16/4/15

Lec 37

X vs R $\quad Z = R + jX$

$$\frac{X}{X_0} + \frac{R}{R_0} = 1$$

$$R = \frac{Z + Z^*}{2} \; ; \quad X = \frac{Z - Z^*}{2j}$$

$$\frac{Z + Z^*}{2 \times R_0} + \frac{Z - Z_0^*}{2jX_0} = 1$$

$$Z\underbrace{\left[\frac{1}{2R_0} + \frac{1}{2jX_0}\right]}_{Y_0} + Z^*\left[\frac{1}{2R_0} - \frac{1}{2jX_0}\right] = 1$$

$$\boxed{ZY_0 + Z^*Y_0^* = 1}$$

equation of a st. line in Z plane

map to Y-plane: $\quad Y = 1/Z$

$$\frac{Y_0}{Y} + \frac{Y_0^*}{Y^*} = 1$$

$$YY^* = Y^*Y_0 + YY_0^*$$

$$YY^* - Y_0Y^* - Y_0^*Y = 0$$

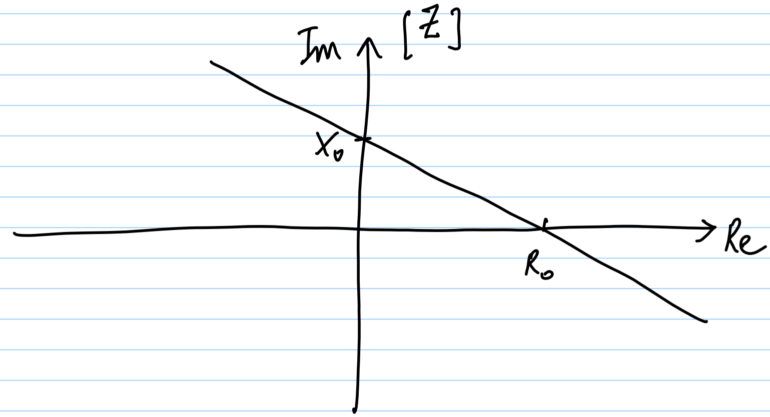
$$+\, Y_0Y_0^* \qquad\qquad + Y_0Y_0^*$$

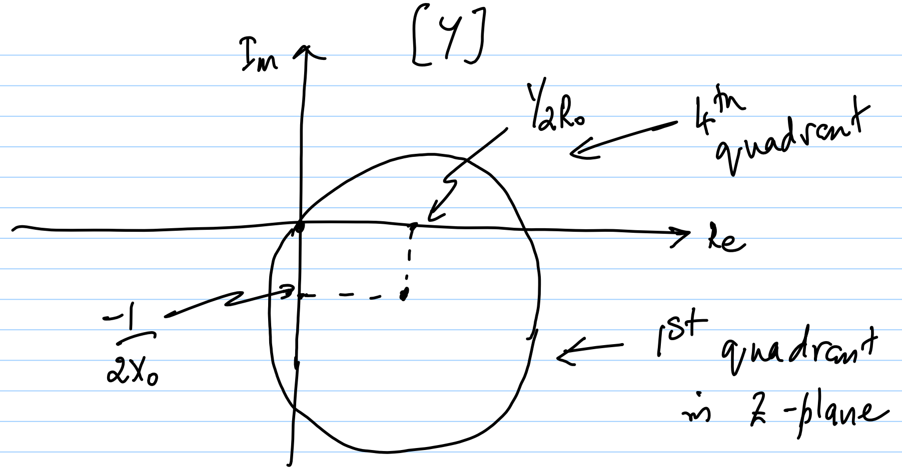
$$YY^* - Y_0Y^* - Y_0^*Y + Y_0Y_0^* = Y_0Y_0^*$$

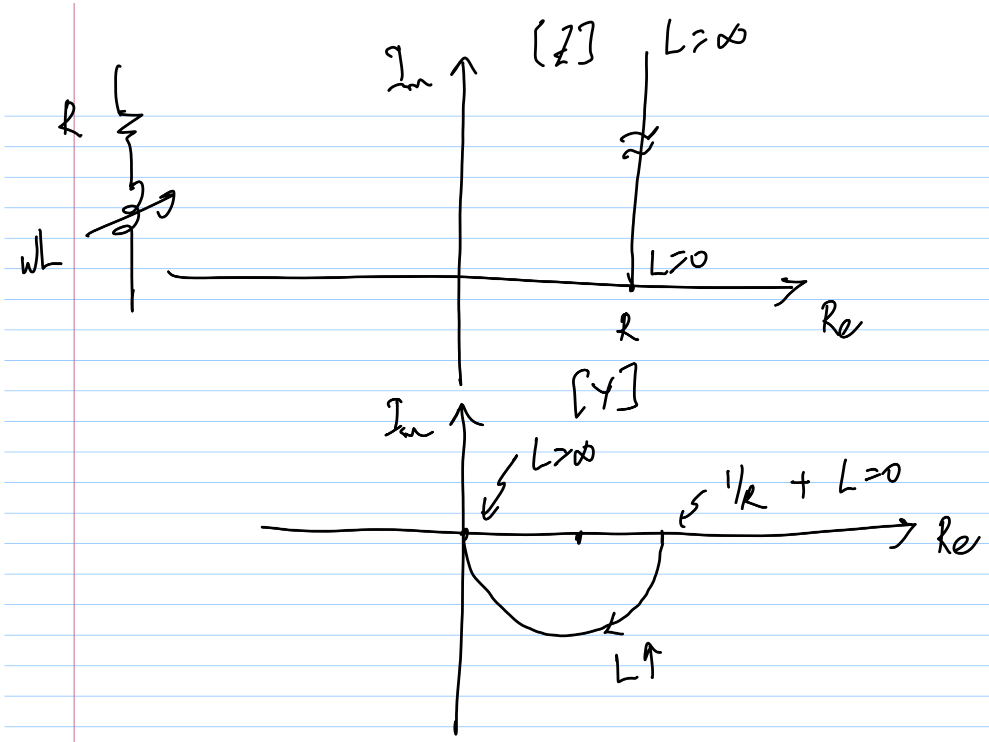
$$(Y - Y_0)(Y - Y_0)^* = |Y_0|^2$$

$$\boxed{|Y - Y_0|^2 = |Y_0|^2}$$

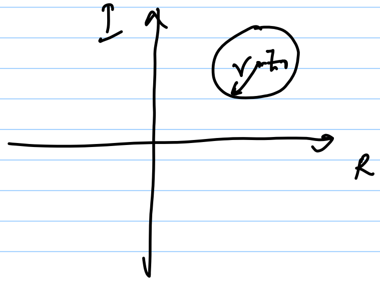
Circle with radius = centre = $|Y_0|$







Circle in Z-plane



$$|Z - Z_0|^2 = |r|^2$$

$$(Z - Z_0)(Z - Z_0)^* = |r|^2$$

$$\left(\frac{1}{Y} - Z_0\right)\left(\frac{1}{Y} - Z_0\right)^* = |r|^2$$

multiply both sides by $\dfrac{YY^*}{Z_0 Z_0^*}$

$$\left(Y - \frac{1}{Z_0}\right)\left(Y - \frac{1}{Z_0}\right)^* = \frac{r^2 \cdot |Y|^2}{|Z_0|^2}$$

$$YY^* - Y \cdot \frac{1}{Z_0^*} - Y^* \frac{1}{Z_0} + \frac{1}{|Z_0|^2} = \frac{r^2 \cdot |Y|^2}{|Z_0|^2}$$

$$\underbrace{|Y|^2\left[1 - \frac{r^2}{|Z_0|^2}\right]}_{k^2} - Y \cdot \frac{1}{Z_0^*} - Y^* \cdot \frac{1}{Z_0} + \frac{1}{|Z_0|^2} = 0$$

$$k^2|Y|^2 - \frac{Y}{Z_0^*} - \frac{Y^*}{Z_0} + \frac{1}{|Z_0|^2} = 0$$

$$|Y|^2 - \frac{Y}{k^2 Z_0^*} - \frac{Y^*}{k^2 Z_0} + \frac{1}{k^2 |Z_0|^2} = 0$$

add & subtract $\frac{1}{k^4 |Z_0|^2}$

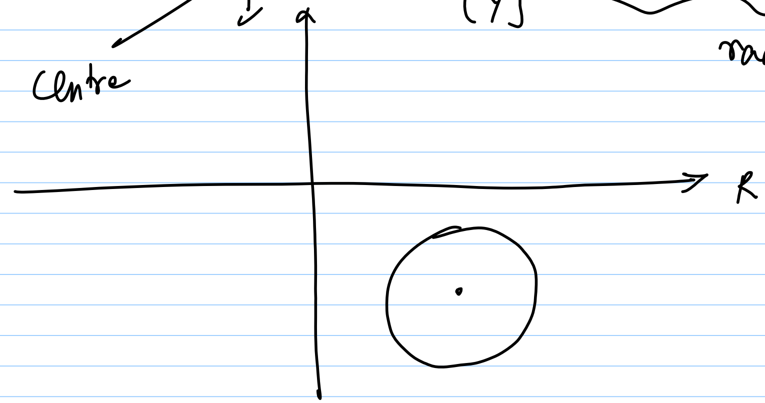
$$YY^* - \frac{Y}{k^2 Z_0^*} - \frac{Y^*}{k^2 Z_0} + \frac{1}{k^4 |Z_0|^2}$$

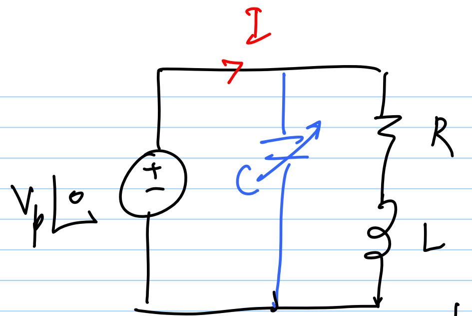
$$= \frac{1}{|Z_0|^2}\left[\frac{1}{k^4} - \frac{1}{k^2}\right]$$

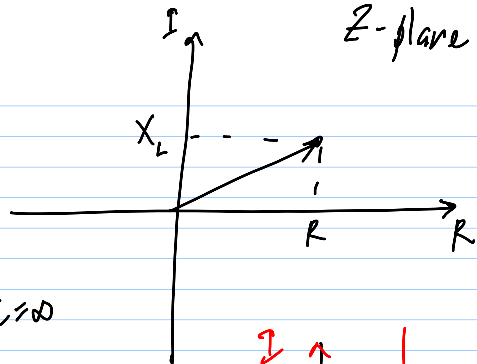
$$\left(Y - \frac{1}{k^2 Z_0}\right)\left(Y - \frac{1}{k^2 Z_0}\right)^* = \frac{1}{|Z_0|^2}\left[\frac{1}{k^4} - \frac{1}{k^2}\right]$$

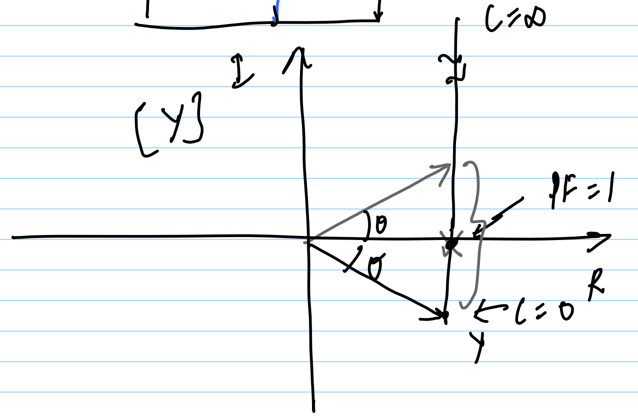
circle $$\left|Y - \frac{1}{k^2 Z_0}\right|^2 = \underbrace{\frac{1}{|Z_0|^2}\left[\frac{1}{k^4} - \frac{1}{k^2}\right]}_{\text{rad}^2}$$

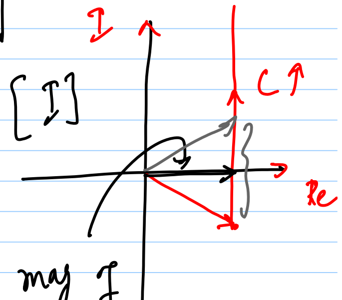
Centre











may J

I is smaller at PF=1