EE5120 Linear Algebra: Tutorial Test 1, 27.08.18A

Give your answers in the space provided. No calculators or smartphones allowed. Please take a few minutes to read the questions carefully and answer only what is asked, keeping your answers brief.

Roll: No:__________________ NAME:____________________________ Time: 15 mins

$\boxed{3}$ 1. The product of three matrices is given as $D = ABC$.
(i) State the conditions on the sizes of A, B, C such that this product is legal, and (ii) Write the expression for the (i, j)th element of D in terms of the elements of A, B, C in compact form. [1+2]

Solution: (i) $A : m \times p$, $B : q \times r$, $C : s \times n$, condition: $p = r$ and $r = s$ [1]
(i) Since $(AB)_{i,j} = \sum_p A_{ip}B_{pj}$, we get $(ABC)_{i,j} = \sum_q(\sum_p A_{ip}B_{pq})\, C_{qj}$ [2]

$\boxed{7}$ 2. Let A and B be $m \times n$ matrices, and A_1, B_1 and B_2 be $(m+1) \times (n+1)$ matrices defined as,

$$A_1 = \begin{bmatrix} 1 & \mathbf{0}_1^T \\ \mathbf{0}_2 & A \end{bmatrix}; B_1 = \begin{bmatrix} 1 & \mathbf{0}_1^T \\ \mathbf{0}_2 & B \end{bmatrix}, B_2 = \begin{bmatrix} \mathbf{0}_2 & B \\ 1 & \mathbf{0}_1^T \end{bmatrix}$$

where $\mathbf{0}_1$ is an $n \times 1$ all-zero vector and $\mathbf{0}_2$ is an $m \times 1$ all-zero vector. Given that A can be transformed to B by elementary row transformations, find the transformation required to transform: (i) A_1 to B_1, and (ii) B_1 to B_2.
Hint: If you can't solve (i) you can still attempt (ii) directly. [3+4]

Solution: (i) Suppose E is the elementary row transformation matrix such that, $B = EA$. Then, it can be verified that $\tilde{B} = \tilde{E}\tilde{A}$, with $\tilde{E}$ being,

$$\tilde{E} = \begin{bmatrix} 1 & \mathbf{0}_2^T \\ \mathbf{0}_2 & E \end{bmatrix},$$

which is clearly an elementary row transformation matrix. [3]
(ii) The required permutation matrix is

$$\begin{bmatrix} 0 & 1 & 0 & \dots \\ 0 & 0 & 1 & \dots \\ \vdots & & & \\ 1 & 0 & 0 & \dots \end{bmatrix}$$

[4]