EE5120 Linear Algebra: Tutorial Test 7, 11.11.18A

Give your answers in the space provided. No calculators or smartphones allowed. Please take a few minutes to read the questions carefully and answer (briefly) only what is asked.

Roll: No:________________ NAME:________________________ Time: 20 mins

$\boxed{2}$ 1. Fill in the blanks in terms of the SVD of $A \in \mathbb{R}^{m\times n} = U\Sigma V^H$, of rank r:

The column space of A has as basis vectors ________________________

The row space of A has as basis vectors ________________________

The null space of A has as basis vectors ________________________

The left null space of A has as basis vectors ________________________

Solution: $\{u_i\}_{i=1}^r$, $\{v_i\}_{i=1}^r$, $\{v_i\}_{i=r+1}^n$, $\{u_i\}_{i=r+1}^m$

$\boxed{2}$ 2. What is the condition number of A^HA if the condition number of A is κ? (Don't just state the answer, show) Now, assume that A is square, full rank and you are presented with two algorithms for solving $Ax = b$. The first one does $x = A^{-1}b$, while the second one does it in two steps: $A^HAx = A^Hb \rightarrow x = (A^HA)^{-1}(A^Hb)$. If b is corrupted by noise, will both algorithms give the same results? If not, which will be more accurate?

Solution: Condition number of A^HA is κ^2 (use the SVD of A and multiply it out), i.e. it is worse than A since κ is always greater than 1.
Answer to second part pending. [1+1]

$\boxed{6}$ 3. Given $F = \frac{9}{4}x^4 + 2x^3 - xy^3 + y$ and a point $(\frac{1}{3}, 1)$. Is this point a critical/stationary point? If so, is it a minima/maxima or a saddle point? If not, why? *Hint*: Consider the multi-variable Taylor's theorem that we discussed in class, and keep only terms up to the second derivative.

Solution: Compute gradient $\nabla F = \begin{bmatrix} \frac{\partial F}{\partial x} \\ \frac{\partial F}{\partial y} \end{bmatrix} = \begin{bmatrix} 9x^3 + 6x^2 - y^3 \\ -3xy^2 + 1 \end{bmatrix}$, which is equal to 0 at the given point. Thus it is a critical point. [2]

The second derivative matrix is $A = \begin{bmatrix} \frac{1}{2}\frac{\partial^2 F}{\partial x^2} & \frac{\partial^2 F}{\partial x \partial y} \\ \frac{\partial^2 F}{\partial x \partial y} & \frac{1}{2}\frac{\partial^2 F}{\partial y^2} \end{bmatrix} = \begin{bmatrix} 0.5 * (27x^2 + 12x) & -3y^2 \\ -3y^2 & -3xy \end{bmatrix} = \begin{bmatrix} 3.5 & -3 \\ -3 & -1 \end{bmatrix}$ at $(\frac{1}{3}, 1)$. [2]

The eigenvalues of this matrix are 5,-2.5, and thus is indefinite. Thus shape of the function in the neighbourhood of $(\frac{1}{3}, 1)$ is like a saddle, implying that the critical point is a saddle point. [2]