NPTEL - Computational Electromagnetics 2019 – Dr. Uday Khankhoje, EE IITM
Assignment 3: Solutions of questions 2 and 3

1. For a string tied at both ends ($x = 0$, $x = l$) in which the displacement is governed by the differential equation $\nabla^2 u(x) = F(x)$, where F is a specified forcing function. The value of Green's function at $x = l/3$ and $x' = 3l/4$ is given by:

Solution: (B) We know that Green's function in 1D can be written as

$$g(x, x') = \begin{cases} \left(\frac{x'-l}{l}\right) x & x < x' \\ \left(\frac{x-l}{l}\right) x' & x > x' \end{cases}$$

Now given that $x < x'$, so we will use the first expression. So the value of Green's function becomes:

$$g(l/3, 3l/4) = \frac{(-l/4)}{l}\frac{l}{3} = -\frac{l}{12}$$

2. For the above question, if the force applied on the string is given by $F(x) = 1\,N$ for all points on the string of length 1 m, then the displacement $u(x)$ is given by

Solution: The displacement $u(x)$ can be written in terms of the Green's function $g(x, x')$ and the forcing function $F(x)$ as

$$u(x) = \int_0^1 g(x, x')F(x')dx' \tag{1}$$

The Green's function for $l = 1m$ is:

$$g(x, x') = \begin{cases} (x-1)x' & x' < x \\ (x'-1)x & x' > x \end{cases}$$

Splitting the integral in Equation (1):

$$u(x) = \int_0^1 g(x, x')F(x')dx' = \int_0^x g(x, x')F(x')dx' + \int_x^1 g(x, x')F(x')dx'$$

Now we substitute the expression for $g(x, x')$ and $F(x) = 1$.

$$\begin{aligned} u(x) &= \int_0^x (x-1)x'dx' + \int_x^1 (x'-1)x\,dx' \\ &= \Big[(x-1)\frac{x'^2}{2}\Big]_0^x + \Big[\Big(\frac{x'^2}{2} - x'\Big)x\Big]_x^1 \\ &= (x-1)\frac{x^2}{2} + \Big(\frac{1}{2} - 1\Big)x - x\Big(\frac{x^2}{2} - x\Big) \\ &= \frac{x^3}{2} - \frac{x^2}{2} - \frac{x}{2} - \frac{x^3}{2} + x^2 \\ &= \frac{x^2 - x}{2} \end{aligned}$$

Therefore, $u(x) = (x^2 - x)/2$.

Observe that this is consistent with the boundary conditions i.e. , $u(0) = u(1) = 0$.