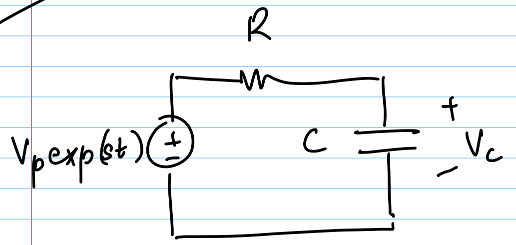
16/3/15

Lec 26



$$RC\frac{dV_c}{dt} + V_c = V_p \exp(st) \quad \text{—— (1)}$$

$$V_{c_1} = V_c - V_p \exp(st)$$

$$RC\frac{dV_{c_1}}{dt} + V_{c_1} = -sCR\,V_p \exp(st) \quad \text{—— (2)}$$

(1) × sCR + (2) gives:

$$RC\frac{d}{dt}\left[V_{c_1} + sCRV_c\right] + \left[V_{c_1} + sCRV_c\right] = 0$$

$$V_{c_2} = V_{c_1} + sCRV_c$$

$$V_{c_2} = V_{c_{20}} \exp(-t/RC)$$

$$= sCRV_c + V_c - V_p\exp(st)$$

$$V_c(t) = \frac{V_p \exp(st)}{1+sCR} + (V_0)\exp\left(\frac{-t}{RC}\right)$$

forced response

natural response (initial condition)

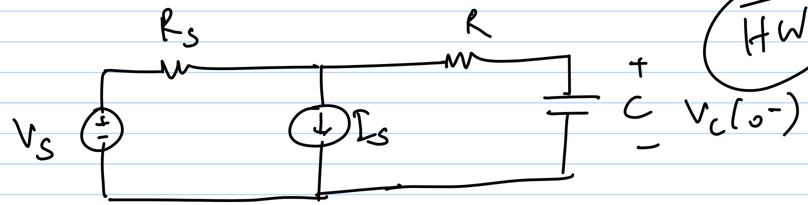
input $c\,(\;)\exp(st)$

output (SSR) = scaled version.

<u>Linear system</u>

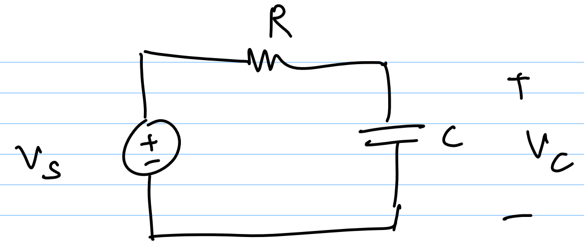
SSR — obeys superposition

TR — does not



HW

apply superposition.



$V_s = V_p\cos(\omega t + \varphi)$

(I) $$RC\frac{dV_c}{dt} + V_c = V_p\cos(\omega t + \varphi)$$

$$V_c = \alpha\cos(\omega t + \varphi) + \beta\sin(\omega t + \varphi)$$

equate sine & cosine coefficients

I

$\sqrt{-1} = i$ X

in EE $\sqrt{-1} = j$

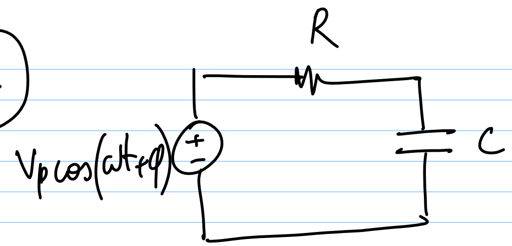
$$e^{j\theta} = \cos\theta + j\sin\theta$$

$$V_p\cos(\omega t+\varphi) = \frac{V_p}{2}\left[\exp(j(\omega t+\varphi)) + \exp(-j(\omega t+\varphi))\right]$$

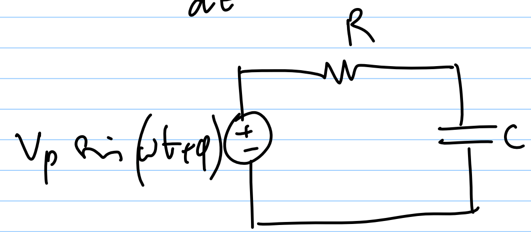
set $s = j\omega$

$$V_p' = \frac{V_p}{2}\exp(j\varphi)$$

II



$$RC\frac{dV_c}{dt} + V_c = V_p\cos(\omega t+\varphi) \quad \text{--- (3)}$$



$$RC\frac{dV_c}{dt} + V_c = V_p\sin(\omega t+\varphi) \quad \text{--- (4)}$$

$$1\times(3) + j\times(4)$$

input $= V_p\exp(j(\omega t+\varphi))$

$$V_p\exp(st) \longrightarrow \frac{V_p}{1+sCR}\cdot\exp(st)$$

$s = j\omega$

$$\text{output} = \frac{V_p\exp(j\varphi)}{1+j\omega CR}\exp(j\omega t)$$

$$V_c = \frac{V_p}{1+j\omega CR}\exp(j(\omega t+\varphi))$$

Output when input is $V_p\cos(\omega t+\varphi)$:

$$V_{c_1} = \mathrm{Re}\{V_c\} \quad \leftarrow \text{corresponds to cosine input}$$

$$x+jy = Ae^{j\theta}$$

$$V_{c_1} = \frac{V_p}{\sqrt{1+(\omega RC)^2}}\cdot\cos\left[\omega t+\varphi-\tan^{-1}(\omega RC)\right]$$

"Sinusoidal steady state response"