Computational Electromagnetics : Summary of Integral Equation Methods

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Topics in this module

1. Surface v/s Volume Integral Approach
2. Finding the Radar Cross-Section (RCS)
3. Computational Considerations

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Quick aside: Surface Integral Equations and PECs

How do we deal with scatters that are made of perfect electric conductors?

Recall boundary conditions for PEC

$\hookrightarrow \; E_z = 0 = \phi$

TM pol: $\phi \rightarrow E_z$, $\nabla\phi \cdot \hat{n} \rightarrow H_{tan}$

If we have a PMC $\rightarrow H_{tan} = 0$, $E_z \neq 0$.

The original system of equations:

$$\left\{\begin{aligned} &\oint [g_1(r,r')\nabla\phi(r)\cdot\hat{n} - \phi(r)\nabla g_1(r,r')\cdot\hat{n}]\,dl = \phi_i(r'), \quad r' \in V_2 \\ &\oint [g_2(r,r')\nabla\phi(r)\cdot\hat{n} - \phi(r)\nabla g_2(r,r')\cdot\hat{n}]\,dl = 0, \quad r' \in V_1 \end{aligned}\right\} \; 2N \times 2N$$

$\phi(r) \rightarrow 0$

$$\left\{\begin{aligned} &\oint (g_1(r,r')\nabla\phi\cdot\hat{n})\,dl = \phi_i(r') \\ &\oint g_2(r,r')\nabla\phi(r)\cdot\hat{n}\,dl = 0 \end{aligned}\right\} \; N \times N \text{ system.}$$

Surface v/s Volume Integral Equations

Surface approach:

For each region:

$$\left.\begin{aligned}\nabla^2\phi_n + k_n^2\phi &= Q_n\\ \nabla^2 g_n + k_n^2 g_n &= -\delta(r,r')\end{aligned}\right\}$$

Each eqn solved separately for each region.

variables: $\bar{E}_{tan}$, $\bar{H}_{tan}$ on S.

Huygen's principle.

$k_1, k_2 \to$ constants

Homogeneous

Volume approach:

$$\left\{\begin{aligned}\nabla^2\phi + k_n^2\phi &= Q_n\\ \nabla^2\phi_i + k_0^2\phi &= Q_n\end{aligned}\right.$$

with & without object.

Eqn in terms of $(\phi - \phi_i)$

$$\nabla^2(\phi-\phi_i) + k_0^2(\phi-\phi_i) = (\quad)$$

$$\nabla^2 g + k_0^2 g = -\delta$$

$\boxed{k_n^2 = k_0^2\varepsilon_r(r)}$ Heterogeneous.

Surface v/s Volume Integral Equations

Surface approach: (Huygen's)

$$\phi(r') = \phi_i(r') + \left[\oint_S [\phi(r)\nabla g_1(r,r') - g_1(r,r')\nabla\phi(r)] \cdot \hat{n}\, dl\right] \rightsquigarrow \text{surface equivalence.}$$

Volume approach:

$$\phi(r') = \phi_i(r') + k_0^2 \int_{V_2} g_1(r,r')[\epsilon_r(r) - 1]\phi(r)\, dr \rightsquigarrow \text{Volume equivalence principle.}$$

Surf faster volume.

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Rx angle

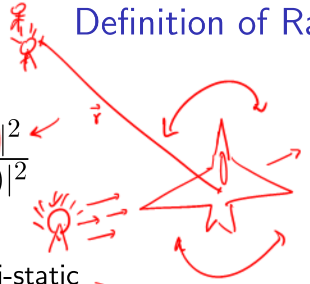
tx angle

Definition of Radar Cross-Section (RCS): σ

far field

$$\sigma_{TM}(\theta, \theta_i) = \lim_{r\to 0} 2\pi r \frac{|E_z^2(r,\theta)|^2}{|E_z^i(0,0)|^2}$$

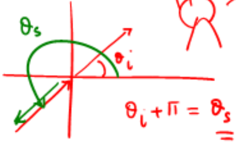
$$\sigma_{TM}(\theta, \phi, \theta_i, \phi_i) = \lim_{r\to 0} 4\pi r^2 \frac{|E_z^2(r,\theta,\phi)|^2}{|E_z^i(0,0,0)|^2}$$



Mono-static and Bi-static

Tx and Rx are different.

Tx and Rx same pos.



$$\sigma = \lim_{r'\to\infty} 2\pi|r'| \frac{|E_z^s(r',\theta)|^2}{|E_z^i(0,0)|^2}$$

$e^{j\omega t}$

$r \to \infty$

Approximations in the RCS

Volume

An integral involving Green's function : $\phi(r') = \phi_i(r') + \boxed{k_0^2 \int_{V_2} g_1(r,r')\chi(r)\phi(r)\,dr}$

In 2D: $g(r,r') = \frac{-j}{4} H_0^{(2)}(k|r-r'|)$ In 3D: $g(r,r') = \frac{1}{4\pi|r-r'|}\exp(-jk|r-r'|)$

phase

for $x \gg 1$, $H_0^{(2)}(k\rho) \approx \sqrt{\frac{2j}{\pi k\rho}}\exp(-jk\rho)$ ← 2D

$|r-r'|$ amp R^2

$$\rho = |r-r'| = \sqrt{(x-x')^2 + (y-y')^2} = \sqrt{x'^2 + y'^2 - 2xx' - 2yy' + y^2 + x^2}$$

$$= R\sqrt{1 - \frac{2xx' - 2yy'}{R^2} + \frac{x^2+y^2}{R^2}}$$

small

$$\simeq R\left[1 - \frac{(2xx' + 2yy')}{R^2}\right]^{1/2}$$

For amp $\rho \approx R$

For phase

$$\rho \approx R\left[1 - \frac{(xx' + yy')}{R^2}\right]$$

Note: RCS independent of r

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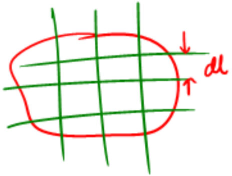
Computational Considerations

inc field wavelength coarse fine

$dl \to \frac{\lambda}{5} - \frac{\lambda}{10} - \frac{\lambda}{15}$

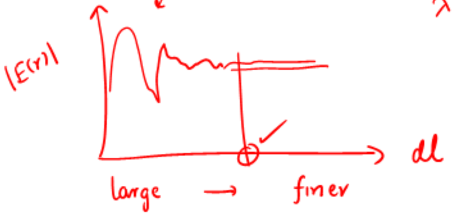
- How fine do you discretize?

(fixed.



Numerical Convergence.

numerical



$\lambda = \frac{\lambda_0}{n}$ ↳ refractive index of obj

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Reference: Ch 1 of Peterson's book on CEM