## EE2025 Engineering Electromagnetics: July-Nov 2019

Tutorial 1: Transmission Lines
Note : All transmission lines can be assumed to be lossless, unless mentioned otherwise.

1. Sinusoidally varying voltages and currents can in general be represented as $V \cos (\omega t+\psi)$ and $I \cos (\omega t+\phi)$, where $V, I$ are real. These can also be written in phasor notation as $R e\left[V e^{j \psi} e^{j \omega t}\right]$ and $\operatorname{Re}\left[I e^{j \phi} e^{j \omega t}\right]$ : we now call the terms accompanying $e^{j \omega t}$ as the phasors corresponding to the voltage and current respectively (i.e. $V e^{j \psi}$ and $I e^{j \phi}$ ). Note that phasors are always time independent. Find an expression for the average power (over a cycle) in terms of these phasors.
2. The length of a microstrip trace line connecting two components on a chip is 50 cm . A sinusoidal signal of frequency 1 GHz is supplied to the trace at one end. Assuming the velocity of propagation of the signal is $2 \times 10^{8} \mathrm{~m} / \mathrm{sec}$ and there are no reflections,
3. Calculate the time taken by the signal to reach the other end of the trace.
4. What is the phase difference between the signal at the two ends of the trace?
5. Using the concepts of electrostatics, find the capacitance per unit length, $C$ of
6. parallel wire line, with each wire of radius $a$ and separated by a distance $2 d$, where $a \ll 2 d$.
7. coaxial cable of inner radius $a$ and outer radius $b$.

(b)

Figure 1: (a)Two wire transmission line. $a \ll d$. (b) Coaxial transmission line with inner radius $a$ and outer radius $b$ and length $l$
4. You are required to buy a cable from an electronics shop to connect your dish antenna to your set top box and your set top box to your TV.

1. Write the name of the cable you would buy.
2. Upto what length do you think you can use this cable, in the lumped circuit model and why ?
3. A transmission line with characteristic impedance $Z_{0}=50-\mathrm{j} 5 \Omega$ and propagation constant $\gamma=$ $0.2+\mathrm{j} 2.5 / \mathrm{m}$ is connected to a load impedance of $100+\mathrm{j} 50 \Omega$. Find

| Frequency Band | RF Channels | Frequency (MHz) |
| :---: | :---: | :---: |
| Very high frequency-Low | $2-6$ | $54-88$ |
| Very high frequency-High | $7-13$ | $174-216$ |
| Ultra high frequency | $14-69$ | $470-806$ |

Table 1: Frequency bands in television

1. Reflection coefficient of the line at the load end.
2. Reflection coefficient of the line 5 m from the load.
3. (a) Show that the impedance along the line will lie between $Z_{0} / \rho$ and $Z_{0} \rho$, where $\rho$ is the VSWR.
(b)A $300 \Omega$ transmission line is connected to a circuit with an input impedance of $75+\mathrm{j} 35 \Omega$. Find
4. $\rho$
5. Maximum impedance seen on the line
6. Minimum impedance seen on the line
7. An RG-59U coaxial cable has a loss of 10 dB per 100 ft of length. A $10 \mathrm{~V}-3 \mathrm{~A}$ signal is generated using a function generator and connected to one end of the 50 ft long cable. On the other side, the cable is impedance matched to a set top box unit. Find the power delivered to the load.
8. According to the maximum power transfer theorem, maximum time averaged power is transferred from a source with internal impedance $Z_{g}$ to a load, $Z_{L}$ when $Z_{g}=Z_{L}^{*}$. A 50 MHz generator with an internal impedance $\left(Z_{g}\right)$ of $50 \Omega$ is connected to an impedance $50-\mathrm{j} 25 \Omega$. How would you ensure maximum power transfer in this case using a lossless transmission line of characteristic impedance $100 \Omega$, and what should be the minimum length of the transmission line element ? Assume $v=2 \times 10^{8} \mathrm{~m} / \mathrm{s}$ as wave velocity.


Figure 2: Impedance matching using a transmission line of length $l$
9. On a $50 \Omega$ BNC cable line, the reflection co-efficient is measured at the load end to be $0.7 \angle 30^{\circ}$. If the propagation constant of the line is $20 \angle 89^{\circ} / \mathrm{m}$, find the impedance seen on the transmission line at a distance of 4 m from the load. (Note: BNC is a very popular type of coaxial cable used for frequencies even up to 4 GHz )
10. Calculate the average power dissipated by each resistor in the circuit shown in Fig. 4.
11. Given the system in (Fig. 5 ) is operating with $\lambda=100 \mathrm{~cm}$ and $Z_{0}=300 \Omega$. If $d_{1}=10 \mathrm{~cm}$, $d=25 \mathrm{~cm}$, and the system is matched to $300 \Omega$, find $Z_{L}$ ?


Figure 3: Impedance matching using a short circuited stub of length $l$ and its equivalent circuit


Figure 4
12. The two-wire lines shown in Fig. 6 are all lossless and have $Z_{0}=200 \Omega$. Find the possible values of $d$ and $d_{1}$ to provide a matched load if $\lambda=100 \mathrm{~cm}$. (Note that the un-shaded and shaded conductor are both parts of the same transmission line, for example they can be the inner and outer conductor of a coaxial cable.)
13. Approximate distributed circuit models of (lossless) a lossless transmission operating in high frequency modes is shown in Fig. 7. Note that $L$ has units $H \cdot m, C$ has units $F \cdot m, L_{0}$ has units $H / m$ and $C_{0}$ has units $F / m$. Obtain expressions for the propagation constant $\beta$ and the characteristic impedance $Z_{0}$ of the line for both circuits at frequency $\omega$.
14. For the transmission line represented in Fig. 8, calculate the potential developed across the $80 \Omega$ resistor for (a) $f=60 \mathrm{~Hz}$, (b) $f=1 \mathrm{MHz}$, (c) Repeat part (a) with length $10^{7} \mathrm{~m}$ instead of 80 m .


Figure 5


Figure 6


Figure 7


Figure 8

