

Introduction:

Erbium doped fiber amplifiers (EDFAs) have been widely employed as inline fibre amplifiers in long haul fibre optic links. The amplifiers are intrinsically noisy and data rates are limited by the “optical SNR”. In these amplifiers, noise cannot be characterized by specifying mean and variance alone. The proper way to analyze their performance is through the probability distribution.

Figure 1 shows the probability of finding n photons in a pulse. In classical channels, with large n , our assumption of a Gaussian probability distribution looks valid. However, the assumption breaks down at low photon numbers, and we must rely on quantum statistics. Classical models for the EDFA are approximations of the underlying quantum reality. The question is under what conditions do the classical and quantum descriptions agree, and when do they diverge?

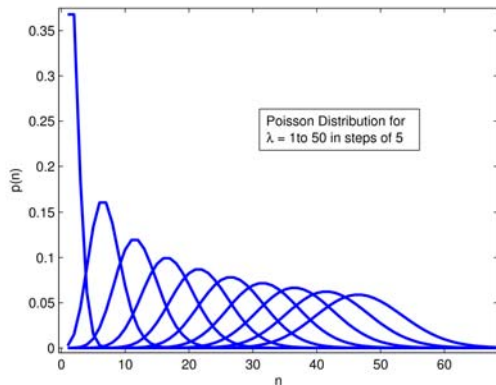


Figure 1: Probability distributions of photon number $p(n)$ given the average photon number n in a pulse

At the output of the EDFA, we will observe a photon number distribution that depends on the input photon statistics. A previous study to demonstrate equivalence between quantum and classical observations, for large photon numbers, showed that the distributions agreed for the most part [1]. However the bit error rate (BER) calculation of the channel, depended on the tail of the distributions where there was poor agreement.

Poisson Transformation:

A good approach to study this problem is to develop a connection between the discrete and continuous distributions. For the distributions found in EDFA, the Poisson transformation

$$Pr(Y = i) = \int \frac{v^i e^{-v}}{i!} f_v(v) dv,$$

relates the discrete probability of finding ‘ i ’ photons to an integral over the continuous chi-square distribution $f_v(v)$. If we look at the integral, it is nothing but a correlation of a window $v^i e^{-v} / i!$ and the continuous distribution. The

window is the Gamma distribution, $I(v)$. This is the source of a low pass smoothing, with both mean and variance ‘ i ’.

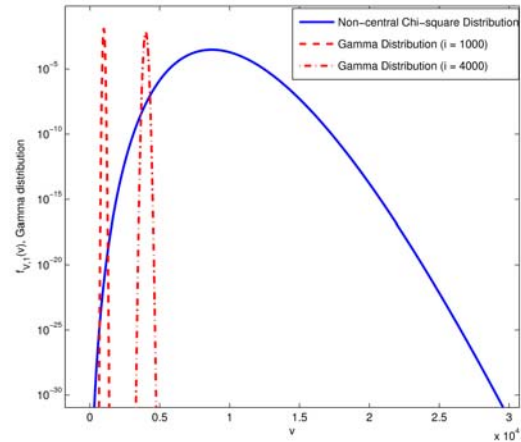


Figure 2: Poisson transformation with the windows for two values of ‘ i ’ for bit ‘1’ transmission, with $G = 101$.

Figure 2 is a plot of pulse energy distribution, corresponding to a ‘1’ bit. Using the Poisson transformation, we extract the probable photon number. Clearly, the quantum and classical descriptions will merge only if the window function (red) has a variance that is much smaller than that of the classical chi-square distribution (blue). We find that this happens only when the EDFA gain $G \gg 1$. This is surprising since we expect quantum systems to approach classical descriptions when either the number of input photons is large or the noise is high. Yet, in this case it is G that matters. A possible reason is that G is stochastic and is also a measure of noise in the system. The effect of gain on the channel BER is summarized in the table, with the quantum model predicting a higher BER at low photon numbers.

Gain	BER (Semi Classical)	BER (Quantum)
3	8.98e - 12	1.897e - 9
4	1.72e - 10	5.312e - 9
11	1.051e - 8	2.743e - 8
50	4.218e - 8	5.102e - 8
101	5.055e - 8	5.5479e - 8

[1] A. Mecozzi, “Quantum and semiclassical theory of noise in optical transmission lines employing in-line erbium amplifiers,” J. Opt. Soc. Am. B 17, 607-617 (2000).