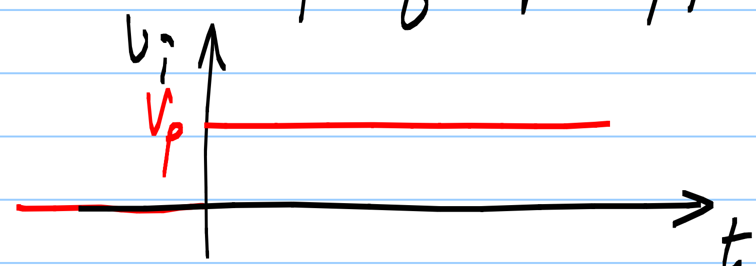
EC1010, Tutorial 3: RC/RL circuit step response calculations

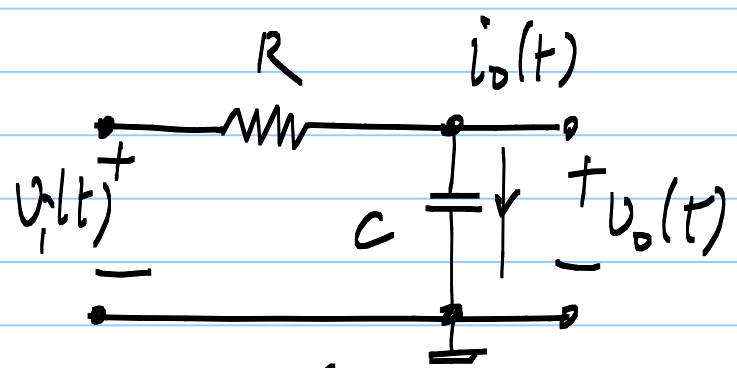
Note Title — 8/16/2011

(1) Evaluate and sketch $v_o(t)$, and when marked, $i_o(t)$ in the following:

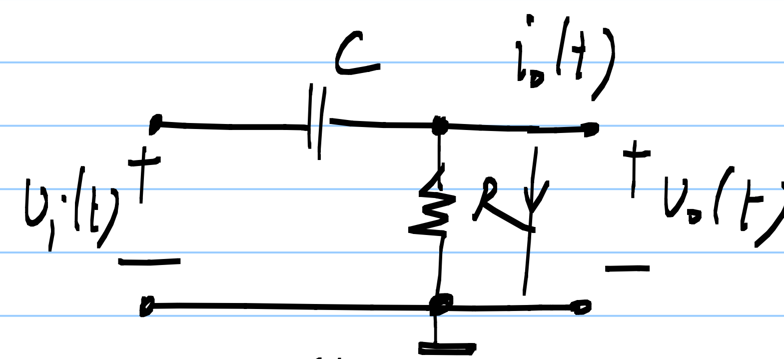
* $v_i(t)$ is a step of V_p applied at $t=0$



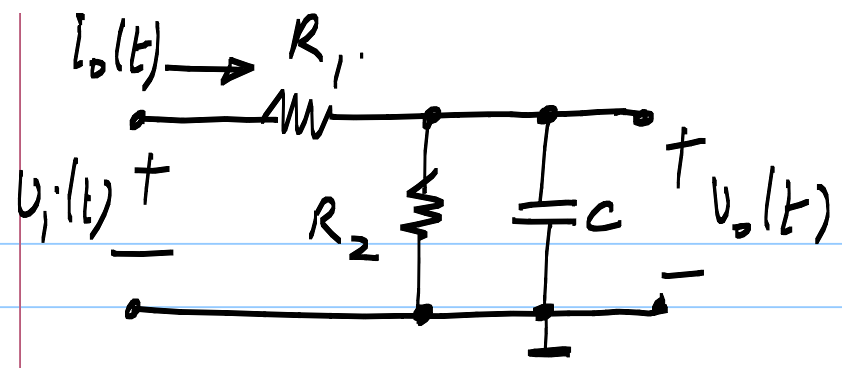
* Capacitors and inductors have zero initial condition



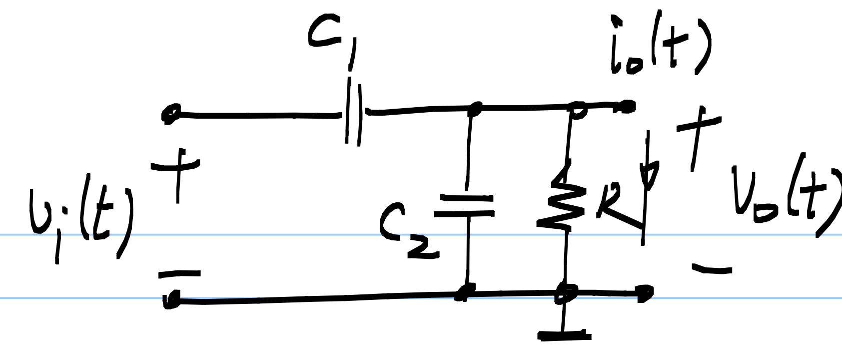
(a)



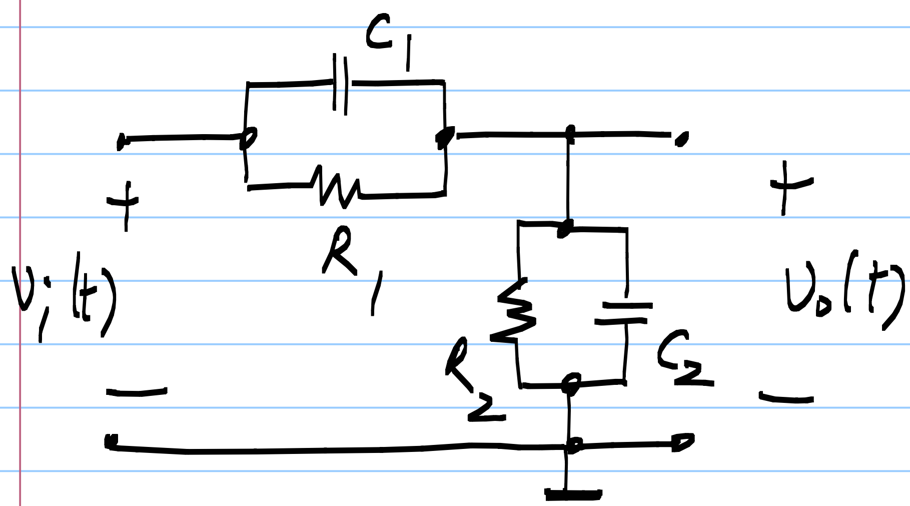
(b)


$i_o(t)$
R_1
R_2
C
$v_i(t)$
+
−
$v_o(t)$
+
−

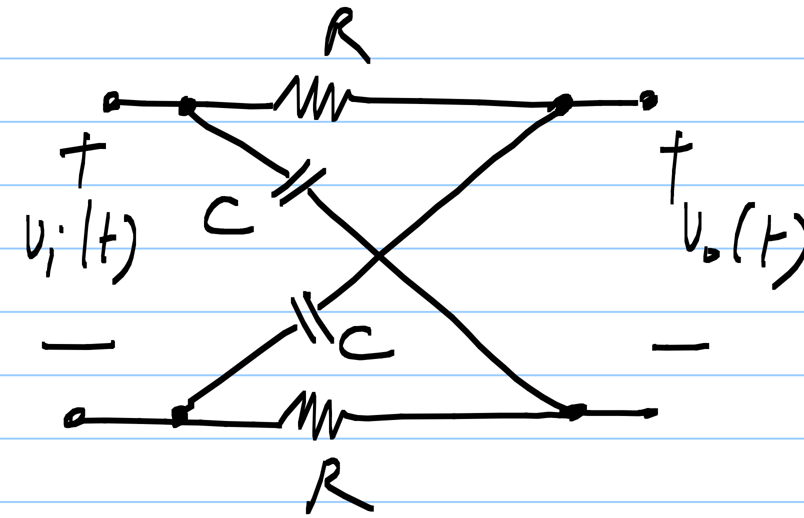
(c)


C_1
C_2
R
$i_o(t)$
$v_i(t)$
+
−
$v_o(t)$
+
−

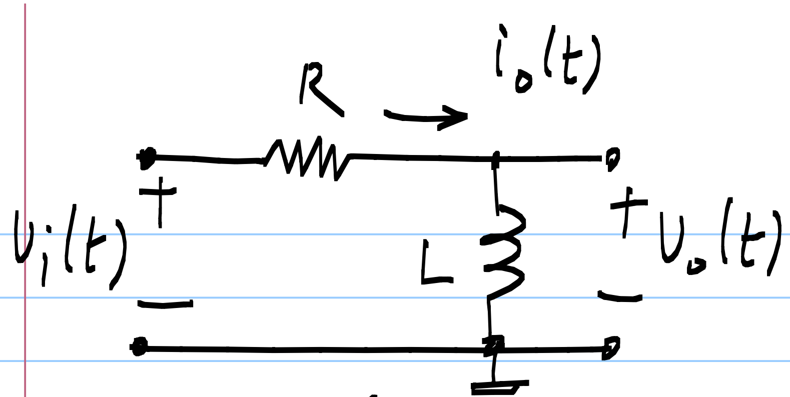
(d)


C_1
R_1
R_2
C_2
$v_i(t)$
+
−
$v_o(t)$
+
−

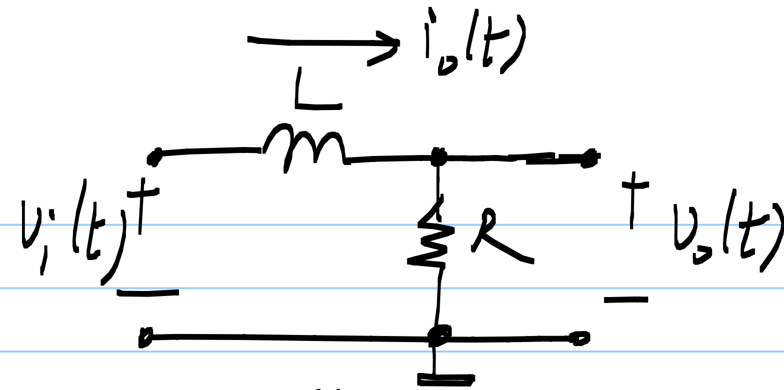
(e)


R
C
C
R
$v_i(t)$
+
−
$v_o(t)$
+
−

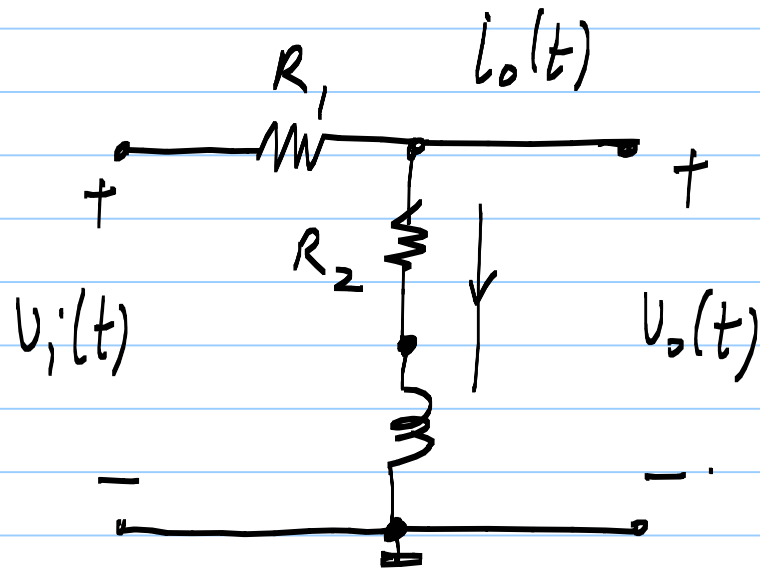
(f)



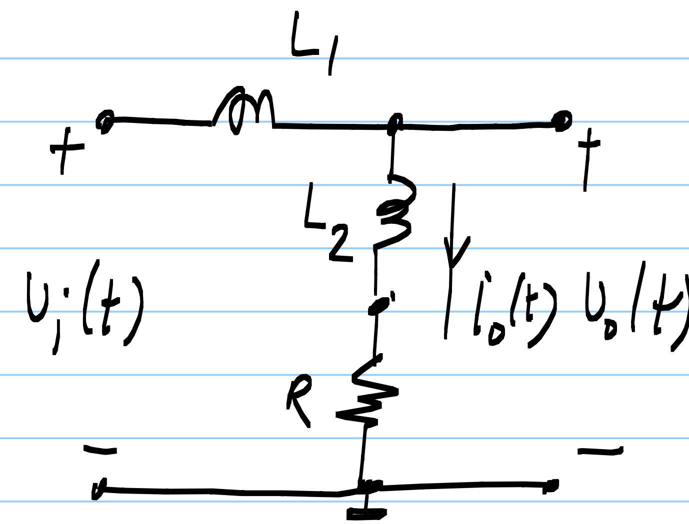
(g)



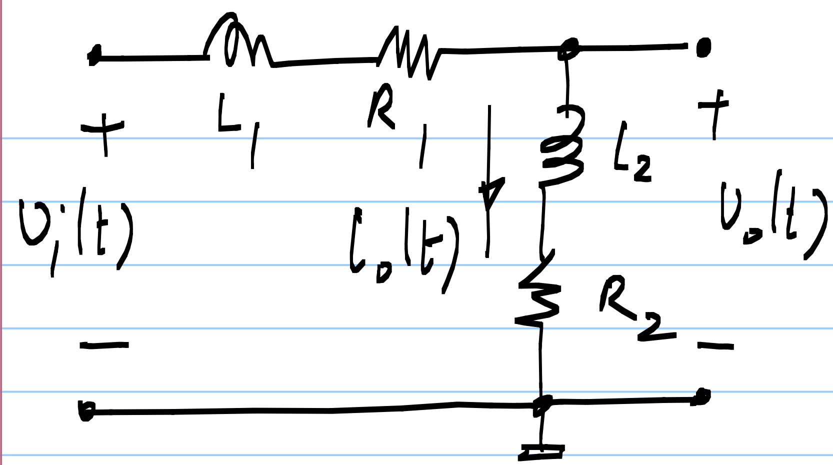
(h)



(i)

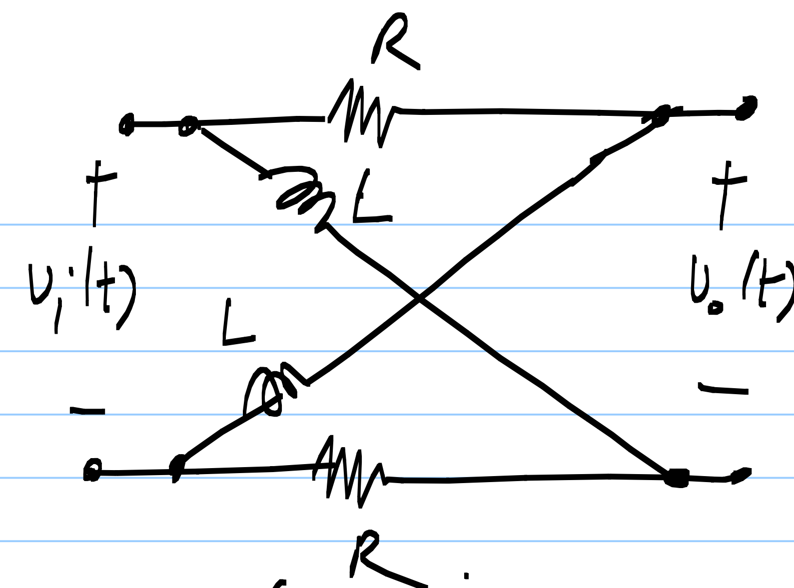


(j)



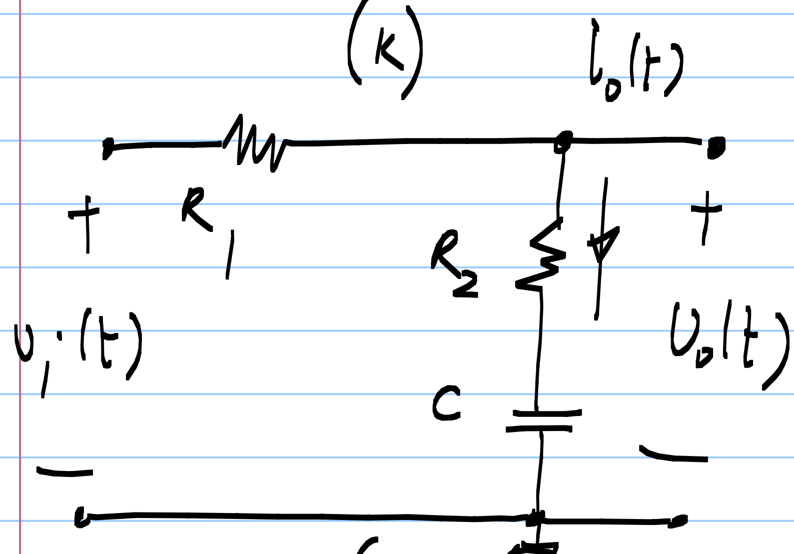
L_1
R_1
L_2
R_2
$i_o(t)$
$v_i(t)$
$v_o(t)$
+
−
+
−

(k)



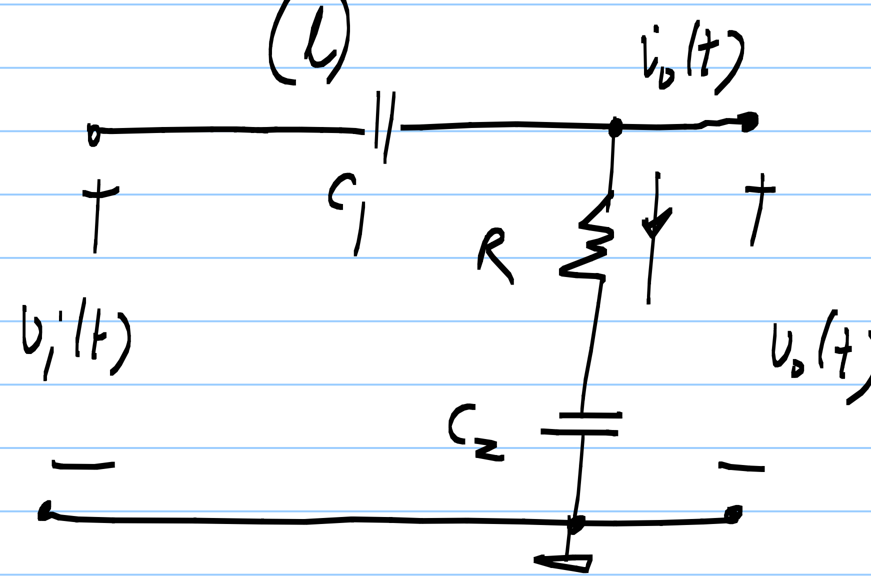
R
L
L
R
$v_i(t)$
$v_o(t)$
+
−
+
−

(l)



R_1
R_2
C
$i_o(t)$
$v_i(t)$
$v_o(t)$
+
−
+
−

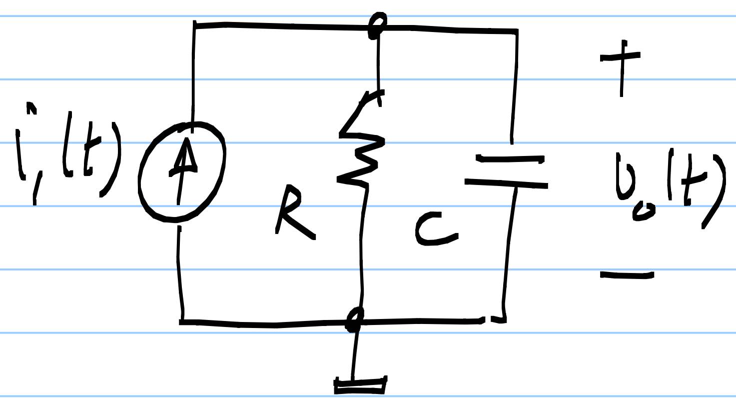
(m)



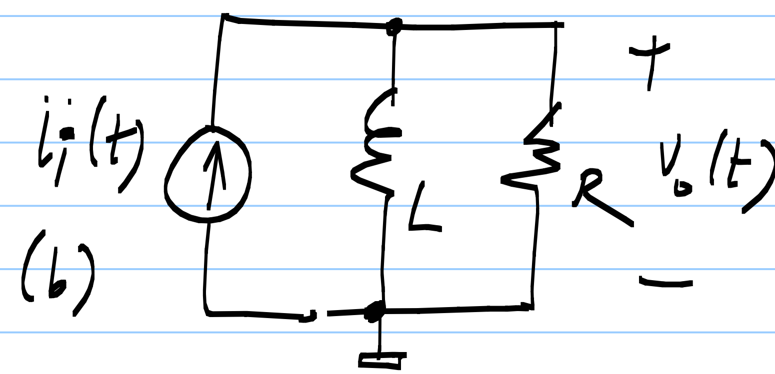
C_1
R
C_2
$i_o(t)$
$v_i(t)$
$v_o(t)$
+
−
+
−

(2) Just so that you get used to a current input, calculate $v_o(t)$ when $i_i(t)$ is a step current of value I_p

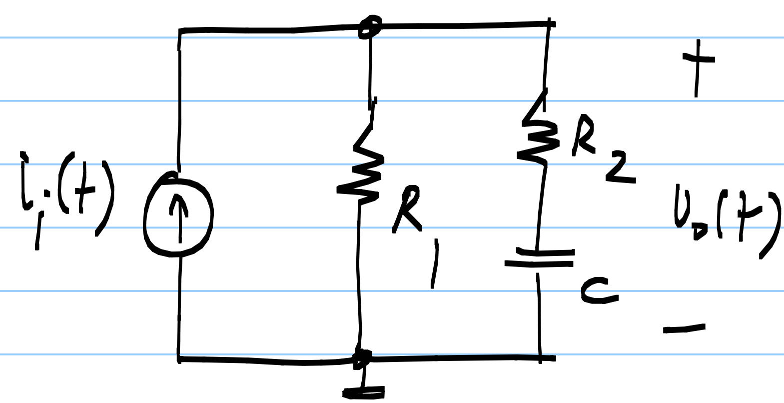
(a)



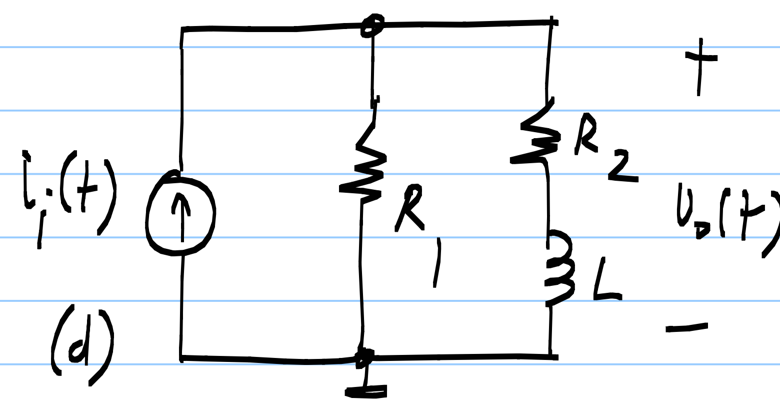
(b)



(c)

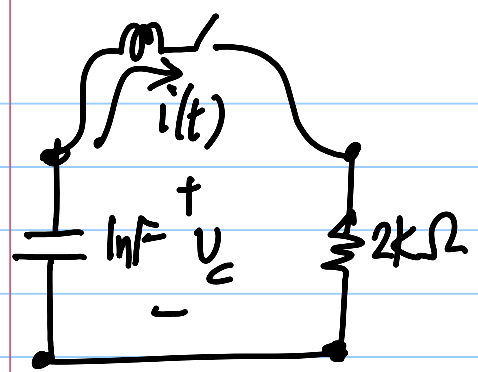


(d)



(3). In all of the previous problems, open circuit the capacitors or short circuit the inductors, and calculate v_o and i_o for an input of V_p or I_p as applicable. How are the values so obtained related to $v_o(t)$ and $i_o(t)$ you calculated earlier.

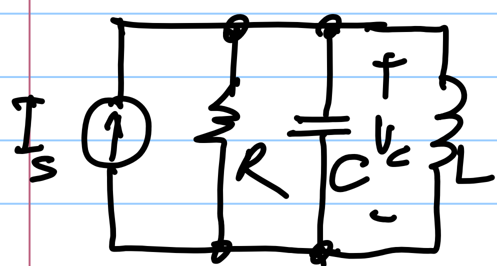
(4)



(a) A 1nF capacitor charged to 1V must be connected to a resistor through a switch to discharge it. The switch is connected using a long wire which has some inductance. Write the expressions for $v_c(t)$ & $i(t)$ when the inductance is zero, 0.1mH, 1mH, and 10mH. What are the damping (or quality) factors in each case.

(b) What are $\int_0^\infty i(t)\,dt$ and $\int_0^\infty i^2(t)\,dt$ in the three cases?

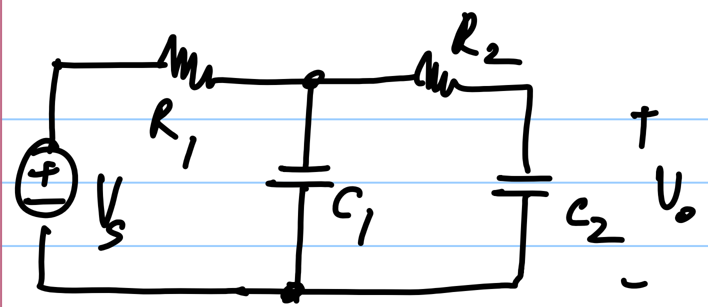
(5)



(i) Write down the differential equation governing this circuit with v_c as the variable.

(ii) Determine the conditions for under damping, critical damping, and over damping. How do these compare to the series RLC case?

(6)

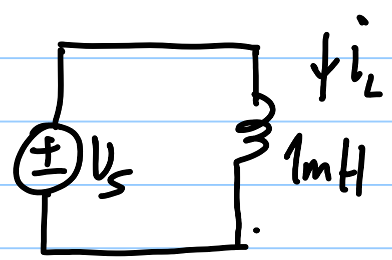


(i) Write down the differential equation with v_o as the variable.

(ii) Determine the natural frequency, damping factor, and the natural response of v_o when $R_2 = R_1 = 1k\Omega$, $C_2 = C_1 = 1nF$. Assume $V_{C_1}(0)$ and $V_{C_2}(0)$ as the initial conditions on the two capacitors.

(iii) What is the lowest damping factor (highest quality factor) that is possible for this circuit? What are the conditions for this to happen?

(7)



$v_s(t) = 1V \sin(10^6 t)$.

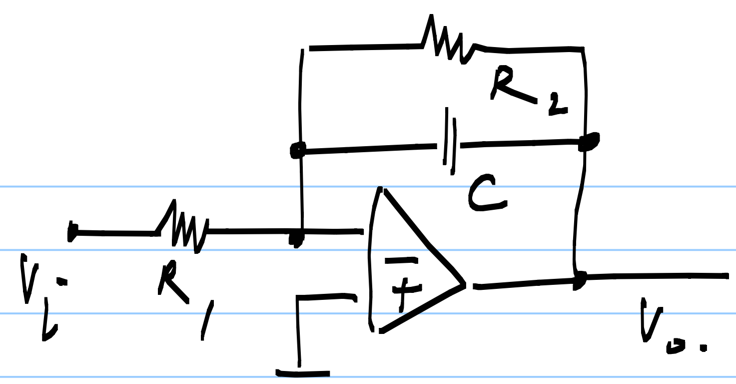
The inductor has zero current at $t=0$.

(a) Determine $i_L(t)$ for $t>0$ using the i–v relationship of the inductor in the time domain.

(b) Determine $i_L(t)$ using phasor analysis.

(c) Explain the results in (a) & (b).

(8)



(i) write down the differential equation for v_o

(ii) For a 1V input step, evaluate the total response