Lecture 33

Note Title 4/3/2008

→ intrinsic/extrinsic LLR

$$L = \log \frac{\Pr(u_i = 0 \mid \underline{r}^{(0)}, \underline{r}^{(1)})}{\Pr(u_i = 1 \mid \underline{r}^{(0)}, \underline{r}^{(1)})}$$

$$\Pr(u_i = 0 \mid r_i^{(0)}, \underline{r}^{(0)}_{\neq i}, \underline{r}^{(1)}) = \frac{f(u_i = 0, r_i^{(0)}, \underline{r}^{(0)}_{\neq i}, \underline{r}^{(1)})}{f(r_i^{(0)}, \underline{r}^{(0)}_{\neq i}, \underline{r}^{(1)})}$$

$$= \frac{f(r_i^{(0)} \mid u_i = 0, \ldots\ldots)\; f(u_i = 0, \underline{r}^{(0)}_{\neq i}, \underline{r}^{(1)})}{f(r_i^{(0)} \mid \underline{r}^{(0)}_{\neq i}, \underline{r}^{(1)}) \cdot f(\underbrace{\underline{r}^{(0)}_{\neq i}, \underline{r}^{(1)}}_{B})}$$

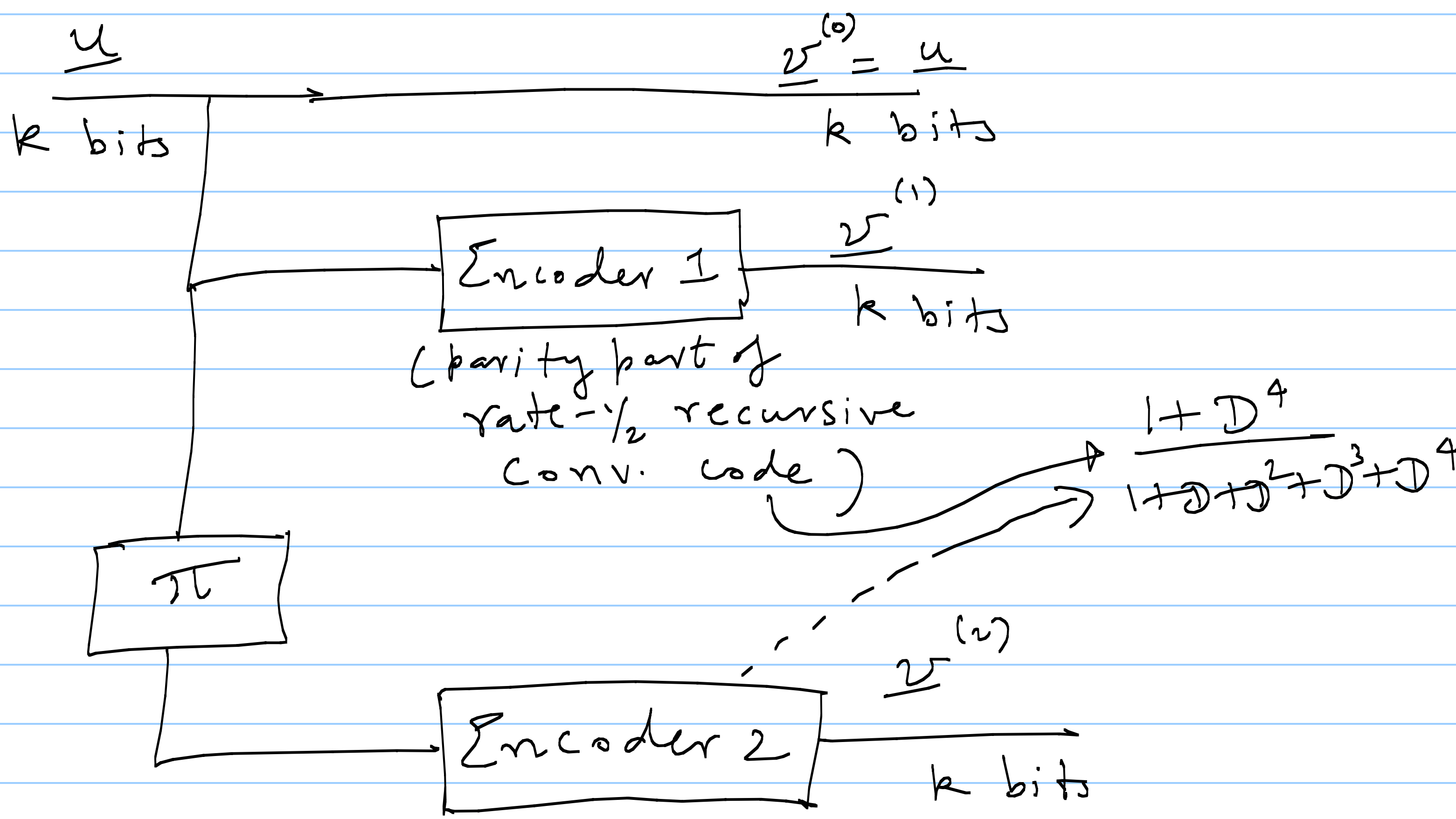
$$\Pr(u_i=0 \mid \underline{r}) = \frac{f(r_i^{(0)} \mid u_i=0) \cdot \Pr(u_i=0 \mid \underline{r}^{(0)}_{\neq i}, \underline{r}^{(1)})}{f(r_i^{(0)} \mid \underline{r}^{(0)}_{\neq i}, \underline{r}^{(1)})}$$

$$L = \log \underbrace{\frac{f(r_i^{(0)} \mid u_i=0)}{f(r_i^{(0)} \mid u_i=1)}} + \log \frac{\Pr(u_i=0 \mid \underline{r}^{(0)}_{\neq i}, \underline{r}^{(1)})}{\Pr(u_i=1 \mid \underline{r}^{(0)}_{\neq i}, \underline{r}^{(1)})}$$

$$\boxed{L_{i,tot} = \frac{2}{\sigma^2} r_i^{(0)} + L_{i,ext}}$$

$$\text{bitwise-MAP}(\underline{r}^{(0)}, \underline{r}^{(1)}) = [L_1 \; L_2 \; \cdots \; L_n]$$

Turbo codes: Encoder (parallel concatenated)



Turbo Decoder:

→ Optimal decoder is complex.

$\Pr(u_i = 0 \mid \underline{r})$: difficult to compute

$$\begin{array}{ccc} \underline{r}^{(0)} & \underline{r}^{(1)} & \underline{r}^{(2)} \\ \updownarrow & \updownarrow & \updownarrow \\ \underline{u} & \underline{v}^{(1)} & \underline{v}^{(2)} \end{array}$$

First decoder:

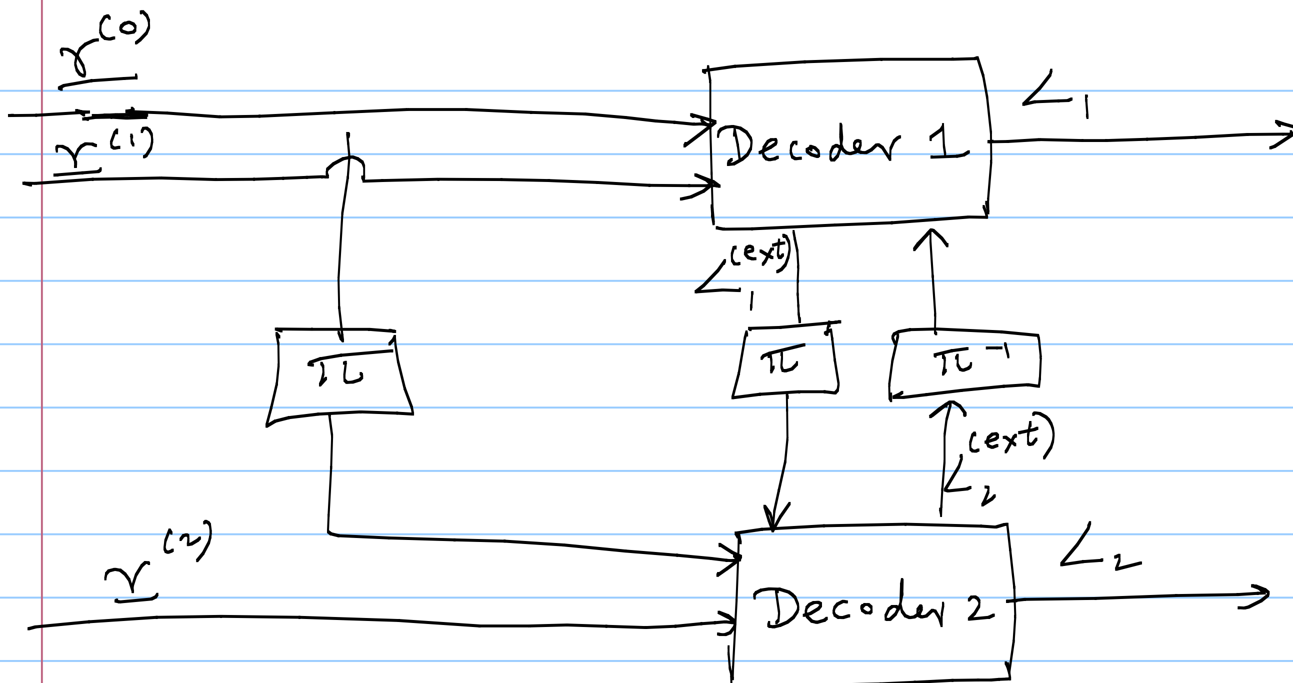
$$L_{1,i} = \Pr(u_i = 0 \mid \underline{r}^{(0)}, \underline{r}^{(1)}) = \frac{2}{\sigma^2} \cdot r_i^{(0)} + \underbrace{L_{1,i}^{(ext)}}_{}$$

Second Decoder:

$L_1^{(ext)} \longrightarrow \Pr\left(u_i = 0 \mid \underline{r}^{(0)}, \underline{r}^{(2)}\right)$

incorporated in branch metric of bitwise-MAP decoder.

LLR produced by second decoder, $L_{2,i} = \underbrace{\frac{2}{\sigma^2} r_i^{(0)}}_{\text{systematic part}} + \underbrace{L_{1,i}^{(ext)}}_{\text{a priori LLR}} + \underbrace{L_{2,i}^{(ext)}}_{\text{extrinsic}}$



$$\underline{L}_1^{(ext)} = \underline{L}_1 - (\underline{r}^{(0)}) \frac{2}{\sigma^2} - \underline{L}_2^{(ext)(prev)}$$

$$\underline{L}_2 = \text{Decoder 2}\left(\pi(\underline{r}^{(0)}), \underline{r}^{(2)}, \pi(\underline{L}_1^{(ext)})\right)$$

$\underline{r}^{(0)}, \underline{r}^{(1)}, \underline{r}^{(2)}, \quad \underline{L}_2^{(ext)(prev)} = \underline{0}$

for $i = 1$ to maxiterations,

$$\underline{L}_1 = \text{Decoder1}\left(\underline{r}^{(0)}, \underline{r}^{(1)}, \underline{L}_2^{(ext)(prev)}\right)$$

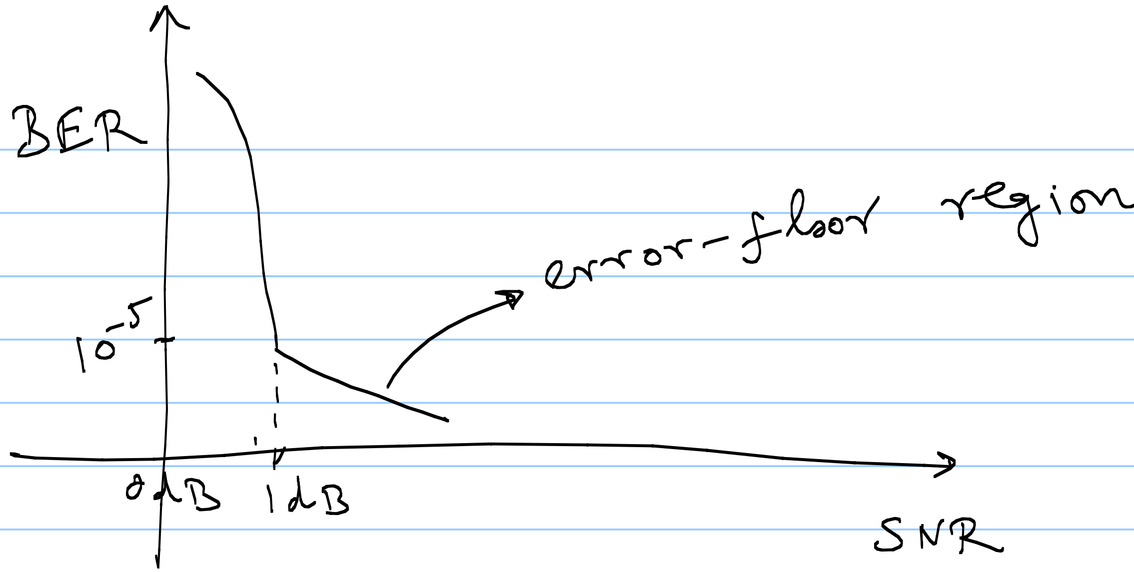
$$\underline{L}_1^{(ext)} = \underline{L}_1 - \left(\underline{r}^{(0)}\right)\frac{2}{\sigma^2} - \underline{L}_2^{(ext)(prev)}$$

$$\underline{L}_2 = \text{Decoder2}\left(\pi\left(\underline{r}^{(0)}\right), \underline{r}^{(2)}, \pi\left(\underline{L}_1^{(ext)}\right)\right)$$

$$\underline{L}_2^{(ext)(prev)} = \pi^{-1}\left(\underline{L}_2 - \frac{2}{\sigma^2}\pi\left(\underline{r}^{(0)}\right) - \pi\left(\underline{L}_1^{(ext)}\right)\right)$$

end

Decide based on $\underline{L}_2$.



Repeat-Accumulate codes: