Stabilization of a Mobile Inverted Pendulum

June 2009

Introduction

Underactuated dynamical systems are those that have more degrees-of-freedom than control inputs. Examples include spacecrafts, underwater autonomous vehicles and mobile robots. Control and stabilization of these systems are challenging tasks and are currently hot topics of research for both engineers and applied mathematicians. New stabilization strategies are validated and tested on classical *benchmark* systems such as the `ball on a beam' system and `pendulum on a cart' system. One such system which has drawn the attention of control researchers in recent times is the mobile inverted pendulum (MIP) which is a two-wheeled robot with a central body that carries a payload. The robot has the advantage of having a small footprint in addition to its ability to turn about its central axis. A commercial variant of the MIP is the well known *Segway* (shown in figure 1).

Figure 1: The Segway i2 personal mobility platform

The MIP has six degrees-of-freedom parameterized by $(x, y, \theta, \alpha, \phi_r, \phi_l)$ as depicted in Figure 2. Here, (x ,y) are the inertial co-ordinates of the robot chassis and θ is its orientation. The pitch angle of the inverted pendulum is denoted by α, while ϕ_r and ϕ_l denote the angular positions of the wheels. The robot is subjected to non-holonomic (non-integrable) constraints arising out of no-slip requirements. The wheels are individually actuated by DC motors, making the system underactuated by a degree four. The resulting equations of motion are of the form

$$dx/dt = f(x) + g_1(x)\tau_l + g_2(x)\tau_r$$

where f , g_1 and g_2 are smooth vector fields and τ_l ,τ_r are the torques applied to the left and right wheels. The control objective is to steer the robot from one point to another in the $X-Y$ plane while stabilizing the pitch angle α of the robot.

Designing the Control System

The system being highly nonlinear poses several control challenges such as the loss of linear controllability, non-existence of smooth time-invariant control laws for asymptotic stabilization of the equilibrium point and lack of complete, state feedback-linearizing property.

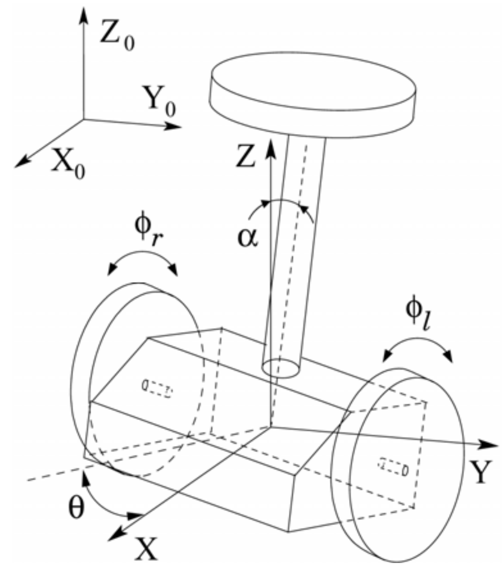

Figure 2: Six degrees of freedom

Control strategy

We have shown that the system is small-time locally controllable (a notion of nonlinear controllability) and have gone on to design a two time-scale discontinuous controller (shown in figure 3) using Lyapunov stability theory. The key strategy in controller design is to use the pitch angle as the throttle for locomotion. The efficacy of the proposed control strategy is validated using simulations. Effects on system stability due to parametric variations have also been investigated and robust control strategies have been proposed to eliminate these effects. In addition to this, an alternate control strategy based on energy-shaping techniques has been proposed to stabilize the robot's pitch dynamics. Future efforts are directed towards building an experimental setup, and devising control strategies to stabilize the MIP moving on an inclined plane.

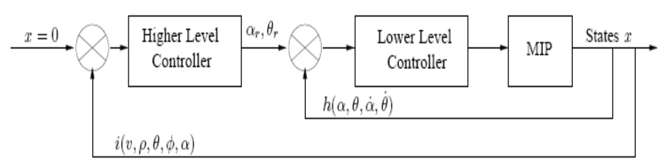

Figure 3: Schematic of the two-time scale controller

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