Computational Electromagnetics : Modes of a structure

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Topics in this module

1. Modes of a structure

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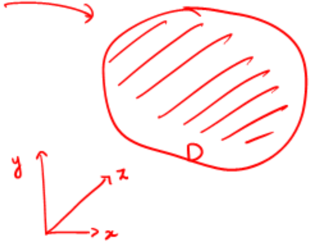
Problem definition

2D structure (cross-section)



$$\epsilon_r(x,y,z) = \epsilon_r(x,y)$$

$$\vec{E} \propto e^{j(\omega t - \beta z)}$$

β → propagation const.

$$\vec{E}_T = \bar{E}(x,y)\, e^{j(\omega t - \beta z)}$$

Q: What is β for a given ω?

Free space

$\omega = c\beta$

→ w.g.?

Solution using the integral equation approach

Assumption : $\nabla \cdot \vec{E}(x,y) = 0$

TM: (H_x, H_y, E_z)

Cross-section → not z-depn.

$\nabla \times \bar{E} = -j\omega\mu\bar{H}$

$\nabla \times \bar{H} = j\omega\epsilon\bar{E}$

$$\vec{E}_T = E(x,y)e^{j(\omega t - \beta z)}$$

$\nabla \times (\nabla \times E) = -j\omega\mu(j\omega\epsilon\bar{E}) \rightarrow \nabla(\nabla \cdot \bar{E}) - \nabla^2\bar{E} = \omega^2\mu\epsilon\bar{E} \Rightarrow \nabla^2\bar{E} + \omega^2\mu\epsilon\bar{E} = 0$

$\nabla^2 = \nabla_T^2 + \frac{\partial^2}{\partial z^2}$

∇_T ↓ (x,y)

$$\nabla_T^2 E(x,y) - \beta^2 E(x,y) + \omega^2\mu\epsilon E(x,y) = 0$$

$$\nabla_T^2 E(x,y) + \omega^2\mu\epsilon E(x,y) = \beta^2 E(x,y) \quad (1)$$

$\omega^2\mu\epsilon \rightarrow k_0^2\epsilon_r(r)$

$$\nabla_T^2 E(x,y) + k_0^2 E(x,y) = [\beta^2 - k_0^2(\epsilon_r(x,y) - 1)]E(x,y) \quad (2)$$

$$\nabla_T^2 G(r,r') + k_0^2 G(r,r') = -\delta(r,r') \quad (3)$$

↓ 2D green's fn

Need to solve (2) using (3)

Solution using the integral equation approach

Soln $E(\underline{r}) = \iint G(\underline{r}, \underline{r}') \left[k_0^2 (\varepsilon_r(\underline{r}') - 1) - \beta^2 \right] E(\underline{r}') \, d\underline{r}'$

where $\varepsilon_r(\underline{r}') - 1 = \chi(\underline{r}')$

Soln via Conv of G with forcing fn in eqn (2).

Integral Eqn + Green's fns.

known? G, ε_r, k_0

unknown? E, β

Convert to discrete

$r \in D$

$$e = k_0^2 G \chi e - \beta^2 G e$$

G: Coupling matrix has integrals inside

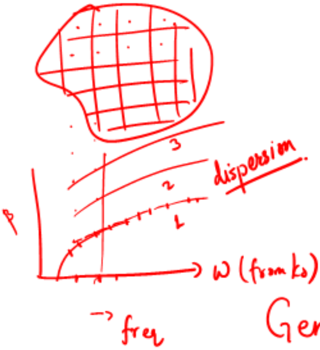
$$\left[I - k_0^2 G \chi \right] e = -\beta^2 G e$$

$I - k_0^2 G \chi$: known; G: known

$$A x = \lambda B x$$

keep purely real β's only → Travelling modes.

Generalized eigenvalue problem.



Topics that were covered in this module

1. Modes of a structure

References: