

EE5120 Linear Algebra: Tutorial Test 5, 18.10.18A

Give your answers in the space provided. No calculators or smartphones allowed. Please take a few minutes to read the questions carefully and answer (briefly) only what is asked.

Roll: No: _____

NAME: _____

Time: 15 mins

- 6 1. You are given the following matrix, $A = \begin{bmatrix} 2 & 2 & 1 \\ 0 & 3 & 1 \\ 0 & 0 & 2 \end{bmatrix}$, and are asked to find out the value of the expression: $A^3 - 7A^2 + 14A - 11I$. You must do so without explicitly computing the power of *any* matrix (no credit if you do this). Show your steps to receive credit.

Solution: This can be solved using the Cayley Hamilton theorem.

1. The characteristic polynomial is $p(\lambda) = |A - \lambda I| = (2 - \lambda)^2(3 - \lambda) = 0$, since the matrix is triangular and its determinant is the product of diagonal values.

2. From the C-H theorem, the matrix also satisfies this and we get: $A^3 - 7A^2 + 16A - 12I = 0$.

3. Given expression simplifies as $A^3 - 7A^2 + 14A - 11I = (-16A + 12I) + (14A - 11I) = -2A + I = \begin{bmatrix} -3 & -4 & -2 \\ 0 & -5 & -2 \\ 0 & 0 & -3 \end{bmatrix}$

- 2 2. Find any one eigenvalue and eigenvector of the matrix $X = I + 2uu^T$, where $u^T u = 1$.

Solution: We can see that $Xu = 3u$, thus u is an eigenvector with value 3.

- 2 3. Is the matrix $A - 3I$ invertible? A is the matrix from question 1. Write your answer/calculation in one line.

Solution: No, since it has 0 as an eigenvalue.