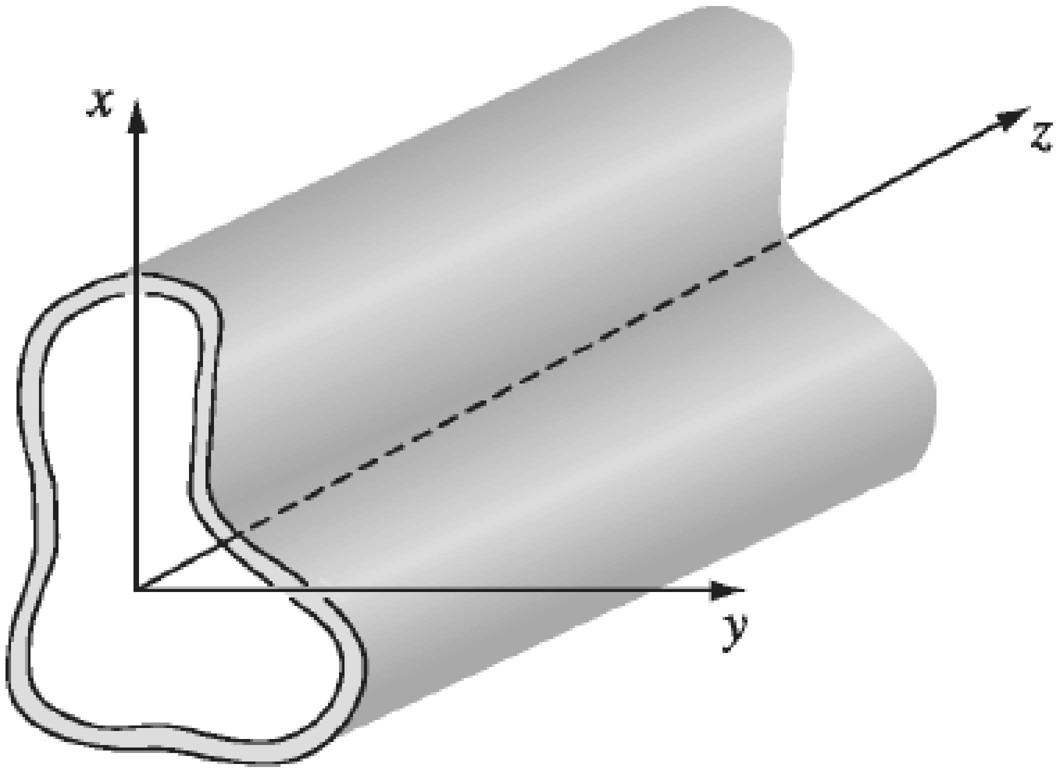
Electromagnetic Resonators

EEL207: Uday Khankhoje
EE, IIT Delhi

Waveguides (recap)



Waveguides (recap) [1]

1. Maxwell:

$$\left.\begin{array}{llll}\text{(i)} & \nabla\cdot\mathbf{E}=0, & \text{(iii)} & \nabla\times\mathbf{E}=-\dfrac{\partial\mathbf{B}}{\partial t}, \\ \text{(ii)} & \nabla\cdot\mathbf{B}=0, & \text{(iv)} & \nabla\times\mathbf{B}=\dfrac{1}{c^2}\dfrac{\partial\mathbf{E}}{\partial t}.\end{array}\right\}$$

2. Expand fields into components:

$$\boxed{\begin{gathered}\vec{E}(x,y,z,t)=\vec{E}_0(x,y)\,e^{j(\omega t-kz)} \\ \vec{E}_0=E_x\ \hat{x}+E_y\ \hat{y}+E_z\ \hat{z}\end{gathered}}$$

Waveguides (recap) [1]

3. Transverse components in terms of Ez, Bz:

$$
\begin{aligned}
&\text{(i)} \quad E_x = \frac{i}{(\omega/c)^2 - k^2}\left(k\frac{\partial E_z}{\partial x} + \omega\frac{\partial B_z}{\partial y}\right),\\
&\text{(ii)} \quad E_y = \frac{i}{(\omega/c)^2 - k^2}\left(k\frac{\partial E_z}{\partial y} - \omega\frac{\partial B_z}{\partial x}\right),\\
&\text{(iii)} \quad B_x = \frac{i}{(\omega/c)^2 - k^2}\left(k\frac{\partial B_z}{\partial x} - \frac{\omega}{c^2}\frac{\partial E_z}{\partial y}\right),\\
&\text{(iv)} \quad B_y = \frac{i}{(\omega/c)^2 - k^2}\left(k\frac{\partial B_z}{\partial y} + \frac{\omega}{c^2}\frac{\partial E_z}{\partial x}\right).
\end{aligned}
$$

(Note: replace i by –j above)

Waveguides (recap) [1]

4. Solve for Ez, Bz from here:

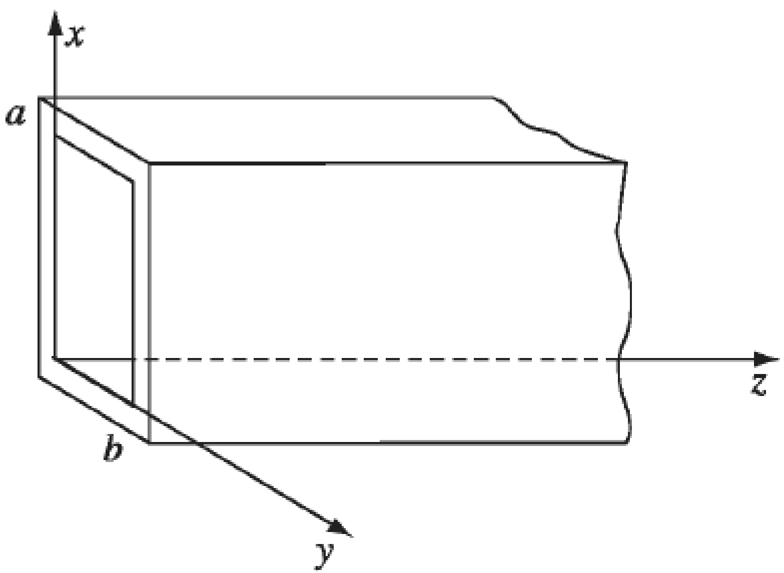
$$\left.\begin{aligned}
&\text{(i)} \quad \left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + (\omega/c)^2 - k^2\right] E_z = 0, \\
&\text{(ii)} \quad \left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + (\omega/c)^2 - k^2\right] B_z = 0.
\end{aligned}\right\}$$

Waveguides (recap) [1]

- TE Solutions of the form

$$B_z(x, y, z, t) = B_0 \cos\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) \ e^{j(\omega t - kz)}$$

i.e. a fwd travelling wave



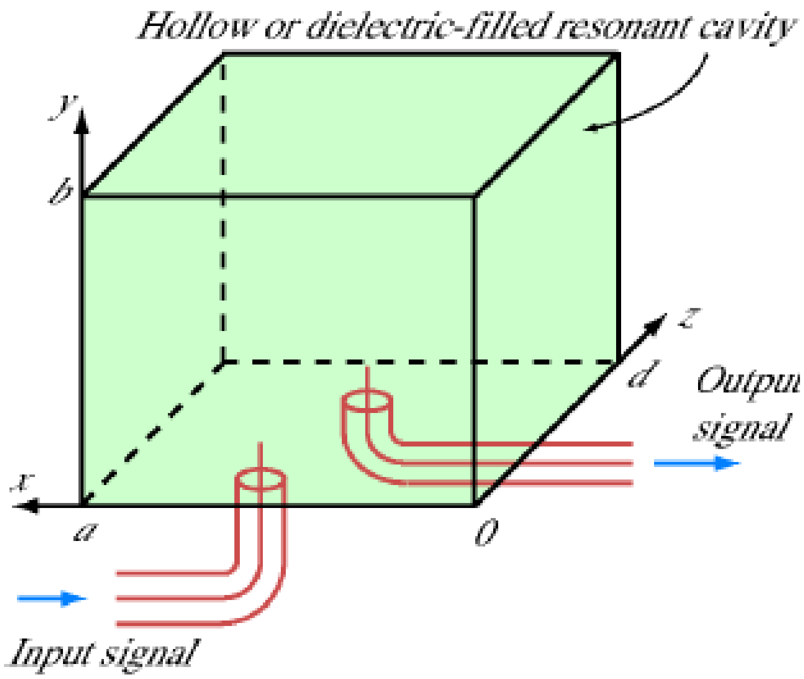
Cut off freqs:

$$\omega_{mn} = c\pi \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{a}\right)^2}$$

Cap the ends with metal plates [2]

$$B_z = \left(B_0^+ e^{-jkz} + B_0^- e^{jkz}\right) \cos\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{j\omega t}$$

Fwd and bkwd waves



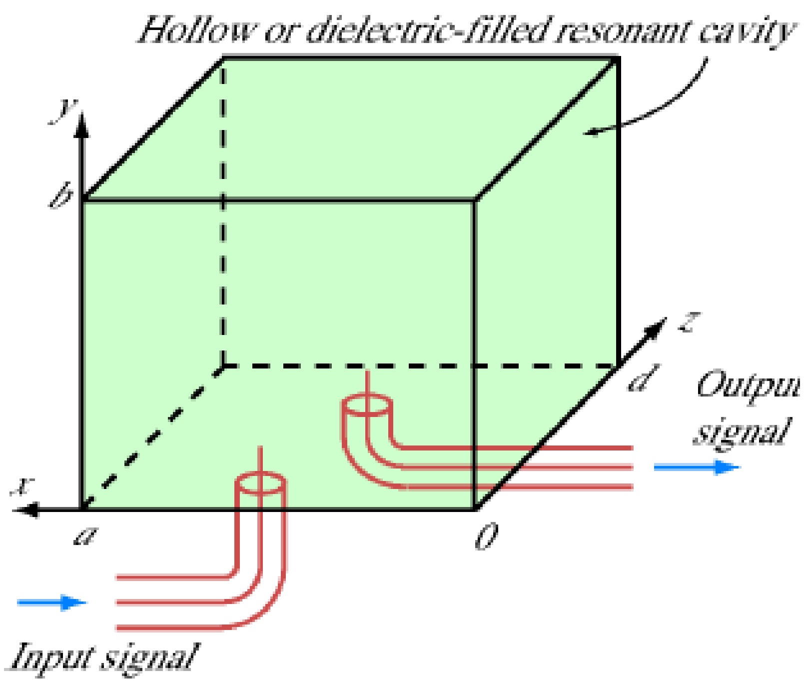
Boundary conditions at $z = 0, z = d$:

$$\left(\nabla \times \vec{E}\right)_z = -j\omega B_z$$
$$= \frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x}$$
$$= 0$$

Cap the ends with metal plates [2]

This implies that $B_0^+ = -B_0^-$, $kd = p\pi$ and :

$$B_z = -2jB_0^+ \sin\left(\frac{\mathrm{p}\pi z}{d}\right) \cos\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{j\omega t}$$



The frequency of a waveguide mode earlier was

$$ck = \sqrt{\omega^2 - \omega_{mn}^2}$$

Now, k is restricted, so

$$\omega_0 = c\pi \sqrt{\left(\frac{p}{d}\right)^2 + \left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

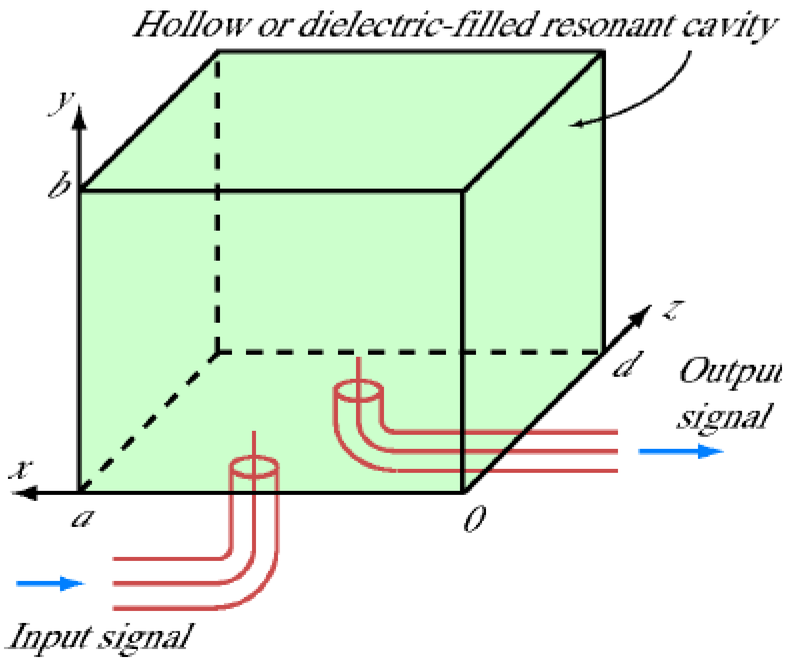
Waveguide frequency

Losses in a resonator

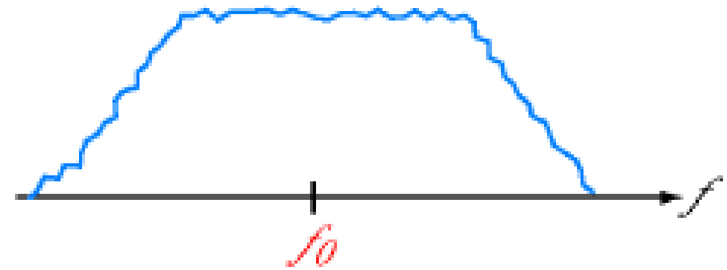
- Perfect metal $\longrightarrow$ No losses

- Finite $\sigma \Rightarrow$ Boundary conditions get slightly modified $\Rightarrow$ System supports small range of frequencies around ω_0

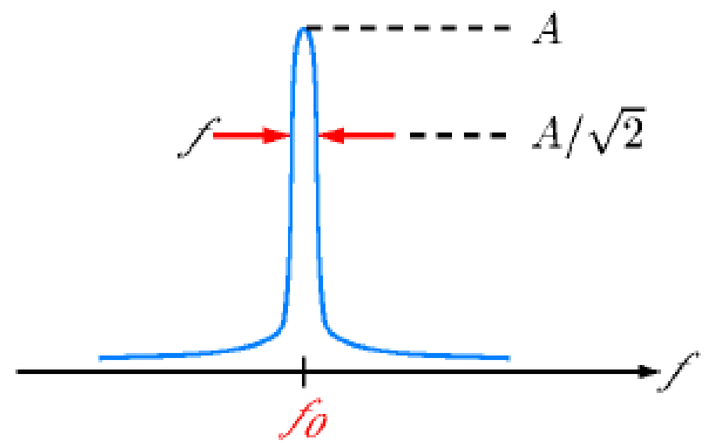
Waveguide resonator [2]



(a) Resonant cavity



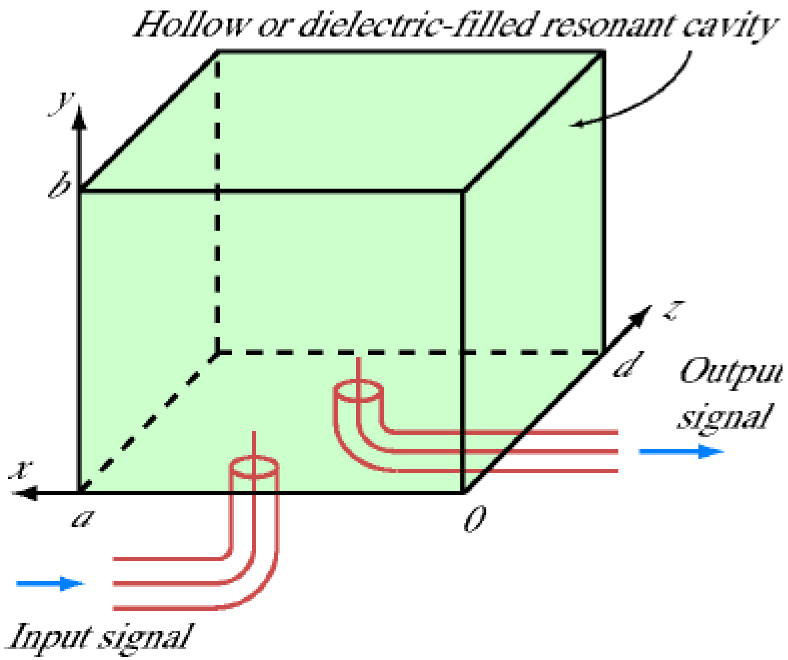
(b) Input spectrum



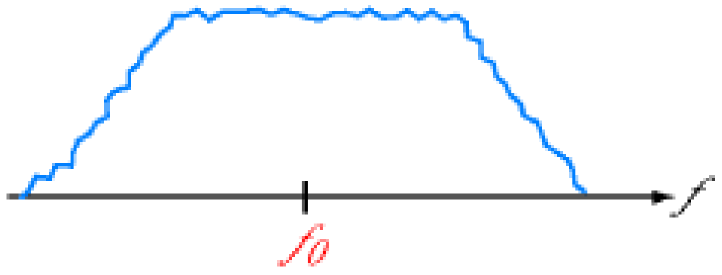
(c) Output spectrum

$$Q = \omega_0 \frac{Energy\ Stored}{Power\ lost}$$

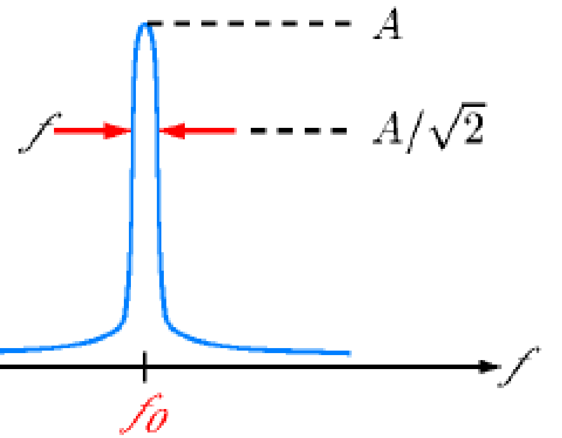
Waveguide resonator [2]



(a) Resonant cavity



(b) Input spectrum



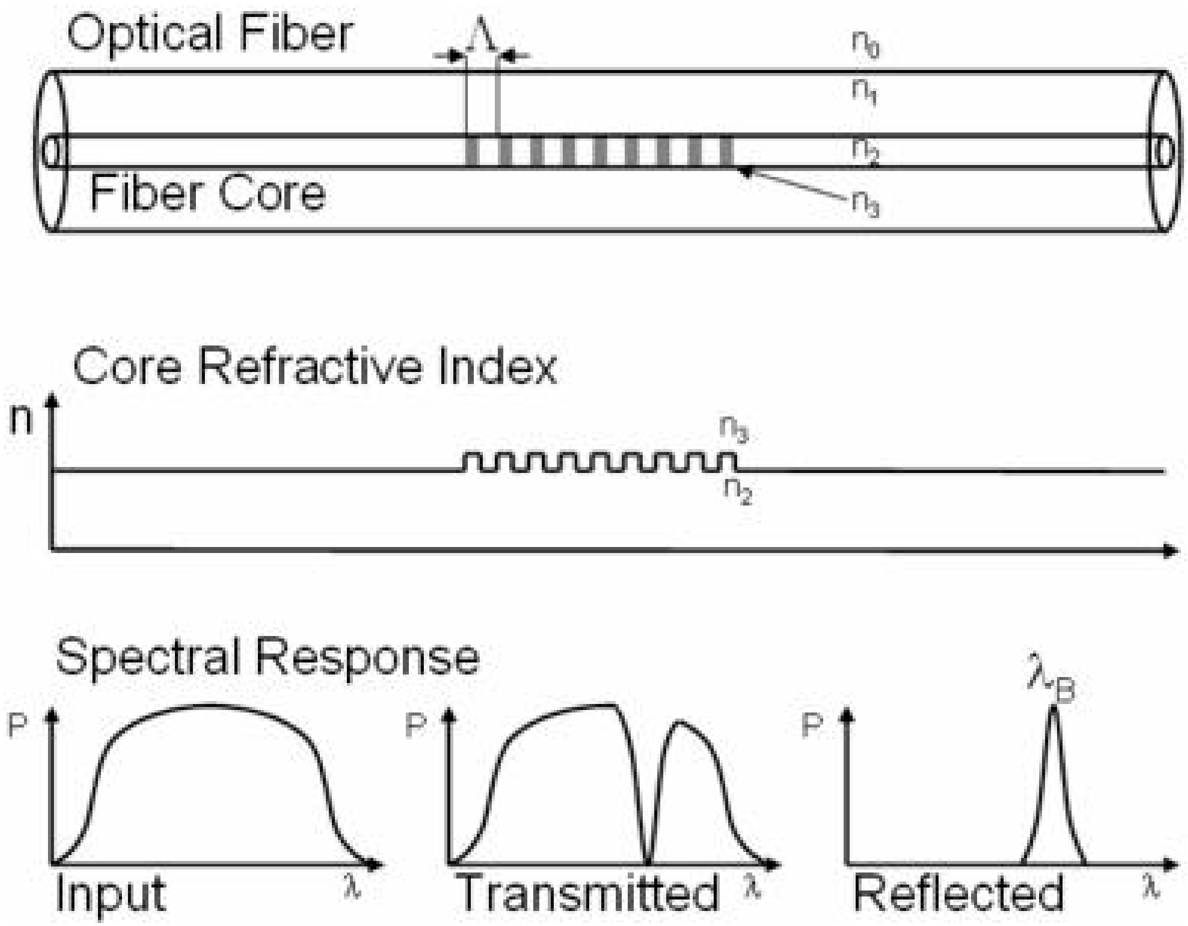
(c) Output spectrum

$$Q \approx \frac{\omega_0}{\Delta\omega}$$

Calculating resonator loss allows Q calc.

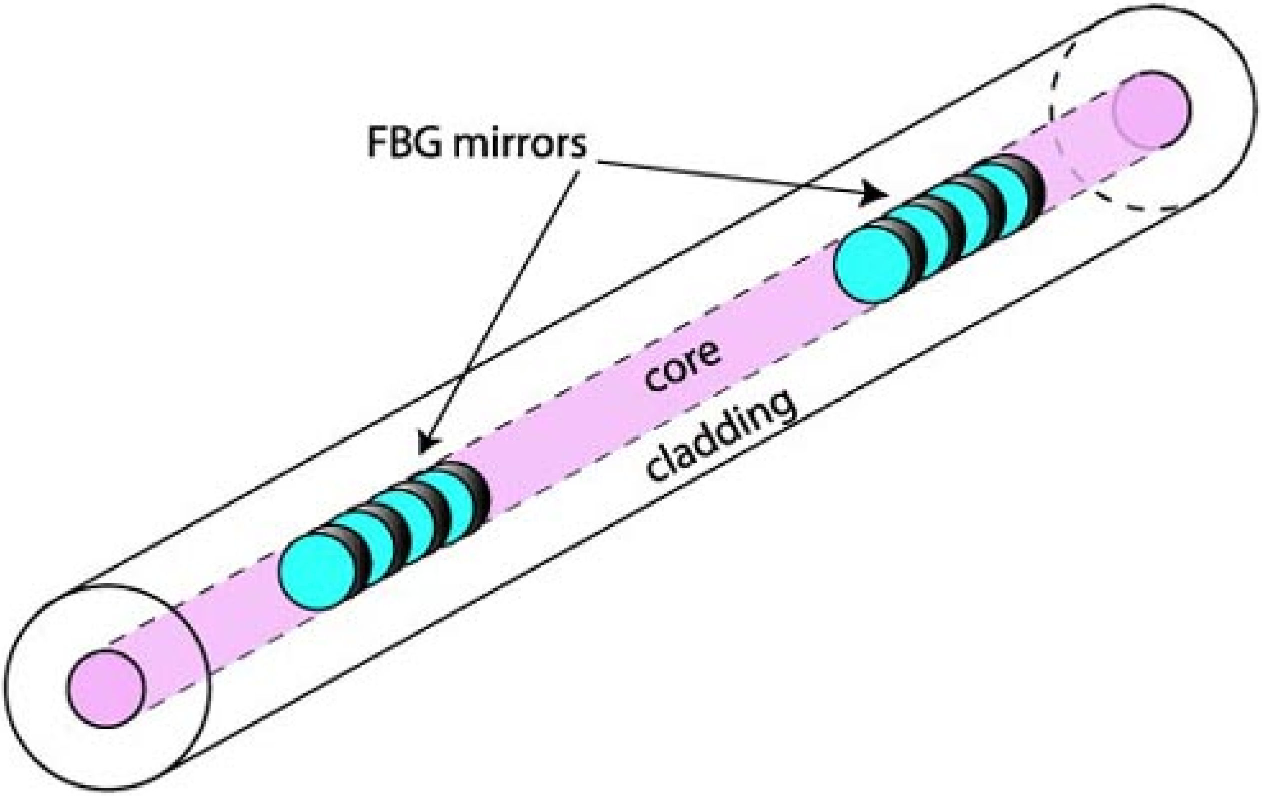
Resonators in the real world

- Fiber-Bragg mirrors [3]

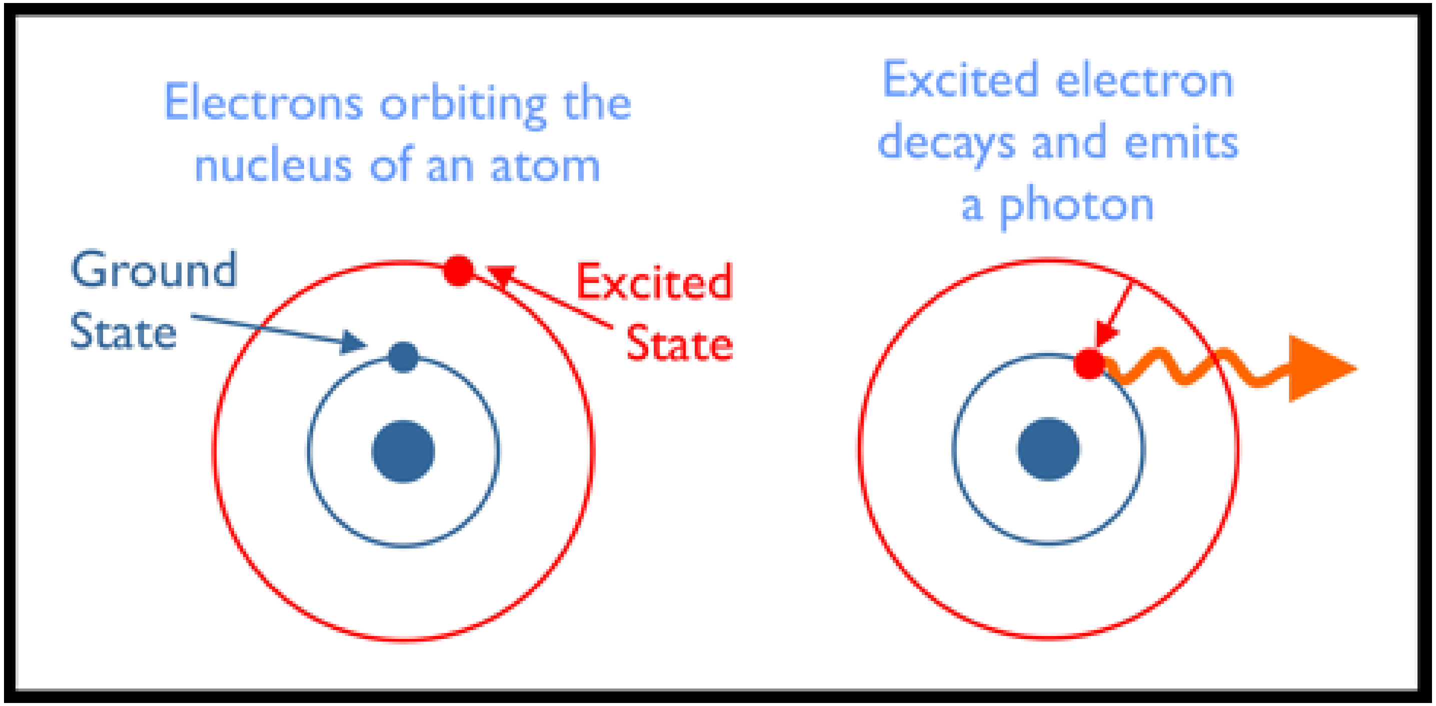


Resonators in the real world

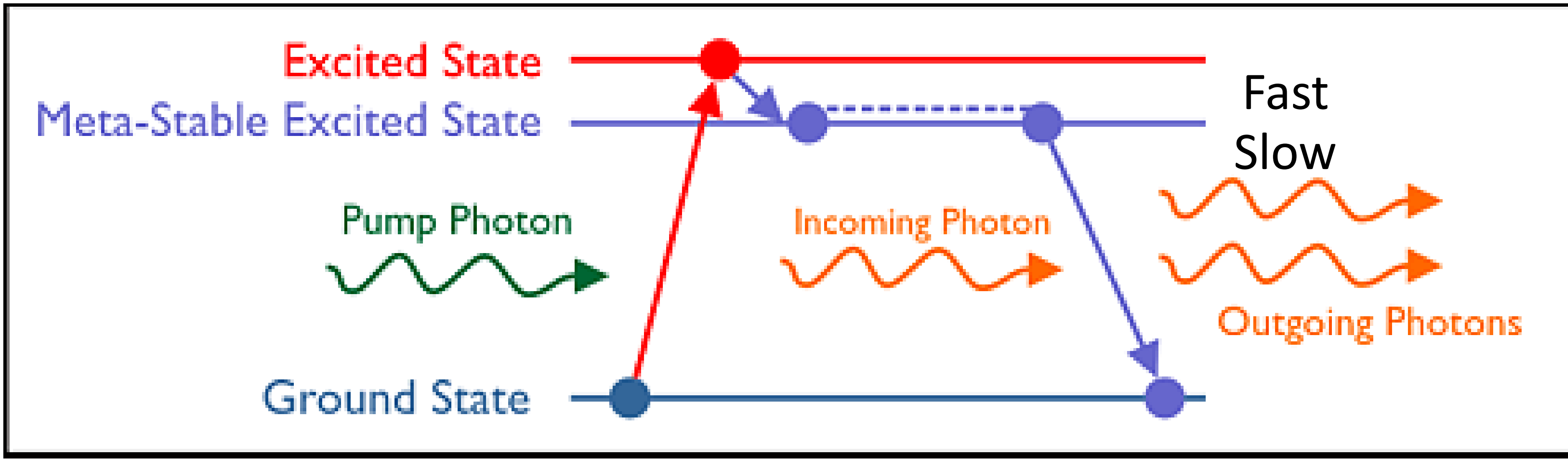
- Fiber-based resonator [4]



Laser Physics in brief [5]



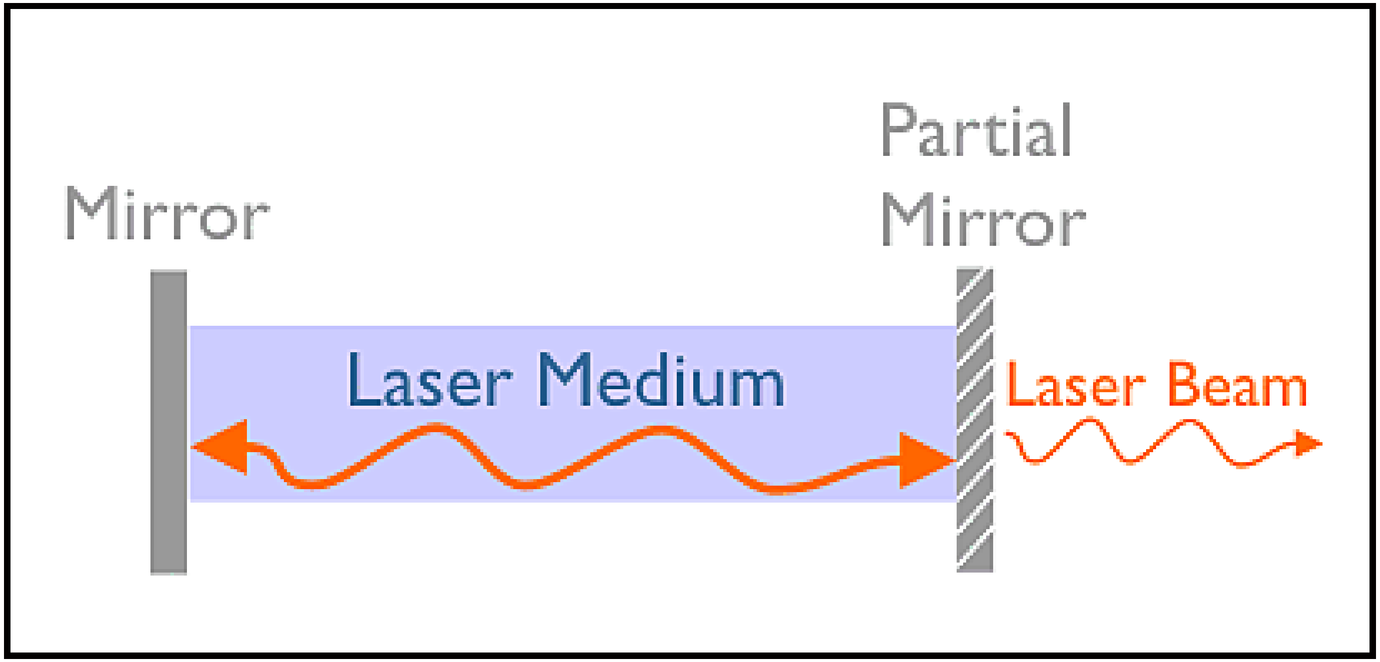
Laser Physics in brief [5]



The presence of a photon encourages another photon to be emitted: Stimulated emission

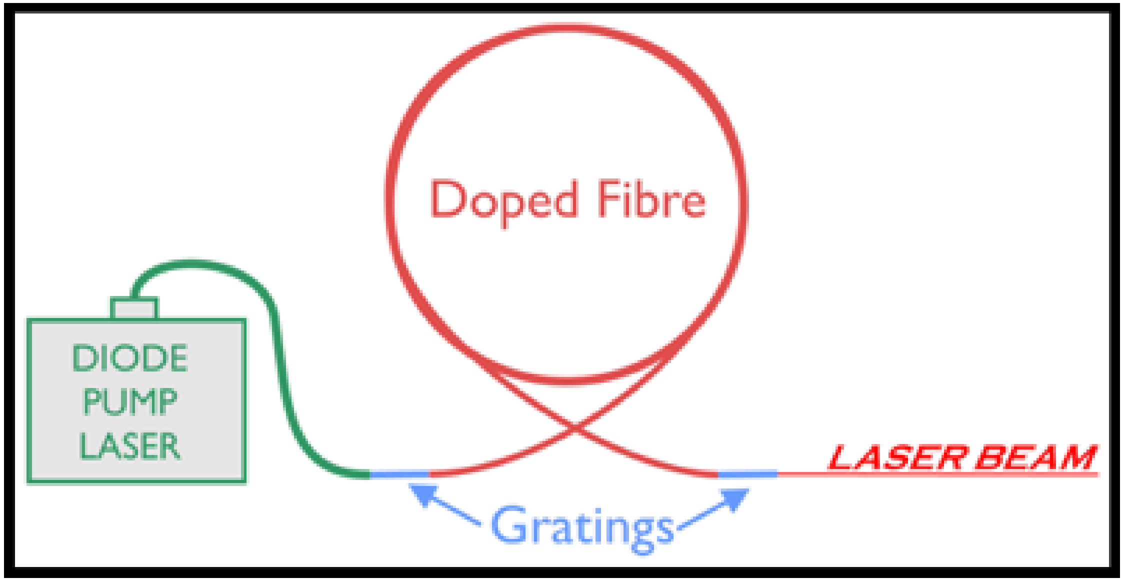
Only 2 level system, $\frac{N_2}{N_1} = \exp(-\frac{\Delta E}{kT})$

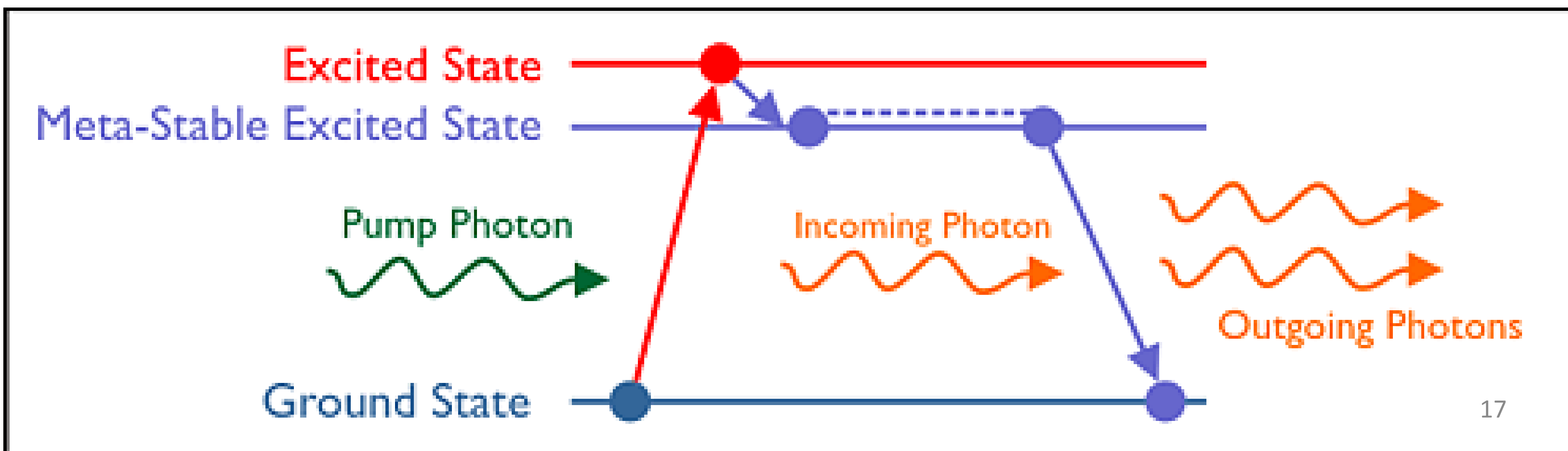
Laser Physics in brief [5]



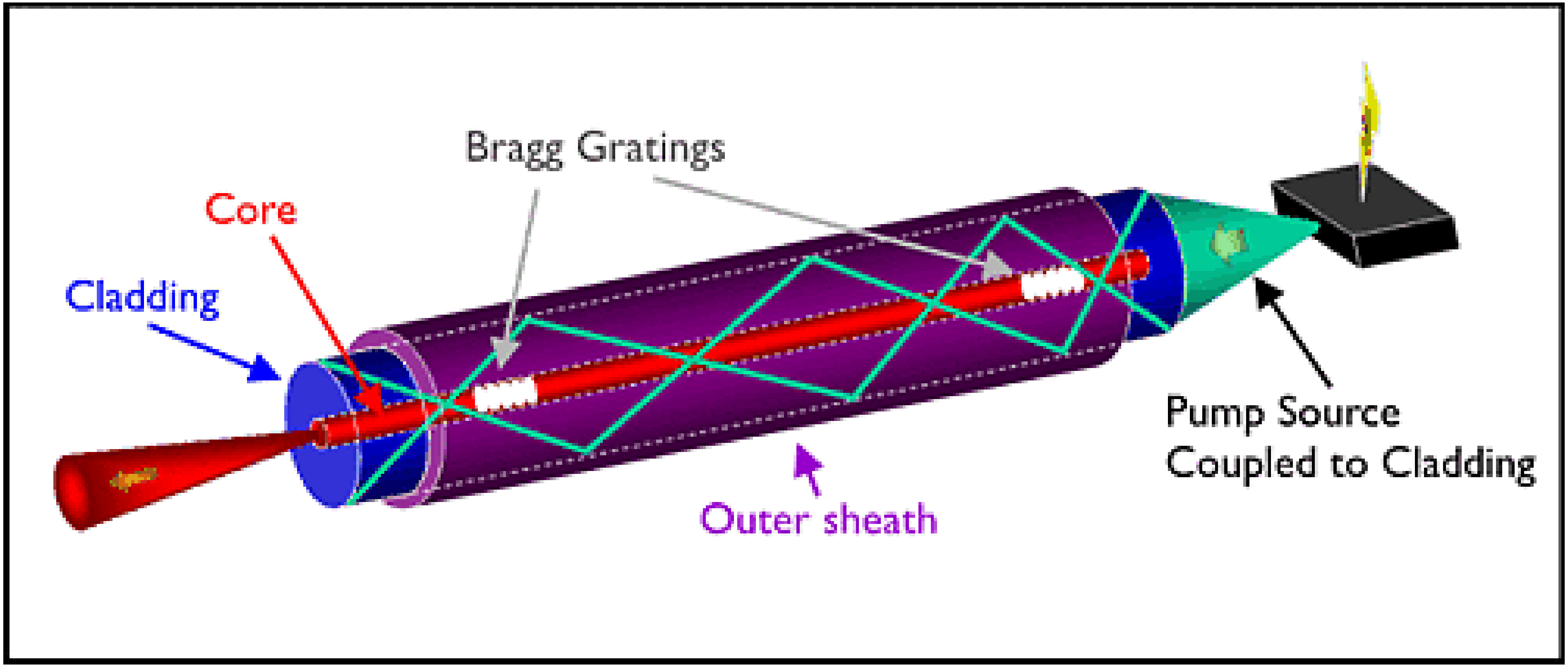
So, the resonant frequency of the resonator must match the energy transition of the gain/laser medium!

Laser Physics in brief [5]





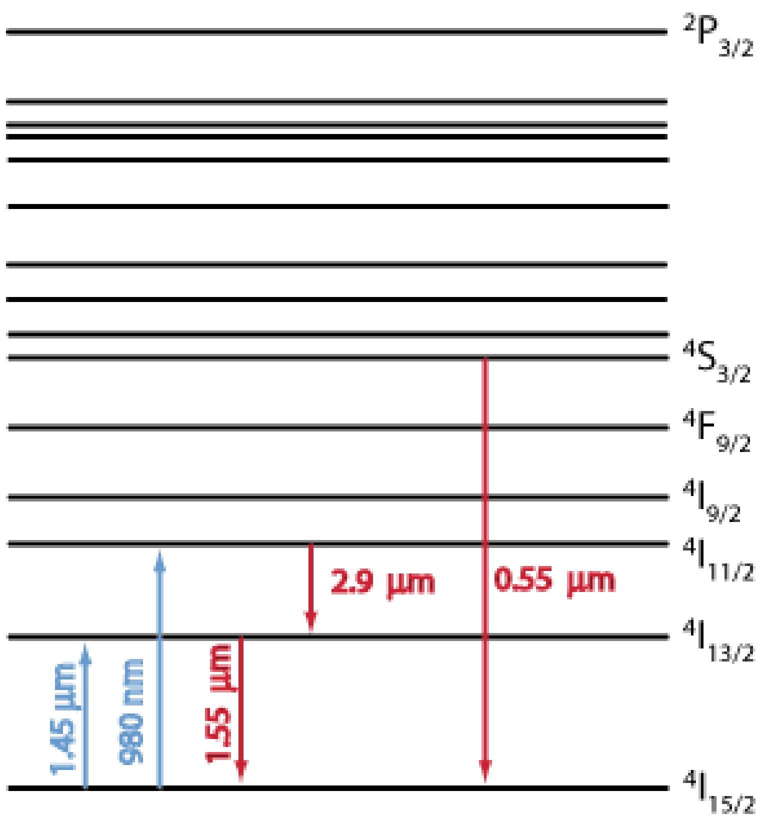
Laser Physics in brief [5]



Gain medium in the fiber? [6]

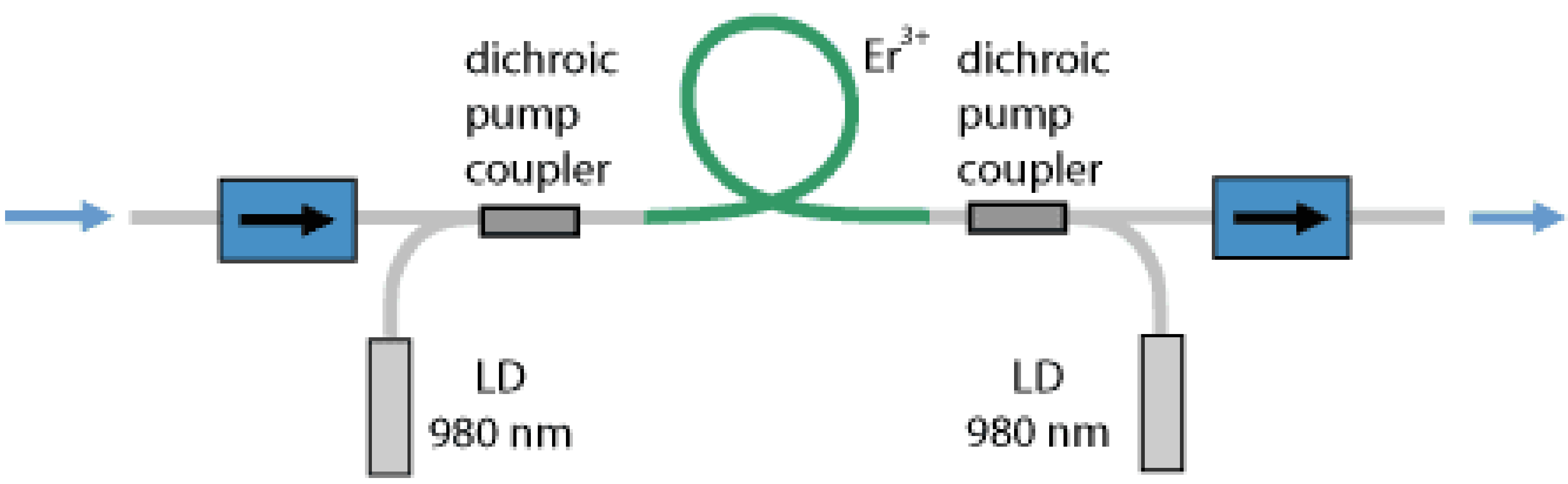
- Introduce a rare Earth ion into the fiber: Called “doping” – Erbium Doped Fiber

Energy band diagram of Er^{3+}



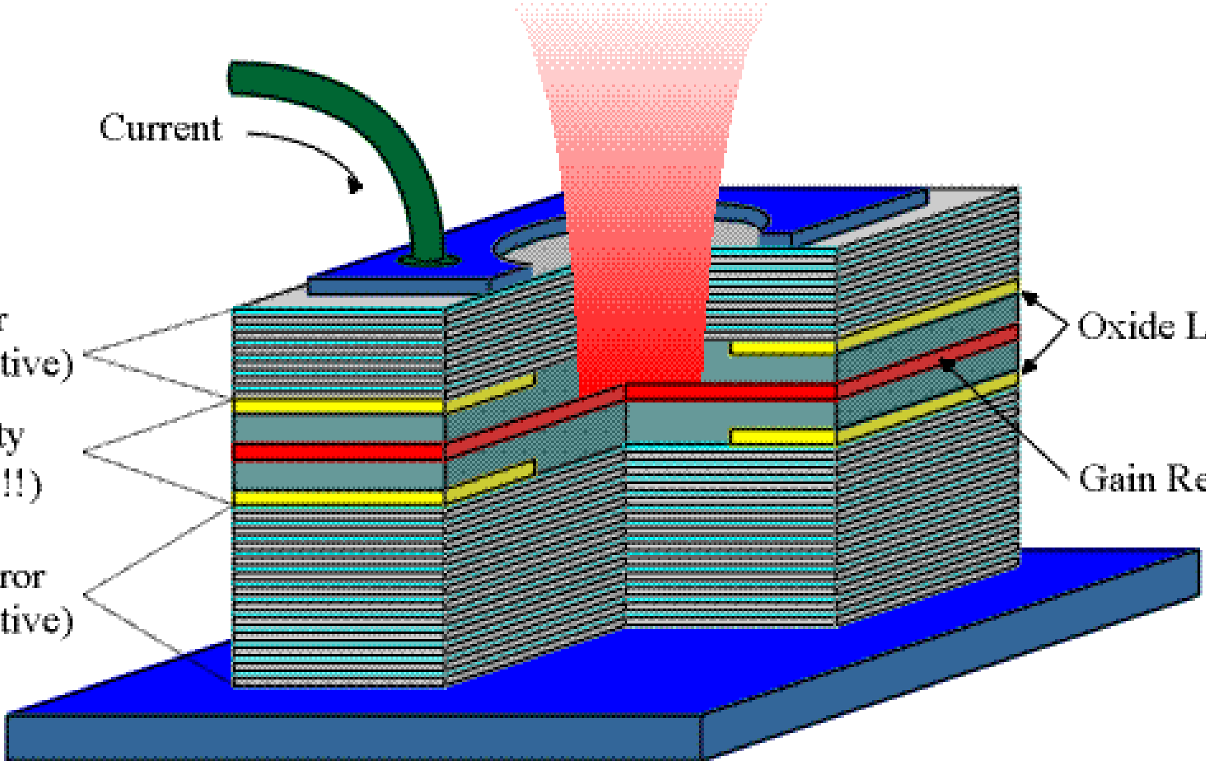
Applications [7]

- As a gain medium in a fiber-laser
- As an amplifier in fiber optics: backbone of ALL telecom!



VCSEL

- Vertical Cavity
  Surface Emitting Laser [8]



References

- [1] Griffiths, Introduction to Electrodynamics, 4th Ed.
- [2] Ulaby et al., Fundamentals of applied electromagnetics, 6th Ed.
- [3] http://www.hoestarinsp.com.sg/FBG1-large.jpg
- [4] http://spie.org/x8609.xml
- [5] http://www.orc.soton.ac.uk/61.html
- [6] http://www.rp-photonics.com/erbium_doped_gain_media.html
- [7] http://www.rp-photonics.com/erbium_doped_fiber_amplifiers.html
- [8] http://japaneseclass.jp/trends/about/VCSEL