Channel Equalization in Analog Domain

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Submitted in partial fulfillment of the
requirements for the degree of
Master of Technology
in the Department of Electrical Engineering

INDIAN INSTITUTE OF TECHNOLOGY, MADRAS

2004

CERTIFICATE

This is to certify that the project report titled **Adaptive Analog Equalizer** submitted by **Shankar T S** to the Indian Institute of Technology, Madras, for the award of the degree of ***Master of Technology* in *Electrical Engineering*** is a bonafide record of the work carried out by him under my supervision .

Date : 24th, April, 2004

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Abstract

Channel Equalization in Analog Domain

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The project deals with a new approach to linear channel equalization. At higher bitrates, the conventional approach to equalization becomes more demanding in terms of power consumption and bandwidth of the devices used to fabricate the equalizer. This project looks at a different approach that leads to the equalizer being implemented in the analog domain using transmission lines, and the corresponding reduction in the power and bandwidth requirements of the devices used.

The initial part includes an introduction to the baseband digital communication links.

The discrete time impulse response obtained from a tapped delay line discrete time FIR filter is shown to be obtainable by using transmission lines in the analog domain.

Next,a generalized framework for the design of the equalizers, both conventional and analog, based on the minimum mean square error (MMSE) criterion is developed. The theory behind the adaptive analog equalizer is developed based on the LMS algorithm.

A particular case, dealing with the implementation of the analog equalizer, using the travelling wave topology with transmission lines is considered for illustration. Then the results of the simulations, comparing the performance of this analog equalizer with that of the conventional equalizers are presented.

Similar approach is employed to arrive at an analog implementation of the decision feedback equaizer (DFE).

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Acknowledgments

Incredible as it may seem, the author worked
under that great analog scholar

Dr. Yendaluri Shanthi Pavan.

Chapter 1

Introduction

1.1

The communication channels are mostly bandlimited and noisy. Signals transmitted over such channels are altered before reception. The physical channel may lead to signal distortion, like echoes and frequency selective filtering of the transmitted signal.

In digital communication systems, this distortion leads to inter symbol interference (ISI), because of which, the symbols transmitted before and after a symbol, corrupt the reception of that symbol.

Linear channel equalization is an approach to counteract the effects of channel distortion by applying a linear filter (called equalizer) to the received signal. In practice, the channel characteristics are mostly unknown (like during startup) or tend to change with time. Hence an adaptive structure is preferred for the equalizer.

Conventional equalization was carried on in the discrete time domain, that is, the linear filter used is a digital filter, applied to the samples of the received signal. But for high bitrate signals, such as those encountered in the optical communications domain, such a design places enormous requirements on power and bandwidth requirements of the devices used to realize these filters. Hence at higher bitrates it becomes lucrative to do the equalization in the analog domain, which reduces the power and bandwidth requirements of the devices used to realize the equalizer.

The development of the theory behind the analog equalization builds on the theory of the conventional equalizer, with some modifications. This report gives a rigorous development of a generalized scheme for designing fixed and adaptive equalizers in the analog domain. It also presents the simulation results comparing the performance of the analog equalizer with that of the conventional equalizers.

The report also deals with extending the idea of equalization in analog domain to the decision feedback equalizer (DFE). The theoretical derivation based on the generalized scheme mentioned above, as well as the simulation results pertaining to the DFE are presented.

Chapter 2

System Model

2.1 Baseband Digital Communication Links

Digital communication links transport information on various physical channels. The information is generally limited to a finite band of frequencies. The centre frequency of this band may be high with respect to the bandwidth, which is the case when the signal is used to modulate a high frequency carrier. The term 'Baseband' is used to signify that no carrier modulation is used. Instead, baseband transmission systems typically use a modulation code to match the characteristics of the data to the transmission channel. At the receiver end, equalization, detection and timing recovery are done to recover the data.

The following sections deal with the various building blocks of the typical baseband communication link, that is considered in the rest of the chapters.

2.2 System Model

The basic building blocks of baseband digital communication link are shown in figure 2.1. The information sourced from a data source is fed to a transmitter that generates the analog information signal $s(t)$. This signal is then transmitted over the physical channel, where it gets corrupted with noise and distotion. The received signal $r(t)$ is then processed to recover the information at the receiver.

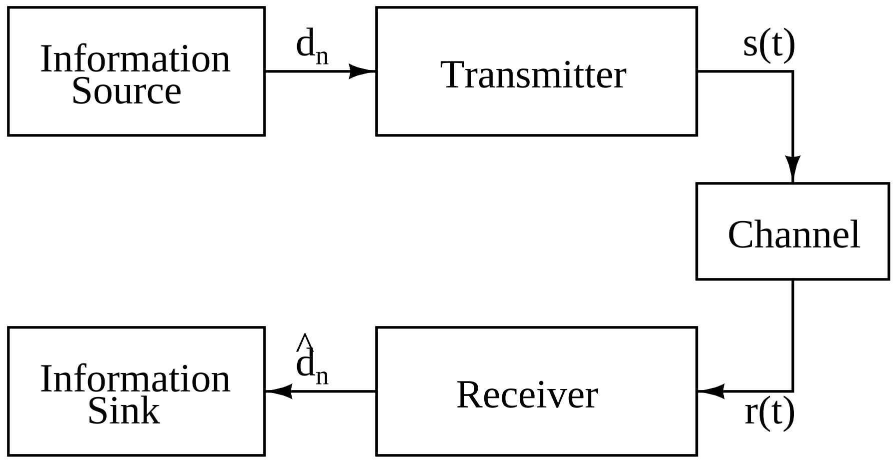

Figure 2.1: Baseband Digital Communication Link

2.3 Transmitter

A transmitter, in general, has an encoder that takes in the information sequence d_n from the data source and outputs a sequence a_k which is partly matched to the channel characteristics. This sequence is then fed into a pulse modulator which generates the analog waveform corresponding to the sequence a_k and matched to the channel characteristics. Figure 2.2 shows the transmitter model that is considered in the following chapters. The encoder is assumed to generate an independent identically distributed bit sequence. The pulse modulator is represented by an impulse response $g(t)$.

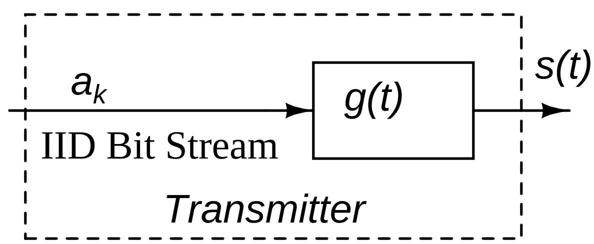

Figure 2.2: Transmitter

2.4 Channel

The physical channel on which the information bearing signal s(t) is transmitted can be represented in the simplest case, by an impulse response $c(t)$ and an additive white gaussian noise(AWGN) signal $n(t)$, that adds to the signal just before the receiver, where, due to attenuation, the signal power will be the lowest. The resulting corrupted signal $r(t)$ is received by the receiver.

In the following chapters, the transmit pulse shaping filter $g(t)$ is considered to be combined with the channel response $c(t)$ and represented by the overall channel response $h(t)$, as shown in the figure 2.4.

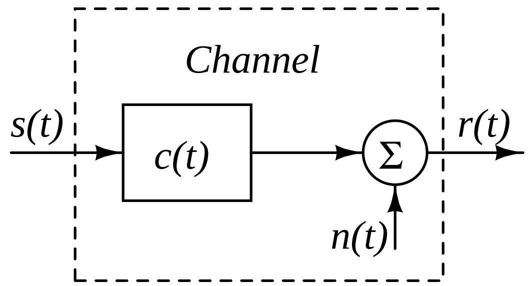

Figure 2.3: AWGN Channel

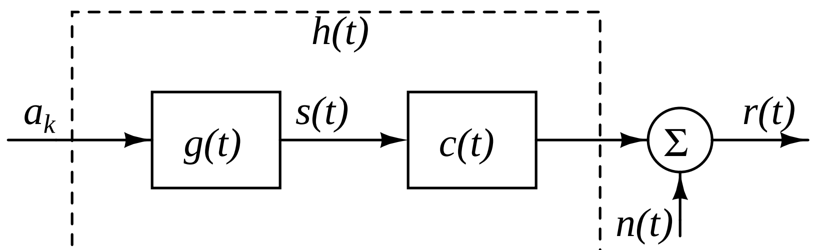

Figure 2.4: Overall Channel Model

2.5 Specific System Model

In the study of the analog equalizers, special emphasis is laid on high bit rate optical channels. Because the analog filters have distinct advantage over digital implementations at higher bitrates. Hence, in all the simulations and analysis, the baseband model of a typical optical link is considered. [3]

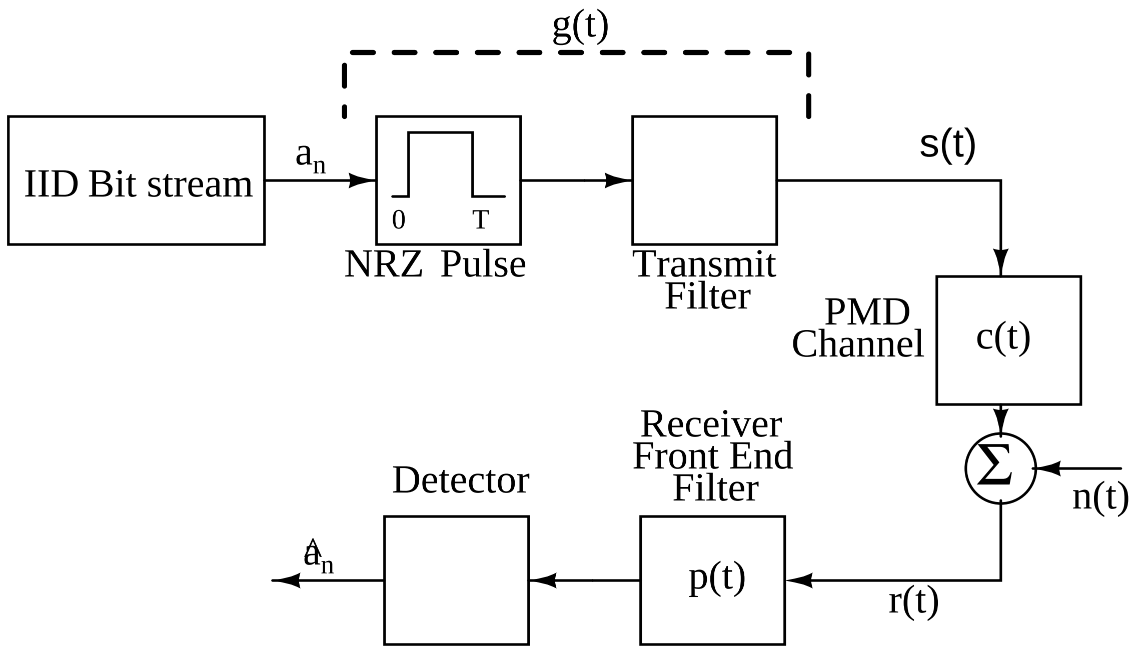

Figure 2.5: System Model

2.5.1 Transmitter Model

The transmitted data is in the form of independent identically distributed bitstream with values 0 or 1.

The non-return to zero (NRZ) coding is used by the encoder. Then there is a pulse shaping filter at the transmitter end. The overall impulse response of the NRZ pulse and the transmit filter is represented as $g(t)$. See figure 2.5

2.5.2 PMD Channel

In the optical channel, the distortion is primarily due to optical phenomena like chromatic dispersion and polarisation mode dispersion (PMD). In the present report, we consider the effects of PMD channels.

Due to polarisation mode dispersion, the signal transmitted over the fiber travels in two different modes of propogation. These modes of propogation have different group velocities. The simplest impulse response to model the PMD channel is

$$c(t) = \alpha\delta(t) + (1-\alpha)\delta(t-T_D) \quad (2.1)$$

where, α depends on the percentage of energy carried in each of the two modes, and T_D depends on the difference between the overall propogation delays for the two modes of propogation.

The worst case scenario is when $\alpha = 0.5$. Such a channel is referred to as an equal power split channel. As T_D gets larger and larger, the channel worsens and the complexity of equalizer needed to counter it varies. For illustration, the rest of the report considers the case when $0 \le T_D \le T$ where T is the bitrate of the system concerned.

2.5.3 Receiver Model

The receiver, in its simplest form contains an antialiasing filter at the front end, to cut off out of band noise, followed by the analog to digital converter (ADC) and equalizer. The receiver front end filter is denoted by its impulse response $p(t)$. See figure 2.5.

Chapter 3

Introduction to Equalization

3.1 Inter Symbol Interference

In digital communication communication systems, a typical manifestation of the distortion caused by the physical channel intes-symbol interference (ISI), whereby, symbols transmitted before and after a given symbol corrupt the detection of that symbol. The problem of ISI becomes worse as the channel is used to transmit higher bitrate streams.

If there should be no ISI, the overall response (Figure 2.5) should be a Nyquist pulse.

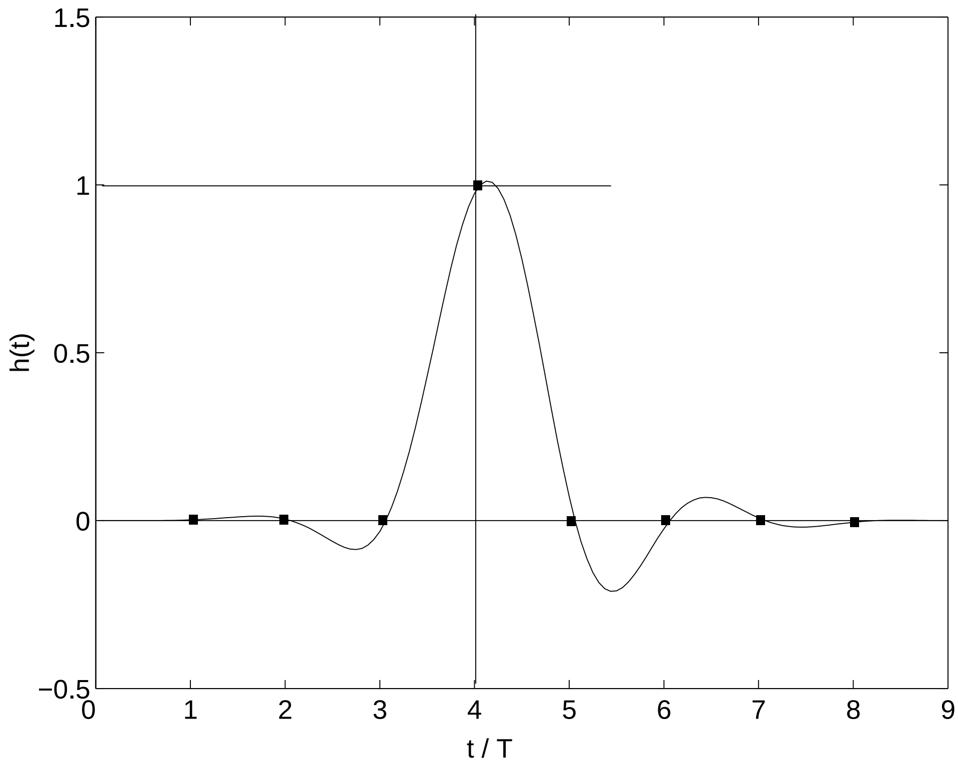

Figure 3.1: Example of a Nyquist Pulse

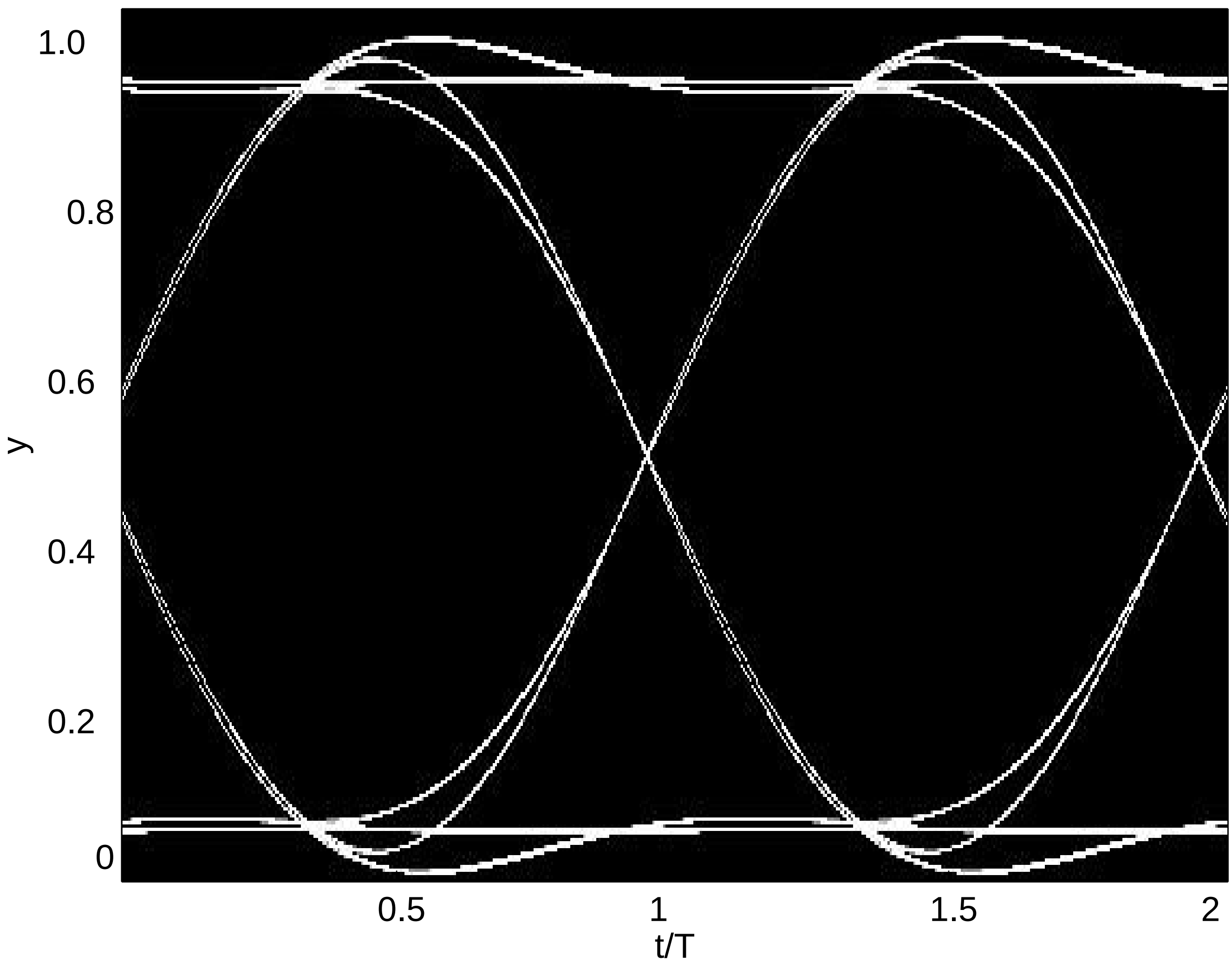

Figure 3.2: Eye-Diagram with no ISI

Figure 3.1 shows one such pulse shape. Note that at the sampling instants, the pulse is constrained to be zero, except at one sampling instant. With one such overall response, the eyediagam of the received signal is shown in Figure 3.2. Note the point of zero ISI, where the 'eye' is completely open. By sampling at this instant, the transmitted bitstream can be recovered.

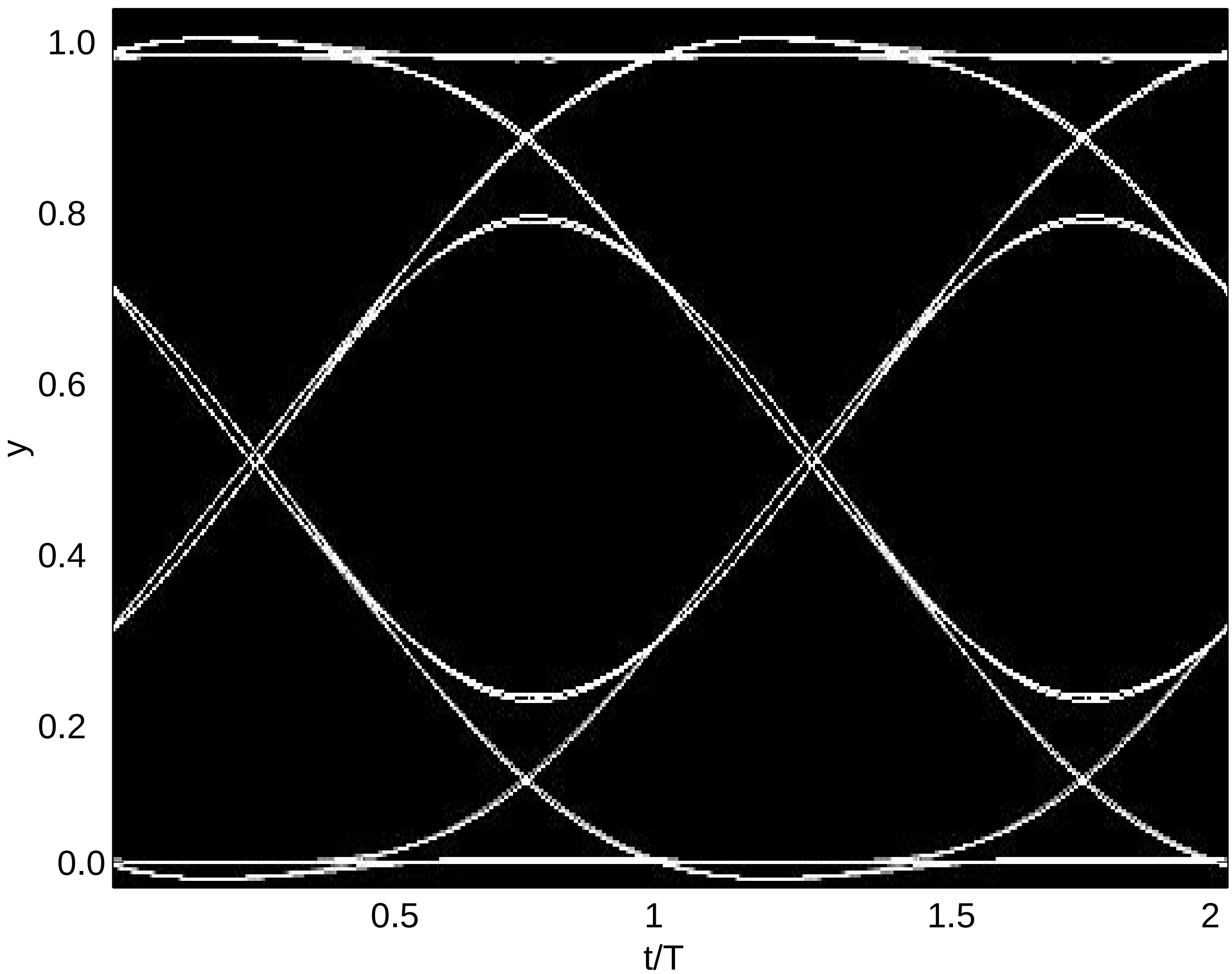

Figure 3.3: Eye-Diagram with ISI

In the presence of ISI, the 'eye' opening is reduced (Figure 3.3). This results in higher bit error rate for the same noise level in the channel.

3.2 Linear Channel Equalization

Linear Channel Equalization is an approach commonly used to counter the effects of ISI. It can be viewed as filtering the received signal with a linear filter(i.e. the equalizer). On applying an equalizer to the received signal with ISI, the ISI can be reduced consider-

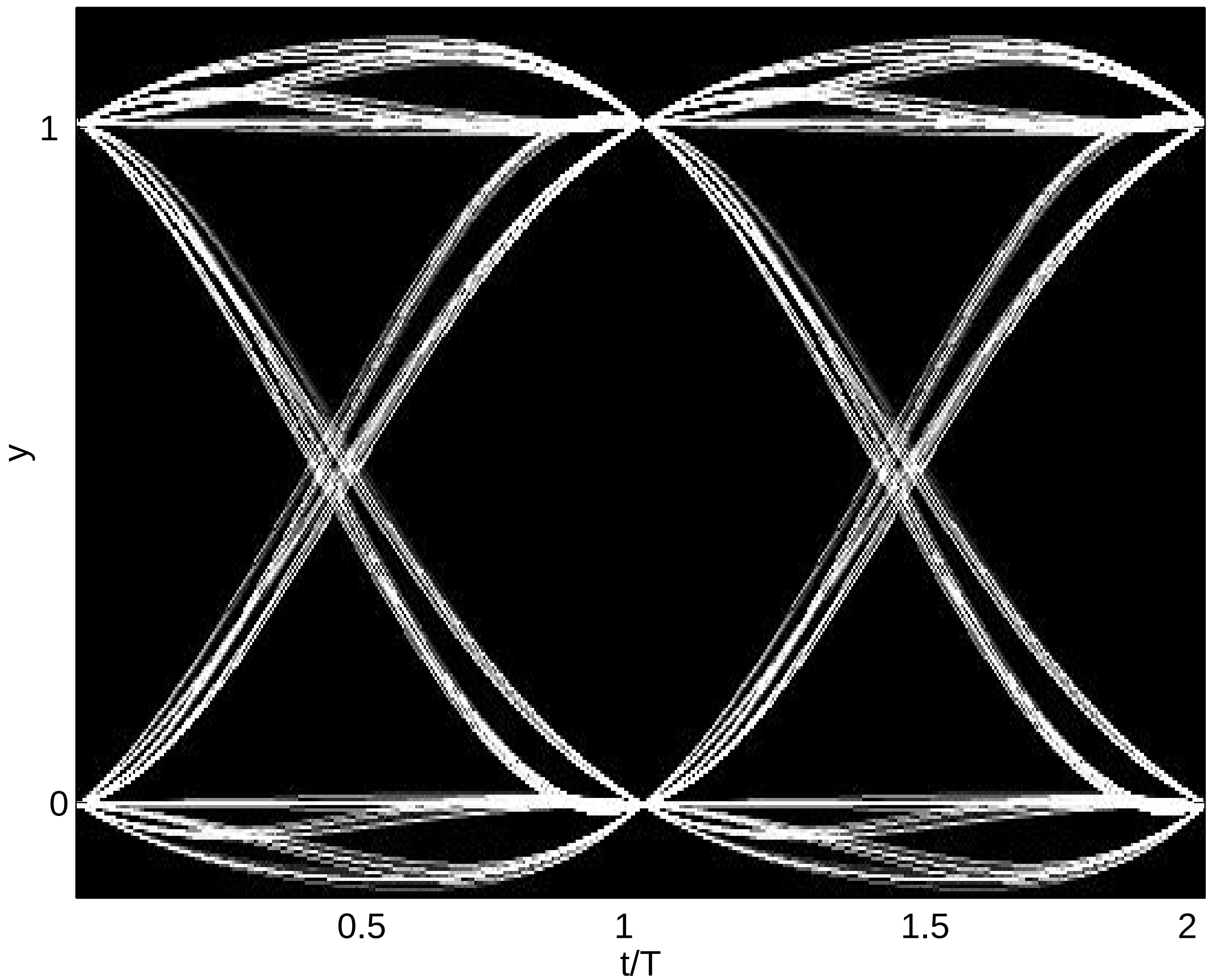

Figure 3.4: Eye-Diagram of the equalized signal

ably, depending on the complexity of the equalizer and the severity of distortion. Figure 3.4 shows the eyediagram after equalization.
(NOTE: in plotting the eyedaigram, the equalizer which in the conventional case is in the discrete time domain, is assumed to work in the analog domain, so that the analog signal is filtered by it, instead of its samples.)

3.3 Adaptive Equalization

It is common for the channel characteristics to be unknown or to change with time. Hence an equalizer that counters such a channel has to be adaptive. The adaptive equalizer might use a training sequence based approach, where a sequence of bits, known to the receiver, are transmitted periodically and the equalizer adapts itself based on the distortion that these undergo. But this leads to a lot of overhead.

Hence, the decision directed approach is used in cases where the bit error rate is very low. Here, the output of the equalizer is assumed to be an exact copy of the transmitted data. Thus the equalizer output is used to adapt the equalizer.In the following chapters, this approach is considered.

3.4 Baud Spaced Equalizer

The equalizer that works on the samples of the received signal, sampled at the symbol rate, is referred to as the symbol spaced or baud spaced equalizer. The Baud spaced equalizer could be fixed or adaptive.

The structure of a baud spaced equalizer is shown in Figure 3.5
The baud spaced equalizer (BSE) is very sensitive to sampling phase offsets. That is, the performance of a BSE depends on the exact instant within a bit interval when the sampling is done.

This can be intuitively understood from Figure 3.6. Here, a data sequence consisting of alternate zeros and ones is considered. Sampling at the symbol rate, the ideal sampling instants are shown. But a sampling phase offset of $\frac{T}{2}$ leads to sampling at the zero crossings. The equalizer, hence tries to invert this null channel and fails.

The solution to this problem is a fractioally spaced equalizer (FSE), which works at higher than bit rate.

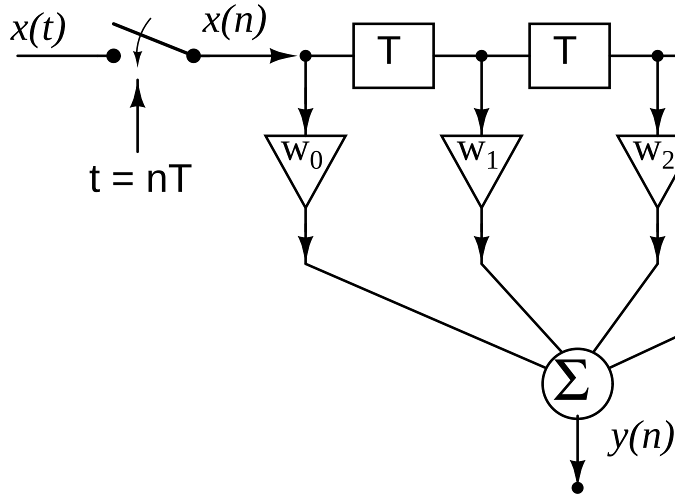

Figure 3.5: Baud Spaced Equalizer

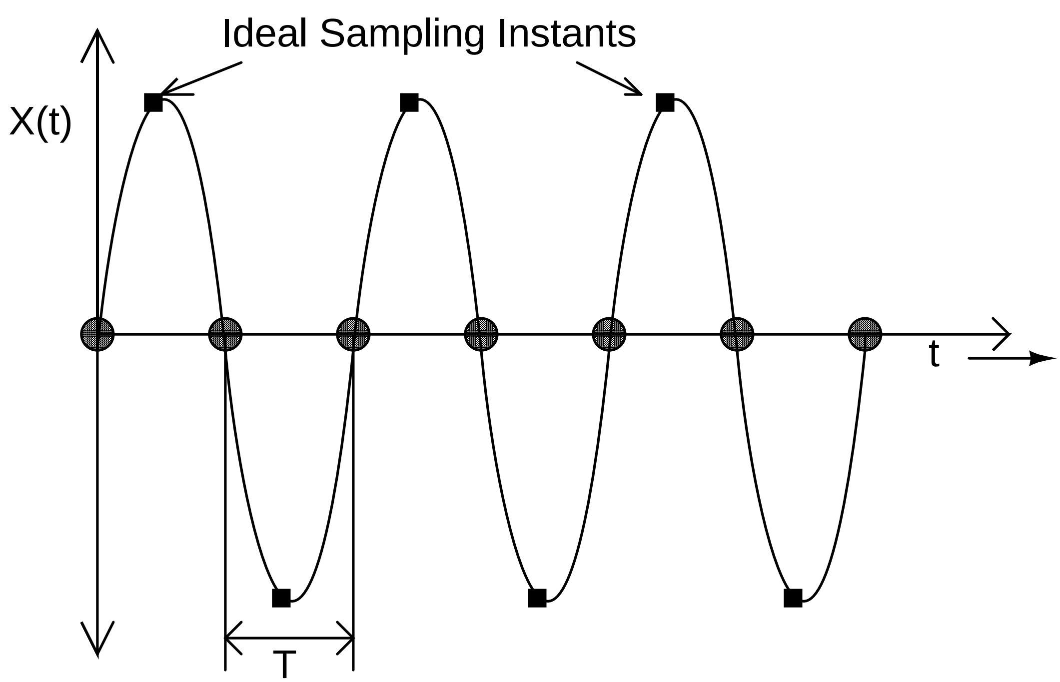

Figure 3.6: Effect of Sampling Phase Offset on BSE

3.5 Fractionally Spaced Equalizer

The fractionally spaced equalizer (FSE) works at higher than symbol rate. The most common case is when two samples are used per symbol. Figure 3.7 shows the structure of such an FSE. The FSE is free of sensitivity to sampling phase offset. The price being paid is the increase in speed of operation. But, since at the output, the samples are being decimated, the filter can be implemented in its polyphase form. But still, the sampler and analog to digital converter have to work at twice the bitrate.

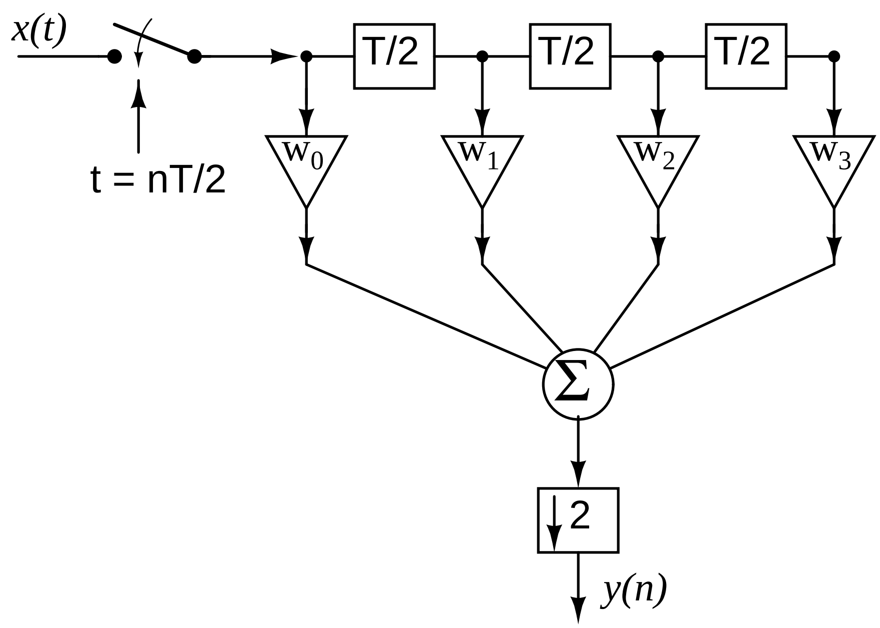

Figure 3.7: Fractionally Spaced Equalizer

3.6 Equalization in Analog Domain

At the very high bitrates of the optical channels (10 Gbps to 40 Gbps), there are significant channel impairments due to optical properties like chromatic dispersion and polarisation mode dispersion (PMD). Well known techniques exist for channel equalization. But at the very high bitrates considered, it becomes more and more difficult to implement these DSP techinques.[**?**]

On the other hand, at the high bit rates considered, it becomes viable to use transmission lines to get the delay element in the analog domain, and hence implement an equivalent tapped delay line filter in analog domain. This filter can then be used to equalize the channel. The following chapters explore this possibility in more detail.

Chapter 4

Transmission Line Filters

4.1 Tapped Delayline Filter

The simplest implementation of a digital FIR filter is a tapped delay line. Figure 4.1 shows the tapped delay line for a four tap digital filter. At higher bitrates, it becomes necessary to have very high bandwith devices to synthesize accurate delay elements, with the very low transition time needed. Figure 4.2 illustrates the effect of using the same devices to build a delay element at two different bitrates. At lower bitrates, the response of the delay element looks more like the ideal response than at the higher bitrate.

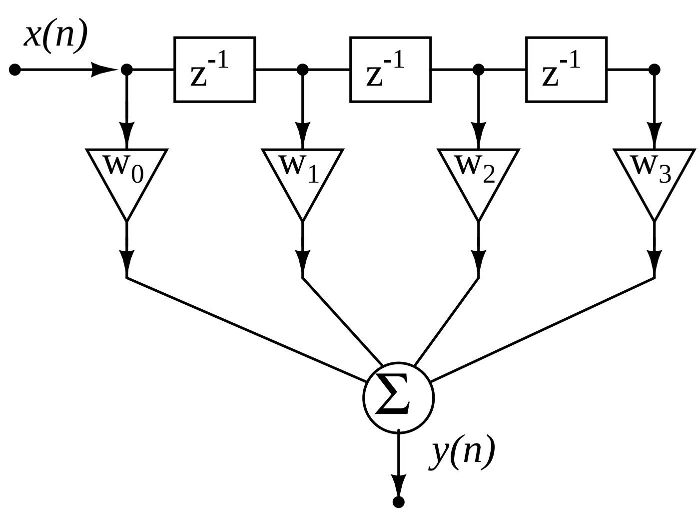

Figure 4.1: Tapped Delayline Filterwith 4 taps

The overall transfer function of a tapped delay line filter can be represented equivalently as a set of filters in parallel, with impulse responses as shown in Figure 4.3.

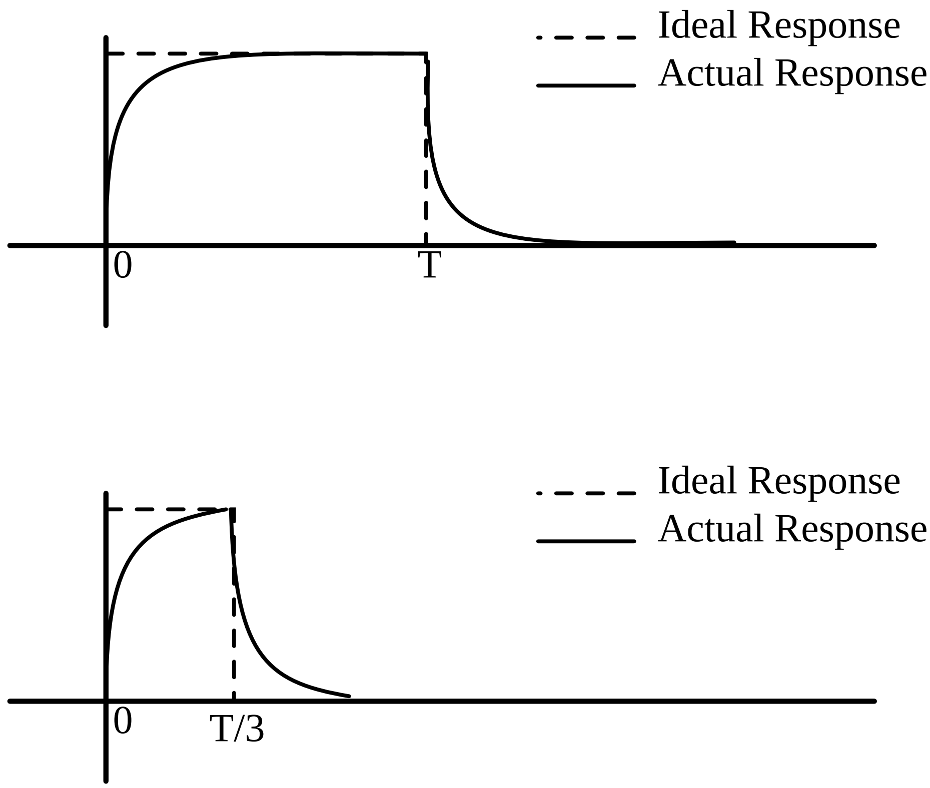

Figure 4.2: Effect of Higher Bitrates

Any FIR filter of length less than or equal to four can be obtained from the tapped delay line shown in figure 4.1. In terms of Figure 4.3 the weights w_i determine the impulse response $\begin{bmatrix} g(0) & g(1) & g(2) & g(3) \end{bmatrix}$ as follows

$$\begin{bmatrix} g(0) \\ g(1) \\ g(2) \\ g(3) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} w_0 \\ w_1 \\ w_2 \\ w_3 \end{bmatrix}$$

4.2 Tapped Delaylines using Transmission lines

In the analog domain, the delay element can be realized by using transmission lines. In VLSI technology, the transmission lines are realized as microstrip lines. At low symbol rates, the length of the microstrip lines needed will make the realisation of such structures in VLSI technology difficult. But as the symbol rates are scaled up, the length of the micro strip lines needed reduces and the filter becomes practical.

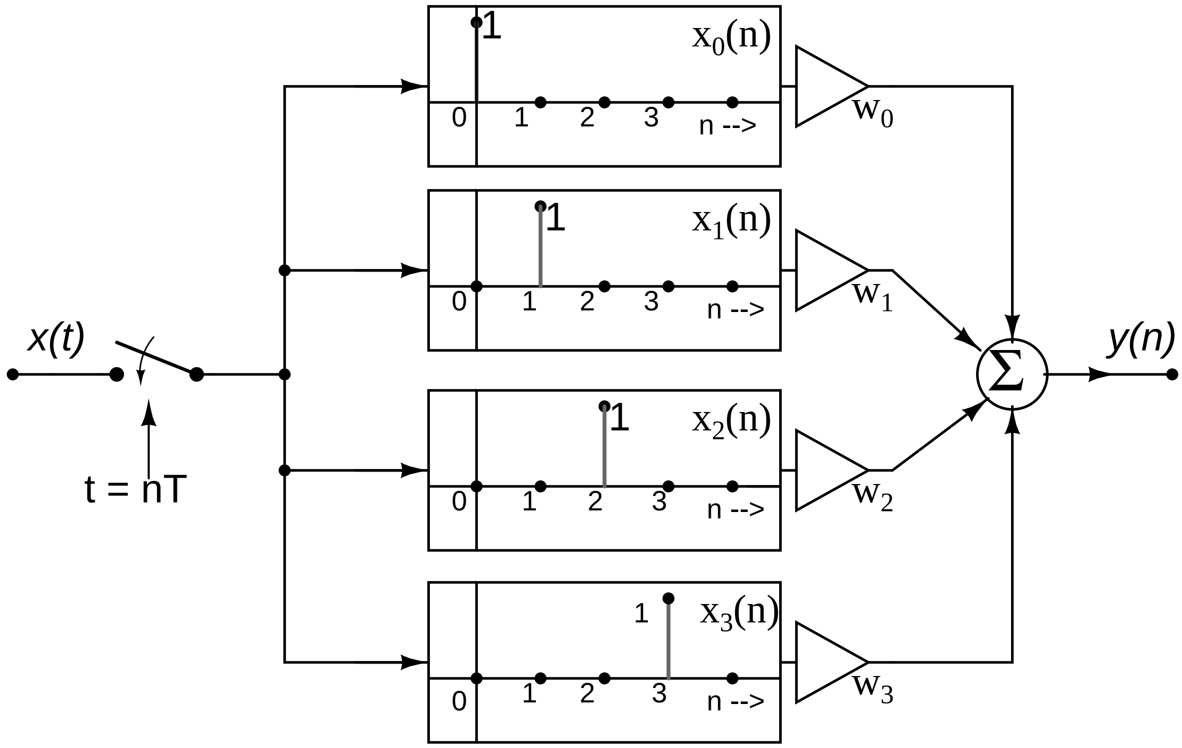

Figure 4.3: Equivalent Representation of Tapped Delayline

Figure 4.4 shows an implementation of a tapped delay line using transmission lines. The impulse responses corresponding to the Figure 4.8 representation of this filter is shown in Figure 4.5. In the absence of non-idealities, this is equivalent to the tapped delay line.

4.3 Equivalence of the Tapped Delayline and its Analog Counterpart

Consider an analog signal $x(t)$ that is sampled at the symbol rate T and the resulting samples are filtered by the tapped delay line in the discrete time domain. For simplicity, infinite precision is assumed in the discrete time domain. Figure 4.3 depicts the situation. The output samples can be expressed in terms of $x(t)$ as

$$y(n) = \sum_{i=0}^{i=3} w_i x(nT - iT)$$

Now consider the structure in figure 4.6. Here, the analog signal is first filtered by the transmission line implementation of the tapped delay line, with the same weights w_i. The output signal can be expressed as

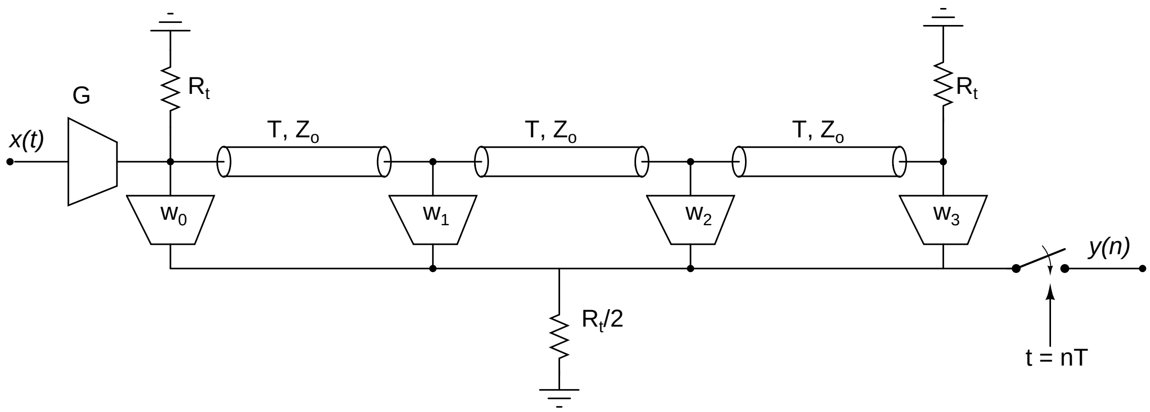

Figure 4.4: Tapped Delayline with Transmission Lines

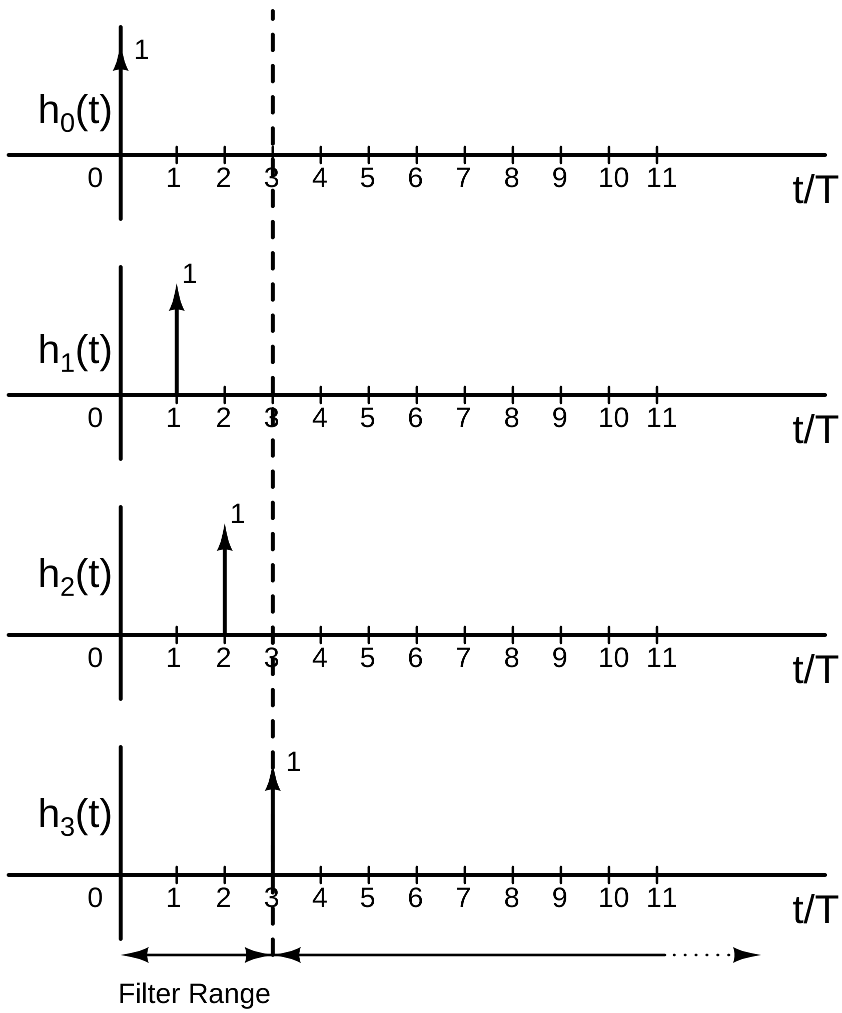

Figure 4.5: Impulse Responses Corresponding to Transversal Filter (Without Non-idealities)

$$y(t) = \sum_{i=0}^{i=3} w_i x(t - iT)$$

$$y(n) = y(t)|_{t=nT} = \sum_{i=0}^{i=3} w_i x(nT - iT)$$

Thus the samples of the output of the analog filter correspond to the output of the digital filter.

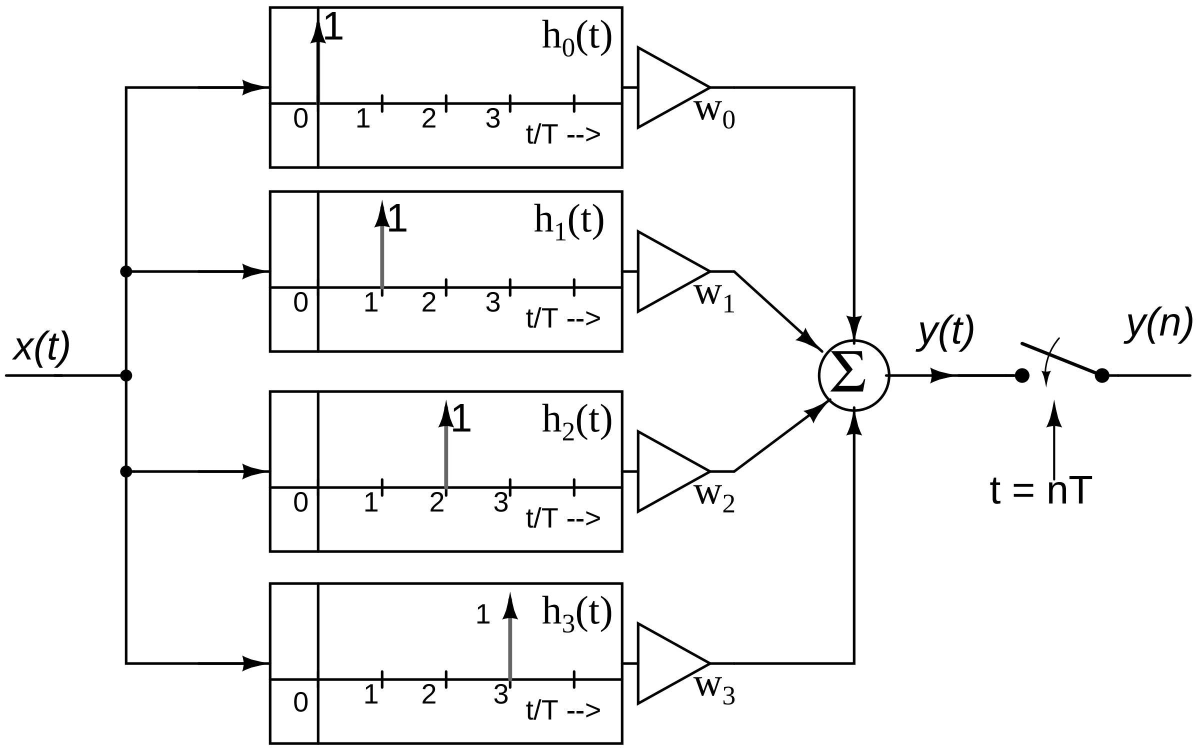

Figure 4.6: Filtering in the Analog Domain

4.4 Non Idealities in the Transmission Lines

The transmission line filter in the ideal case corresponds exactly to the tapped delay line. Bu in practice, the transmission lines have termination mismatches,and series loss and the transconductors used have a finite imput capacitance.

4.4.1 Termination Mismatch

The wave propogation in a transmission line will not get reflected at a termination if it is terminated by an impedence equal to its characteristic impedence Z_o. For a lossless transmission line the characteristic impedence is real and the ideal termination is hence a resistance of value Z_o.

In practice due to tolerances in the resistor values, the terminations are not exactly matched. This leads to terminal reflections. The reflected wave is Γ_t times the incident wave. Here Γ_t is the reflection coefficient.

$$\Gamma_t = \frac{R_t - Z_o}{R_t + Z_o}$$

Thus the primary reflections in figure 4.9 are of value Γ_t and the reflections of these reflected waves (seconadry reflections) are of value Γ_t^2.

4.4.2 Series Loss

In the presence of series loss, the impulse response has exponential tails as shown in the figure 4.9. The simulation of transmission lines with series loss is carrried out by breaking it up into m sections with the series loss distributed in between as shown in figure 4.7.

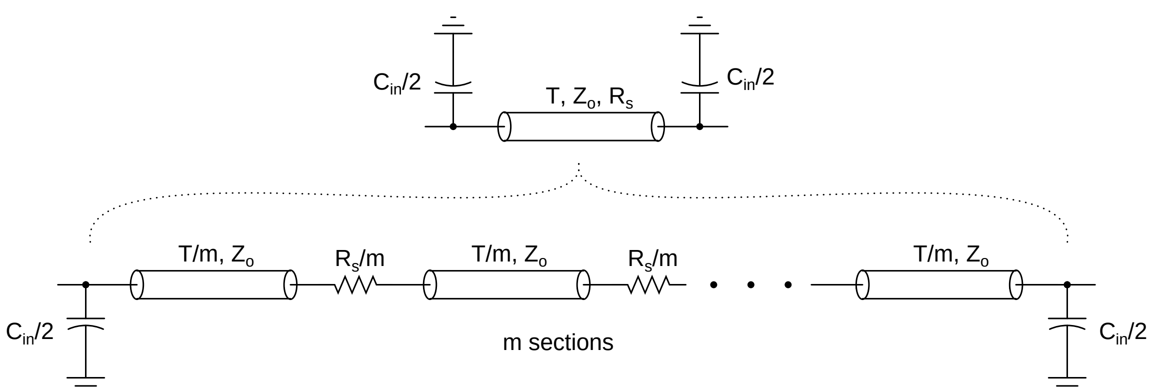

Figure 4.7: Incorporating Series Loss in the Transmission Lines

4.4.3 Parasitic Capacitance due to Transconductor

The transconductors used in the filter have parasitic capacitances at the input and the output. This parasitic capacitance reduces the bandwith of the transmission line and increases the reflection coefficients for high frequency compunents. Thus the impulse response gets further elongated.

4.5 Travelling Wave Filter

A more general case of the filter bank representation of the tapped delayline filter is shown in figure 4.8, where the indivdual filters $h_i(n)$ are not exactly shifted scaled unit impulse functions. Here the overall impulse response can be represented as

$$
\begin{bmatrix} g(0) \\ g(1) \\ g(2) \\ g(3) \\ \vdots \end{bmatrix} = \begin{bmatrix} h_{00} & h_{10} & h_{20} & h_{30} \\ h_{01} & h_{11} & h_{21} & h_{31} \\ h_{02} & h_{12} & h_{22} & h_{32} \\ h_{03} & h_{13} & h_{23} & h_{33} \\ h_{04} & h_{14} & h_{24} & h_{34} \\ \vdots & \vdots & \ddots & \vdots \end{bmatrix} \begin{bmatrix} w_0 \\ w_1 \\ w_2 \\ w_3 \end{bmatrix}
$$

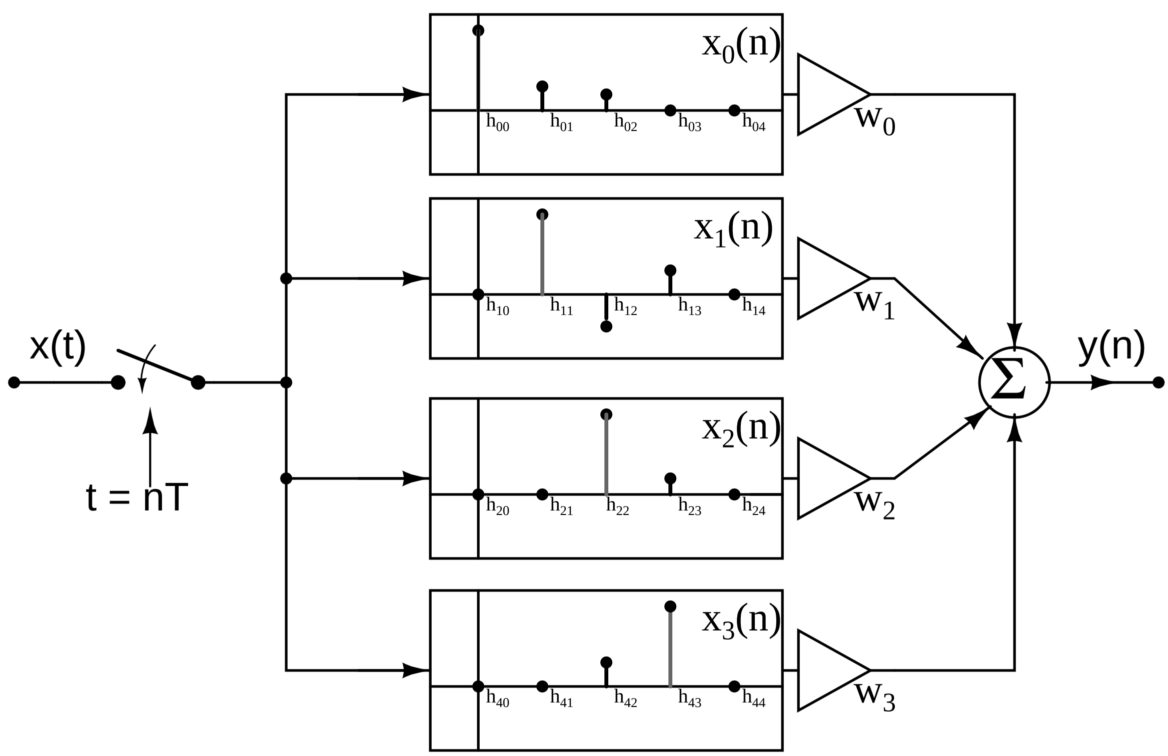

Figure 4.8: Another Possibility

The Analog filter, in the presence of reflections at the terminations and series loss in the transmission lines, behaves in this way. In the transversal filter considered so far, the impulse responses spread outside the range of the filter response (Figure 4.9).

In the absence of non-idealities, an alternate structure that realizes the same impulse response is the travelling wave structure, shown in figure 4.10.
But in the presence of terminal reflections and series loss, this structure performs better because the first order reflections are within the range of the filter response. So, neglecting the secondary reflections, the effects of non-idealities can be corrected for. The corresponding set of impulse responses for the filter bank representation of the travelling wave filter is shown figure 4.11.

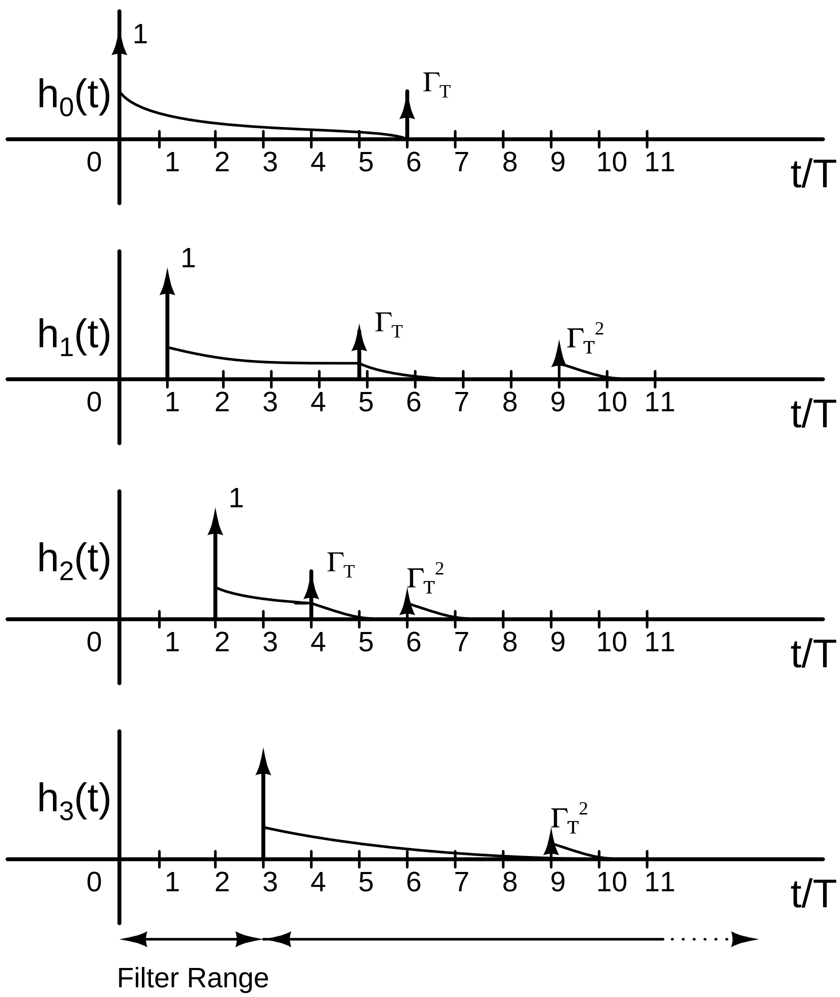

Figure 4.9: Impulse Responses Corresponding to Transversal Filter (terminal reflections and series loss present)

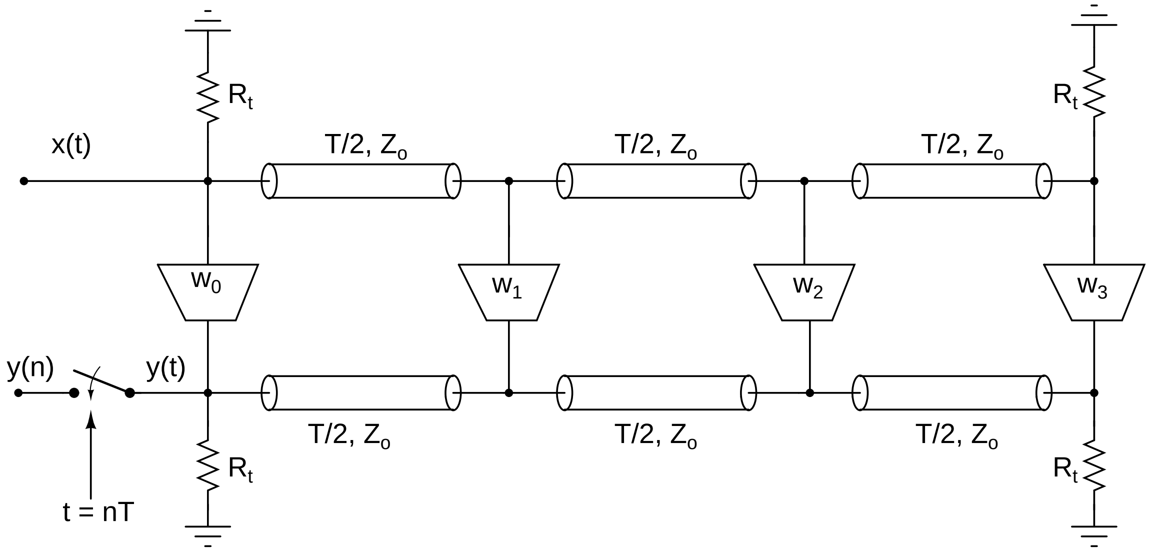

Figure 4.10: Travelling Wave Filter Structure

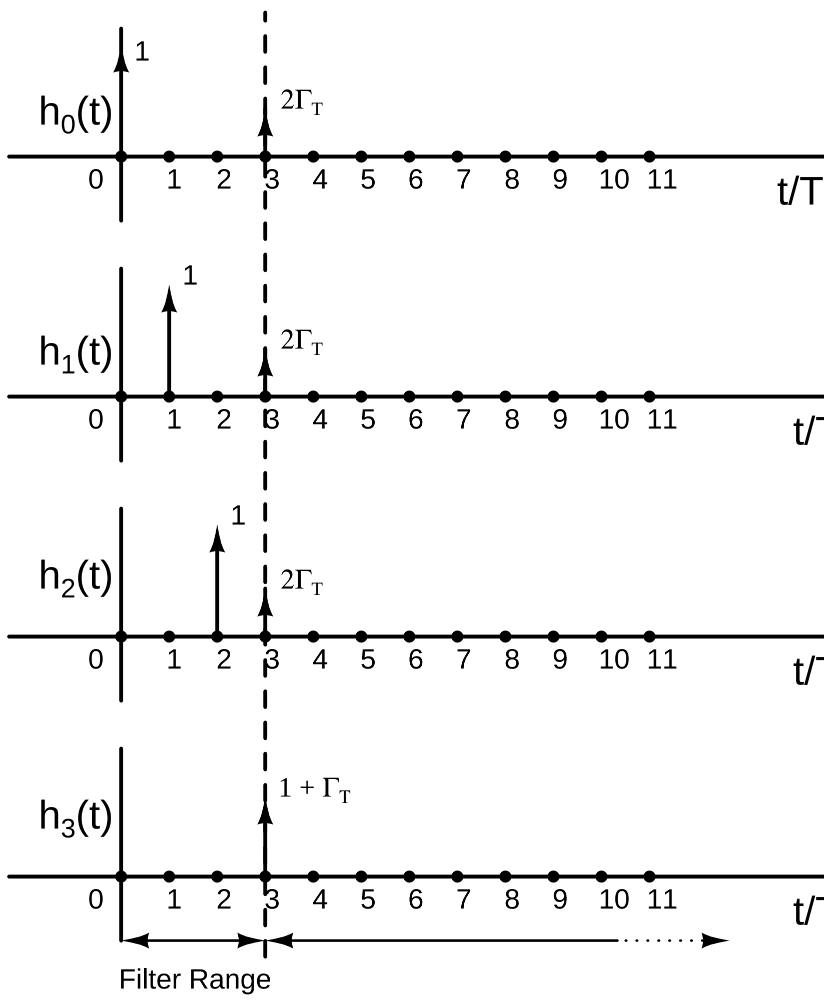

Figure 4.11: Impulse Responses Corresponding to Travelling Wave Filter

4.6 Correction of Weights for Travelling Wave Filter

In the presence of terminal reflections in a travelling wave filter, the primary reflections are within the range of the filter. Assuming that the secondary reflections are negligible, we can predistort the tap weights to get any desired impulse response of suitable length.

Let the desired tap weights be w_i. Let the predistorted tap weights to be used in the non-ideal case be $\overline{w}_i$.

$$\begin{bmatrix} \overline{w}_0 \\ \overline{w}_1 \\ \overline{w}_2 \\ \overline{w}_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & \Gamma_T \\ 0 & 1 & 0 & \Gamma_T \\ 0 & 0 & 1 & \Gamma_T \\ 0 & 0 & 0 & (1+\Gamma_T^2) \end{bmatrix} \begin{bmatrix} w_0 \\ w_1 \\ w_2 \\ w_3 \end{bmatrix}$$

$$\overline{\mathbf{w}} = \mathbf{A}\mathbf{w}$$

$$\mathbf{w} = \mathbf{A}^{-1}\overline{\mathbf{w}}$$

Thus in the case of the travelling wave filter, the non idealities can be corrected for, while the out of range distortion introduced in the conventional filter cannot be corrected.

The travelling wave filter is hence an order of magnitude better than the conventional filter structure. The simulation reslts in the following chapters illustrate the same.

Chapter 5

Theoretical Derivation of the Optimum Equalizer Coefficients

5.1 Goal

An ideal communication link is expected to transmit the information sequence fed into it faithfully, with the possible inclusion of a delay. But practical comunication links are far from this. Consider figure 5.1. The Digital communication link is not ideal.
The goal of the equalizer is to counter the non idealities of the link and produce aqs faithful a reproduction of the transmitted sequence as possible at its output.
In the presence of AWGN and ISI in the channel and using an FIR filter to counter it, the optimum FIR filter coefficients can be derived, aiming to minimize the mean squared error $E|e(n)|^2$. This is the minimum mean square error approach to design an equalizer.

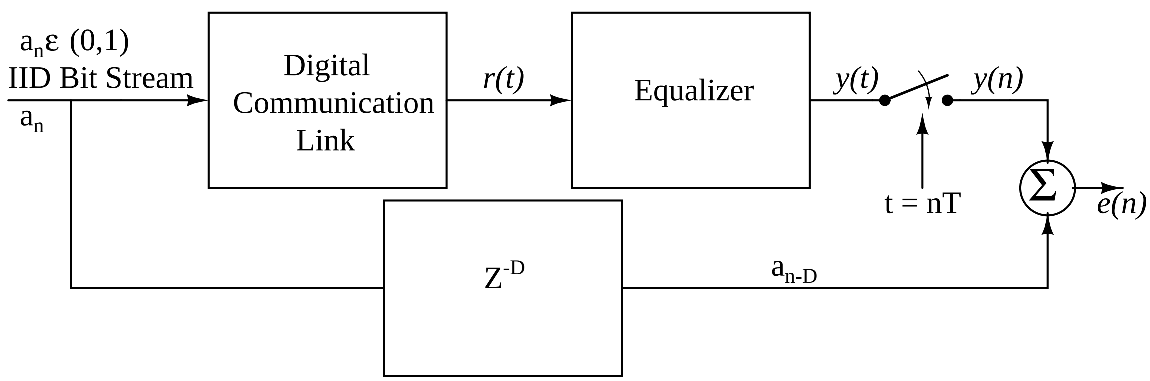

Figure 5.1: Goal of Channel Equalization

5.2 MMSE Approach

Consider the system diagram in figure 5.1. The aim is to derive an expression for the values of the equalizer taps that will minimize the mean squared error at the output of the equalizer.

The output of the equalizer is $y(n)$.

The transmitted data is $a(n)$.

The desired output is $a(n-D)$, where D represents a delay to take care of causality constraints on the filters and channel.

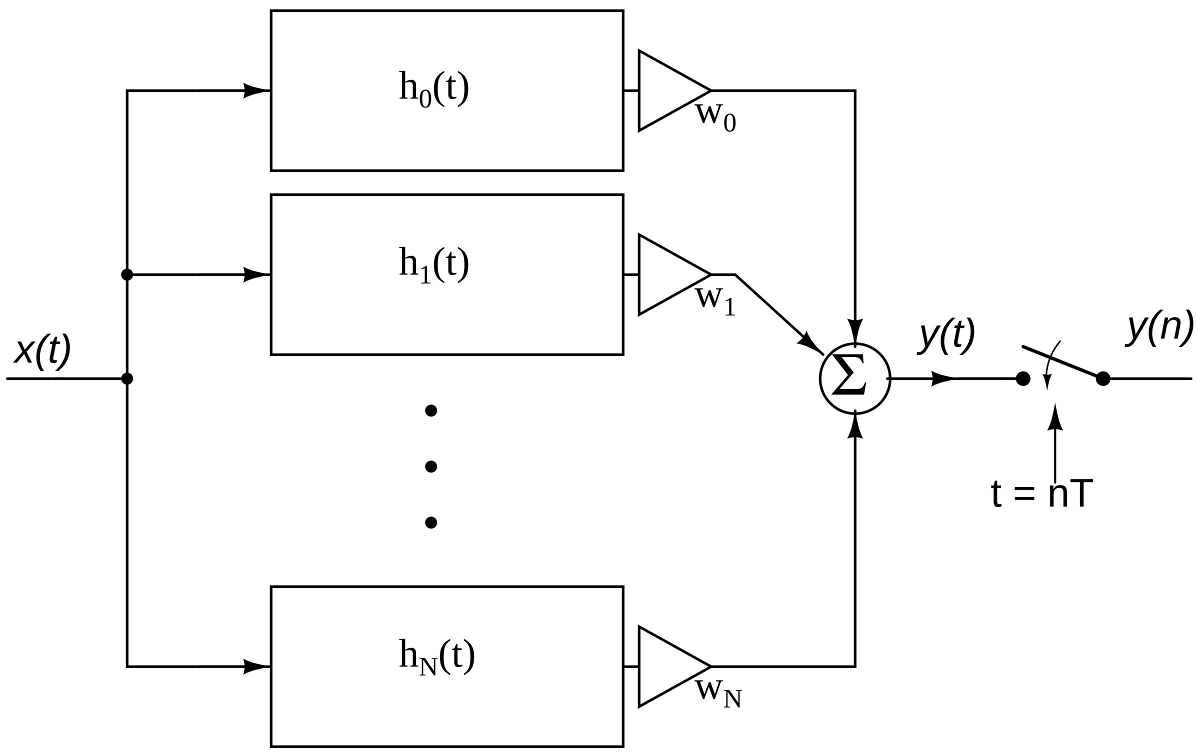

Figure 5.2: Equalizer Block Diagram

In figure 5.2, let $g(t)$ represent the channel response, that includes the transmit pulse shaping filter,$c(t)$ the channel.

Let $r(t)$ represent the receiver front end filter.

Let the transfer function from the equalizer filter input to the i^{th} state be $X_i(j\omega)$ and the corresponding impulse response $x_i(t)$.

The Noise at the receiver front end is assumed to be white.

The noise process is represented as $n(t)$ and the corresponding samples as $\eta(n)$.

The output of the equalizer filter can be written as,

$$y(t) = \sum_{i=1}^{N} \sum_{k=-\infty}^{+\infty} a(k)c_i(t - kT)w_i + \sum_{i=1}^{N} n(t) * f_i(t)w_i \tag{5.1}$$

Where,$c_i(t) = g(t) * r(t) * x_i(t)$ and $f_i(t) = r(t) * x_i(t)$.

are identical if $\frac{1}{T}$ exceeds the nyquist rate for $f_i(t)$

The samples of $y(t)$ can then be expressed as

$$y(n) = \sum_{i=1}^{N} \sum_{k=-\infty}^{+\infty} a(k)c_i(nT - kT)w_i + \sum_{i=1}^{N} \eta_i(n)w_i \tag{5.2}$$

Let the samples of $c_i(t)$ be represented in the matrix form, such that $C_{ij} = c_j(iT)$. Assuming that the responses $c_i(t)$ are significant over L+1 bit intervals, we see that the size of C and F matrices is $N \times (L + 1)$.

$$y(n) =$$

$$\begin{bmatrix} a(n) & a(n-1) & \dots & a(n-L) \end{bmatrix} \begin{bmatrix} c_1(0.T) & c_2(0.T) & \dots & c_N(0.T) \\ c_1(T) & c_2(T) & \dots & c_N(T) \\ \vdots & \vdots & \ddots & \vdots \\ c_1(L.T) & c_2(L.T) & \dots & c_N(L.T) \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_N \end{bmatrix}$$

$$+ \begin{bmatrix} \eta_1(n) & \eta_2(n) & \dots & \eta_N(n) \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_N \end{bmatrix}$$

Equivalently,

$$y(n) = \mathbf{a}(n)\mathbf{C}\mathbf{w} + \eta(n)\mathbf{w}$$

Let $\mathbf{h}_D = \begin{bmatrix} 0 & 0 & \dots & 1 & 0 & \dots & 0 \end{bmatrix}_{1\times(L+1)}$ with only the $(D + 1)^{th}$ element being unity.

So that, the deired output ,$a(n - D) = \mathbf{a}(n)\mathbf{h}_D^T$

The error at the output of the equalizer can be expressed as,

$$\begin{aligned} e(n) &= y(n) - a(n-D) \\ &= \mathbf{a}(n)\left(\mathbf{C}\mathbf{w} - \mathbf{h}_D^T\right) + \eta(n)\mathbf{w} \\ E\left[|e(n)|^2\right] &= \left(\mathbf{C}\mathbf{w} - \mathbf{h}_D^T\right)^T E\left[\mathbf{a}(n)^T\mathbf{a}(n)\right]\left(\mathbf{C}\mathbf{w} - \mathbf{h}_D^T\right) + \mathbf{w}^T E\left[\eta(n)^T\eta(n)\right]\mathbf{w} \end{aligned}$$

Assuming that the transmitted sequence is i.i.d and that the noise at the receiver front end is white, with a noise spectral density of $\frac{N_o}{2}$,
$E\left[\mathbf{a}(n)^T\mathbf{a}(n)\right] = \sigma_a^2\mathbf{I}$ and
$E\left[\eta_i(n)\eta_j(n)\right] = \frac{N_o}{2}\int_{-\infty}^{+\infty} f_i(t)f_j(t)dt$
where, $\sigma_a^2 = E[|a(n)|^2]$.
let $\lambda = \frac{N_o}{2\sigma_a^2}$ and the matrix F defined by,
$F_{ij} = \int_{-\infty}^{+\infty} f_i(t)f_j(t)dt$
The source power normalized mean squared error cost function,

$$J = \left(\mathbf{C}\mathbf{w} - \mathbf{h}_D^T\right)^T\left(\mathbf{C}\mathbf{w} - \mathbf{h}_D^T\right) + \lambda\mathbf{w}^T\mathbf{F}\mathbf{w}$$

$$J = \mathbf{w}^T\mathbf{C}^T\mathbf{C}\mathbf{w} - \mathbf{w}^T\mathbf{C}^T\mathbf{h}_D^T - \mathbf{h}_D^T\mathbf{C}\mathbf{w} + \mathbf{h}_D\mathbf{h}_D^T + \lambda\mathbf{w}^T\mathbf{F}\mathbf{w}$$

To minimise J with respect to w,

$$\begin{aligned} \frac{\partial J}{\partial \mathbf{w}} &= \mathbf{C}^T\mathbf{C}\mathbf{w}_{opt} - \mathbf{C}^T\mathbf{h}_D^T + \lambda\mathbf{F}\mathbf{w}_{opt} = 0 \\ \mathbf{w}_{opt} &= \left(\mathbf{C}^T\mathbf{C} + \lambda\mathbf{F}\right)^{-1}\mathbf{C}^T\mathbf{h}_D^T \end{aligned}$$

The optimal MSE is

$$J_{opt} = \mathbf{h}_D\left(\mathbf{I}_N - \mathbf{C}\left(\mathbf{C}^T\mathbf{C} + \lambda\mathbf{F}\right)^{-1}\mathbf{C}^T\right)\mathbf{h}_D^T$$

which is still a function of the delay D. The optimal delay corresponds to the minimum diagonal element of $\mathbf{I}_N - \mathbf{C}\left(\mathbf{C}^T\mathbf{C} + \lambda\mathbf{F}^T\mathbf{F}\right)^{-1}\mathbf{C}^T$. Therefore,

$$D_{opt} = argmin\left\{\left[\mathbf{I}_N - \mathbf{C}\left(\mathbf{C}^T\mathbf{C} + \lambda\mathbf{F}\right)^{-1}\mathbf{C}^T\right]_{D,D}\right\}$$

5.3 Theory of the Adaptive Equalizer Based on LMS Algorithm

5.3.1 Introduction

The channel charcteristics are not always known and keeps changing with time and environmental conditions. Hence the optimal equalizer taps keep varying. Hence an adaptive equalizer is needed that keeps changing the equalizer settings depending on the changes in the channel.

5.3.2 LMS Algorithm

The LMS algorithm belongs to a class of algorithms called gradient descent algorithms. The output error of a system is a function of several parameters. Thus we have an error surface. The aim is to raech the minimum of this surface iteratively. The gradient descent algorithms do this by starting at some point and moving in small steps opposite to the gradient of the error surface at that point.

In the present case, as explained in the following subsection, the mean square error is a quadratic function of the equalizer taps. By choosing to work with the instantaneous estimate of the gradient, instead of the actual gradient, we land up with the LMS algorithm in the case.

5.3.3 Derivation of Tap Update Expressions in the Analog Equalizer

Consider the equalizer represented in figure 5.3. The aim is to derive an expression for updating the values of the equalizer taps that will minimize the mean squared error at the output of the equalizer,

The output of the equalizer is $y(n)$.

The transmitted data is $a(n)$.

The desired output is $a(n-D)$, where D represents a delay to take care of causality constraints on the filters and channel.

Let the inputs to the taps be $x_i(t)$

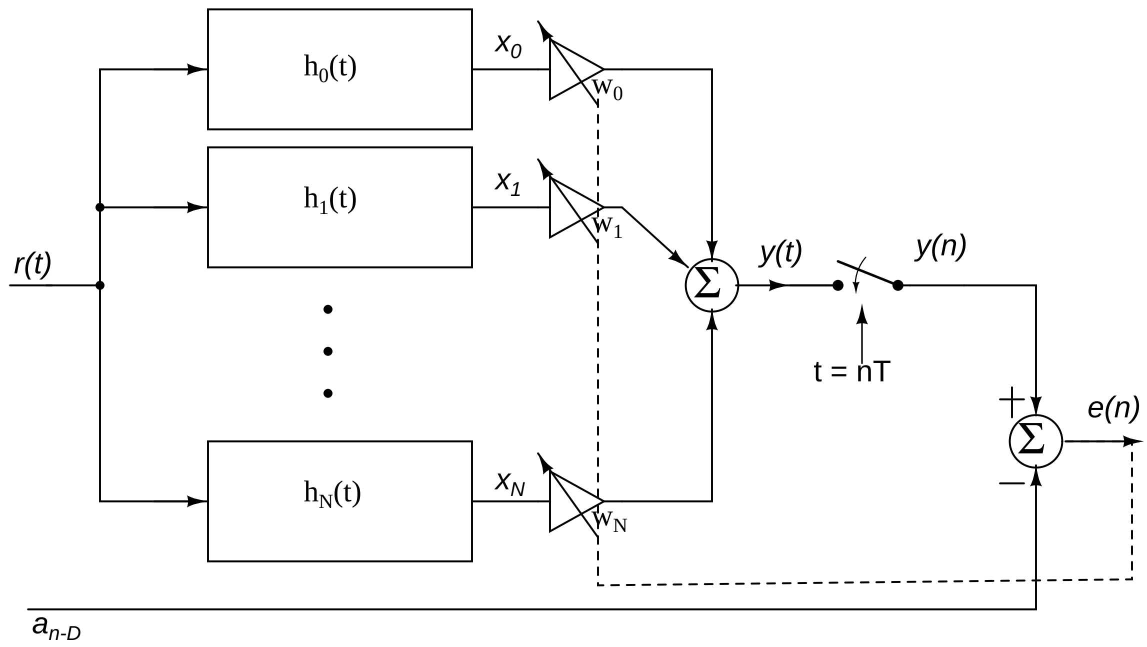

Figure 5.3: Adaptive Equalizer Block Diagram

The output of the equalizer filter can be written as,

$$y(t) = \sum_{i=1}^{N} x_i(t)w_i(t) \tag{5.3}$$

The samples of $y(t)$ can then be expressed as

$$y(n) = \sum_{i=1}^{N} x_i(nT)w_i(nT) \tag{5.4}$$

Equivalently,

$$y(n) = \mathbf{x}(n)\mathbf{w}(n)$$

The error at the output of the equalizer can be expressed as,

$$\begin{aligned} e(n) &= a(n-D) - y(n) \\ &= a(n-D) - \mathbf{x}(n)\mathbf{w}(n) \end{aligned}$$

The mean squared error cost function,

$$J = E\left[|e(n)|^2\right]$$

J is a function of the weights $\mathbf{w}$ of the equalizer. The Least Mean Square (LMS) algorithm aims to recursively minimise J by updating the equalizer weights along the negative of the gradient of the error surface. Hence the LMS algorithm belongs to the class of gradient descent algorithms. The stochastic gradient is hard to compute. The LMS algorithm actually minimises the instantaneous estimate of the gradient The gradient of J with respect to $\mathbf{w}$ is given by,

$$\begin{aligned}
\frac{\partial J'}{\partial \mathbf{w}} &= \frac{\partial |e(n)|^2}{\partial \mathbf{w}} \\
&= 2e(n)\frac{\partial e(n)}{\partial \mathbf{w}} \\
&= -2e(n)\frac{\partial y(n)}{\partial \mathbf{w}} \\
&= -2e(n)\mathbf{x}(n)
\end{aligned}$$

Thus the update equation for the equalizer weights is

$$\begin{aligned}
\mathbf{w}(n+1) &= \mathbf{w}(n) - \mu\frac{\partial |e(n)|^2}{\partial \mathbf{w}} \\
\mathbf{w}(n+1) &= \mathbf{w}(n) + \mu e(n)\mathbf{x}(n) \\
w_i(n+1) &= w_i(n) + \mu e(n)x_i(n)
\end{aligned}$$

5.4 Theory of the Decision Feedback Equalizer

5.4.1 Introduction

The equalizer structures we considered till now are FIR filters. There is no feedback path in the equalizer filter structure. The next stucture considers the inclusion of a feedback path in the filter, along with the FIR part. This leads to the decision feedback equaliser. This structure is helpful in cases where the channel response is long.
Consider the structure of figure 5.4
In the absence of errors, the decisions at the slicer are equivalent to a_{n-D}, the delayed data sequence. Thus the system can equivalently be represented as in figure 5.5.

5.4.2 Theory

Consider the figure 5.5. The derivation here assumes that there are no bit errors. In practice,in the presence of errors, the phenomenon of error propogation is observed in a DFE. This is not dealt with here.

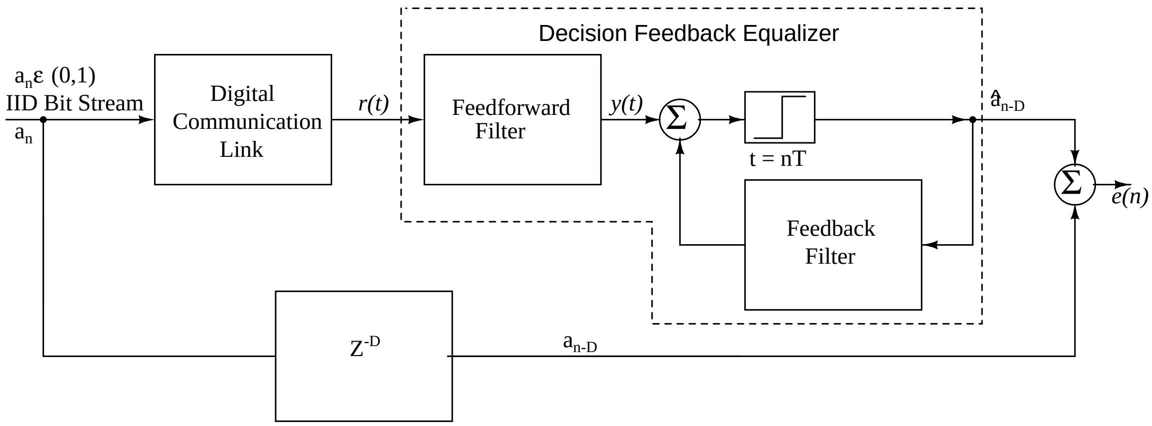

Figure 5.4: Decision Feedback Equalizer Block Diagram

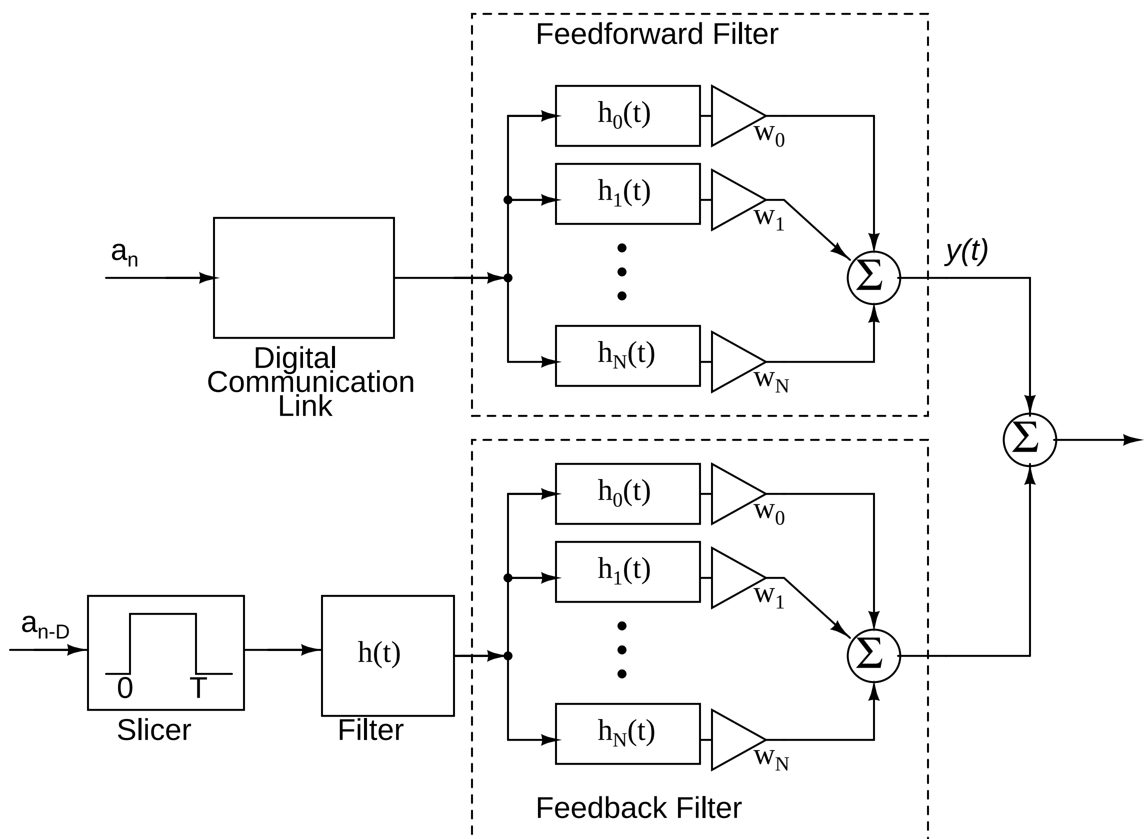

Figure 5.5: Decision Feedback Equalizer Structure

The aim is to derive an expression for the values of the forward and feedback filter taps, that will minimize the mean squared error at the output of the DFE.

The output of the equalizer is $y(n)$.

The transmitted data is $a(n)$.

The desired output is $a(n-D)$, where D represents a delay to take care of causality constraints on the filters and channel.

Let $g(t)$ represent the channel response, that includes the transmit pulse shaping filter, the channel.

Let $r(t)$ represent the receiver front end filter.

Let the transfer function from the equalizer filter input to the i^{th} state be $X_i(j\omega)$ and the corresponding impulse response $x_i(t)$.

The Noise at the receiver front end is assumed to be white.

The noise process is represented as $n(t)$ and the corresponding samples as $\eta(n)$.

The output of the feedforward equalizer filter can be written as,

$$y_1(t) = \sum_{i=1}^{N} \sum_{k=-\infty}^{+\infty} a(k) c_i(t - kT) w_i + \sum_{i=1}^{N} n(t) * f_i(t) w_i$$

Where, $c_i(t) = g(t) * r(t) * x_i(t)$ and $f_i(t) = r(t) * x_i(t)$.

The samples of $y_1(t)$ can then be expressed as

$$y_1(n) = \sum_{i=1}^{N} \sum_{k=-\infty}^{+\infty} a(k) c_i(nT - kT) w_i + \sum_{i=1}^{N} \eta_i(n) w_i$$

Let the samples of $c_i(t)$ be represented in the matrix form, such that $C_{ij} = c_j(iT)$. Assuming that the responses $c_i(t)$ are significant over L+1 bit intervals, we see that the size of C and F matrices is $N \times (L+1)$.

$$y_1(n) =$$

$$\begin{bmatrix} a(n) & a(n-1) & \dots & a(n-L) \end{bmatrix} \begin{bmatrix} c_1(0.T) & c_2(0.T) & \dots & c_N(0.T) \\ c_1(T) & c_2(T) & \dots & c_N(T) \\ \vdots & \vdots & \ddots & \vdots \\ c_1(L.T) & c_2(L.T) & \dots & c_N(L.T) \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_N \end{bmatrix}$$

$$+\begin{bmatrix} \eta_1(n) & \eta_2(n) & \dots & \eta_N(n) \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_N \end{bmatrix}$$

Equivalently,

$$y(n) = \mathbf{a}(n)\mathbf{C}\mathbf{w} + \eta(n)\mathbf{w}$$

Let D be the delay introduced by the DFE. That is, the output at sampling instant n is an estimate of a_{n-D}. Note that the feedback path is noise free.

Let the filter coeffecients for the feedback filter be b_i , $0 \le i \le N-1$. The output from the feedback path can then be represented as

$$y_2(n) = \begin{bmatrix} a(n) & a(n-1) & \dots & a(n-D) & a(n-D-1)\dots & a(n-L) \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ p_1(0.T) & p_2(0.T) & \dots & p_N(0.T) \\ p_1(T) & p_2(T) & \dots & p_N(T) \\ \vdots & \vdots & \ddots & \vdots \\ p_1(\overline{L-D}.T) & p_2(\overline{L-D}.T) & \dots & p_N(\overline{L-D}.T) \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_N \end{bmatrix}$$

Where, $p_i(t) = h(t) * h_i(t)$ is the overall impulse response of the feedback path. Equivalently,

$$y_2(n) = \mathbf{a}(n)\mathbf{P}\mathbf{b}$$

The overall output $y(n)$ can then be expressed as

$$\begin{aligned} y(n) &= y_1(n) - y_2(n) \\ &= \mathbf{a}(n)\mathbf{C}\mathbf{w} + \eta(n)\mathbf{w} - \mathbf{a}(n)\mathbf{P}\mathbf{b} \\ &= \mathbf{a}(n)\left[\mathbf{C}\mathbf{w} - \mathbf{P}\mathbf{b}\right] + \eta(n)\mathbf{w} \end{aligned}$$

For any given feedforward tap settings, the ideal feedback filter should subtract the residual ISI after the Dth instant. Hence the matrix $\mathbf{Cw} - \mathbf{Db}$ reduces to

$$\mathbf{Cw} - \mathbf{Pb} =$$

$$\begin{bmatrix} c_0(0.T) & c_1(0.T) & \dots & c_N(0.T) \\ \vdots & \vdots & \ddots & \vdots \\ c_1(\overline{D-1}.T) & c_2(\overline{D-1}.T) & \dots & c_N(\overline{D-1}.T) \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_N \end{bmatrix}$$

Let the matrix $\mathbf{C}$ that is truncated as shown above be represented as $\mathbf{C_D}$ to indicate the delay D.

Let $\mathbf{h}_D = \begin{bmatrix} 0 & 0 & \dots & 1 & 0 & \dots & 0 \end{bmatrix}_{1\times(L+1)}$ with only the $(D+1)^{th}$ element being unity.

So that, the deired output ,$a(n-D) = \mathbf{a}(n)\mathbf{h}_D^T$

The error at the output of the equalizer can be expressed as,

$$\begin{aligned} e(n) &= y(n) - a(n-D) \\ E\left[|e(n)|^2\right] &= \left(\mathbf{C_D w} - \mathbf{h}_D^T\right)^T E\left[\mathbf{a}(n)^T\mathbf{a}(n)\right]\left(\mathbf{C_D w} - \mathbf{h}_D^T\right) + \mathbf{w}^T E\left[\eta(n)^T\eta(n)\right]\mathbf{w} \end{aligned}$$

Assuming that the transmitted sequence is i.i.d and that the noise at the receiver front end is white, with a noise spectral density of $\frac{N_o}{2}$,

$E\left[\mathbf{a}(n)^T\mathbf{a}(n)\right] = \sigma_a^2\mathbf{I}$ and

$E\left[\eta_i(n)\eta_j(n)\right] = \frac{N_o}{2}\int_{-\infty}^{+\infty} f_i(t)f_j(t)dt$

where, $\sigma_a^2 = E[|a(n)|^2]$.

let $\lambda = \frac{N_o}{2\sigma_a^2}$ and the matrix F defined by,

$F_{ij} = \int_{-\infty}^{+\infty} f_i(t)f_j(t)dt$

The source power normalized mean squared error cost function,

$$J = \left(\mathbf{C_D w} - \mathbf{h}_D^T\right)^T\left(\mathbf{C_D w} - \mathbf{h}_D^T\right) + \lambda\mathbf{w}^T\mathbf{F}\mathbf{w}$$

$$J = \mathbf{w}^T\mathbf{C_D}^T\mathbf{C_D w} - \mathbf{w}^T\mathbf{C_D}^T\mathbf{h}_D^T - \mathbf{h}_D^T\mathbf{C_D w} + \mathbf{h}_D\mathbf{h}_D^T + \lambda\mathbf{w}^T\mathbf{F}\mathbf{w}$$

To minimise J with respect to w,

$$\frac{\partial J}{\partial \mathbf{w}} = \mathbf{C_D}^T \mathbf{C_D} \mathbf{w}_{opt} - \mathbf{C_D}^T \mathbf{h}_D^T + \lambda \mathbf{F} \mathbf{w}_{opt} = 0$$

$$\mathbf{w}_{opt} = \left(\mathbf{C_D}^T \mathbf{C_D} + \lambda \mathbf{F}\right)^{-1} \mathbf{C_D}^T \mathbf{h}_D^T$$

The optimal MSE is

$$J_{opt} = \mathbf{h}_D \left(\mathbf{I}_N - \mathbf{C_D} \left(\mathbf{C_D}^T \mathbf{C_D} + \lambda \mathbf{F}\right)^{-1} \mathbf{C_D}^T\right) \mathbf{h}_D^T$$

which is still a function of the delay D. The optimum MSE is calculated for the delay in the range of $0 \leq D \leq N-1$ and the optimum delay D_opt is chosen to be that which gives the minimum MSE. For this D_{opt}, the optimal weights are calculated.

What remains now is to compute the feedback taps such that the residual tail is cancelled out as assumed in the derivation.

Let $(\mathbf{C} - \mathbf{C_D})\mathbf{w}$ represent the residual tail. The condition to be satisfied by the feedback filter is

$$\mathbf{P b_{opt}} = (\mathbf{C} - \mathbf{C_D})\mathbf{w}$$

Let

$$\mathbf{P b} - (\mathbf{C} - \mathbf{C_D})\mathbf{w} = \mathbf{e}$$

On minimising $||e||^2$, we get,

$$\mathbf{b_{opt}} = \left(\mathbf{P}^T \mathbf{P}\right)^{-1} \mathbf{P^T} (\mathbf{C} - \mathbf{C_D})\mathbf{w}$$

5.5 Modelling of Non-Idealities in Transmission Line Filters

The Previous sections have dealt with the derivation of the optimum equalizer taps, assuming the various transfer functions to be known.

This section deals with the modelling of the transmission line filters in the presence of the non-idealities discussed in chapter 4.

5.5.1 ABCD matrix formulation

Consider an 2-port network, shown in figure 5.6. The input/output relationship can be expressed in the well known ABCD matrix form. Accordingly,

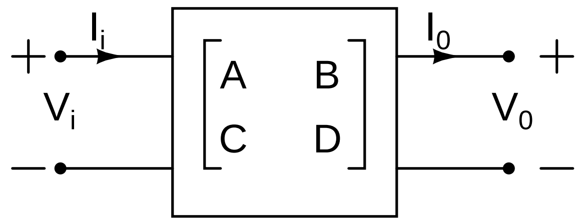

Figure 5.6: General 2 Port Network

$$\begin{bmatrix} V_i \\ I_i \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_o \\ I_o \end{bmatrix}$$

In the present context, we work in the frequency domain and hence V_i, V_o, I_i, I_o are the frequency domain representations of the voltages and currents. Hence the terms A, B, C, D are also functions of frequency.
If two 2-port networks are in cascade, the overall ABCD matrix for the composite system is the product of the ABCD matrices of the individual 2-ports.
The transmission line filters can be viewed as a cascade of several 2-port networks. Hence the overall ABCD matrix representation of the filter can be obtained.

5.5.2 ABCD matrix representation of important blocks

5.5.2.1 Ideal Transmission Line

An ideal transmission line with characteristic impedance Z_o and delay T can be represented in the ABCD matrix form, at a frequency ω as

$$\begin{bmatrix} V_i \\ I_i \end{bmatrix} = \begin{bmatrix} cos(wT) & Z_o sin(wT) \\ \frac{sin(wT)}{Z_o} & cos(wT) \end{bmatrix} \begin{bmatrix} V_o \\ I_o \end{bmatrix}$$

5.5.2.2 Shunt Resistance

A shunt resistance of value R_t can be represented in the ABCD matrix form, at any frequency as

$$\begin{bmatrix} V_i \\ I_i \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{1}{R_t} & 1 \end{bmatrix} \begin{bmatrix} V_o \\ I_o \end{bmatrix}$$

5.5.2.3 Shunt Capacitance

A shunt capacitance of value C can be represented in the ABCD matrix form, at a frequency ω as

$$\begin{bmatrix} V_i \\ I_i \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ j\omega C & 1 \end{bmatrix} \begin{bmatrix} V_o \\ I_o \end{bmatrix}$$

5.5.2.4 Series Resistance

A series resistance of value R_s can be represented in the ABCD matrix form, at any frequency as

$$\begin{bmatrix} V_i \\ I_i \end{bmatrix} = \begin{bmatrix} 1 & R_s \\ 0 & 1 \end{bmatrix} \begin{bmatrix} V_o \\ I_o \end{bmatrix}$$

5.5.2.5 Lossy Transmission Line

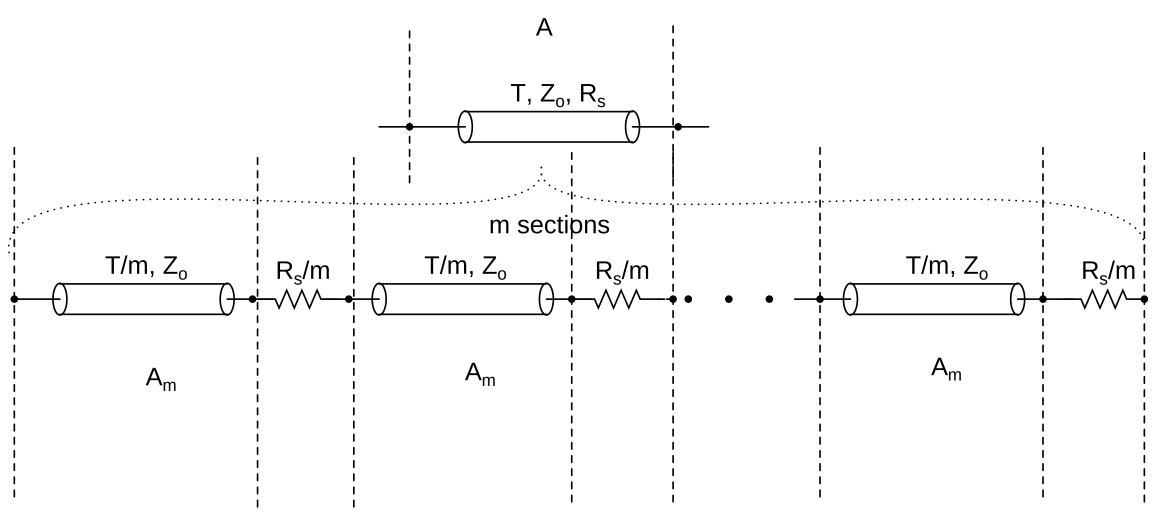

Figure 5.7: ABCD Representation of Lossy Transmission Lines

A lossy section of a transmission line of delay T and overall series loss of R_s ohms is split up m subsections as shown in figure 5.7. If $\mathbf{A}_m$ represents the ABCD matrix of the subsection and $\mathbf{R}_m$ represents the ABCD matrix of the series resistance per subsection, then the overall ABCD matrix for the whole Transmission Line, A is given by

$$\mathbf{A} = (\mathbf{A}_m \mathbf{R}_m)^m$$

5.5.3 Transfer Functions for the Transversal Filter

The transversal filter is a cascade of various 2 ports as shown on figure 5.8.

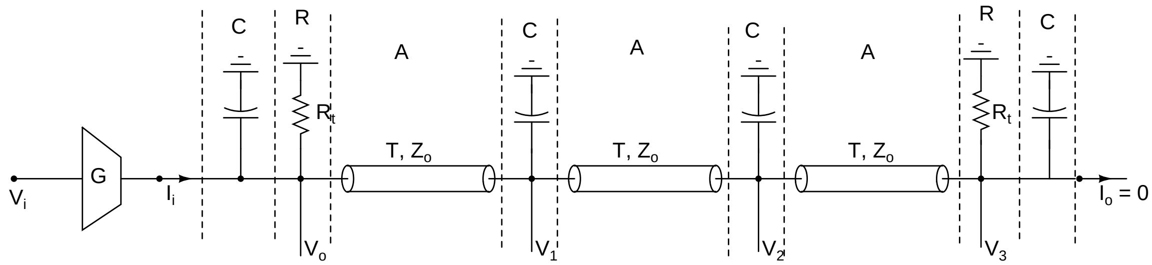

Figure 5.8: ABCD Representation of Transversal Filter Topology

The overall ABCD matrix ($\mathbf{A}_total$) representation is given by

$$\mathbf{A}_{total} = \mathbf{CR}(\mathbf{AC})^N \mathbf{R}$$

The transfer function to the first tap is given by

$$H_o = \frac{V_o}{V_i} = \frac{V_o}{GI_i} = \frac{A_{total}}{GC_{total}}$$

Where A_{total} and C_{total} correspond to the A and C elements of the overall ABCD matrix.

The transfer function to the last tap is given by

$$H_N = \frac{V_N}{V_i} = \frac{V_N}{GI_i} = \frac{1}{GC_{total}}$$

The transfer function to the $(N-1)^{th}$ tap is given by

$$H_{N-1} = A_1 H_N$$

Where A_1 corresponds to the A element of the ABCD matrix $\mathbf{ARC}$ that represents the 2-port between the $(N-1)^{th}$ tap and the N^{th} tap.

Proceeding likewise we can find all the transferfunctions.

The time domain impulse responses can be eveluated by inverse Fourier transform.

5.5.4 Transfer Functions for the Travelling Wave Filter

The Travelling wave topology as shown in figure 5.9 can be seen as two transversal filters with the transmission lines of half the length.

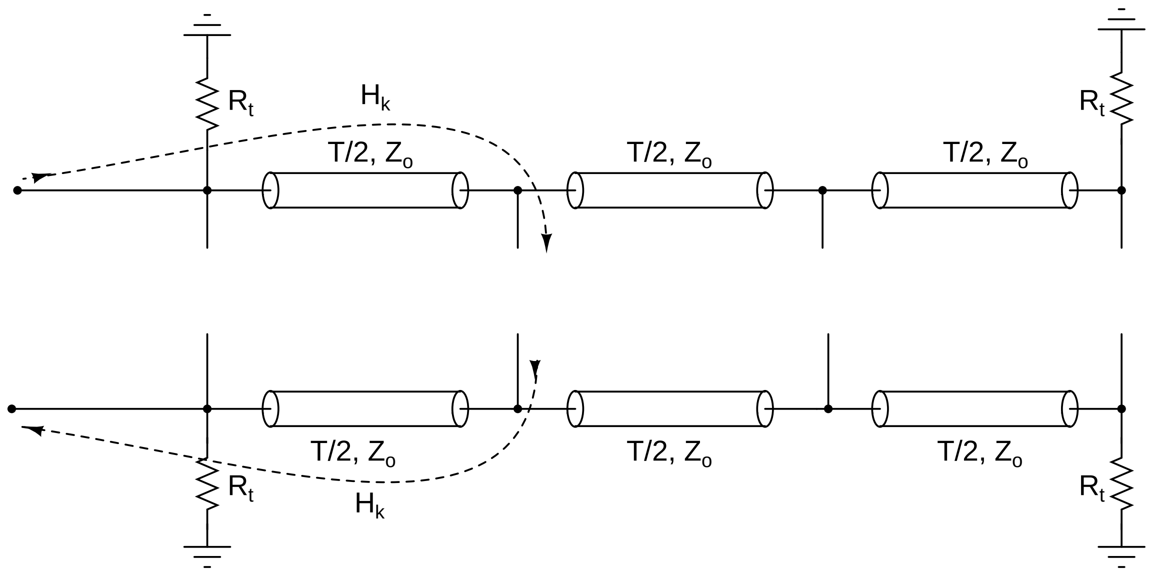

Figure 5.9: Travelling Wave Filter Topology

By reprocity theorem, The transfer function from the input to the k^{th} tap on the first line is same as the transfer function from the k^{th} tap to the output on the second line. Thus, the overall transfer function is the square of the transfer function H_k.

The theory developed for the transversal filter case can now be applied to get the transfer functions and they are squared to get the transfer functions in the travelling wave case.

(Note: At the output of the transconductors, there will be parasitic capacitance and this is assumed to be equal to the input capacitance to ensure symmetry.)

Chapter 6

Simulation Results

6.1 System Details

The simulations assume the system model described in section 2.5. Refer figure 2.5 The details of individual blocks is as follows

Transmit filter 4th order Bessel filter with 3dB bandwidth of 0.5 times bitrate.

Channel Equal power split PMD channel with AWGN.

Receiver front end filter 4th order Butterworth filter with 3dB bandwidth of 0.7 times bitrate.

6.2 Conventional Discrete Time Equalizers

This section presents the performance results of the conventional equalizers.

6.2.1 Baud Spaced Equalizer

Figure 6.1 shows the mean square error at the output of a 5 tap baud spaced equalizer at various noise levels for PMD channel with $0 \leq T_d \leq 0.875T$.

6.2.2 Fractionally Spaced Equalizer

Figure 6.2 shows the mean square error at the output of a 10 tap fractionally spaced equalizer at various noise levels for PMD channel with $0 \leq T_d \leq 0.875T$.

6.2.3 Timing Phase Sensitivity

Figure 6.3 shows the mean square error at the output of a 5 tap baud spaced equalizer and 10 tap fractionally spaced equalizer for PMD channel with $T_d = 0.75T$.

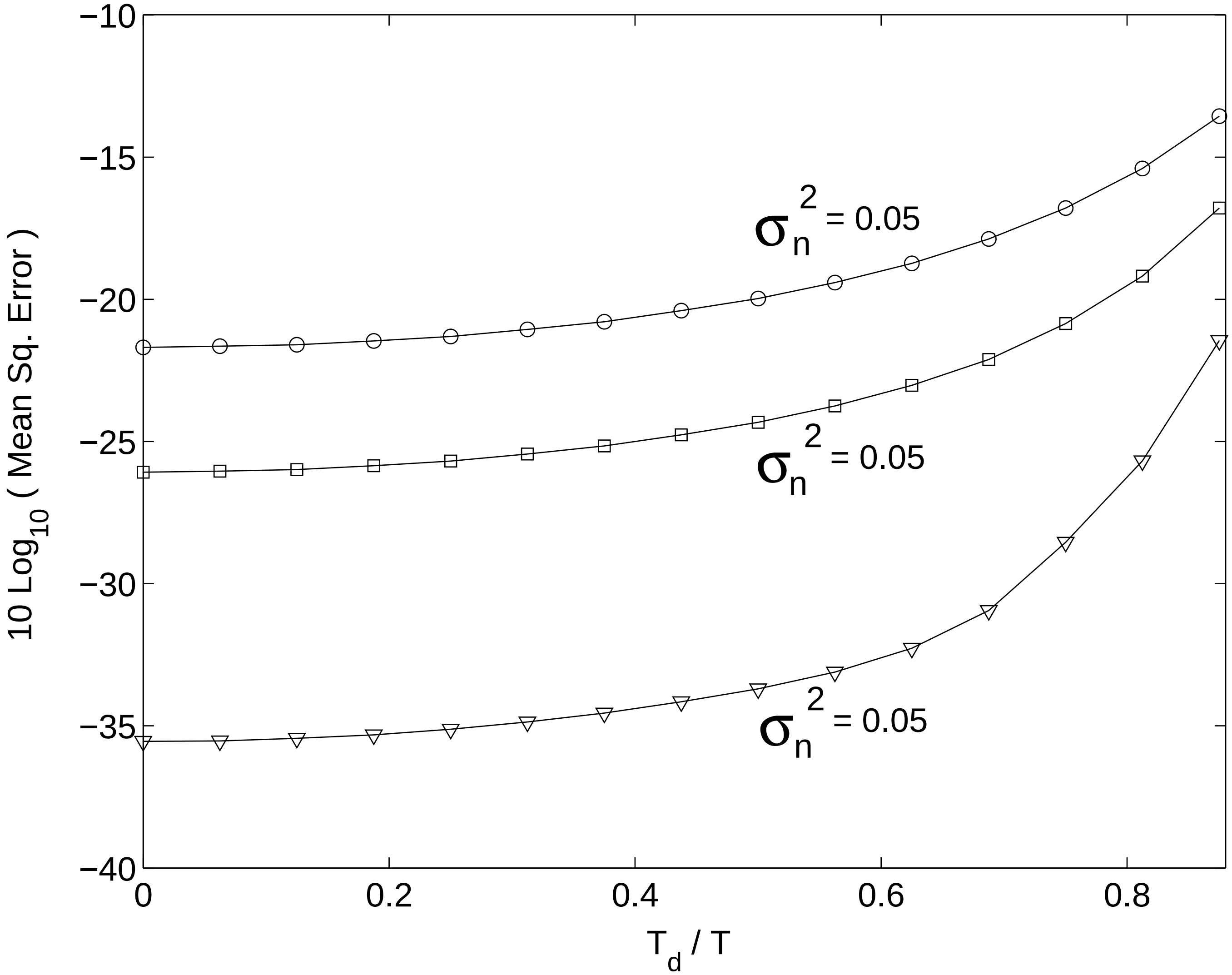

Figure 6.1: MSE of BSE for different channel conditions

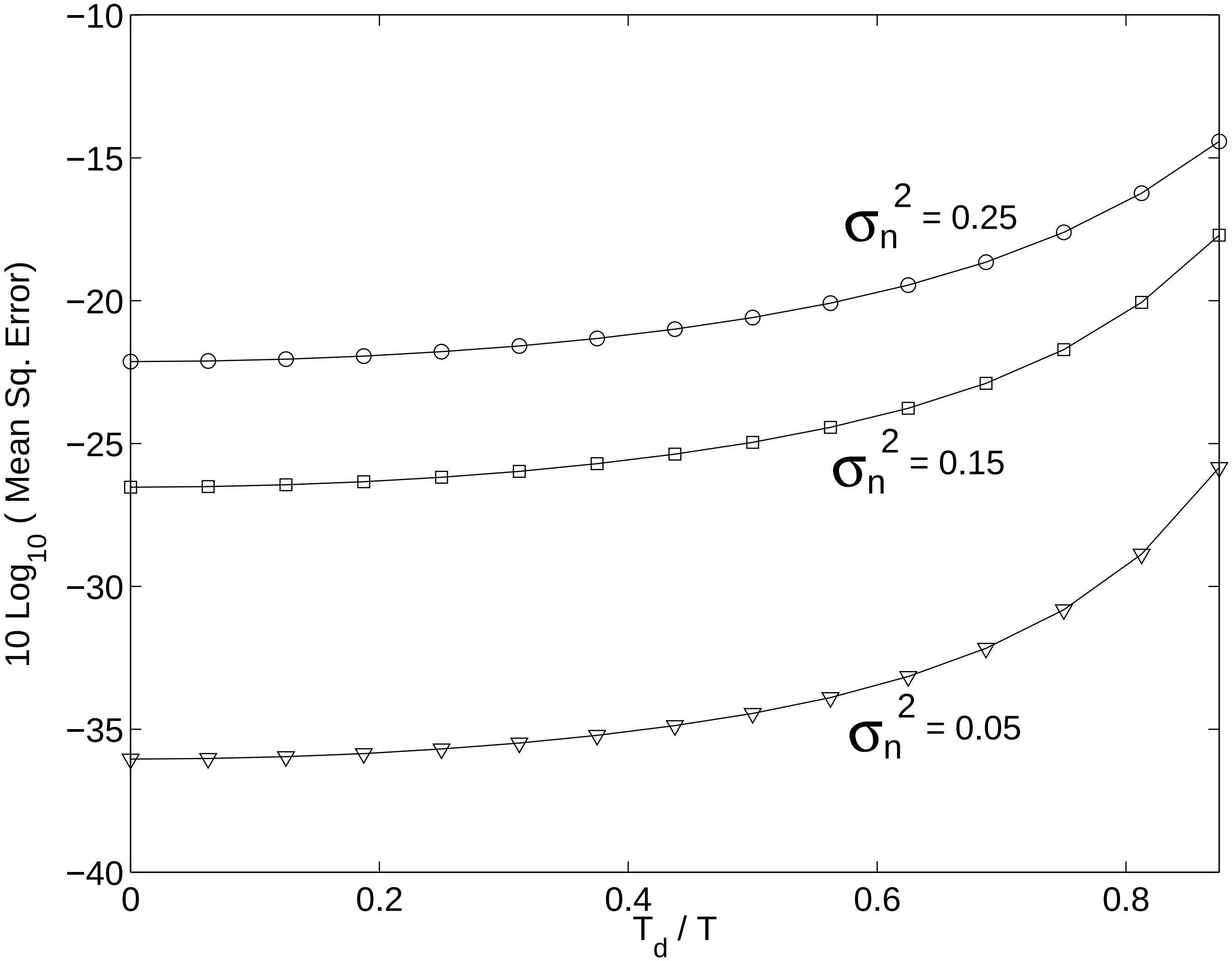

Figure 6.2: MSE of FSE for different channel conditions

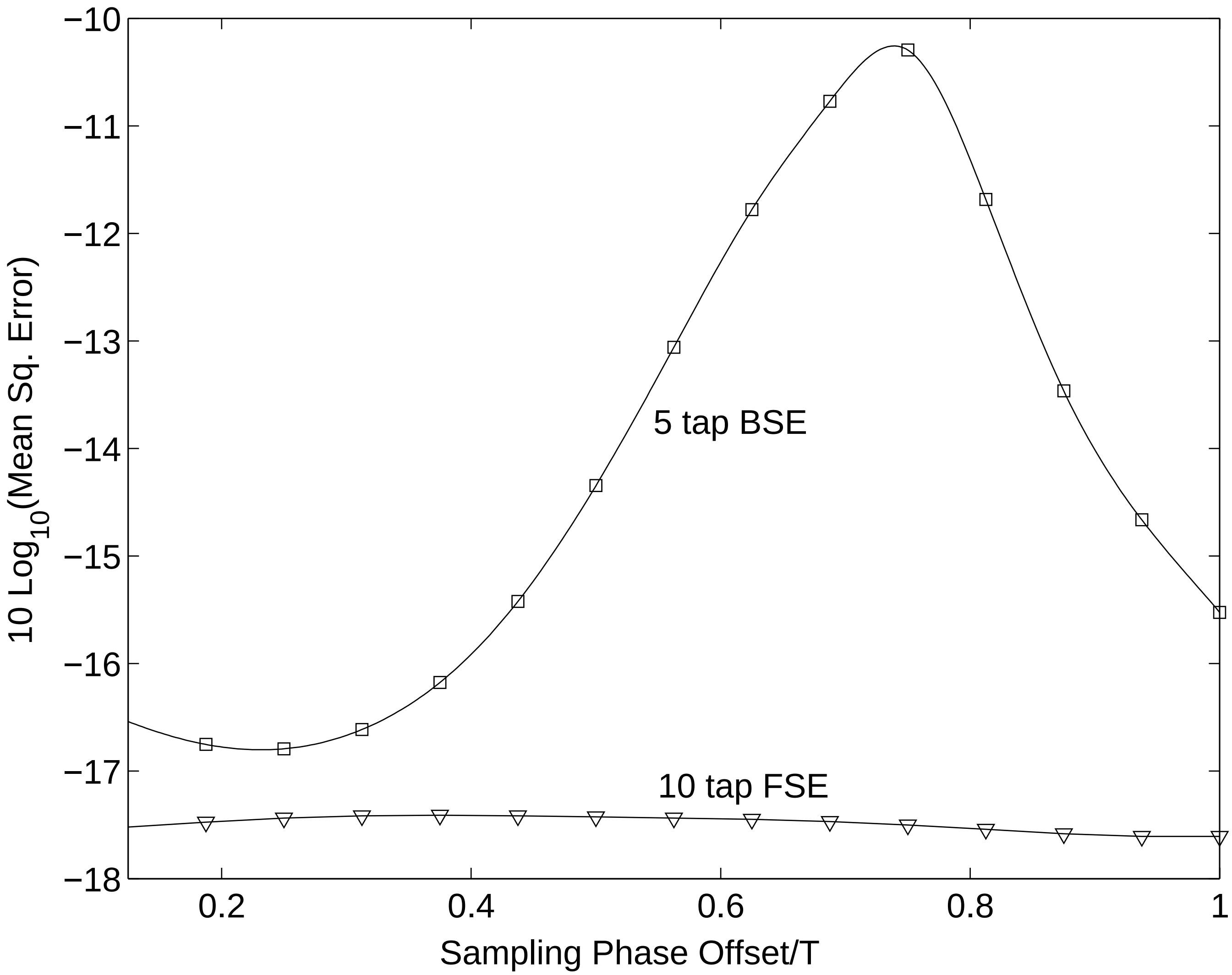

Figure 6.3: Sampling Phase Sensitivity of BSE and FSE

6.3 Transmission Line Filters

This sections presents the performance results of the two topologies of transmission line equalizers. The change in performance with the inclusion of non-idealities is also shown. For both the structures, the filters are arranged to get an analog implementation of a 10 tap FSE with an oversampling ratio of 2.

6.3.1 Ideal Case

In the absence of non idealities, both the topologies are identical to each other and to the conventional 10 tap FSE. see figure 6.4

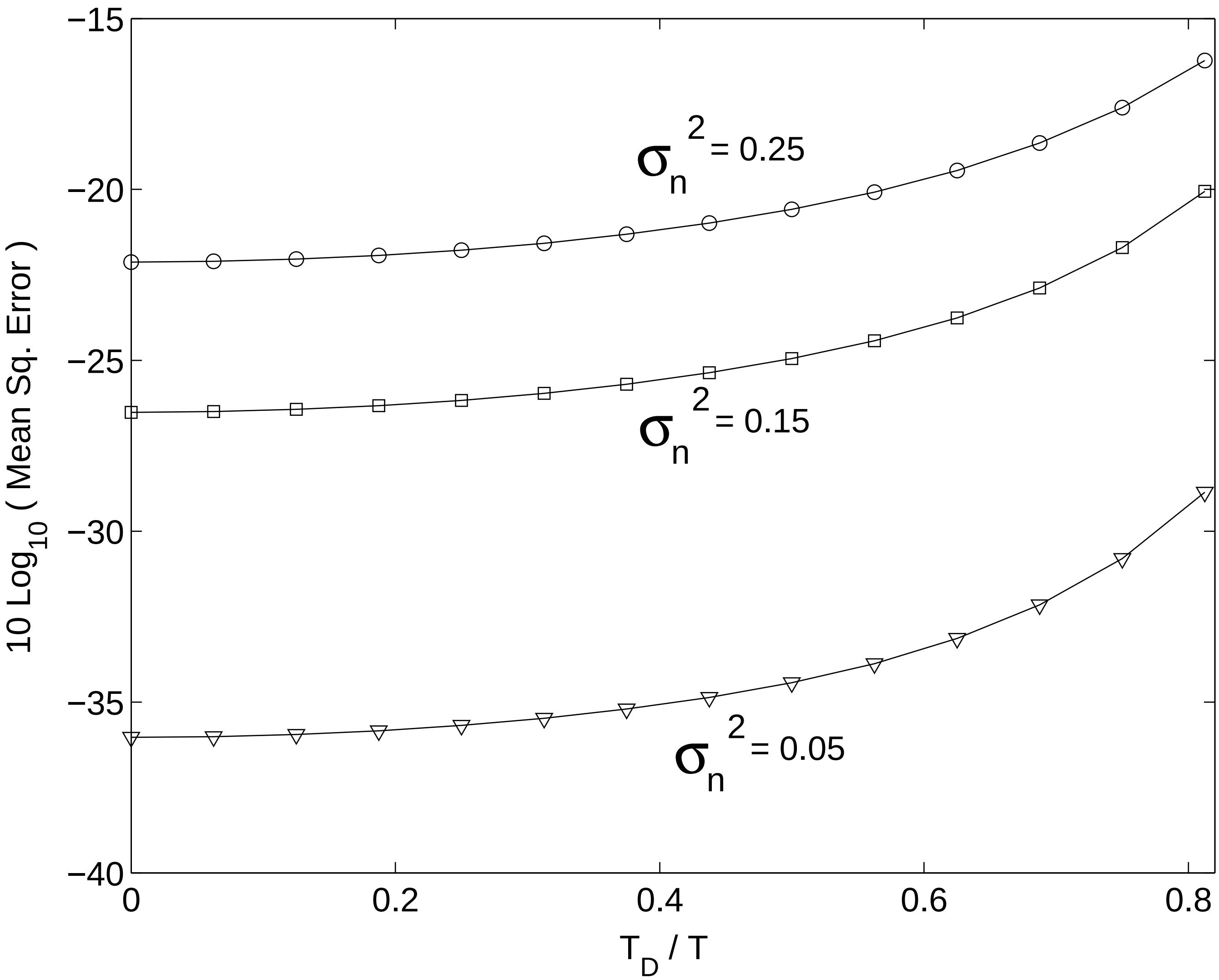

Figure 6.4: M S E in the absence of non-idealities

6.3.2 Termination Mismatch

The effect of mismatch in the terminating resistance and the transmission line characteristic impedance is shown here. see figures 6.5 and 6.6

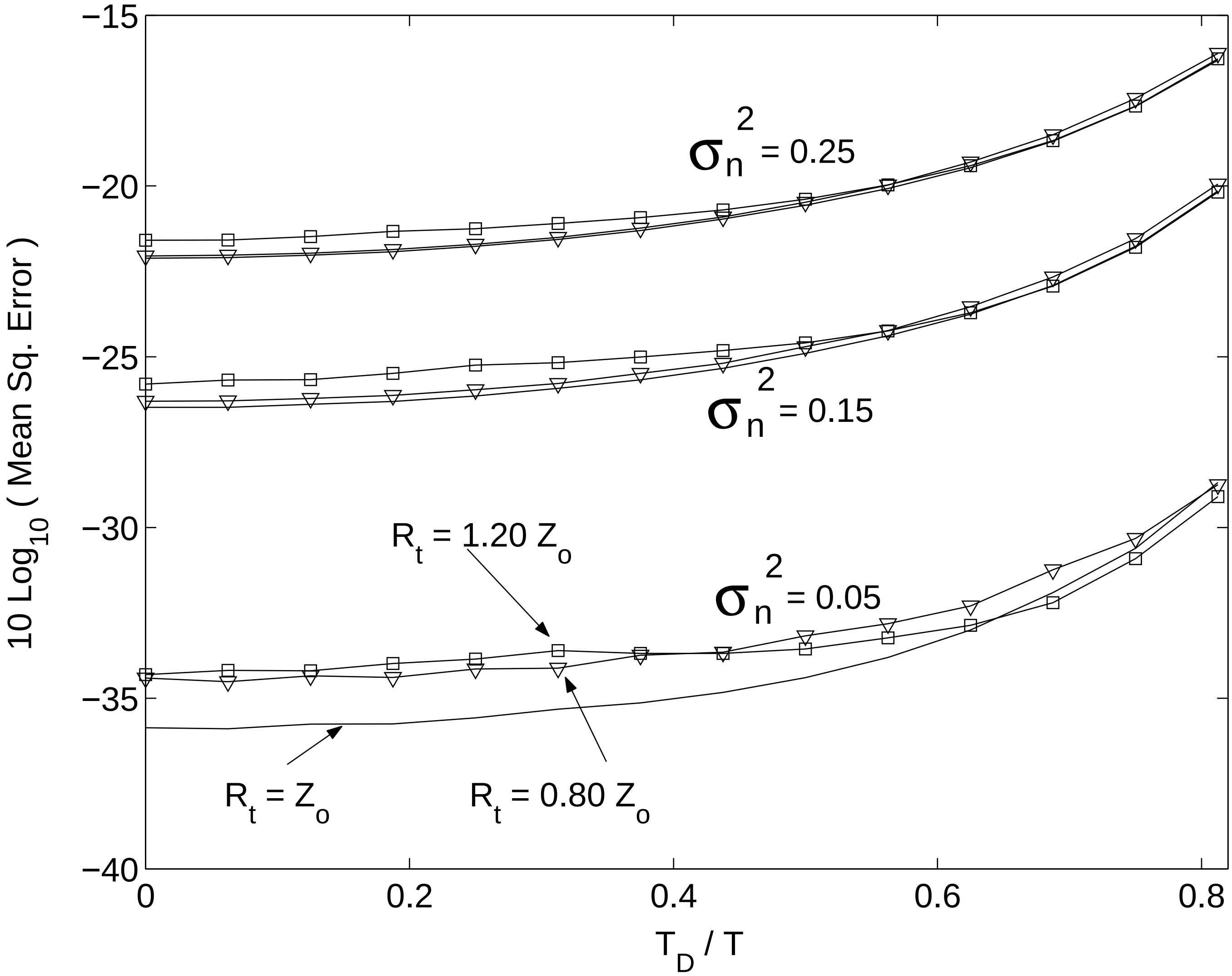

Figure 6.5: Effect of Termination Mismatch in Transversal Filter Topology

6.3.3 Series Loss

The effect of series loss in transmission lines is shown here. see figures 6.7 and 6.8

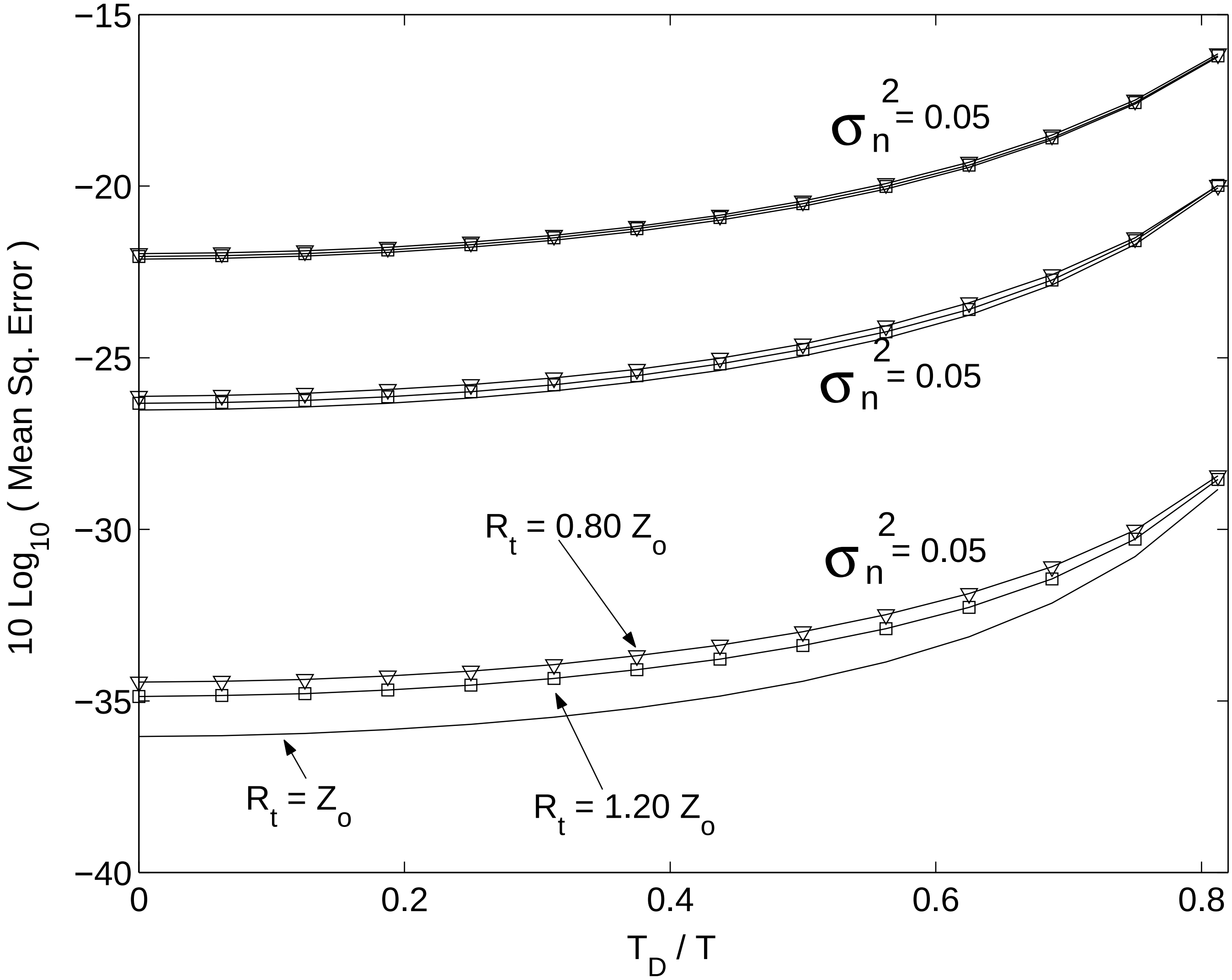

Figure 6.6: Effect of Termination Mismatch in Travelling Wave Filter Topology

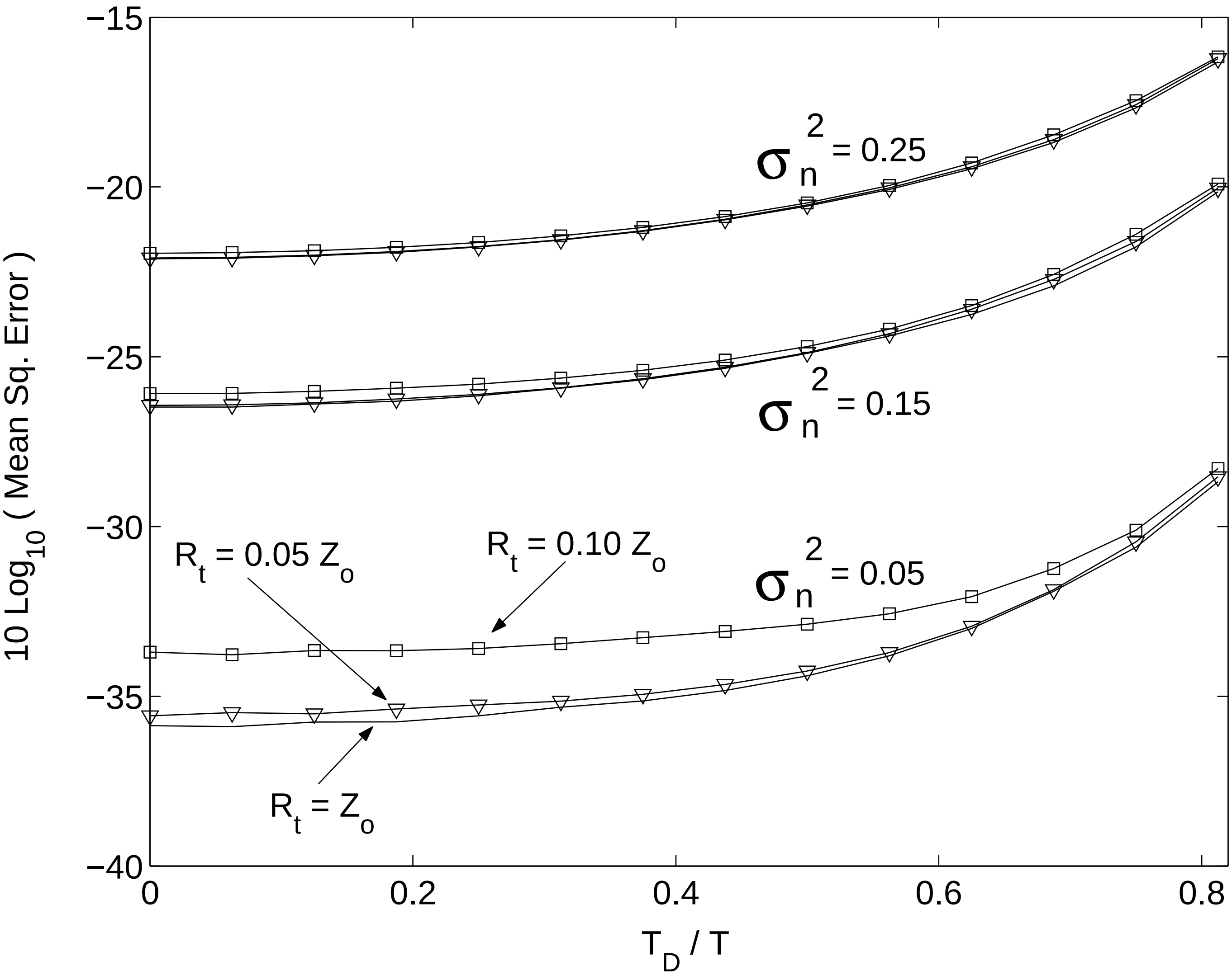

Figure 6.7: Effect of Series Loss in Transversal Filter Topology

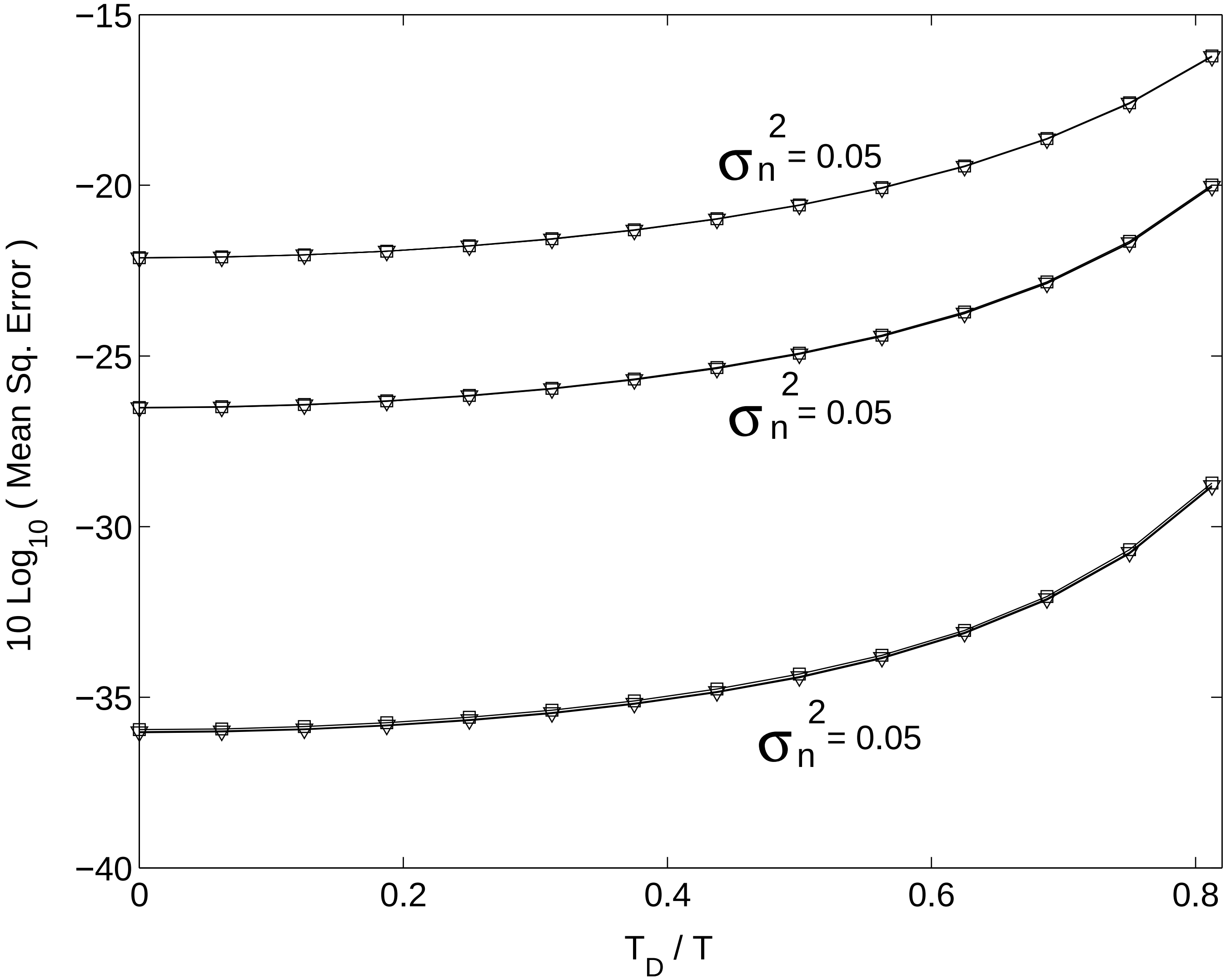

Figure 6.8: Effect of Series Loss in Travelling Wave Filter Topology

6.3.4 Parasitic Capacitances

The effect of Parasitic capasitance at the input and output terminals of the transconductor is shown here. see figures 6.9 and 6.10

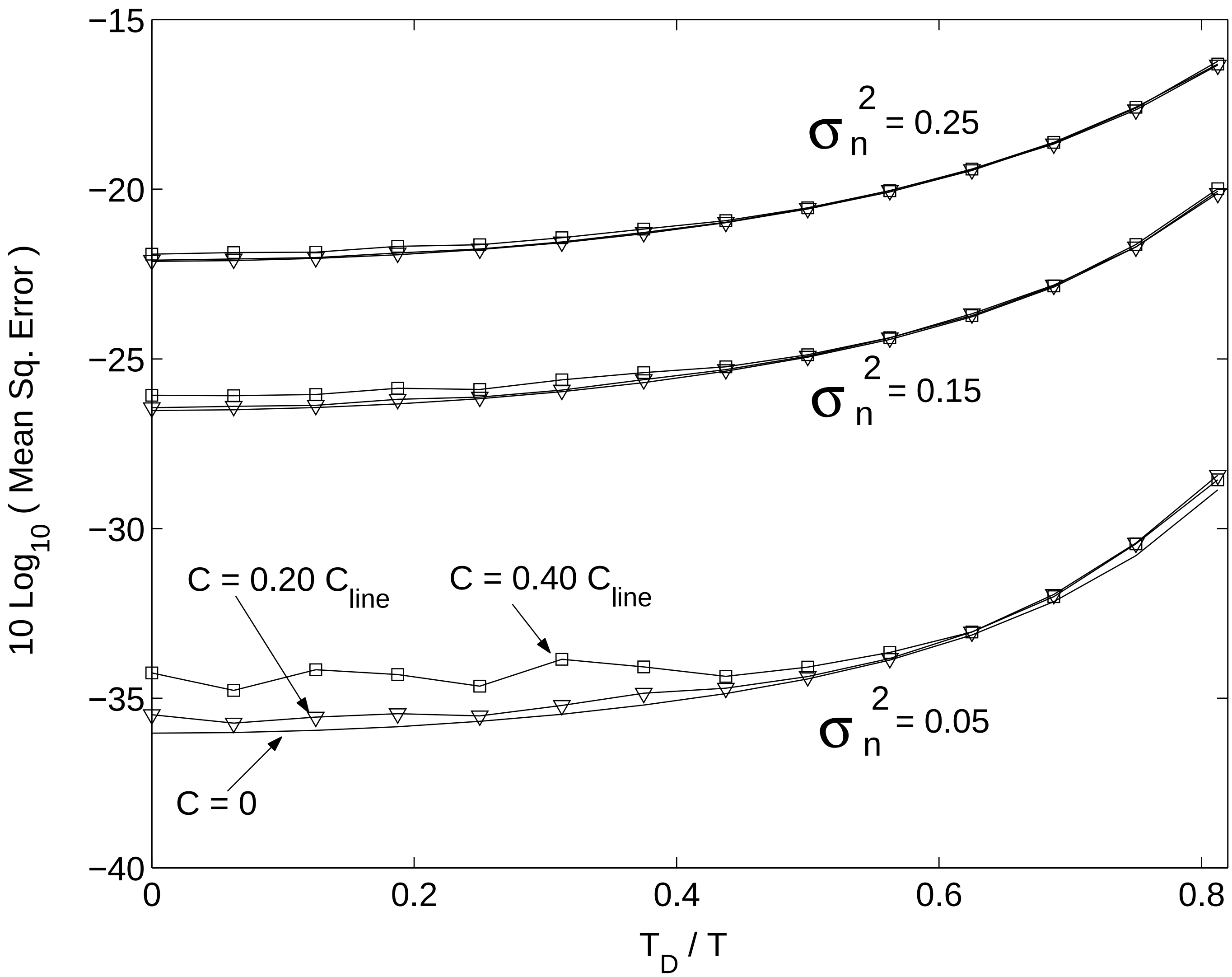

Figure 6.9: Effect of Parasitic Capacitance in Transversal Filter Topology

6.3.5 Reducing the Terminating Resistance

The effect of reducing the terminating resistance in the presence of series lossis shown here. see figures 6.11 and 6.12

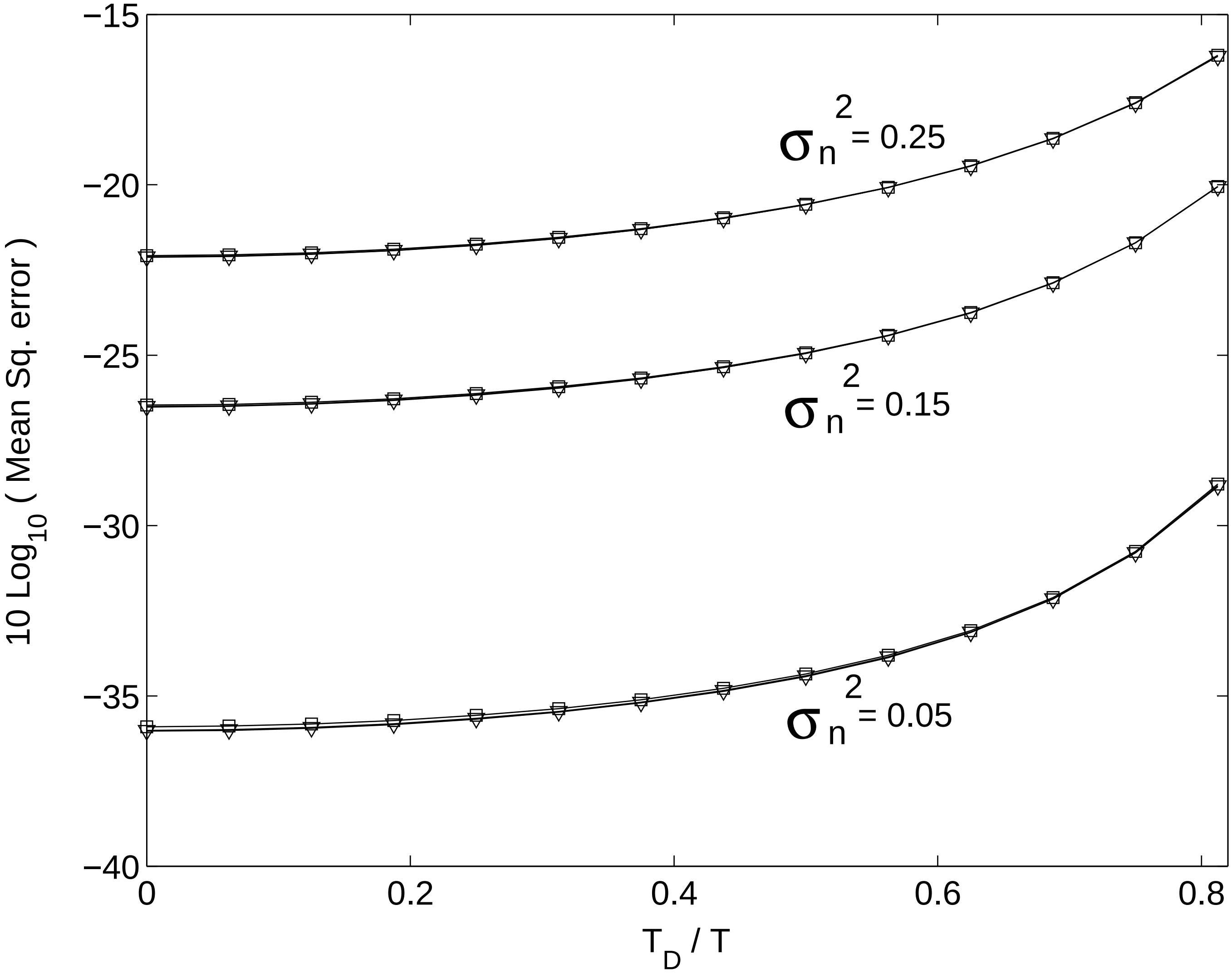

Figure 6.10: Effect of Parasitic Capacitance in Travelling Wave Filter Topology

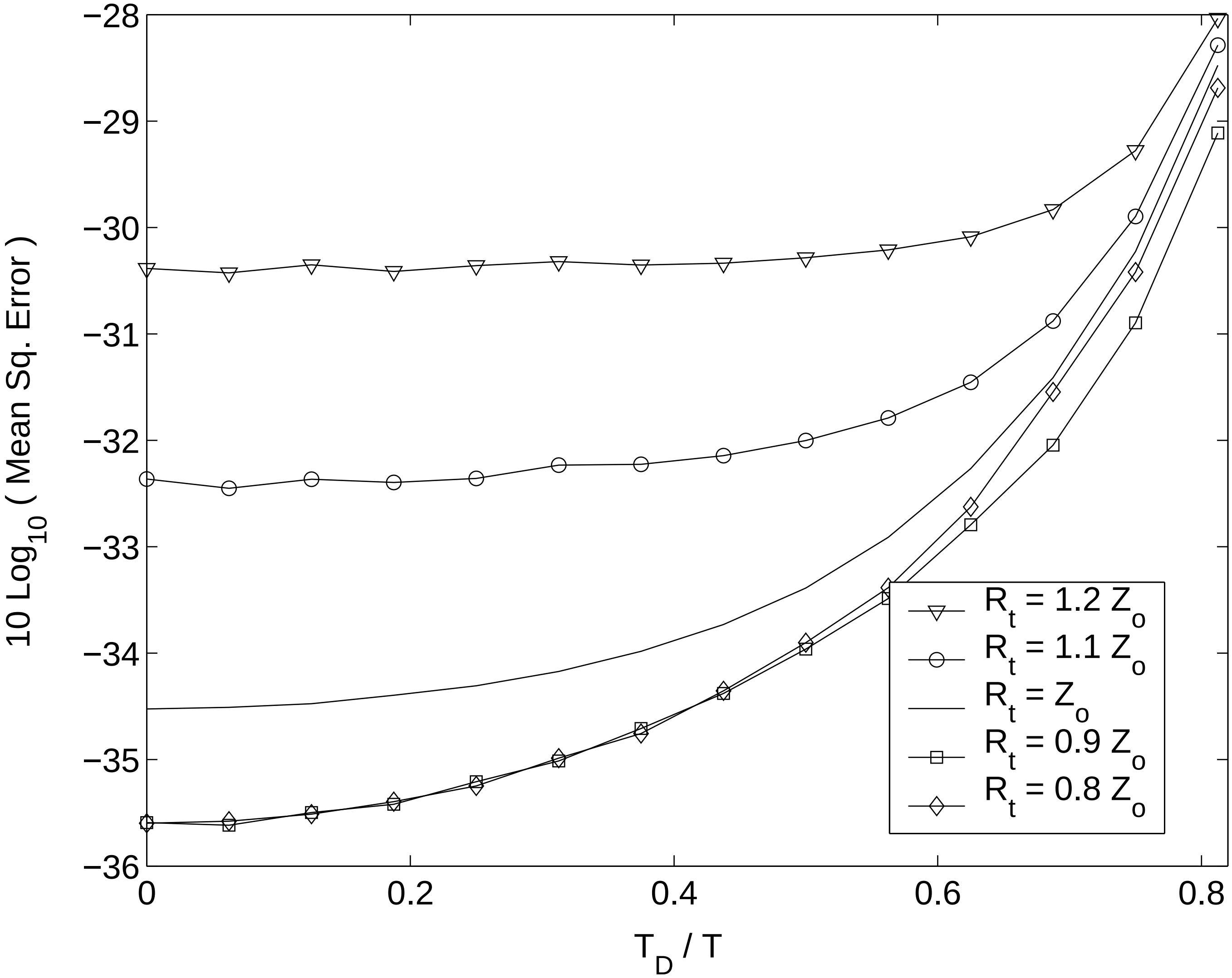

Figure 6.11: Effect of varying R_t in Transversal Filter Topology

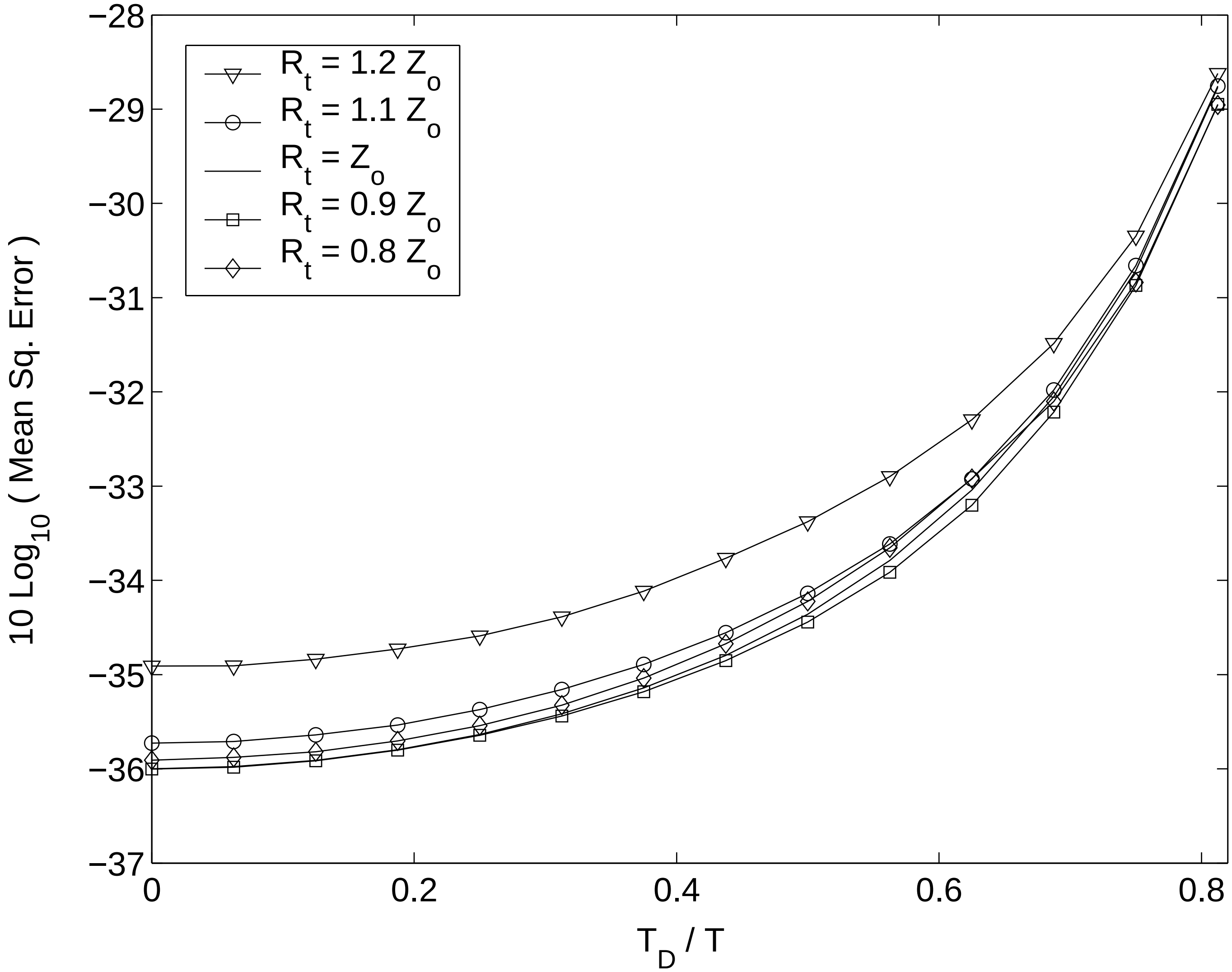

Figure 6.12: Effect of varying R_t in Travelling Wave Filter Topology

6.4 Decision Feedback Equalizer

The performance of the DFE, in comparison to an FSE of similar complexity is shown in figure 6.13. Note that the DFE is able to equalize even those channel conditions where the FSE fails completely, as in the case of $T_d = T$.

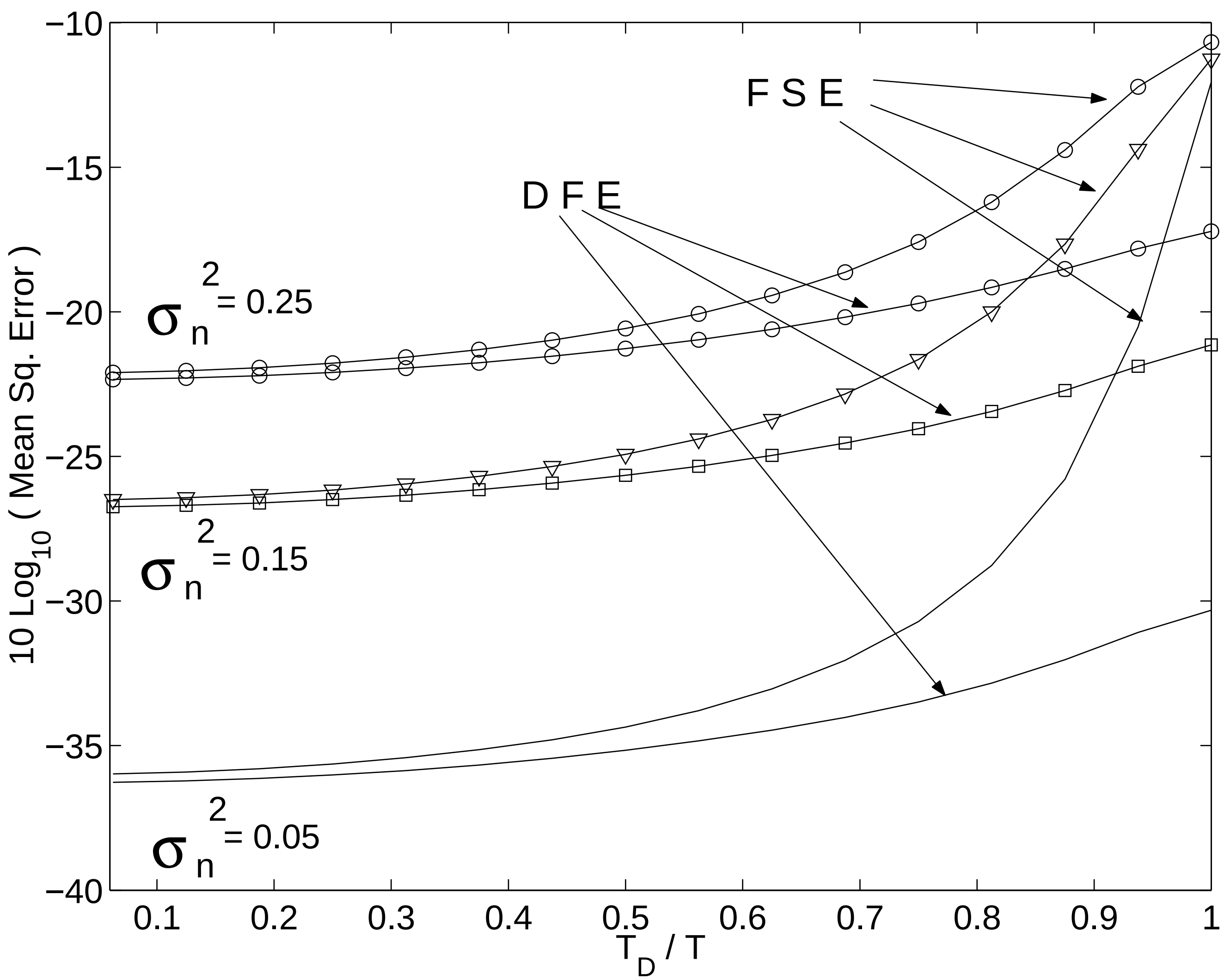

Figure 6.13: MSE of DFE and FSE for different channel conditions

Chapter 7

Conclusion

7.1 Conclusion

From the simulation results we can see that a Fractionally spaced equalizer can be implemented in the analog domain for high bit rate channels therby achieving considerable savings in power and making the fabrication possible in cheaper VLSI technology. The non idealities associated with the analog components can be minimised by choosing the travelling wave filter topology which has a marked advantage over the transversal filter topology.

7.1.1 Future Possibilities

The line of reasoning followed in the theoretical derivation of the analog domain equalizers is very general and can be directly applied to implementing the FIR filters in the analog domain, using different elements to generate equivalent impulse responses for the filter banks considered. This opens the possibility of having analog FIR filters, without using transmission lines.

Bibliography

[1] John M. Cioffi, *EE379A - Digital Communication: Signal Processing Course Reader*, http://www.stanford.edu/class/ee379a/reader.html.

[2] John G Proakis, *Digital Communications*, The McGraw-Hill Companies, Edition 4, Aug 2000.

[3] Enrico Forestieri, *Evaluating the error probability in lightwave systems with chromatic dispersion, arbitrary pulse shape and pre and post detection filtering* , Journal of Lightwave Technology, Vol. 18, No. 11, Nov 2000.

[4] Rajiv Ramaswami, Kumar Sivarajan, *Optical Communications-A Practical Perspective*, Morgan Kaufmann, 2nd edition, Oct 2001.

[5] C Richard Johnson, Jr., Philip Schniter, Thomas J Endres, James D Behm, Donald R Brown and Raul Casas, *Blind equalization using the constant modulus criterion : A review*, Proceedings of the IEEE, Oct 1998.

Dedicated to the memory of those who,
having been called upon to give or take a course
in analog circuits,
know that it is more blessed to give.