Computational Electromagnetics:
Summary of Integral Equation Methods

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Topics in this module

1. Surface v/s Volume Integral Approach
2. Finding the Radar Cross-Section (RCS)
3. Computational Considerations
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2 Finding the Radar Cross-Section (RCS)
3 Computational Considerations
Quick aside: Surface Integral Equations and PECs

How do we deal with scatters that are made of perfect electric conductors?

Recall boundary conditions for PEC

\[ E_z = 0 = \phi \]

If we have a PMC \( \Rightarrow H_{tan} = 0 \), \( E_z \neq 0 \).

The original system of equations:

\[
\begin{align*}
\oint [g_1(r, r')\nabla \phi(r) \cdot \hat{n} - \phi(r)\nabla g_1(r, r') \cdot \hat{n}] \, dl &= \phi_i(r'), \quad r' \in V_2 \\
\oint [g_2(r, r')\nabla \phi(r) \cdot \hat{n} - \phi(r)\nabla g_2(r, r') \cdot \hat{n}] \, dl &= 0, \quad r' \in V_1 \\
\oint (g_i (r, r') \nabla \phi(r)) \, dl &= \phi_i (r') \\
\oint g_c (r, r') \nabla \phi (v) \cdot \hat{n} \, dl &= 0
\end{align*}
\]

\( 2N \times 2N \) system.
Surface v/s Volume Integral Equations

Surface approach:

For each region:

\[ \nabla^2 \phi_n + k_n^2 \phi_n = Q_n \]

\[ \nabla^2 g_n + k_n^2 g_n = -\delta(x, r') \]

Each eqn solved separately for each region.

Variables: \( E_{\tan}, H_{\tan} \) on \( S \).

Huygen's principle.

\( k_1, k_2 \rightarrow \) constants

Homogeneous

Volume approach:

\[ \nabla^2 \phi + k_n^2 \phi = Q_n \]

\[ \nabla^2 \phi_i + k_o^2 \phi = Q_n \]

Eqn in terms of \( \phi - \phi_i \)

\[ \nabla^2 (\phi - \phi_i) + k_o^2 (\phi - \phi_i) = \frac{Q_n}{\epsilon_r(r)} \]

\[ k_n^2 = k_o^2 \epsilon_r(r) \]

Heterogeneous
Surface v/s Volume Integral Equations

Surface approach:

\[ \phi(r') = \phi_i(r') + \oint_S [\phi(r) \nabla g_1(r, r') - g_1(r, r') \nabla \phi(r)] \cdot \hat{n} dl \]

Volume approach:

\[ \phi(r') = \phi_i(r') + k_0^2 \int_{V_2} g_1(r, r') [\varepsilon_r(r) - 1] \phi(r) \, dr \]

\text{Surface faster volume.}

Huygen's surface equivalence principle.
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Definition of Radar Cross-Section (RCS): \( \sigma_{TM}(\theta, \theta_i) = \lim_{r \to 0} 2 \pi r \frac{|E_z^2(r, \theta)|^2}{|E_{z}^i(0,0)|^2} \)

\( \sigma_{TM}(\theta, \phi, \theta_i, \phi_i) = \lim_{r \to 0} 4 \pi r^2 \frac{|E_z^2(r, \theta, \phi)|^2}{|E_{z}^i(0,0,0)|^2} \)

Mono-static and Bi-static

\( \theta_i + \pi = \theta_s \)
Approximations in the RCS

An integral involving Green’s function:

$$\phi(r') = \phi_i(r') + k_0^2 \int_{V_2} g_1(r, r') \chi(r) \phi(r) \, dr$$

In 2D:

$$g(r, r') = -\frac{j}{4} H_0^{(2)}(k|r - r'|)$$

for $x \gg 1$, $H_0^{(2)}(k\rho) \approx \sqrt{\frac{2j}{\pi k\rho}} \exp(-jk\rho)$

In 3D:

$$g(r, r') = \frac{1}{4\pi|\mathbf{r} - \mathbf{r}'|} \exp(-jk|\mathbf{r} - \mathbf{r}'|)$$

For amp $P \propto R$

For phase $p \propto R \left[ 1 - \frac{(xx' + yy')}{R^2} \right]$

Note: RCS independent of $r$
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Computational Considerations

• How fine do you discretize?

Numerical Convergence
Topics that were covered in this module

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Reference: Ch 1 of Peterson’s book on CEM