EE2025 Engineering Electromagnetics: July-Nov 2019

Tutorial 6: Waveguides

1. A section of a rectangular waveguide of cross-section 2 cm × 1.5 cm is to be used as a delay line in a radar at 10 GHz. What should be the length of the section to realize a delay of 10 nsec ?

> **Solution:**
>
> Cutoff frequency of TE_{10},
>
> $$f_c = \frac{c}{2a} = \frac{3 \times 10^8}{2 \times 2 \times 10^{-2}} = 7.5GHz$$
>
> Cutoff frequency of the next higher order mode TE_{01} (Since, $a < 2b$ can't be TE_{20}),
>
> $$f_c^{'} = \frac{c}{2b} = \frac{3 \times 10^8}{2 \times 1.5 \times 10^{-2}} = 10GHz$$
>
> Thus only possible mode of transmission is TE_{10}
>
> Group velocity
>
> $$v_g = c \times \sqrt{1 - (\frac{f_c}{f_o})^2} = 3 \times 10^8 \times \sqrt{1 - (\frac{7.5}{10})^2} = 1.984 \times 10^8 ms^{-1}$$
>
> Length for a delay of 10 ns,
>
> $$l = v_g \times 10 \times 10^{-9} = 1.984m = 198.4cm$$

2. (a) Derive an expression for the conductive loss in a rectangular waveguide for the fundamental mode.
(b) In a 5 cm × 3 cm waveguide the TE_{10} mode is propagating at 10 GHz. The total power carried by the waveguide is 10 W. If the conductivity of the waveguide walls is 2 $\times 10^7 \mho/m$, find the attenuation constant of the waveguide in dB/m.

> **Solution:**
>
> (a) Refer to section 6.8.2 of R.K.Shevgaonkar's "Electromagnetic waves".
>
> (b) Conductive loss in TE_{10} mode in a rectangular waveguide is given by,
>
> $$\alpha_c = \frac{R_s(1 + \frac{2b}{a}(f_c/f)^2)}{\eta b\sqrt{1 - (f_c/f)^2}} \tag{1}$$
>
> $R_s = \sqrt{\frac{\omega\mu}{2\sigma}} = \sqrt{\frac{2\pi \times 10 \times 10^9 \times 4\pi \times 10^{-7}}{2 \times 2 \times 10^7}} = 0.044\,\Omega$
>
> Considering dielectric as air,
>
> $f_c = \frac{v}{2a} = \frac{3 \times 10^8}{2 \times 0.05} = 3\,GHz$
>
> $\eta = 377 \times \sqrt{1 - 0.3^2} = 359.63\Omega$
>
> $\alpha_c = \frac{0.044(1+\frac{0.06}{0.05}(0.3)^2)}{359.63 \times 0.03\sqrt{1-0.3^2}} = 4.74 \times 10^{-3}\,Np/m = 0.041\,dB/m$

3. In an air-filled rectangular waveguide with a = 2.286 cm and b = 1.016 cm, the y component of the TE mode is given by

$$E_y = \sin(2\pi x/a)\cos(3\pi y/b)\sin(10\pi \times 10^{10} t - \beta z) V/m$$

Find
(a) the operating mode
(b) the propagation constant γ
(c) the intrinsic impedance η
(d) Find the dielectric loss α_d of the mode when the waveguide is filled with water(Complex permittivity of water at 50 GHz is $\epsilon_0(16.40 - j26.31)$)

Solution: (a) m = 2, n= 3, so the mode is TE_{23}

(b) $\gamma = j\beta$

$$\beta = \beta_1 \sqrt{1 - \left(\frac{f_c}{f}\right)^2}, f = 50GHz$$

$$f_c = \frac{1}{2\pi\sqrt{\mu\epsilon}} \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2} = 46.19GHz$$

$\beta = 400.9 rad/m$, $\gamma = j400.9m^{-1}$

(c) Intrinsic impedance for the TE mode

$$\eta = \frac{\eta_1}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}} = 984.6\Omega$$

(d)

$$\epsilon'' = \frac{\sigma}{\omega}$$

$$\sigma = 2\pi \times 50 \times 10^9 \times 8.85 \times 10^{-12} \times 26.31 = 73\mho/m$$

$$\alpha_d = \frac{\sigma\eta'}{2\sqrt{1 - (f_c/f)^2}}$$

$$\eta' = \sqrt{\frac{\mu}{\epsilon}} = 93\Omega$$

$$\alpha_d = 8.865 \times 10^3 Np/m$$

4. A frequency of 9.5 GHz is used to excite all possible modes in a hollow rectangular waveguide of dimensions 3 cm × 2 cm. The length of the waveguide is 100 m. Find the difference between time of arrivals of the fastest mode and the slowest mode.

Solution:

Given, Operating frequency = 9.5 GHz, a=3 cm and b=2 cm

length of the waveguide=100 m

cutoff frequency, $f_c = \frac{1}{2\pi\sqrt{\mu_0\epsilon_0}} \sqrt{(\frac{m\pi}{a})^2 + (\frac{n\pi}{b})^2} = 15 \times 10^9 \sqrt{(\frac{m^2}{9}) + (\frac{n^2}{4})}$

For, $f > f_c \Rightarrow 9.5 \times 10^9 > 15 \times 10^9 \sqrt{(\frac{m^2}{9}) + (\frac{n^2}{4})} \Rightarrow (\frac{m^2}{9}) + (\frac{n^2}{4}) < 0.4$

Therefore, possible operating modes are $TE_{10}, TE_{01}, TE_{11}$ and TM_{11}.

Group velocity of fastest mode (TE_{11}) $= c\sqrt{1-(\frac{f_c}{f})^2} = c\sqrt{1-(\frac{9}{9.5})^2} = 2.55 \times 10^8$ m/s

Group velocity of slowest mode (TE_{10}) $= c\sqrt{1-(\frac{f_c}{f})^2} = c\sqrt{1-(\frac{5}{9.5})^2} = 0.96 \times 10^8$ m/s

Difference of time of arrival of slowest and fastest mode $= (\frac{100}{0.96\times10^8}) - (\frac{100}{2.55\times10^8}) = 0.649\ \mu s$

5. A 240 degree phase shift is produced by a 4 GHz signal when traveling along a dielectric filled waveguide 3 cm long. If the cutoff frequency of the waveguide when air-filled is 10 GHz, calculate the relative permittivity of the dielectric.

Solution: Cut-off frequency in air-filled waveguide = 10GHz
Cut-off frequency in dielectric waveguide $f_c = \frac{10GHz}{\sqrt{\epsilon_r}}$
Given phase shift $\beta l = 240°$ and $l = 0.03m$
$\beta = \frac{400\pi}{9} rad/m$
$\beta = \frac{\omega\sqrt{\epsilon_r}}{c}\sqrt{1-\left(\frac{f_c}{f}\right)^2}$
Given $f = 4GHz$, substituting the values in above equation,
$\epsilon_r = 9.0046$

6. For a square waveguide, show that attenuation α_c is minimum for TE_{10} mode when $f = 2.962 f_c$

Solution: In a rectangular waveguide, for TE_{10} mode

$\alpha_c = \frac{2R_s}{\eta b}\left[\frac{\frac{1}{2}+\frac{b}{a}\left(\frac{f_c}{f}\right)^2}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}\right]$, where $R_s = \sqrt{\pi\mu f/\sigma}$

For square waveguide $a = b$, we get

$\alpha_c = \frac{2R_s}{\eta a}\left[\frac{\frac{1}{2}+\left(\frac{f_c}{f}\right)^2}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}\right]$

For the frequency at which α_c is minimum, $\frac{d\alpha_c}{df} = 0$

Solving the equation, we get $f^4 - 9f_c^2 f^2 + 2f_c^4 = 0$

substituting $f = kf_c$, in the above equation we obtain

$k^4 - 9k^2 + 2 = 0 \Rightarrow r^2 - 9r + 2 = 0$; $(r = k^2)$

solving the quadratic equation for r we get $r = k^2 = 8.772, 0.228 \Rightarrow k \approx 2.962, 0.477$.

Even though we get two solutions (numerically), $k < 1$ is not a physically realizable solution because for the operating frequency $f < f_c$ the mode of interest itself is not supported by the waveguide.

Therefore, for minimum attenuation $f = 2.962 f_c$

7. A 4 cm square waveguide is filled with a dielectric with complex permittivity $\epsilon_c = 16\epsilon_o(1 - j10^{-4})$ and is excited with the TM_{21} mode. If the waveguide operates at 10 % above the cutoff frequency, calculate attenuation α_d. How far can the wave travel down the guide before its magnitude is reduced by 20 % ?

Solution:

$$\epsilon_c = \epsilon' - j\epsilon'' = \epsilon - j\frac{\sigma}{\omega} \tag{2}$$

Comparing this with

$$\epsilon_c = 16\epsilon_o(1 - j10^{-4}) = 16\epsilon_o - j16\epsilon_o \times 10^{-4} \tag{3}$$

$$\epsilon = 16\epsilon_o, \quad \frac{\sigma}{\omega} = 16\epsilon_o \times 10^4 \tag{4}$$

For TM_{21} mode, (with $c' = c/\sqrt{\epsilon_r}$)

$$f_c = \frac{c'}{2}\left[\frac{m^2}{a^2} + \frac{n^2}{b^2}\right]^{\frac{1}{2}} = 2.096\ GHz \tag{5}$$

$$f = 1.1f_c = 2.306\ GHz \tag{6}$$

$$\sigma = 16\epsilon_o\omega \times 10^{-4} = 16 \times 2\pi \times 4.6123 \times 10^9 \times \frac{10^{-9}}{36\pi} \times 10^{-4} = 2.05 \times 10^{-4} \tag{7}$$

$$\eta' = \sqrt{\frac{\mu}{\epsilon}} = 30\pi \tag{8}$$

$$\alpha_d = \frac{\sigma\eta'}{2\sqrt{1 - (f_c/f)^2}} = 0.0320\ Np/m \tag{9}$$

$$E_o e^{-\alpha_d z} = 0.8E_o \rightarrow z = 6.97m \tag{10}$$