Computational Electromagnetics : Finite Difference Time Domain Methods – Sources

Uday Khankhoje

Electrical Engineering, IIT Madras

Topics in this module

1. Current Sources
2. Indirect Sources: Scattering problems
3. Summary of FDTD

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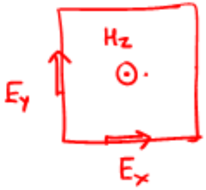
Volume current excitation

↳ $\vec{J}(r,t)$

(2D) TE case → (E_x, E_y, H_z)

$$\nabla \times \bar{H}^{n-1/2} = \varepsilon \dot{\bar{E}}^{n-1/2} + \bar{J}^{n-1/2}$$



- We need to know $\vec{J} = (J_x, J_y, J_z)$ as fn (r,t)
- at time instances : $n - 1/2$
- at space instances : E field locations.

Easy to implement.

Set $\Delta x, \Delta t, \alpha$

Relation between current source and Δt, Δx?

$\vec{J}(t) \xleftrightarrow{\mathcal{F}} \tilde{\vec{J}}(f)$, say band limited.

$\tilde{\vec{J}}(f) = 0, \; f > f_0$

Nyquist thm: Correctly represent $\vec{J}(t)$

$$\Delta t \leq \frac{1}{2 f_0}$$

⇒ High BW current source

⇒ space discretization fixed.

At the same time,

Courant factor: $\alpha = \dfrac{c \Delta t}{\Delta x}$

f_0 fixed ⇒ Δt fixed ⇒ Δx fixed.

meep

Other implementation issues

1) Gaussian current source.

$$g(t) = \exp\left(-\left(\frac{t-t_0}{t_w}\right)^2\right) \xleftrightarrow{\mathcal{F}} t_w\sqrt{\pi}\,\exp[-(\pi t_w f)^2]\,\exp[-j2\pi f t_0]$$

what is $f_{bw} = \frac{1}{\pi t_w}$. To be safe $f_0 = 2f_{bw} \Rightarrow f_0 = f_{max} = \frac{2}{\pi t_w}$

$\Rightarrow$ fixes Δx.

2) At start, $t=0$, $g(0) = e\left(-\left(\frac{t_0}{t_w}\right)^2\right)$. Minimize high values of $g(0)$

Make t_0 large. eg. $t_0 \approx 4t_w \Rightarrow$ longer simulations.

3) How long to run the sim? long enough e.g. $4t_w \times 2$.

Common mistake eg. $T \approx 2t_w$.

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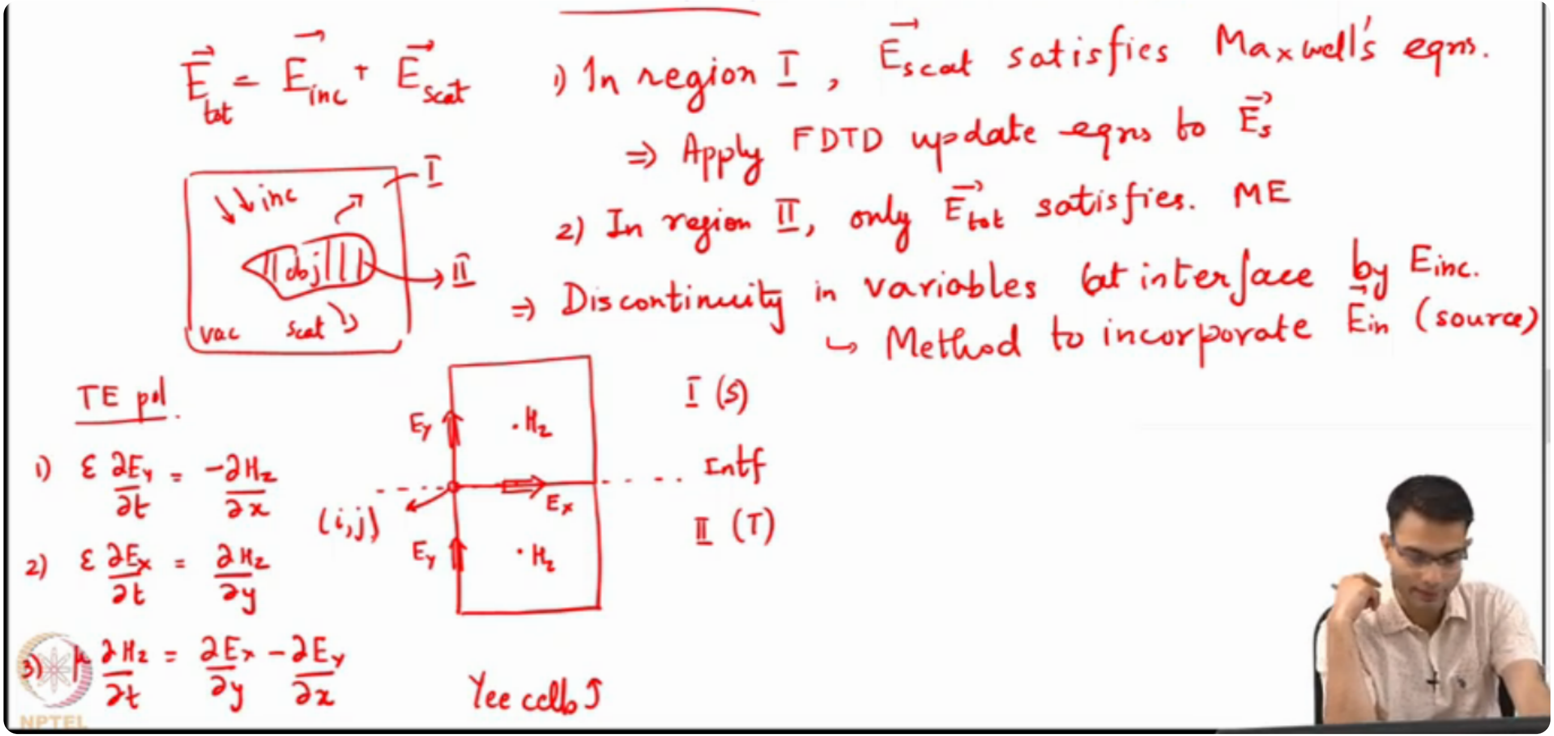
No “$J(r,t)$” term in scattering problems



In s cell.

Fix via total/scattered formulation

Eqn ① $\Rightarrow \varepsilon\left(\frac{E_{y,s}^{n} - E_{y,s}^{n-1}}{\Delta t}\right) = \frac{H_{z,s}^{n-1/2}(i-1/2, j+1/2) - H_{z,s}^{n-1/2}(i+1/2, j+1/2)}{\Delta x}$ unchanged.

Eqn ② $\Rightarrow \varepsilon\left(\frac{E_{x,t}^{n} - E_{x,t}^{n-1}}{\Delta t}\right) = \frac{H_{z,t}(i+1/2, j+1/2) - H_{z,t}(i+1/2, j-1/2)}{\Delta y}$

But we are storing $H_{z,s}$ & not $H_{z,t}$ in s.

" $= \frac{H_{z,s}(i+1/2, j+1/2) - H_{z,t}(i+1/2, j-1/2)}{\Delta y} + \frac{H_{z,i}(i+1/2, j+1/2)}{\Delta y}$

interpret as "current" term

① ∴ incident field gets introduced at the interface

② Choose for $E_x \rightarrow$ scat or total on interface.

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Summary of FDTD

Flow of FDTD.

- ↳ Maxwell's eqns: finite differences (Taylor's thm)
- ↳ Stencil (Yee cell) → update eqns
- ↳ Analysis – Convergence, stability (courant)
  - – Accuracy, dispersion (numerical)
- ↳ Materials – simple (Ohm's law), dispersive media (Debye)
  - $D = \varepsilon E$ (freq domain)
- ↳ Absorbing boundary conds
- ↳ PMLs (streched coordinates)
- ↳ Sources

MEEP..

Topics that were covered in this module

1. Current Sources
2. Indirect Sources: Scattering problems
3. Summary of FDTD

References:

* Ch 12 of Computational Methods for Electromagnetics - Peterson, Ray, Mitra
* Computational Electrodynamics: The Finite-Difference Time-Domain Method – Allen Taflove (the 'Bible' for FDTD)