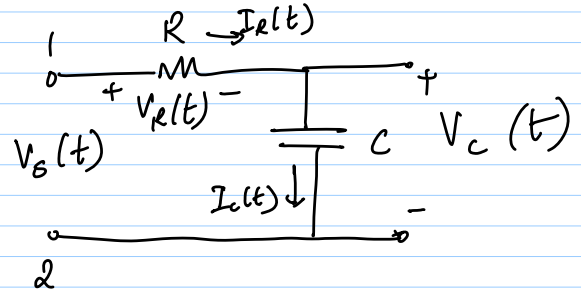
10/3/15

Lec 23

RC ckts



$V_s(t) = V_s$ constant DC voltage

DC current

KVL $V_s(t) = V_R(t) + V_c(t)$

KCL $I_R(t) = I_c(t)$

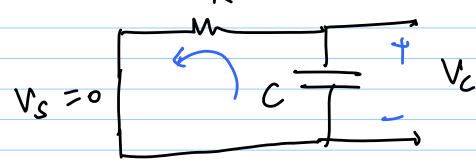
$I_R(t) = \frac{V_R(t)}{R}$

$I_c(t) = C\frac{dV_c(t)}{dt}$

$$RC\frac{dV_c}{dt} + V_c = V_s$$ ← 1st order LDE

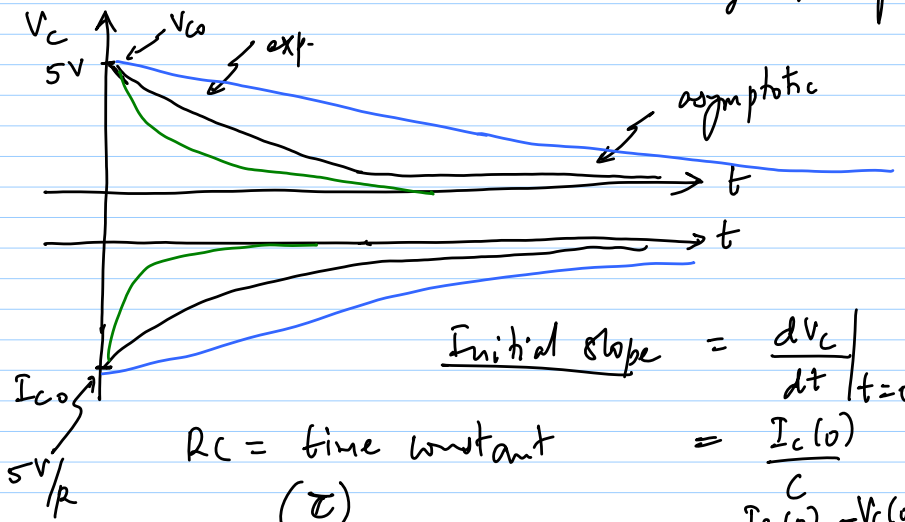
$V_s = 0$

$$RC\frac{dV_c}{dt} + V_c = 0$$ ← Homogeneous eq'n



$$\frac{dV_c}{dt} = -\frac{1}{RC}V_c$$ ← Some kind of exp.

$V_c = V_{c0}\exp(-t/R_c)$; V_{c0} = initial voltage on cap



RC = time constant (τ)

Initial slope $= \left.\frac{dV_c}{dt}\right|_{t=0}$

$= \frac{I_c(0)}{C}$

$= \frac{I_R(0)}{C} = \frac{-V_c(0)}{RC}$

$$RC\frac{dV_c}{dt} + V_c = V_s$$ ← Constant

$$\frac{dV_c}{dt} + \frac{V_c}{RC} = \frac{V_s}{RC}$$

$$\frac{dV_c}{dt} + \frac{V_c - V_s}{RC} = 0$$

$V_{c_1} = V_c - V_s$; $\frac{dV_{c_1}}{dt} = \frac{dV_c}{dt}$

$$v_{c_1} = v_{c_{10}} \exp(-t/RC)$$

$$RC\frac{dv_{c_1}}{dt} + v_{c_1} = 0 \leftarrow$$

$$v_{c_{10}} = v_{c_0} - v_s$$

$$v_{c_1}(t) = v_c(t) - v_s$$

$$v_c(t) = v_s + (v_{c_0} - v_s)\exp(-t/RC)$$

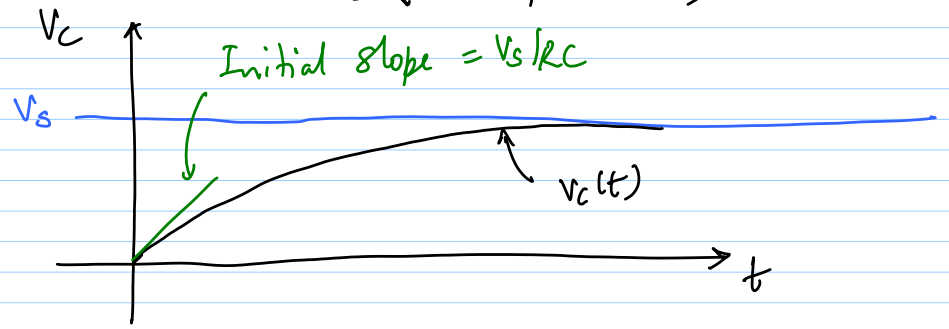
$v_s = 0$

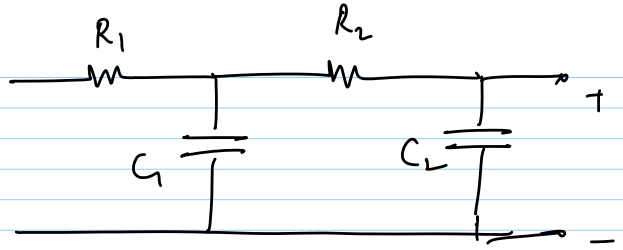
$$v_c(t) = v_{c_0}\exp(-t/RC)$$

$$v_{c_0} = v_c(0) = 0$$

$$v_c(t) = v_s - v_s\exp(-t/RC)$$

$$= v_s(1 - \exp(-t/RC))$$





$$v_c(t) = \underbrace{v_s}_{\text{steady state response}} + \underbrace{(v_c(0) - v_s)\exp(-t/RC)}_{\text{transient response}}$$

$$= \underbrace{v_s(1 - \exp(-t/RC))}_{\text{zero-state response}} + \underbrace{v_c(0)\exp(-t/RC)}_{\text{zero-input response}}$$

Forced response + natural response

particular solution + solution to homogeneous equation