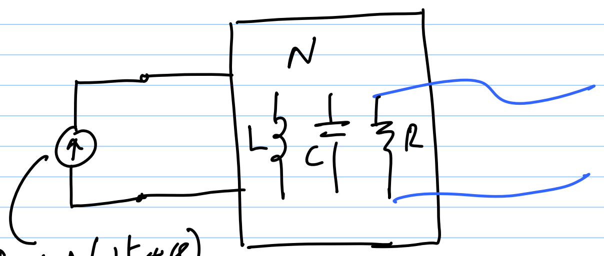
24/3/15

Lec 29



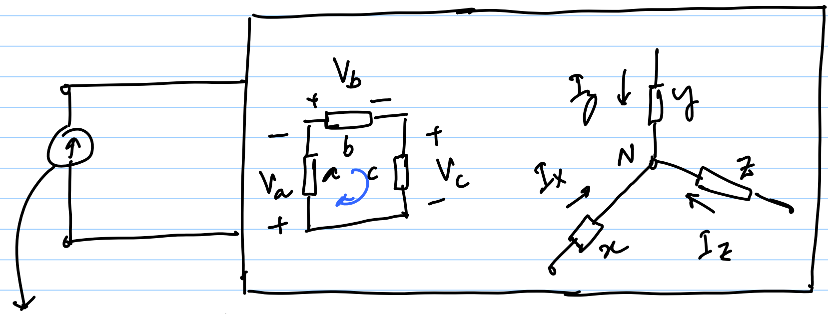
complex no.

$A_1 \exp(j\omega t)$

$I_p \cos(\omega t + \varphi)$

$I_p \exp(j(\omega t + \varphi))$

$I_p \exp(j\varphi) \exp(j\omega t)$



$I_p \exp(j\varphi) \exp(j\omega t)$

phasors

$V_a = \bar{V}_a \exp(j\omega t)$

$V_b = \bar{V}_b \exp(j\omega t)$

$V_c = \bar{V}_c \exp(j\omega t)$

$I_x = \bar{I}_x \exp(j\omega t)$

$I_y = \bar{I}_y \exp(j\omega t)$

$I_z = \bar{I}_z \exp(j\omega t)$

KVL around loop:

$V_a + V_b + V_c = 0$

$(\bar{V}_a + \bar{V}_b + \bar{V}_c) \exp(j\omega t) = 0$

$\bar{V}_a + \bar{V}_b + \bar{V}_c = 0$

phasors

KCL @ N

$I_x + I_y + I_z = 0$

$(\bar{I}_x + \bar{I}_y + \bar{I}_z) \exp(j\omega t) = 0$

$\bar{I}_x + \bar{I}_y + \bar{I}_z = 0$

$\bar{V}_a = V_{a_r} + j V_{a_i}$

$= \hat{V}_a \exp(j\theta)$

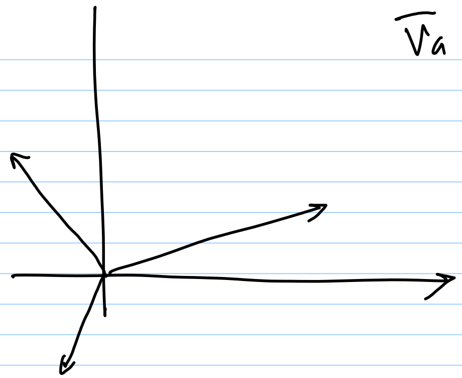
$= \hat{V}_a \angle \theta$

magnitude not complex.

$10\angle 20^\circ \exp(j\omega t)$

All V's & I's $\rightarrow$ same ω

$$\bar{V}_a = \hat{V}_a \exp(j\theta)$$



Node 1 : $\bar{I}_a + \bar{I}_b + \dots = 0$

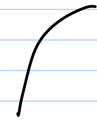
⋮

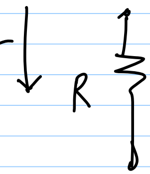
Node k : $\bar{I}_x + \bar{I}_y + \dots = \bar{I}_p$

⋮

Elemental relationships

1) R



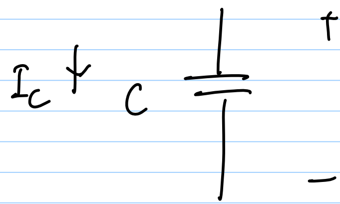


$\bar{I}_R = \hat{I}_R \angle\varphi$

$$\frac{\bar{V}_R}{\bar{I}_R} = R \Rightarrow \hat{V}_R = R \cdot \hat{I}_R$$

$$\theta = \varphi$$

2) C :

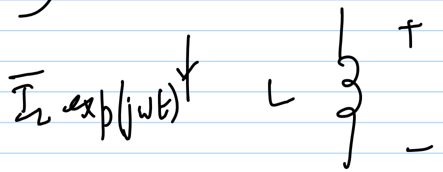


$$I_c = C \frac{dV_c}{dt} = j\omega C \cdot \bar{V}_c \exp(j\omega t)$$

$$\bar{I}_c = j\omega C \bar{V}_c$$

$$\frac{\bar{V}_c}{\bar{I}_c} = \frac{1}{j\omega C} = \frac{-j}{\omega C}$$

3) L



$$V_L = L \frac{dI_L}{dt}$$

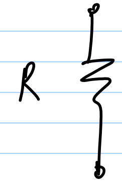
$$V_L = j\omega L \bar{I}_L \exp(j\omega t)$$

$$\frac{\bar{V}_L}{\bar{I}_L} = j\omega L$$

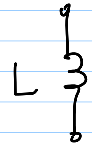
* Algebraic eq. instead of differential eq.
* Ratio of V & I is a complex no.
* ratio of V & I can depend on ω

Impedance

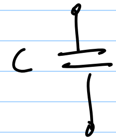
$$\bar{Z}_k = \frac{\bar{V}_k}{\bar{I}_k} = \underbrace{R_k}_{\text{resistance}} + j\underbrace{X_k}_{\text{reactance}}$$



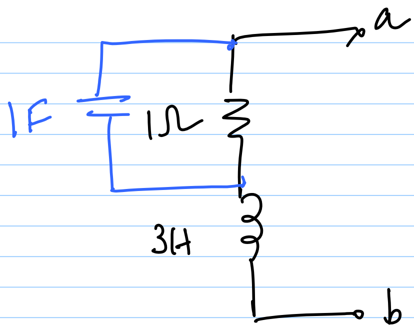
$$\bar{Z} = R + j0 \rightarrow X = 0$$



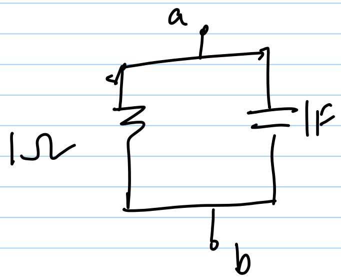
$$\bar{Z} = 0 + j\omega L \rightarrow X = \omega L$$



$$\bar{Z} = 0 - \frac{j}{\omega C} \rightarrow X = \frac{-1}{\omega C}$$



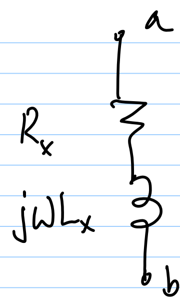
$$Z_{ab} = 1 + j\omega \cdot 3$$



$$Y_{ab} = 1 + j\omega$$

$$\bar{Y} = \frac{\bar{I}_k}{\bar{V}_k} = \underbrace{G}_{\text{conductance}} + j\overbrace{B}^{\text{Susceptance}}$$

Admittance



$$\bar{Y}_{ab} = \frac{1}{Z_{ab}} = \frac{1}{R_x + j\omega L_x}$$

$$= \underbrace{\frac{R_x}{R_x^2 + (\omega L_x)^2}}_{G} - \underbrace{\frac{j\omega L_x}{R_x^2 + (\omega L_x)^2}}_{jB}$$