Tutorial 4: RKS sections 5.2-5.10

1. A uniform plane wave is incident from air onto glass at an angle from the normal of $30^{\circ}$. Determine the fraction of the incident power that is reflected and transmitted for (a) parallel polarization and (b) perpendicular polarization. Glass has refractive index $n_{2}=1.45$.
2. The effect of total internal refection is observed by shining the green laser pointed ( $\lambda=532 \mathrm{~nm}$, $n_{1}=1.5$ ) under $45^{\circ}$ internal angle onto the base of the prism. (see figure 3) At what distance from the surface in the air is the amplitude of the evanescent wave $1 / e$ of its value at the surface?
3. An optical fiber is made up of a core, where light travels, made of glass of refractive index $n_{1}=$ 1.5 surrounded by another layer of glass of lower refractive index $n_{2}$.


Figure 1: Optical fiber
a) Find the refractive index $n_{2}$ of the cladding so that the critical angle at the interface of core cladding is $80^{\circ}$.
b) Let, $\alpha$ be the angle made by the ray with the axis of the fiber. For what values of $\alpha$, the incident angle $\theta_{i}$ is larger than that of the critical angle found in part a) above.
4. Consider a circularly polarized plane wave incident on a medium at an angle $\theta_{i}$.
(a) Derive a condition for the reflected wave to be circularly polarized.
(b) Derive a condition for the transmitted wave to be circularly polarized.
(Hint: Solve Example 5.8 in R.K.Shevgaonkar)
5. A plane wave in region 1 is normally incident on the planar boundary separating lossless regions 1 and 2. If their relative permittivities and permeabilities are related as $\epsilon_{1}=\mu_{1}^{3}$ and $\epsilon_{2}=\mu_{2}^{3}$, find the ratio $\epsilon_{2} / \epsilon_{1}$ such that $20 \%$ of the energy in the incident wave is reflected at the boundary.
6. A $10-\mathrm{MHz}$ uniform plane wave having an initial average power density of $5 \mathrm{~W} / \mathrm{m}^{2}$ is normally incident from free space onto the surface of a lossy material in which $\epsilon_{2}^{\prime \prime} / \epsilon_{2}^{\prime}=0.05, \epsilon_{r 2}^{\prime}=5$, and $\mu_{2}=\mu_{0}$. Calculate the distance into the lossy medium at which the transmitted wave power density is down by 10 dB from the initial $5 \mathrm{~W} / \mathrm{m}^{2}$ :
7. a) Consider a $100 \mathrm{~V} / \mathrm{m}, 3 \mathrm{GHz}$ wave that is propagating in a dielectric material having $\epsilon_{r 1}=4$ and $\mu_{r 1}=1$. The wave is normally incident on another dielectric material having $\epsilon_{r 2}=9$ and $\mu_{r 2}=1$ as shown in the figure below. Find out the locations of maxima and minima of the electric field and the standing wave ratio in the Region 1.
b)If region 2 is free space, at what angle of incidence will the wave in region 1 (dielectric) will undergo total internal reflection assuming parallel polarization and the propagation in x-z plane. Will there be any electric field in the region 2 (free space) under TIR, if yes why?
c) What will be the standing wave ratio if the material in region 2 is a perfect conductor? Apply the boundary conditions to find out electric field expression in region 1.


Figure 2: interface
8. A linearly polarized wave is incident on an isosceles right triangle(prism) of glass as shown in Figure 3, and it exists as shown in figure. Assuming that the dielectric constant of the prism is 2.25 , find the ratio of the exited average power density $S_{e}$ to that of the incident $S_{i}$.


Figure 3: Prism
9. You are asked to measure the distance from an antenna to a reflecting conducting surface. A plane wave is transmitted to the surface (normal incidence) and a zero (minimum reception) in the standing wave pattern is recorded using a second antenna at a distance 10 m from the sending antenna, as shown in Setup diagram. The frequency of the wave is 100 MHz . Now, the frequency is decreased until the receiving antenna reads the next maximum in the electric field at the same location. If the frequency for the maximum reading is 99.9 MHz , calculate the distance between the transmitting antenna to the conducting surface. Use the properties of free space without attenuation.
10. Consider an air-medium interface. Determine the value of $n$ of the medium for which Brewster's angle is equal to the critical angle.
11. Show how a single block of glass can be used to turn a parallel polarized light through $180^{\circ}$ (i.e change the direction of propagation by 180 degrees), with the light suffering (in principle) zero reflective loss. The light is incident from air and returning beam (also in air) may be displaced sideways from the incident beam. Specify all pertinent angles and use $\mathrm{n}=1.45$ for glass. (More than one design is possible here).


Figure 4: Setup diagram
12. Given three materials fiberglass $(n=1.6)$, rain erosion paint $\left(n=1.6\right.$, thickness $\left.=\lambda_{3} / 16\right)$ and primer $\left(n=2.56\right.$, thickness $\left.=\lambda_{2} / 2\right)$. Where $n$ is the relative refractive index of the material.

Design a cascade of these three materials (propose an arrangement and the thickness of the fiberglass), such that overall transmission coefficient for a normally incident wave is unity. The wave propagates through the air $(n=1)$ into the cascade and then leaves back into the air. Suppose now you use this configuration for a radome which must transmit at least $95 \%$ of the incident signal power. Find the value of $n$ for the atmosphere ( $n$ of atmosphere varies with height). Assume that the air between the aircraft antenna and the radome walls has $n=1$ always, i.e air at sea level.
13. A 1 GHz parallel polarized plane-wave is incident from medium 1 onto the medium 1 and 2 interface with an angle greater than critical angle as shown in Fig.5. Given the thickness of medium 2 is 5 cm . (Info: Materials are non-magnetic $\left(\mu_{r}=1\right)$ and magnetic fields are in + ve y-direction).


Figure 5: There will be infinite multiple reflections at each interface. For simplicity we show the steady state fields.
(a) Derive the expressions for fields in all the three medium when $\theta_{\text {inc }}=60^{\circ}$.
(b) What is the average power density reflected and transmitted from medium 1 and medium 3 respectively when the incident average power density is $5 \mathrm{~W} / \mathrm{m}^{2}$.
(c) Is law of conservation of energy satisfied? If not, why?

