$$

$$e^{j\theta} = \cos\theta + j\sin\theta$$

$$\theta = (2k+1)\pi$$

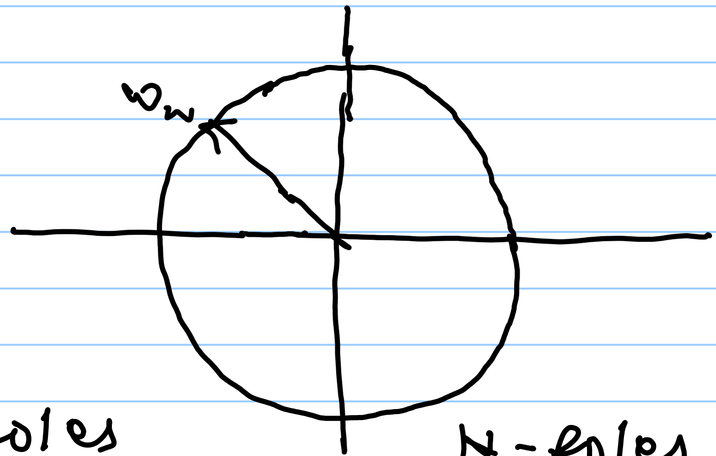
$$\cos\theta = -1, \quad \sin\theta = 0$$

$$\left(\frac{s}{j\omega_n}\right)^{2N} = \exp j(2k+1)\pi \qquad k = 0, 1 \cdots 2N-1$$

$$\frac{s}{j\omega_n} = \exp \frac{j(2k+1)\pi}{2N}$$

$$s = j\omega_n \exp \frac{j(2k+1)\pi}{2N} = \omega_n \exp j\left[\frac{(2k+1)\pi}{2N} + \frac{\pi}{2}\right]$$

$$\theta = \frac{2k+1}{2N}\pi + \frac{\pi}{2}, \qquad k = 0, 1 \cdots 2N-1$$



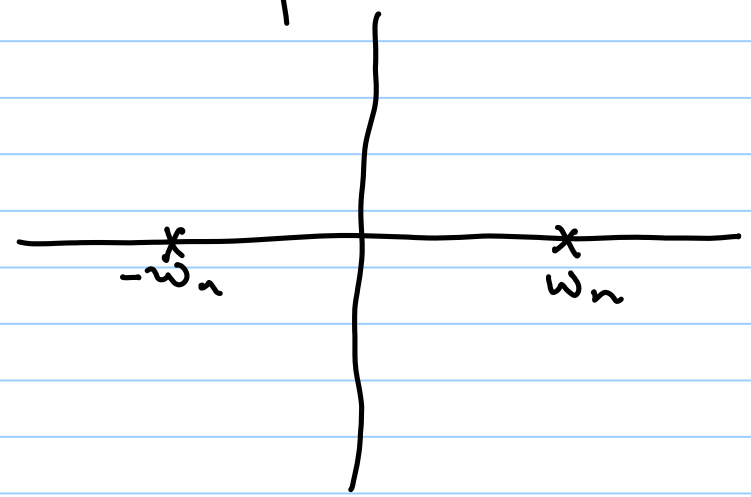
N - poles in L.H.P.

N - poles in R.H.P

$$s = \omega_n \exp j\left[\frac{(2k+1)\pi}{2N} + \frac{\pi}{2}\right]$$

$N = 1 \Rightarrow k = 0, 1$

$K = 0$	$K = 1$
$s = \omega_n \exp j(\pi)$	$s = \omega_n$
$s = -\omega_n$



Poles of butterworth filters are all poles in L.H.P.

$\Rightarrow$ discard all pole in R.H.P.

$s = -\omega_n \Rightarrow H(s) = \dfrac{1}{1 + s/\omega_n}$

2^{nd} order filter.

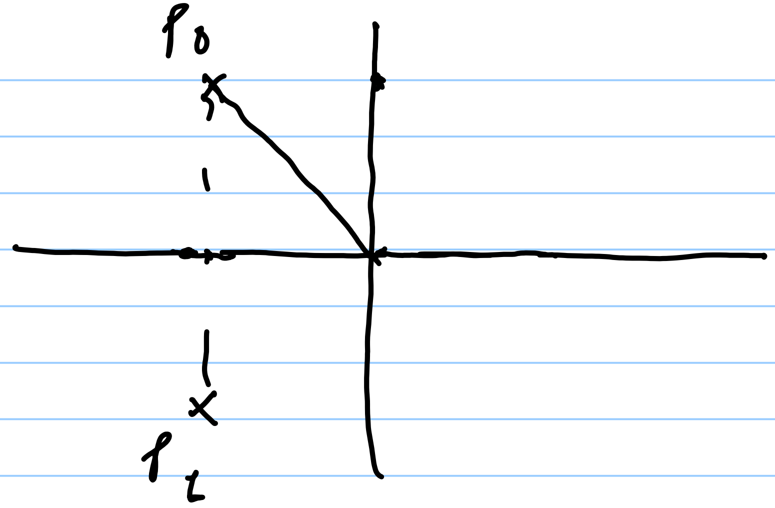
$N = 2, \quad k = 0, 1, 2, 3$

$$s = \omega_n \exp j\left[\frac{(2k+1)\pi}{2N} + \frac{\pi}{2}\right]$$

$$\theta_k = \frac{2k+1}{2N}\pi + \frac{\pi}{2} = \frac{2k+1}{4}\pi + \frac{\pi}{2}$$

$\theta_0 = \dfrac{\pi}{4} + \dfrac{\pi}{2}$ $\qquad \theta_2 = \dfrac{5}{4}\pi + \pi/2$

$\theta_1 = \dfrac{3}{4}\pi + \dfrac{\pi}{2}$ $\qquad \theta_3 = \dfrac{7}{4}\pi + \pi/2$



$k = 0$ to $N-1$ L.H.P.

$k = N$ to $2N-1$ R.H.P.

L.H.P. roots of $D(s)$

$$p_1 = s = -\cos\frac{\pi}{4} + j\sin\frac{\pi}{4} = -\frac{1}{\sqrt{2}} + \frac{j}{\sqrt{2}}$$

$$p_2 = s = -\cos\pi/4 - j\sin\pi/4 \qquad -\frac{1}{\sqrt{2}} - j/\sqrt{2}$$

$$H(s) = \frac{1}{(s-p_1)(s-p_2)} = \frac{1}{\left(s+\frac{1}{\sqrt{2}}-\frac{j}{\sqrt{2}}\right)\left(s+\frac{1}{\sqrt{2}}+\frac{j}{\sqrt{2}}\right)}$$

normalized across ω_n ⊖ $s \to \frac{s}{\omega_n}$

$$H(s) = \frac{1}{s^2+\sqrt{2}s+1} = \frac{1}{s^2+\frac{s}{Q}+1}$$

$$Q = \frac{1}{\sqrt{2}}$$

for 3rd order.

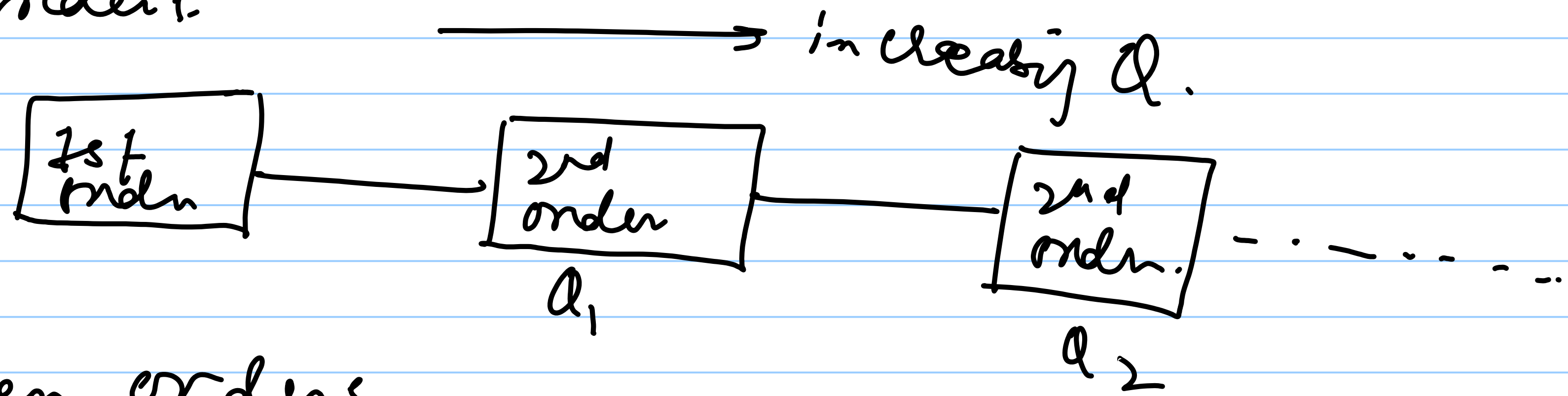
$$H(s) = \frac{1}{(s+1)\left(s^2 + \frac{s}{Q} + 1\right)}$$

for 4th order

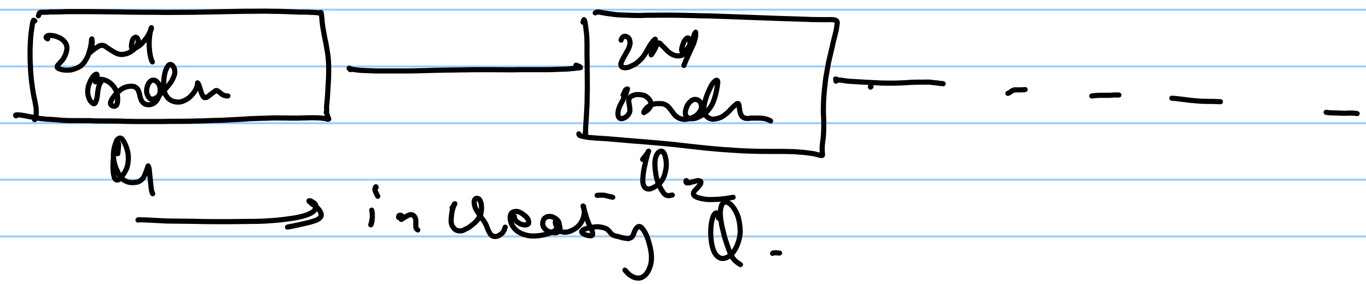
$$H(s) = \frac{1}{\left(s^2 + \frac{s}{Q_1} + 1\right)\left(s^2 + \frac{s}{Q_2} + 1\right)}$$

Any order filter can be designed with cascading 1st & 2nd order filters.

Odd orders:



Even orders



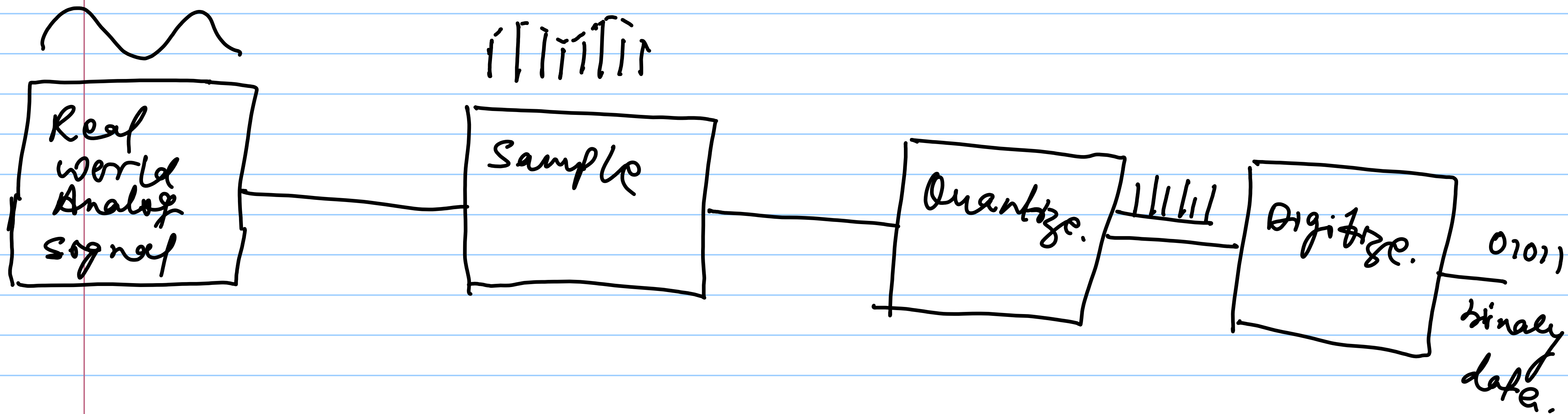
A/D converters

Analog to Digital converter.

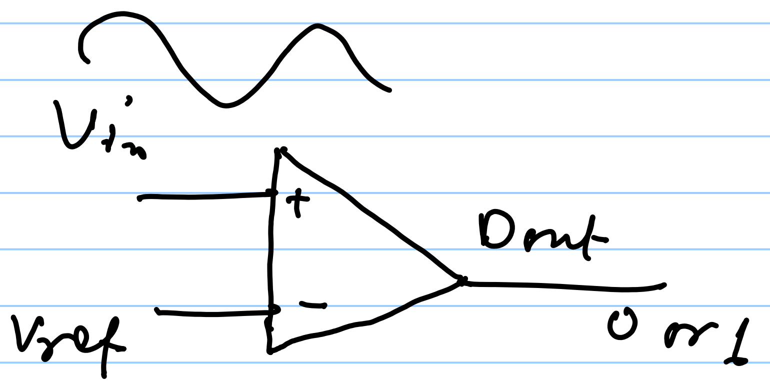
Why do we need A/D ?

1. Digital data can be stored in memory
2. computer can accept only digital data

Processor or DSP.



1-bit A/D



$V_{in} > V_{ref}$: $D_{out} = 1$

$V_{in} < V_{ref}$: $D_{out} = 0$