EE5120 Linear Algebra: Tutorial Test 3, 12.09.18A

Give your answers in the space provided. No calculators or smartphones allowed. Please take a few minutes to read the questions carefully and answer (briefly) only what is asked.

Roll: No:_____   NAME:_____   Time: 15 mins

1 1. Imagine a chess board of size $n \times n$ and now visualize it as a matrix, with white and black representing the values 0, 1 respectively. What is the rank of such a matrix (with a one line reason)?

> **Solution:** 2 linearly independent columns, rank = 2.

5 2. Given a linear transformation T from $\mathbb{R}^2 \to \mathbb{R}^2$ such that:
$T[(x,y)^T] = (2x + 3y, 3x - 2y)^T$. Find the matrix corresponding to this transformation when both the input basis and output basis are the same and represented as $\{(1,1)^T, (1,-1)^T\}$. If you find this tricky, you may solve this in the canonical basis (both i/p and o/p) for [3] points.

> **Solution:** (i) $T[(1,1)^T] = (5,1)^T$ and $T[(1,-1)^T] = (-1,5)^T$.
>
> (ii) But the output must be presented in the same basis, and we find that $\begin{bmatrix} 5 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \end{bmatrix}$, similarly, $\begin{bmatrix} -1 \\ 5 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 2 \\ -3 \end{bmatrix}$, so the final matrix is $\begin{bmatrix} 3 & 2 \\ 2 & -3 \end{bmatrix}$.

4 3. If a rectangular matrix A has full row rank, which of $(AA^T, A^T A)$ is invertible? Why?

> **Solution:** Refer to tutorial 3 for solution.