EE6506 Computational Electromagnetics: Jan - May 2018, Uday Khankhoje
Assignment 1, 01.02.2018, due 16.02.2018 (at start of class)

Clearly state the CONCEPT involved in solving each problem. Do NOT write lengthy answers. Make reasonable assumptions and STATE them. You are free to discuss with your classmates and look at reference books. But, you must write your solutions yourself and mention who you collaborated with for your solution. Looking for solutions online is not allowed.

1. For a volume V with a surface S and scalar functions ψ, ϕ, prove:
   (a) $\oint_S \phi \frac{\partial \psi}{\partial n} ds = \int_V (\phi \nabla^2 \psi) dv + \int_V (\nabla \phi \cdot \nabla \psi) dv$: Green's first identity
   (b) $\oint_S (\phi \frac{\partial \psi}{\partial n} - \psi \frac{\partial \phi}{\partial n}) ds = \int_V (\phi \nabla^2 \psi - \psi \nabla^2 \phi) dv$: Green's second identity
2. Given that the instantaneous electric field inside a source-free, homogeneous, isotropic, and linear medium is given by $\mathcal{E} = [\alpha(x+y)\hat{x} + \beta(x-y)\hat{y}] \sin(\omega t)$, find a relation between α, β.
3. An electric line source of infinite length and constant current along the z axis radiates in free space, and at large distances from the source the expression for the magnetic field is as follows: $\mathbf{H} = \hat{\phi} H_0 \exp(-j\beta_0 \rho)/\sqrt{\rho}$, ($H_0$ is a constant). Find the corresponding electric field.
4. Give a short proof of the uniqueness theorem, being sure to point out why the medium needs to be lossy.
5. Derive a continuity relation for the normal components of $\mathbf{D}$ at an interface between two media in the presence of a surface charge.
6. Derive (but do not solve) the form of the differential equation obeyed by a time-harmonic wave travelling along the z axis of a waveguide at a point away from the exciting current. The cross-section of the waveguide is not a function of the z co-ordinate, but an arbitrary $\epsilon(x, y)$. Hint: What is the logical and simplest form of the z-dependence of the field? Warning: In addition to the four equations of Maxwell, there is also the continuity equation $\nabla \cdot \mathbf{J} = -\frac{d\rho}{dt}$, so zero current does not imply zero charge.
7. (a) The textbook (Balanis) uses the theory of Sturm-Liouville problems to derive general Green's functions, whereas in class we derived them directly. Follow the class approach to derive in closed and series form, the Green's function for the 1D wave equation: $\phi''(x) + k^2\phi(x) = f(x)$ subject to $\phi(0) = \phi(l) = 0$.
   (b) (This one is simpler than it looks) Derive the Green's function for the scalar 3D Helmholtz equation, $\nabla^2\phi(\mathbf{r}) + k^2\phi(\mathbf{r}) = f(\mathbf{r})$. Clearly show the use of boundary conditions at ∞.
   Hint: Use separation of variables. Which coordinate system is simplest given the symmetry of the problem? Hint2: When faced with a long differential equation, a change of variables of the form $t(r) = r\phi(r)$ might help you.
8. In class we derived the Huygen's principle and the extinction theorem in a 2D setting for the E_z polarization. Extend the analysis and solve the following:
   (a) Derive the expressions the theorems above take when the scattering object is a perfect electric conductor (PEC). Hint: Start by listing the boundary conditions on this surface and seeing if it leads to any simplifications.
   (b) Derive a general expression for the theorems when the H_z polarization is chosen. As in class, assume the object to be a dielectric object $(\epsilon, \mu) = (\epsilon(r), \mu_0)$